ANSWER FOR 1B 2016

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(1)

$$\frac{x^3-3}{x^3-x^2-x+1} = \frac{x^3-x^2-x+1+(x^2+x-4)}{x^3-x^2-x+1}$$
$$= 1 + \frac{x^2+x-4}{x^3-x^2-x+1}$$

ここで

$$x^3-x^2-x+1 = (x-1)^2(x+1)$$

だから

$$\frac{x^2+x-4}{x^3-x^2-x+1} = \frac{A}{(x-1)^2} + \frac{B}{x-1} + \frac{C}{x+1}$$

とおけて,

$$x^2+x-4 = A(x+1) + B(x+1)(x-1) + C(x-1)^2$$
$$x^2+x-4 = (B+C)x^2 + (A-2B)x + A - B + C$$

係数を比較して,

$$\begin{cases} B+C=1 \\ A-2B=1 \\ A-B-C=-4 \end{cases}$$

これを拡大係数行列にして連立方程式を解くと,

$$\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & -2 & 0 & 1 \\ 1 & -1 & -1 & -4 \end{bmatrix} \to \begin{bmatrix} 1 & -1 & -1 & -4 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$\therefore C=3, B=-2, A=-3$$

$$I = \int 1dx + \int \left(\frac{-3}{(x-1)^2} + \frac{-2}{x-1} + \frac{1}{x+1} \right) dx$$
$$= x + \frac{3}{x-1} + \log \left| \frac{x+1}{(x-1)^2} \right| + \text{const.}$$

(2)

$$\mathcal{D} := \{(x,y) \mid 0 \le y \le 1, y^3 \le x \le 2-y^2\}$$

これをグラフに書くと

$$\mathcal{D}' = \left\{ (x,y) \middle| 0 \le x \le 2, \begin{cases} 0 \le y \le \sqrt[3]{x} & (0 \le x \le 1) \\ 0 \le y \le\le \sqrt{2-x} & (1 \le x \le 2) \end{cases} \right\}$$

従って定積分は

$$I = \int_0^1 \left(\int_0^{\sqrt[3]{x}} \frac{y^2}{x\sqrt{2-x}} dy \right) dx + \int_1^2 \left(\int_0^{\sqrt{2-x}} \frac{y^2}{x\sqrt{2-x}} dy \right) dx$$

$$= \int_0^1 \frac{1}{3\sqrt{2-x}} dx + \int_1^2 \frac{2-x}{3x} dx = \left[-\frac{2}{3}\sqrt{2-x} \right]_0^1 + \left[\frac{2}{3}\log x \right]_1^2 - \left[\frac{1}{3}x \right]_1^2$$

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$$= \frac{2}{3}(\sqrt{2} + \log 2) - 1$$

(3)