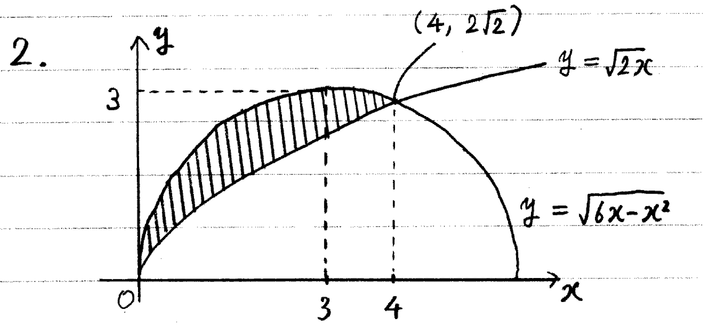
数学BI 期末試験 解答例と配点

各10点、計90点満点、

1. (1)
$$\int \frac{x^2+2x-1}{x^2+4x+5}\,dx = \int \left(1-\frac{2x+6}{x^2+4x+5}\right)dx$$
$$= \int dx - \left(\int \frac{2x+4}{x^2+4x+5}\,dx + 2\int \frac{dx}{x^2+4x+5}\right)$$
$$= \int dx - \int \frac{(x^2+4x+5)'}{x^2+4x+5}\,dx - 2\int \frac{dx}{(x+2)^2+1}$$
$$= x - \log(x^2+4x+5) - 2\,\mathrm{Tan}^{-1}(x+2) + C$$

(2) $t=\tan\frac{x}{2}$ とおくと、$\cos x = \frac{1-t^2}{1+t^2}$, $dx = \frac{2}{1+t^2}\,dt$ なので、
$$I = \int_0^{1/\sqrt{3}} \frac{1}{1+2\,\frac{1-t^2}{1+t^2}}\cdot\frac{2}{1+t^2}\,dt$$
$$= 2\int_0^{1/\sqrt{3}} \frac{dt}{3-t^2} = \int_0^{1/\sqrt{3}} \left(\frac{1}{t+\sqrt{3}} - \frac{1}{t-\sqrt{3}}\right)dt$$
$$= \frac{1}{\sqrt{3}}\left[\log\frac{t+\sqrt{3}}{t-\sqrt{3}}\right]_0^{1/\sqrt{3}} = \frac{1}{\sqrt{3}}\log 2$$

2.



$$I = \int_0^{2\sqrt{2}} \left[\int_{3-\sqrt{9-y^2}}^{y^2/2} f(x,y)\,dx\right]dy + \int_{2\sqrt{2}}^{3} \left[\int_{3-\sqrt{9-y^2}}^{3+\sqrt{9-y^2}} f(x,y)\,dx\right]dy$$

3.
$$I = \int_0^1 \left[\int_0^{1-x} \left[\int_0^{1-x-y} (x+y+z)^2\,dz\right]dy\right]dx$$
$$= \int_0^1 \left[\int_0^{1-x} \left[\tfrac{1}{3}(x+y+z)^3\right]_0^{1-x-y} dy\right]dx$$
$$= \frac{1}{3}\int_0^1 \left[\int_0^{1-x} \{1-(x+y)^3\}\,dy\right]dx$$
$$= \frac{1}{3}\int_0^1 \left[y - \tfrac{1}{4}(x+y)^4\right]_0^{1-x} dx$$
$$= \frac{1}{3}\int_0^1 \left\{(1-x) - \frac{1}{4} + \frac{1}{4}x^4\right\}dx$$
$$= \frac{1}{3}\left[\frac{1}{20}x^5 - \frac{1}{2}x^2 + \frac{3}{4}x\right]_0^1$$
$$= \frac{1}{3}\left(\frac{1}{20} - \frac{1}{2} + \frac{3}{4}\right) = \frac{1}{10}$$

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4. (1) $\begin{cases} x = \frac{3}{5}u + \frac{2}{5}v \\ y = \frac{u}{5} - \frac{v}{5} \end{cases}$ より $J(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{vmatrix} = -\frac{1}{5}$

(2) $I = \int_0^{2\pi} \left[\int_0^{5\pi} \sin^2 u \cdot \left| -\frac{1}{5} \right| dv \right] du$

$= \frac{1}{5} \cdot 5\pi \cdot \int_0^{2\pi} \sin^2 u \, du$

$= \pi \int_0^{2\pi} \frac{1}{2} (1 - \cos 2u) \, du = \pi^2$

5. $-1 \leq z = -\frac{1}{2}(x^2+y^2)$ より, $x^2+y^2 \leq 2$

$S = \iint_{x^2+y^2 \leq 2} \sqrt{1 + (-x)^2 + (-y)^2} \, dx dy$

$= \iint_{x^2+y^2 \leq 2} \sqrt{1 + x^2 + y^2} \, dx dy$

$= \int_0^{2\pi} \left[\int_0^{\sqrt{2}} \sqrt{1+r^2} \cdot r \, dr \right] d\theta$ $\quad (x = r\cos\theta,\ y = r\sin\theta,\ |J(r,\theta)| = r)$

$= 2\pi \cdot \left[\frac{1}{3} (1+r^2)^{\frac{3}{2}} \right]_0^{\sqrt{2}}$

$= \frac{2}{3}\pi \left(3\sqrt{3} - 1 \right)$

6. (1) $f(x,y,z) = x+y+z+xyz$

(2) $f(x,y,z) = x+y+z+xyz \Rightarrow \nabla f = (1+yz,\ 1+xz,\ 1+xy)$

∴ 勾配と線積分の関係より,

$I = \int_\Gamma \nabla f \cdot ds = f(10,10,20) - f(1,2,10) = 2007$.

[別解] $(x(t), y(t), z(t)) = (1,2,10) + t\{(10,10,20) - (1,2,10)\}$

$= (1+9t,\ 2+8t,\ 10+10t), \quad 0 \leq t \leq 1$

∴ $I = \int_0^1 [\{1+(2+8t)(10+10t)\} 9 + \{1+(1+9t)(10+10t)\} \cdot 8 + \{1+(1+9t)(2+8t)\} \cdot 10] dt$

$= \int_0^1 (2160t^2 + 1960t + 307) \, dt$

$= 2007$.