Question 1: Theoretical Questions [12 points]

Are the following typing statements true or false? Explain why. **[4x3 points]**

a. {f : [T2 → T3], g : [T1 → T2], a : Number} ⊢ (f (g a)) : T3

b. {f : [T1 → [T2 → Boolean]], x : T1, y : T2} ⊢ (f x y) : Boolean

c. {f : [T1 × T2 → T3], y : T2} ⊢ (lambda (x) (f x y)): [T1 → T3]

d. {f : [T2 → T1], x : T1, y : T3} ⊢ (f x) : T1

a.

$$\left\{\begin{array}{l} a : number \\ g : T1 \to T2 \end{array}\right. \longrightarrow g : number \to t_2 \Rightarrow (g\ a) = T_2 \ (*)$$

$F : t_2 \to T_3$

From (*) F recieve T_2 thefore F will output T_3,

hence the answer is true //

b.

$F : t_1 \to [t_2 \to Boolean]$, $x : t_1$, $g : t_2$

as we see F recieve one var and here we handing two vars to f, hence the ans is false //

c. $F: T_1 \times T_2 \to T_3, \; y : t_2$

$x, y \in T_1 \times T_2$, therefore F recieve valid input

also, F will output T_3, and the lambda

get input $x : T_1$. hence the typing statment

is true //

d. $F: t_2 \to t_1, \; X : T_1, \; y : T_3$

if $(F\,x)$ is valid then $t_1 = T_2$ and because we

don't have any information that contradicts

that t_1 is equal to T_2, forthere we

F will output T_1 as required, hence

the statment is true //.

2.1

a. Never

b. String

c. any

d. number

e. never

f. boolean

2.2

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[a] = string
[b] = string
[c] (if (is boolean z)
        ("boolean")
        ("number"))
```

2.3

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(Union string (Union boolean number))
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