Modelling Complex Systems: Project Sheet 2015

March 23, 2015

The deadline for this first exercise sheet is midnight Tuesday 21st of April.

Please submit hand-ins on Studentportalen. All code should be submitted as an appendix and not as part of the answer to the hand-ins. Please feel free to submit videos illustrating your results where appropriate, via Studentportalen or uploaded elsewhere.

1 Cellular automata

This exercise will be covered in lab session on Thursday March 26th. Please start the questions before you come to the lab session.

Go to
http://ccl.northwestern.edu/netlogo/models/CA1DElementary
and try running the one dimensional cellular automata with a variety of rules. Find rules which are always the same independent of starting conditions; rules which produce periodic patterns repeating indefinitely; rules which produce random or chaotic patterns; and rules that produce 'pretty' patterns (you don't need to report your findings).

Now answer the following questions

1. Implement your own one-dimensional elementary cellular automata simulator. The input should be a vector of length 8 or a number between 0 and 255, indicating the different rules. The output should be the change in cells through time in a two dimensional array. Simulate and provide examples of each of the four complexity classes discussed in the lectures. **(5 points)**

2. In this exercise you implement a two-dimensional cellular automata model. Consider a 'painter' who behaves as follows. It starts living in a world of only white squares. Then on each time step

   (a) If it is on a white square, it paints the square black, turns 90 degrees right, and moves forward one square.

   (b) If it is on a black square, it paints the square white, turns 90 degrees left, and moves forward one square.

   Simulate this painter for 11,000 steps. What type of pattern emerges? **(5 points)**.

3. Now create a painter who has n coloured paints, labeled $0, ..., n-1$. When the painter arrives at a square of colour k it changes it to colour $k+1$ (modular n, so that $n = 0$). It then turns by following a rule-string defined by n symbols which are each 0 or 1. If the kth symbol is 1 it turns right, and if it is 0 it turns left. It moves one square on and

repeats. Under the above definition the black and white painter of the previous question was defined as 10.

Implement this model, and find some pretty pictures painted by this painter. **(5 points)**

2 Fashion and fads

This exercise will be covered in lab session on Friday April 17th. Please attempt the questions before you come to the lab.

We will look at two different models of phone usage and investigate various models for how iPhone ownership changes over time. Consider a group of students Y of whom own iPhones and $X = N - Y$ others who own another brand.

1. First consider the case where people make decisions independently of each other. On each time step of the model we choose one student at random and her phone breaks down, irrespective of if she currently has an iPhone or another brand. The probability the student buys an iPhone is then p and the probability she buys another brand is $1 - p$, again irrespective of her earlier phone type. Write a simulation of this model and plot the equilibrium distribution of the number of iPhone owners for $N = 15$ and $N = 100$ and for $p = 0.5$ and $p = 0.7$. **(2 points)**

2. Write down a Master equation for the model for how the probability $\pi(i, t)$ that i students have iPhones at time t changes through time, i.e. how $\pi(i, t+1)$ changes with $\pi(i, t)$, $\pi(i+1, t)$ and $\pi(i-1, t)$. Solve the Master equation for $t \to \infty$ to show that

$$\pi(i) = \binom{N}{i} p^i (1-p)^{N-i}$$

   Compare this distribution to that you find by simulating the model by plotting the two on the same figure. **(3 points)**

3. Now consider the following model of how the students change phone ownership. On each time step we choose one individual whose phone breaks down. This individual does one of two things.

   - With probability q she will choose a iPhone with probability 1/2 or another brand with probability 1/2.
   - With probability $1 - q$ she looks at the phone choice of two other individuals. If these two both have the same phone type she adopts that type. Otherwise, she just keeps the phone she had before.

Implement this as a simulation model for $N = 15$ individuals. Investigate the role of q in phone choice dynamics. In particular, for values of q ranging between 0 and 1. Find values of q where you get switching backwards and forwards between nearly all individuals owning an iPhone to only small numbers owning one. For this value plot a histogram of the distribution of Iphone owners. **(4 points)**

4. Run the simulation for a large number of time steps for q in the range $[0 : 0.01 : 1]$ and plot a 'phase transition' diagram showing how the distribution of the number of iPhone owners depends on p. Repeat the same exercise, but now for $N = 1000$. Discuss the similarities and differences between the diagrams for $N = 15$ and $N = 1000$. **(2 points)**

5. Write down a mean-field version of the model. Let x_t denote the average proportion of individuals with an iPhone and write an equation for how the expected value of x_t changes through time. Find the three steady states for this equation and draw a bifurcation diagram for how p affects the steady state of Iphone ownership. Compare your bifurcation diagram to the phase transition diagram for simulations of $N = 1000$ individuals. In deriving the mean-field equation you can assume that the probability of choosing the same individual twice is negligible. **(4 points)**