CSE 242 Assignment 4, Fall 2022

3 Questions, 100 pts, due: 23:59 pm, Nov 11th, 2022

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Instruction

- Submit your assignments onto **Gradescope** by the due date. Upload a zip file containing:
 - (1) The saved/latest ipynb file, please **rename this file with your name** included.
 - (2) Also save your file into a pdf version, if error appears, save an html version instead (easy to grade for written questions).

For assignment related questions, please reach TA or grader through Slack/Piazza/Email.

 This is an individual assignment. All help from others (from the web, books other than text, or people other than the TA or instructor) must be clearly acknowledged.

Objective

- Task 1: EM algorithm (Mathematical Derivation)
- Task 2: K-Means implementation (Coding)
- Task 3: Kernel Methods with Noisy Setting (Coding)

Question 1. (EM algorithm, 20 pts)

Derive the E-step and M-step update equations of EM algorithm for estimating the Gaussian mixture model $p(X;\theta) = \sum_{k=1}^K \pi_k N(x;\mu_k,\sigma_k^2)$ where π_k is the mixture weight with $\pi_k \geq 0$ and $\sum_{k=1}^K \pi_k = 1$, and μ_k , σ_k^2 are the mean and variance of the gaussian distribution corresponding to cluster k.

For the E-step, first prove that
$$z_{ik} = P(z_i = k|X, \mu, \sigma, \pi) = \frac{\pi_k N(x_i; \mu_k, \sigma_k^2)}{\sum_{k=1}^K \pi_k N(x_i; \mu_k, \sigma_k^2)}.$$

Then, for the M-step, show the derivation to compute the updates for (μ_k, π_k) . Note that, you don't need to show the derivation for σ_k . For each derivation step, mention the concept applied (e.g. just 2-3 keywords, e.g. formula for expectation, independence of datapoints, (f+g)' = f' + g', etc ...).

Hint: For the M-step, you need to solve for $\mu_k^t = \underset{\mu^k}{argmax} \ E_{p(Z|X,\mu^{(t-1)},\sigma^{(t-1)},\pi^{(t-1)})}[\log p(X,Z|\mu,\sigma,\pi)] \ (\text{and similarly for } \pi_k) \ \text{by applying the first order conditions for function optimization (take derivative and set it to zero). Note that the term <math>p(Z|X,\mu^{(t-1)},\sigma^{(t-1)},\pi^{(t-1)})$ is the one computed in the E-step, and uses fixed values for μ,σ,π from the previous iteration (t-1).

```
In [1]: from IPython.display import Image
    # Replace the figure name
    Image(filename='1.1.jpg')
```

Out[1]:

()	Expectation Step:			
	The probability of assigning the ith data point to the cluster k is given by:			
	$Z_{ik} = P(Z_{i=k} X = x_i, u_k, \sigma_k, \pi_k)$			
	$P(X Y) = \frac{P(Y X) P(X)}{P(Y)}$ By Bayes Rule			
	Thus $Z_{ik} = P(Z_{i=k}) P(X_i, M_k, \sigma_k, T_k Z_{i=k})$ $P(X_i, M_k, \sigma_k, T_k)$			
	Since $P(Z_{i=k}) = T_k$ — (1) N-date points			
	and $P(X_i, M_k, \sigma_k, T_k Z_i : k) = N(x_i M_k, \sigma_k^2)$ — (2) - likelihood of datapoint given the			
	Unter k			
	P(x) = & P(x, y) = & P(x y) P(y)			
	$P(x_i, \mu_k, \sigma_k, \pi_k) = \sum_{k=1}^{k} P(x_i, \mu_k, \sigma_k, \pi_k z_i = k) \times P(z_i = k)$			
	$ \geqslant P(x_i, M_K, \sigma_K, T_K) = \sum_{k=1}^{K} N(x_i M_K, \sigma_k^2) * T_K - (3) $			
	Hence: $\frac{1}{2ik} = \frac{\pi_k \times N(\pi_i M_k, \sigma_k^2)}{\sum_{k=1}^k N(\pi_i M_k, \sigma_k^2)}$ from (1) (2) A (3)			

In [2]: Image(filename='1.2.jpg')

Out[2]:

:	
	Maninization Step
	Assuming $\Omega = \mathbb{E}_{P[Z X, T^{t-1}, M^{t-1}, \sigma^{t-1})} \log(P(X, Z T, M, \sigma))$
	Since $E[f(x)] = \xi f(x) p(x)$
	Hence $\Omega = \mathbb{Z} \log (P(x, z \pi, M_{\odot})) P(z x, \mu^{t}, \sigma^{t}, \pi^{t-1})$
	$\Omega = \underbrace{\mathcal{E}}_{Z_1 \nmid Z_2, \cdots, Z_N \mid X_1, X_2, \cdots} X_{N, M^{t-1}, \sigma^{t-1}, \pi^{t-1}})$
	13(P(Z, ×1 π,μ,σ))
	= Z P(Z1, Z2, ZN/ 1, 1/2, XN, Mt1, ot1, 1) X
	2 thon (P(Z K) x P(X 6, M,Z))
	of we consider data points to be independent
	$ \Omega = \mathbb{Z} \left[\left(\mathbb{Z}_{1} \cdot \mathbb{Z}_{N} \mathbb{X}_{1} \cdot \mathbb{X}_{N}, \mathbb{M}^{1-1} \circ \mathbb{H}^{1}, \mathbb{K}^{1} \right) \log \left(\frac{N}{N} P(\mathbb{Z}_{1} \mathbb{K}) \times P(\mathbb{X} \mathbb{Z}_{1}, \mathbb{M}_{16}) \right) \right] $
	= E P(Z, -Z,) x, -X, , M - ot x = 1) (wg [P(Z; M) x Z N Z N Z N Z N N N

In [3]: Image(filename='1.3.jpg')

Out[3]:

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		D= E P(Z, ZN X, XN, Mt , ot), xt -1) x 67 [P(Z, 1x) x [= 1 = 1 for) P(x Z, M, o)]
		[= = 1 P(x12i, M, E)]
		Z1, Z2 Zp are all independent
		Phus: D=
		Σ ε ρ(z, x, μ σ χ ρ(z, x, μ σ χ σ
		x [[[[] x P (X Z ; M, 0)]
	0	1 = 2 = P(Z; x , u, r x) x lg[P(Z; x), P(x; Z; , u, o)] x
	1 <i>L</i>	i=1 2;
		Σ P(Z, X, μ,σ, π). P(Z, X,μ,σ,π). P(ZN X,ν,σ,μπ)
		-1
	Ω	= Z = P(Z; [x; N, 6, r') x lg[P(Z; 1x). P(x; 12; N,0)]-2
		i=1 K
		$\mu^{\text{new}} = \operatorname{argman} \Omega(\mu)$
		$\mu^{\text{new}} = \underset{\text{argman}}{\text{argman}} \Omega(\mu)$ $= \underset{\mu}{\text{argman}} \sum_{i>1} \sum_{k'} P(Z_i K_i, \mu^{t-1}) \times log(P(Z_i > k' K))$
		m is k
		P(x; 12; k', M, o, 2))]
		$\frac{\partial h''}{\partial \sigma(n)} = 0$
		J Mr.

In [4]: Image(filename='1.4.jpg')

Out[4]:

Thus
$$\frac{\partial \Omega(\mu)}{\partial \mu_{k}} = \frac{\partial}{\partial \mu_{k}} \left[\sum_{i=1}^{N} \mathcal{E} P(\mathcal{Z}_{i} | x_{i}, \mu^{t-1}) \times \log \left(P(\mathcal{Z}_{i} = \mu^{t} | x_{i}) \right) \cdot P(x_{i} | \mu_{i}, \sigma_{i}, \sigma_{i}^{2}) \right] = 0$$

Derivative of turn in the sum of derivatives

$$\sum_{i=1}^{N} \mathcal{E} \left(\mathcal{Z}_{i=k} | x_{i}, \mu^{t-1} \right) \frac{\partial}{\partial \mu_{k}} \log \left(P(\mathcal{Z}_{i} = k^{t} | x_{i}) \cdot P(x_{i} | \mathcal{Z}_{i=k}, \mu_{i}, \sigma_{i}^{2}) \right) \\
= 0$$

$$\sum_{i=1}^{N} P(\mathcal{Z}_{i=k} | x_{i}, \mu^{t-1}) \frac{\partial}{\partial \mu_{k}} \left[\log \left(N(x_{i} | \mu_{k'}, \sigma_{k'}, \mu^{t}) \right) \right] = 0$$

$$\sum_{i=1}^{N} P(\mathcal{Z}_{i=k} | x_{i}, \mu^{t-1}) \frac{\partial}{\partial \mu_{k}} \left[\log \left(\frac{1}{|x_{i} - x_{k}|} \times e^{-\frac{(x_{i} - \mu_{k})^{2}}{2\sigma^{2}}} \right) \right] = 0$$

$$\sum_{i=1}^{N} P(\mathcal{Z}_{i=k} | x_{i}, \mu^{t-1}) \frac{\partial}{\partial \mu_{k}} \left[\log \left(\frac{1}{|x_{i} - x_{k}|} \times e^{-\frac{(x_{i} - \mu_{k})^{2}}{2\sigma^{2}}} \right) \right] = 0$$

$$\sum_{i=1}^{N} P(\mathcal{Z}_{i=k} | x_{i}, \mu^{t-1}) \frac{\partial}{\partial \mu_{k}} \left[-\frac{(x_{i} - \mu_{k})^{2}}{2\sigma^{2}} \right] = 0$$

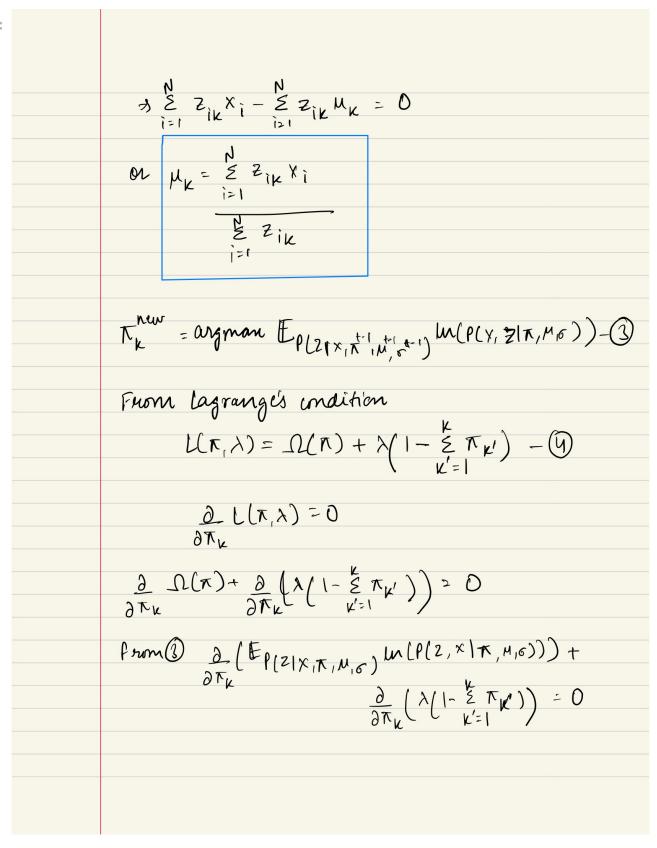
$$\sum_{i=1}^{N} \mathcal{Z}_{i=k} \left(-\frac{\mathcal{Z}}{|x_{i} - \mu_{k}|} \times e^{-\frac{(x_{i} - \mu_{k})^{2}}{2\sigma^{2}}} \right) = 0$$

$$\sum_{i=1}^{N} \mathcal{Z}_{i=k} \left(-\frac{\mathcal{Z}}{|x_{i} - \mu_{k}|} \times e^{-\frac{(x_{i} - \mu_{k})^{2}}{2\sigma^{2}}} \right) = 0$$

$$\sum_{i=1}^{N} \mathcal{Z}_{i=k} \left(-\frac{\mathcal{Z}}{|x_{i} - \mu_{k}|} \times e^{-\frac{(x_{i} - \mu_{k})^{2}}{2\sigma^{2}}} \right) = 0$$

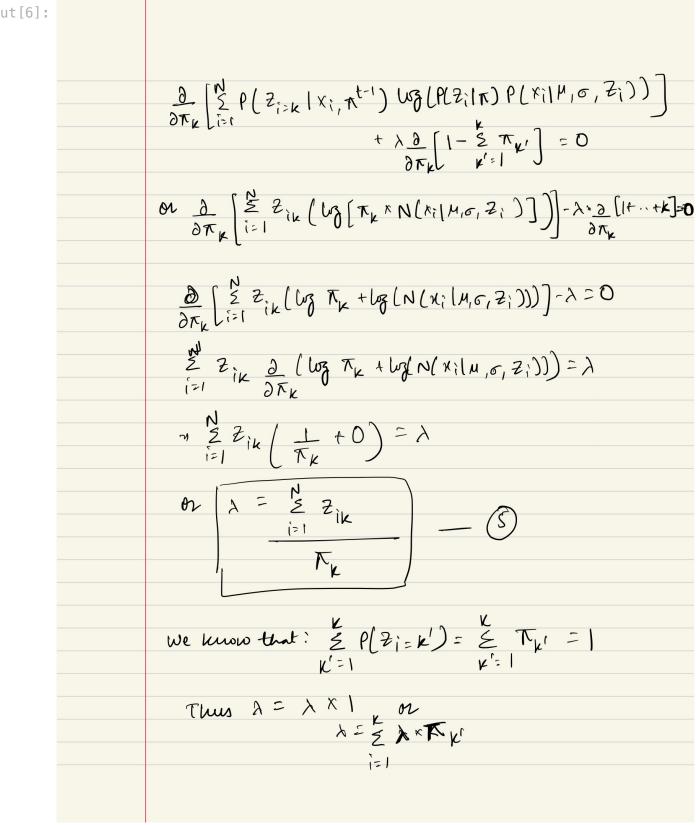
In [5]: Image(filename='1.5.jpg')

Out[5]:



In [6]: Image(filename='1.6.jpg')

Out[6]:



Image(filename='1.7.jpeg') In [7]:

Out[7]:

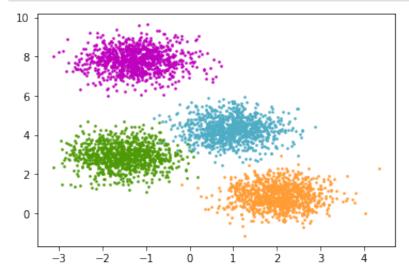
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	$\lambda = \frac{1}{2} \left(\frac{2}{121} \frac{2}{121} \frac{2}{121} \frac{1}{121} \right) \times \frac{1}{121} \frac{1}$			
	$\mathcal{K}^{2} \left(\frac{1^{2} 1}{2} \right)^{N}$			
	N			
	N $\lambda = \xi \xi \xi_{ik}$ $\lambda = 1$ because we are			
	1. C 11 11 mm 101 200			
zs A	adding all the probabilities = $\leq 1 = N - 6$ of in datapoint belonging to all the clusters.			
-,	i=1 to all the clusters.			
Eva	m (5) 1 (6)			
110	M J D D			
	T _K = \(\frac{\times}{2}\) ik			
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https:	//www.youtube.com/watch?v=XGRXH9Fcc-w			
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Question 2. (K-Means implementation, 20 pts)

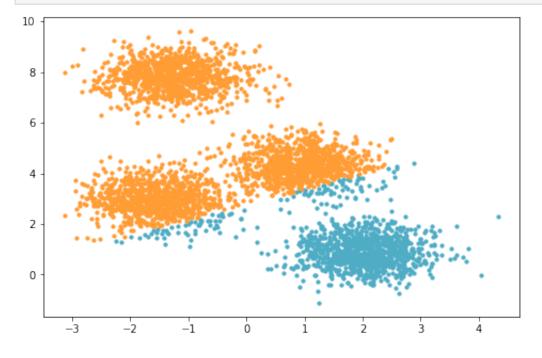
Question 2.1. Implement K-means in Python from scratch. Complete following sub-functions update_centroids and update_assignments.

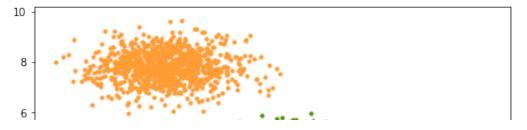
```
In [8]: import numpy as np
        from scipy.spatial.distance import cdist
        def update assignments(data, centroids):
          ############################
          #### YOUR CODE HERE ####
          row, col = data.shape
          assignments = np.empty([row])
          distances = cdist(data, centroids)
          assignments=np.argmin(distances,axis=1)
          ## you will get cluster#
          ##assignments here #####
          #############################
          return assignments
        def update centroids(data,centroids,assignments):
          ##########################
          #### YOUR CODE HERE ####
          ###########################
          K = centroids.shape[0]
          centroids=np.empty(centroids.shape)
          for i in range(K):
            centroids[i]=np.mean(data[assignments ==i], axis=0)
          return centroids
        def kmeans(data, centroids, max iterations):
            for j in range(max iterations):
                # update cluter assignments
                assignments = update assignments(data,centroids) # WRITE CODE FOR
                 # update centroid locations
                centroids = update_centroids(data,centroids,assignments) # WRITE CC
            # final assignment update
            assignments = update_assignments(data,centroids)
            return centroids, assignments
```

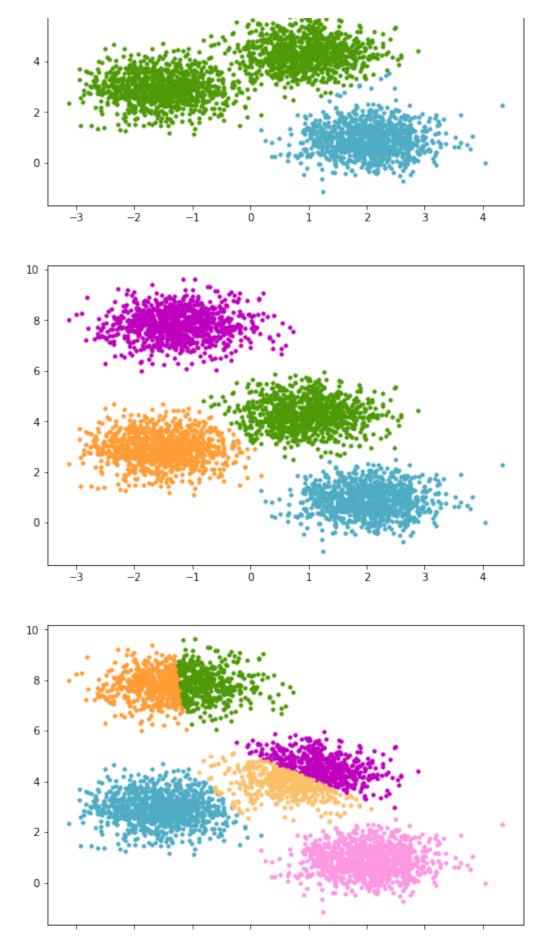
Question 2.2. Run your code on following toy dataset for different k-values, where $k = \{2, 3, 4, 6, 10\}$ and plot the cluster assignments for different k's as shown in following diagram.



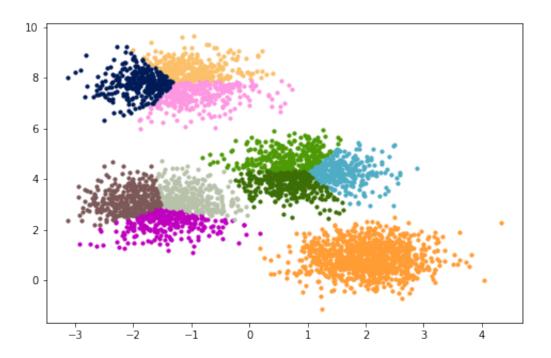
```
In [10]: import numpy as np
         # function to get initial cluster centroids; we randomly choose k points from
         def get initial clusters(k, X):
          random indices = np.random.randint(0, X.shape[0], k)
          initial centroids = X[random indices]
          return initial_centroids
         # your code here.
         clusters=[2,3,4,6,10]
        fig, axs = plt.subplots(len(clusters), figsize=[8,30])
         for idx, i in enumerate(clusters):
            centroids=get initial clusters(i, X)
            centroids,assignments=kmeans(X,centroids,100)
            #print(assignments.shape)
            for k in range(i):
                cluster data = assignments == k
                axs[idx].scatter(X[cluster_data, 0], X[cluster_data, 1], color=color
```











Question 3. (Kernel Methods with Noisy Setting, 60 pts)

SVM on synthetic dataset generated as follows:

- ullet Draw $1000~(x_0,x_1)$ feature vectors from the 2-D Gaussian distribution with mean $\mu_+=(1,1)$ and $\Sigma_+=[1,0;0,1]$ and label them as +1.
- Draw $1000~(x_0,x_1)$ feature vectors from the 2-D Gaussian distribution with mean $\mu_-=(-1,-1)$ and $\Sigma_-=[3,0;0,3]$ and label them as -1.
- \bullet This gives you a 2000 example training set. Repeat the above to draw a test set the same way.

Use a SVM package (scikit-learn svm.SVC class) to learn SVMs with a variety of parameter settings.

```
In [11]: # Your code here
         import numpy as np
         import pandas as pd
         from sklearn.utils import shuffle
         np.random.seed(220)
         mean1 = (1,1)
         cov1=[[1,0],[0,1]]
         mean2=(-1,-1)
         cov2=[[3,0],[0,3]]
         df1 = pd.DataFrame(np.random.multivariate normal(mean1,cov1,1000),columns=[
         df1['label']=+1
         df2 = pd.DataFrame(np.random.multivariate normal(mean2,cov2,1000),columns=[
         df2['label']=-1
         df_train = pd.concat([df1,df2], ignore_index=True)
         #Shuffling to avoid consistency
         # df train = shuffle(df train)
         df train = df train.reset index(drop=True)
         df train
```

Out[11]:

	X	У	label
0	-0.324352	-0.465870	1
1	-0.249991	1.654616	1
2	2.235651	-0.122390	1
3	1.414549	0.806798	1
4	-0.747681	2.687990	1
•••			
1995	-2.454008	-3.098631	-1
1996	-5.584851	-0.248678	-1
1997	2.095982	0.153912	-1
1998	1.639896	-3.109557	-1
1999	0.186215	-2.850072	-1

2000 rows × 3 columns

```
In [12]: df1 = pd.DataFrame(np.random.multivariate_normal(mean1,cov1,1000),columns=['
    df1['label'] = +1
    df2 = pd.DataFrame(np.random.multivariate_normal(mean2,cov2,1000),columns=['
    df2['label'] = -1
    df_test = pd.concat([df1,df2], ignore_index=True)
    df_test
```

Out[12]:

	Х	У	label
0	-1.898160	1.556240	1
1	1.784113	-0.436965	1
2	1.986831	2.236635	1
3	-0.213913	1.240479	1
4	2.431783	0.760874	1
•••			
1995	-1.012130	0.737504	-1
1996	0.199889	1.865863	-1
1997	-2.578212	1.371016	-1
1998	-0.800268	-1.951012	-1
1999	-2.646145	-1.403640	-1

2000 rows × 3 columns

(a -- 20 pts)

- ullet Use an RBF kernel with parameters C=1, $\gamma=0.01$.
- For each training data with +1 label, randomly flip their label to -1 with probability
 0.35.
- For each training data with -1 label, randomly flip their label to +1 with probability
 0.20.
- Train with the above noisy training examples.
- Random flipping introduces the randomness. You can repeat multiple times (e.g. 20)
 and then report the average accuracy on the testing dataset (clean) in the noise
 parameter setting.

```
In [13]: from sklearn.model_selection import train_test_split
          from sklearn.metrics import accuracy score
          from sklearn.svm import SVC
          import random
          X_train =df_train[['x', 'y']]
          y train =df train['label'].values
          X_{\text{test}} = df_{\text{test}[['x', 'y']]}
          y_test = df_test['label'].values
          model = SVC(kernel='rbf',C=1, gamma=0.01, random_state = 21)
          model = model.fit(X train, y train)
          y pred = model.predict(X test)
          accuracy = accuracy_score(y_test, y_pred)
          print("Accuracy score before flip: ",accuracy)
         Accuracy score before flip: 0.8585
In [14]: def flip labels(pos,neg,prob pos=0.35,prob neg=0.20):
              pos_prob=np.random.random(size=pos.shape[0])
              flip pos=np.where(pos probprobprobpos,-1,pos)
              neg prob=np.random.random(size=neg.shape[0])
              flip neg=np.where(neg probprobobneg,1,neg)
              return np.hstack([flip pos,flip neg])
          y train flipped= flip labels(y train[:1000],y train[1000:])
```

```
In [15]: model1 = SVC(kernel='rbf', probability=True, C=1, gamma=0.01)
model1 = model1.fit(X_train, y_train_flipped)

y_pred = model1.predict(X_test)
accuracy = accuracy_score(y_test, y_pred)
print("Accuracy score after flip: ",accuracy)
```

Accuracy score after flip: 0.7895

(b -- 20 pts) Open question

- Try using K-Nearst Neighbors to correct wrong labels before training.
- Then train the model with the newly processed training dataset.
- Report the accuracy on the testing dataset in the noise parameter setting. Do you observe performance improvement?

```
In [16]: # Your code here
         import matplotlib.pyplot as plt
         %matplotlib inline
         from sklearn.neighbors import KNeighborsClassifier
         from sklearn.metrics import accuracy score
         best knn svm=0
         best_k_svm=0
         best knn=0
         best k=0
         best_knn_model=None
         for k in range(1,100):
             knn_neigh=KNeighborsClassifier(n_neighbors=k)
             knn neigh.fit(X train,y train flipped)
             y neigh predict test=knn neigh.predict(X train)
             if(accuracy score(y test,y neigh predict test)*100>best knn):
                 best knn=accuracy score(y test,y neigh predict test)*100
                 best k=k
                 best knn model=knn neigh
         #print(best k)
         svm knn=SVC(kernel='rbf',random state=0,gamma=.01,C=1)
         y_train_knn=best_knn_model.predict(X_train)
         svm_knn.fit(X_train,y_train_knn)
         y_svm_knn_predict_test=svm_knn.predict(X_test)
         best knn svm=accuracy score(y test,y svm knn predict test)*100
             #if(accuracy score(y test,y svm knn predict test)*100>best knn svm):
                  best knn svm=accuracy score(y test,y svm knn predict test)*100
                  best k svm=k
         print(f"Accuracy with knn+svm: {best knn svm}")
         #print(f"Accuracy with just knn: {best knn},k neighbors:{best k}")
```

After correcting the noisy labels using knn, and then running svm on that- the accuracy has improved

(c -- 20 pts) Open question

- Try using **clustering (i.e., K-means, EM-clustering)** to correct wrong labels before training.
- Then train the model with the newly processed training dataset.
- Report the accuracy on the testing dataset in the noise parameter setting. Do you observe performance improvement?

Accuracy with kmeans+svm: 82.25

The accuracy after k-means is comparitively less as it is unsupervised and the labels are labelled based on the cluster. But the accuracy has still improved a little.

```
In [18]: from sklearn.mixture import GaussianMixture
    EM_cluster= GaussianMixture(n_components=2,random_state=0).fit(X_train)
    Y_EM=EM_cluster.predict(X_train)
    #print(Y_EM)

for i,ele in enumerate(Y_EM):
    if Y_EM[i] ==1:
        Y_EM[i]=-1
    elif Y_EM[i]==0:
        Y_EM[i]=1
    #print(Y_EM)
    svm_EM=SVC(kernel='rbf', random_state=0,gamma=.01,C=1)
    svm_EM.fit(X_train,Y_EM)
    y_svm_EM_predict_test=svm_EM.predict(X_train)
    print(f"Accuracy with EM+svm: {accuracy_score(y_test,y_svm_EM_predict_test)})
```

Accuracy with EM+svm: 83.3500000000001

The accuracy has improved compared to the one with noisy labels. Still its less than knn because EM is again unsupervised.