HOMEWORK ASSIGNMENT #9

Due Fri. Nov. 15, 2019 (in class)

1. **Polyphase Filter Design**: There are two different approaches to the design of a polyphase interpolation filter bank. In the "indirect" method, we design the "master" filter $h[n]$ for interpolation-by-U and then downsample it to create the polyphase filters, $h_p[n]$ for $p = 0, \ldots, U-1$. In the "direct" method, we individually design the U polyphase filters $h_p[n]$ to have DTFTs $H_p(e^{j\omega}) \approx e^{-j\frac{d-p}{U}\omega}$.

   We can compare these two methods by examining the group delays and DTFT magnitude response of the polyphase filters, or by examining the DTFT magnitude response of the corresponding master filter. (Note that we can recover a master filter from the direct design by interleaving the polyphase filters.)

   In this problem, we will assume interpolation factor $U = 15$, polyphase filter length $L = 11$ (i.e., master filter length UL), and an input signal bandlimited to $\omega_0 = 0.8\pi$ radians.

   (a) For the indirect method, what are the desired passbands and stopbands of $H(e^{j\omega})$? Specify your answer for $\omega \in [0, \pi)$.

   (b) Derive an expression for the desired polyphase impulse response, $d_p[n] = \mathcal{F}^{-1}_{\mathsf{DTFT}}(e^{-j\frac{d-p}{U}\omega})$. Use $d = \frac{UL-1}{2}$ for compatibility with the master filters designed earlier. Express your answer using $\text{sinc}(x) \triangleq \frac{\sin(\pi x)}{\pi x}$, as defined in Matlab (type "`help sinc`").

   (c) Using `firls`, design a causal, length-UL, generalized-linear-phase weighted-least-squares approximation to the desired $h[n]$. Then downsample it to create the U polyphase filters $h_p[n]$. Finally, plot

      i. The resulting DTFT magnitude response in dB versus "matlab normalized frequency" $\omega/\pi \in [0, 1]$. Use "`axis`" to zoom into the range between -70 and +30 dB.

      ii. The polyphase filters' DTFT magnitude responses in dB versus $\omega/\pi \in [0, 1]$. Use "`axis`" to zoom into the range between -0.5 and +0.5 dB.

      iii. The polyphase filters' group delays (use "`grpdelay`"). Use "`axis`" to zoom into the delay range between 4 and 6.

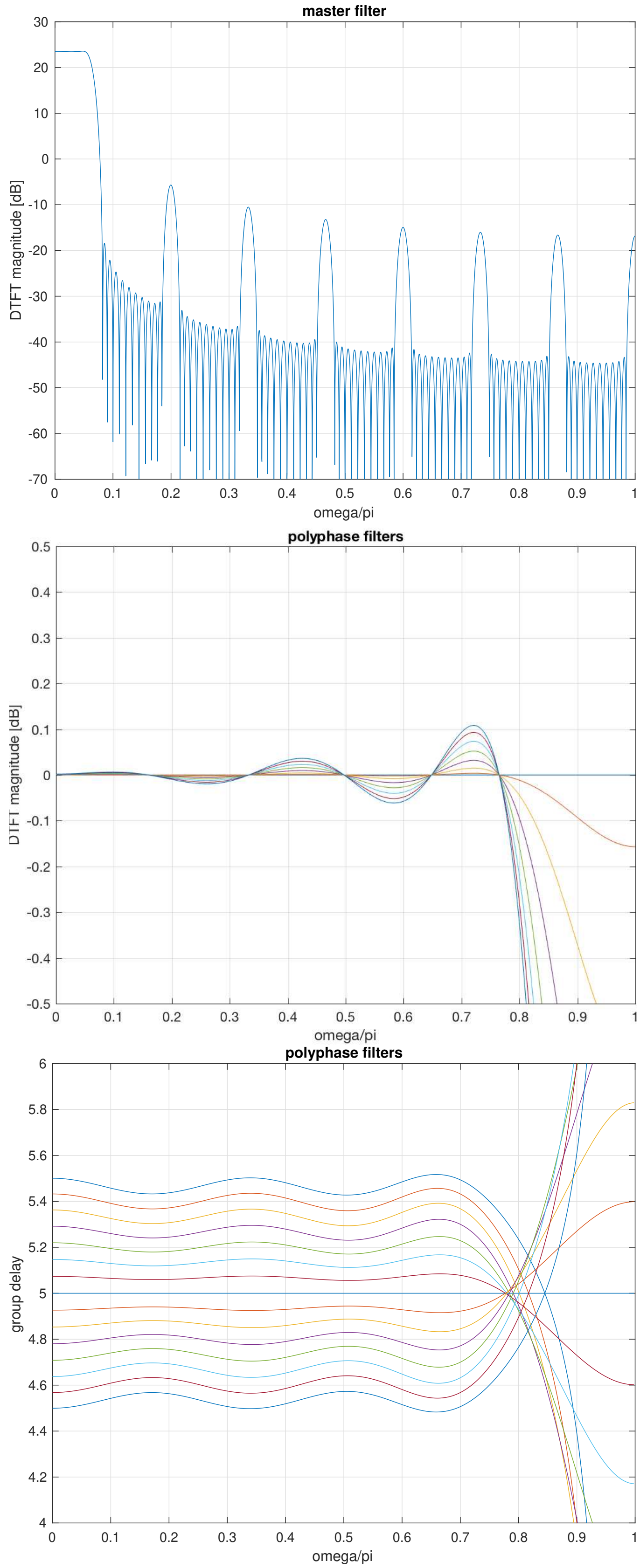
      Use a total of three plots. (See example on next page.)

   (d) Design an equiripple approximation to desired $h[n]$ via `firpm` and give the same plots as (c).

   (e) Use the (Hamming) window design method to directly design each of the length-L polyphase filters $h_p[n]$. (To do this, simply multiply $d_p[n]$ from (b) by the length-L Hamming window.) Next, interleave the polyphase filters $h_p[n]$ into a corresponding master filter $h[n]$. Finally, give the same plots as (c) and (d).

   (f) Based on your plots, which approach do you expect to yield the best interpolator?

Example plots for problem 1(c):



2. **Polyphase Interpolation**: We will now implement a polyphase filterbank in MATLAB using the filters designed in Problem 1.

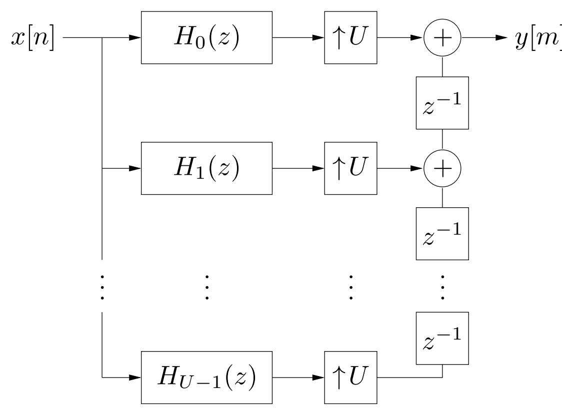
   (a) First, generate a length-1050 signal $x[n]$ bandlimited to $\omega_0 = 0.8\pi$ radians as follows:

      i. Generate a random full-bandwidth signal $u[n]$ using "`rng(0); u = randn(1,1000)`" and verify that it is full-bandwidth by plotting `20*log10(abs(fft(u,8192)))`. The command "`rng(0)`" resets the seed on the random number generator so that results are repeatable.

      ii. Using `firls`, design a length-51 lowpass filter $g[n]$ with passband $\omega \in [0, 0.6\pi)$ and stopband $\omega \in [0.8\pi, \pi)$.

      iii. Filter the full-bandwidth signal $u[n]$ using `conv`, and verify that the resulting signal $x[n]$ has bandwidth ω_0 by plotting `20*log10(abs(fft(x,8192)))`.

   (b) Generate polyphase-interpolated $y[m]$ using the structure below with $U = 15$ and the filters designed via the indirect weighted-least-squares approach from Problem 1(c). Demonstrate successful interpolation by plotting $x[n]$ for $n = [500 : 510]$ superimposed on the interpolates $y[m]$ (for the suitable range of m).



   (c) Repeat (b) for the indirect equiripple design of 1c.

   (d) Repeat (b) for the direct Hamming-windowed design of 1e.

   (e) Describe the performance of the interpolators. Are they consistent with your observations of the filters from Problem 1?

Hint: For debugging, recall the equivalence between polyphase and standard interpolation.