ANSWER FOR 2015 1B EXAM

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(1) $\tan\dfrac{x}{2} = t$ とおくと

$$dx = \frac{2dt}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}.$$

積分範囲は

$$0 \leq t \leq \sqrt{3}$$

従って

$$\begin{aligned}
I &= \int_0^{\sqrt{3}} \frac{1}{5+4\dfrac{1+t^2}{1-t^2}} \cdot \frac{2}{1+t^2} dt \\
&= \int_0^{\sqrt{3}} \frac{2}{9+t^2} = \left[\frac{2}{3}\mathrm{Tan}^{-1}\frac{t}{3}\right]_0^{\sqrt{3}} \\
&= \frac{2}{3}\left(\mathrm{Tan}^{-1}\frac{1}{\sqrt{3}} - \mathrm{Tan}^{-1}0\right) = \frac{2}{3}\frac{\pi}{6} \\
&= \frac{\pi}{9}
\end{aligned}$$

(2) 積分順序を交換すると,

$$\begin{aligned}
I &= \int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} dx \int_{-\sqrt{4x^2-1}}^{\sqrt{4x^2-1}} \frac{dy}{\sqrt{(1-4x^2)(x^2-1)}} \\
&= \int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} \frac{2dx}{\sqrt{1-x^2}} = 2\big[\mathrm{Tan}^{-1}x\big]_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} = 2\left(\frac{\pi}{4} - \frac{\pi}{6}\right) \\
&= \frac{\pi}{6}
\end{aligned}$$

(3)

$$\begin{cases} x = \frac{1}{2}u \\ y = v - u \end{cases}$$

とすれば積分範囲は

$$D' = \{(u,v) \mid 0 \leq v \leq 1, u \geq 0, v-u \geq 0\}$$

ヤコビ行列式は

$$J = \begin{vmatrix} \frac{1}{2} & 0 \\ -1 & 1 \end{vmatrix} = \frac{1}{2}$$

被積分関数は

$$\exp\left(\frac{2x}{2x+y+1}\right) = \exp\left(\frac{u}{v+1}\right)$$

従って求める定積分は

$$I = \frac{1}{2}\int_0^1 dv \int_0^v du \exp\left(\frac{u}{v+1}\right)$$

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$$= \int_0^1 dv \left[(v+1) \exp\left(\frac{u}{v+1}\right) \right]_0^v = \int_0^1 dv \left\{ (v+1) \exp\left(\frac{v}{v+1}\right) - \exp v \right\}$$

(4)

(5)

(6)