数学B2, H19. 

[1]

$$\left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ 2 & a & 10 & 3a-4 \\ 3 & 7 & 3-a & 5 \end{array}\right)\begin{array}{l}-①\\-②\\-③\end{array} \xrightarrow[③\leftarrow ③-①\times 3]{②\leftarrow ②-①\times 2} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ & a-4 & 6 & 3a-6 \\ & 1 & -a-3 & 2 \end{array}\right)$$

$$\xrightarrow[②\leftrightarrow ③]{} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ & 1 & -a-3 & 2 \\ & a-4 & 6 & 3a-6 \end{array}\right) \xrightarrow[③\leftarrow ③-(a-4)\times ②]{} \left(\begin{array}{ccc|c} 1 & 2 & 2 & 1 \\ & 1 & -a-3 & 2 \\ & & 6+(a-4)(a+3) & a+2 \end{array}\right)$$

ここで,

$$6+(a-4)(a+3) = a^2-a-6 = (a+2)(a-3) = 0 \iff a=-2,\ 3$$

$$a+2 = 0 \iff a=-2.$$

より.

(1) $a \neq -2, 3$ (2) $a = 3$, (3) $a = -2$

となる. (∵(2): $0z = 5$ なる$z$は存在しない, (3): $0z = 0$となり. $z$は任意).

[2].

$$\begin{array}{cccc} \left(\begin{array}{cccc} 1 & 3 & 2 & 4 \\ 1 & 4 & 3 & b \\ 2 & c & 5 & 9 \end{array}\right) \\ ①\ ②\ ③\ ④ \end{array} \xrightarrow[\substack{③\leftarrow ③-2\times ① \\ ④\leftarrow ④-4\times ①}]{②\leftarrow ②-3\times ①} \left(\begin{array}{cccc} 1 & & & \\ 1 & 1 & 1 & b-4 \\ 2 & c-6 & 1 & 1 \end{array}\right) \rightarrow \left(\begin{array}{cc} 1 & \\ 1 & 1 \\ 2 & 1 \end{array}\right)$$

(∵$\dim \mathrm{Im} f_A = 2$)

より. $c-6=1$, $b-4=1$ ∴ $b=5$, $c=7$

よって,

$$\left(\begin{array}{cccc} 1 & 3 & 2 & 4 \\ 1 & 4 & 3 & 5 \\ 2 & 7 & 5 & 9 \end{array}\right)\begin{array}{l}-①\\-②\\-③\end{array} \xrightarrow[③\leftarrow ③-2\times ①]{②\leftarrow ②-①} \left(\begin{array}{cccc} 1 & 3 & 2 & 4 \\ & 1 & 1 & 1 \\ & 1 & 1 & 1 \end{array}\right) \xrightarrow[③\leftarrow ③-②]{} \left(\begin{array}{cccc} 1 & 3 & 2 & 4 \\ & 1 & 1 & 1 \end{array}\right)$$

より.

$$\mathrm{Ker} f_A = \mathrm{Span}\left\{ \begin{pmatrix} s-t \\ -s-t \\ s \\ t \end{pmatrix} \middle| s, t \in \mathbb{R} \right\} = \mathrm{Span}\left\{ s\begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \middle| s, t \in \mathbb{R} \right\}$$

よって. $\mathrm{Ker} f_A$ の基底は

$$\left\{ \begin{pmatrix} 1 \\ -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix} \right\}$$

MB2H1902

[3]

$$\det\begin{pmatrix} \alpha & \alpha & \alpha & \alpha+\beta \\ \alpha & \alpha & \alpha+\beta & \alpha \\ \alpha & \alpha+\beta & \alpha & \alpha \\ \alpha+\beta & \alpha & \alpha & \alpha \end{pmatrix} = \det\begin{pmatrix} 4\alpha+\beta & \alpha & \alpha & \alpha+\beta \\ 4\alpha+\beta & \alpha & \alpha+\beta & \alpha \\ 4\alpha+\beta & \alpha+\beta & \alpha & \alpha \\ 4\alpha+\beta & \alpha & \alpha & \alpha \end{pmatrix}$$

$$= (4\alpha+\beta)\det\begin{pmatrix} 1 & \alpha & \alpha & \alpha+\beta \\ 1 & \alpha & \alpha+\beta & \alpha \\ 1 & \alpha+\beta & \alpha & \alpha \\ 1 & \alpha & \alpha & \alpha \end{pmatrix} = (4\alpha+\beta)\det\begin{pmatrix} 1 & \alpha & \alpha & \alpha+\beta \\ 0 & 0 & \beta & -\beta \\ 0 & \beta & 0 & -\beta \\ 0 & 0 & 0 & -\beta \end{pmatrix}$$

$$= (4\alpha+\beta)\det\begin{pmatrix} 0 & \beta & -\beta \\ \beta & 0 & -\beta \\ 0 & 0 & -\beta \end{pmatrix} = (4\alpha+\beta)\cdot(-1)\beta\det\begin{pmatrix} \beta & -\beta \\ 0 & -\beta \end{pmatrix}$$

$$= \underline{\beta^3(4\alpha+\beta)}$$

[4]

$$\left(\begin{array}{cccc|cccc} 1 & 3 & -2 & 2 & 1 & 0 & 0 & 0 \\ 2 & 4 & -2 & 3 & 0 & 1 & 0 & 0 \\ -2 & -7 & 5 & -4 & 0 & 0 & 1 & 0 \\ 5 & -1 & 5 & 4 & 0 & 0 & 0 & 1 \end{array}\right)\begin{array}{l} -① \\ -② \\ -③ \\ -④ \end{array} \xrightarrow[\substack{③\leftarrow③+② \\ ④\leftarrow④-①\times5}]{②\leftarrow②-①\times2} \left(\begin{array}{cccc|cccc} 1 & 3 & -2 & 2 & 1 & 0 & 0 & 0 \\ 0 & -2 & 2 & -1 & -2 & 1 & 0 & 0 \\ 0 & -3 & 3 & -1 & 0 & 1 & 1 & 0 \\ 0 & -16 & 15 & -6 & -5 & 0 & 0 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{①\leftarrow①+③ \\ ②\leftarrow②-③ \\ ④\leftarrow④-5\times③}]{} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & -2 & 0 & -1 & 0 \\ 0 & -3 & 3 & -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -5 & -5 & -5 & 1 \end{array}\right) \xrightarrow[③\leftarrow③+3\times②]{} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & -6 & 1 & -2 & 0 \\ 0 & -1 & 0 & -1 & -5 & -5 & -5 & 1 \end{array}\right)$$

$$\xrightarrow[\substack{④\leftarrow④-③ \\ ①\leftarrow①+③ \\ ③\leftarrow-③}]{} \left(\begin{array}{cccc|cccc} 1 & 0 & 1 & 0 & -5 & 2 & -1 & 0 \\ 0 & 1 & -1 & 0 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 6 & -1 & 2 & 0 \\ 0 & -1 & 0 & 0 & 1 & -6 & -3 & 1 \end{array}\right) \xrightarrow[\substack{①\leftarrow①+②+④ \\ ②\leftarrow②+④ \\ ④\leftarrow-④}]{} \left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -6 & -4 & -5 & 1 \\ 0 & 0 & -1 & 0 & -1 & -6 & -4 & 1 \\ 0 & 0 & 0 & 1 & 6 & -1 & 2 & 0 \\ 0 & 1 & 0 & 0 & -1 & 6 & 3 & -1 \end{array}\right)$$

$$\xrightarrow[\substack{②\leftarrow④ \\ ③\leftarrow-② \\ ④\leftarrow-②}]{} \left(\begin{array}{cccc|cccc} 1 & & & & -6 & -4 & -5 & 1 \\ & 1 & & & -1 & 6 & 3 & -1 \\ & & 1 & & 1 & 6 & 4 & -1 \\ & & & 1 & 6 & -1 & 2 & 0 \end{array}\right) \qquad \therefore\ \underline{A^{-1} = \begin{pmatrix} -6 & -4 & -5 & 1 \\ -1 & 6 & 3 & -1 \\ 1 & 6 & 4 & -1 \\ 6 & -1 & 2 & 0 \end{pmatrix}}$$

[5]

固有値をλとすると. 固有方程式は.

$$\det\begin{pmatrix} \lambda-6 & 1 & -5 \\ 3 & \lambda-2 & 3 \\ 7 & -1 & \lambda+6 \end{pmatrix} = (\lambda-6)(\lambda-2)(\lambda+6) + 21 + 15 + 35(\lambda-2) - 3(\lambda+6) + 3(\lambda-6)$$

$$= (\lambda-2)(\lambda^2-36) + 36 + 35(\lambda-2) + 18 - 18$$

$$= (\lambda-2)(\lambda^2-1)$$

$$= (\lambda-2)(\lambda-1)(\lambda+1)$$

$$= 0$$

$$\therefore \lambda = -1, 1, 2$$

[6]

(1) $\nabla g = 0 \iff \begin{cases} g_x = (3x^2-3y)\sin z = 0 \\ g_y = (3y^2-3x)\sin z = 0 \\ g_z = (x^3-3xy+y^3-1)\cos z = 0 \end{cases}$ $(x, y \in \mathbb{R}, 0<z<\pi)$

・$\cos z = 0$ i.e., $z = \frac{\pi}{2}$ のとき.

$\begin{cases} 3x^2 - 3y = 0 & \therefore y = x^2 \\ 3y^2 - 3x = 0 & \therefore x = y^2 \end{cases}$ $\therefore y = y^4$ となり. $y = 1, 0$.

したがって, $(x, y, z) = (0, 0, \frac{\pi}{2}), (1, 1, \frac{\pi}{2})$.

・$\cos z \neq 0$ のとき, $0<z<\frac{\pi}{2}$ より. $\sin z \neq 0$.

$\begin{cases} 3x^2 - 3y = 0 & \cdots① \\ 3y^2 - 3x = 0 & \cdots② \\ x^3 - 3xy + y^2 - 1 = 0 & \cdots③ \end{cases}$

①,②をみたす$(x, y)$は. $(0,0), (1,1)$だが, そのいずれも③を満たさない。

よって, 結局, 停留点は. $(0, 0, \frac{\pi}{2}), (1, 1, \frac{\pi}{2})$

(2) ヘシアンHは、

$$H = \begin{pmatrix} g_{xx} & g_{xy} & g_{xz} \\ g_{yx} & g_{yy} & g_{yz} \\ g_{zx} & g_{zy} & g_{zz} \end{pmatrix} = \begin{pmatrix} 6x\sin z & -3\sin z & (3x^2-3y)\cos z \\ -3\sin z & 6y\sin z & (3y^2-3x)\cos z \\ (3x^2-3y)\cos z & (3y^2-3x)\cos z & -(x^3-3xy+y^3-1)\sin z \end{pmatrix}$$

より.

$$H(0,0,\tfrac{\pi}{2}) = \begin{pmatrix} 0 & -3 & 0 \\ -3 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad H(1,1,\tfrac{\pi}{2}) = \begin{pmatrix} 6 & -3 & 0 \\ -3 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix}$$

(3) ヘシアンの定値性を調べる.

$(0,0,\frac{\pi}{2})$ において.

$$\det\begin{pmatrix} \lambda & 3 & 0 \\ 3 & \lambda & 0 \\ 0 & 0 & \lambda-1 \end{pmatrix} = (\lambda-1)(\lambda^2-9) = (\lambda-1)(\lambda-3)(\lambda+3) = 0$$

$\therefore \lambda = -3, 1, 3$ より. 定値性なし。

したがって, この点では極値をもたない。

$(1,1,\frac{\pi}{2})$ において.

$$\begin{aligned} \det\begin{pmatrix} \lambda-6 & 3 & 0 \\ 3 & \lambda-6 & 0 \\ 0 & 0 & \lambda-2 \end{pmatrix} &= (\lambda-2)\{(\lambda-6)^2-9\} \\ &= (\lambda-2)(\lambda-6+3)(\lambda-6-3) \\ &= (\lambda-2)(\lambda-3)(\lambda-9) = 0 \end{aligned}$$

$\therefore \lambda = 2, 3, 9$ より. 正定値。

したがって, この点では極小となり, 極小値は.

$$g(1,1,\tfrac{\pi}{2}) = -2.$$

となる。

以上より. $g(x,y,z)$ の極値は, $(1,1,\frac{\pi}{2})$ で極小値 $-2$ となる.