ICCG 2019

Approximate Curve-Restricted Simplification of Polygonal Curves

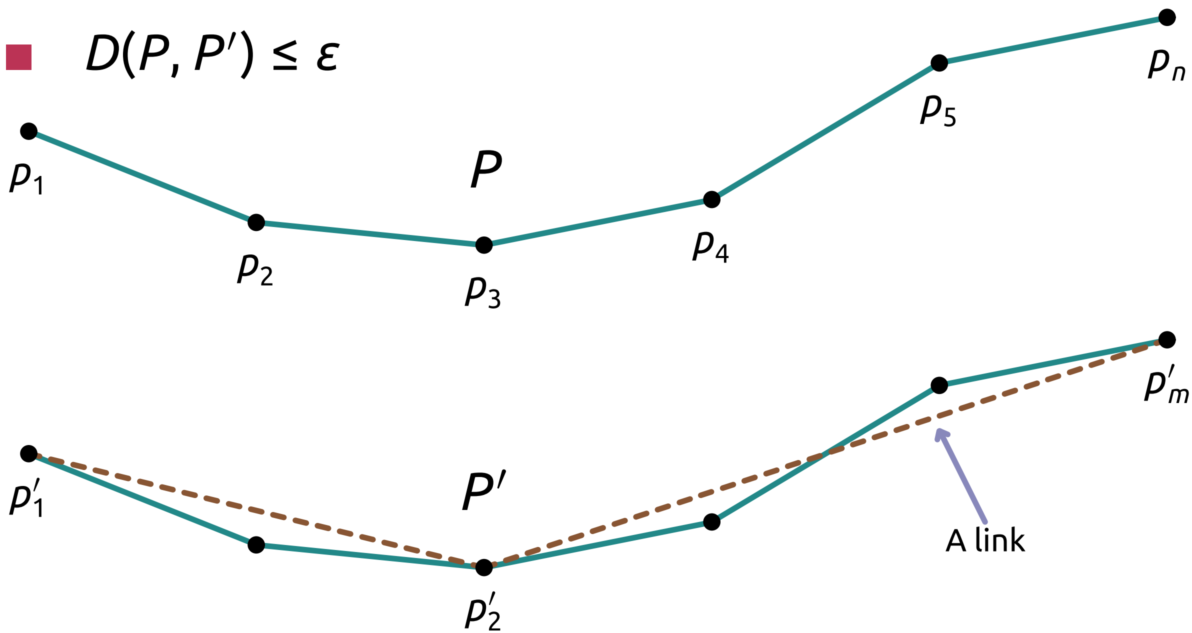
Ali Gholami Rudi

Babol Noshirvani University of Technology

Curve Simplification

$P' = \langle p'_1, \cdots, p'_m \rangle$ is a simplification of $P = \langle p_1, \cdots, p_n \rangle$ if:

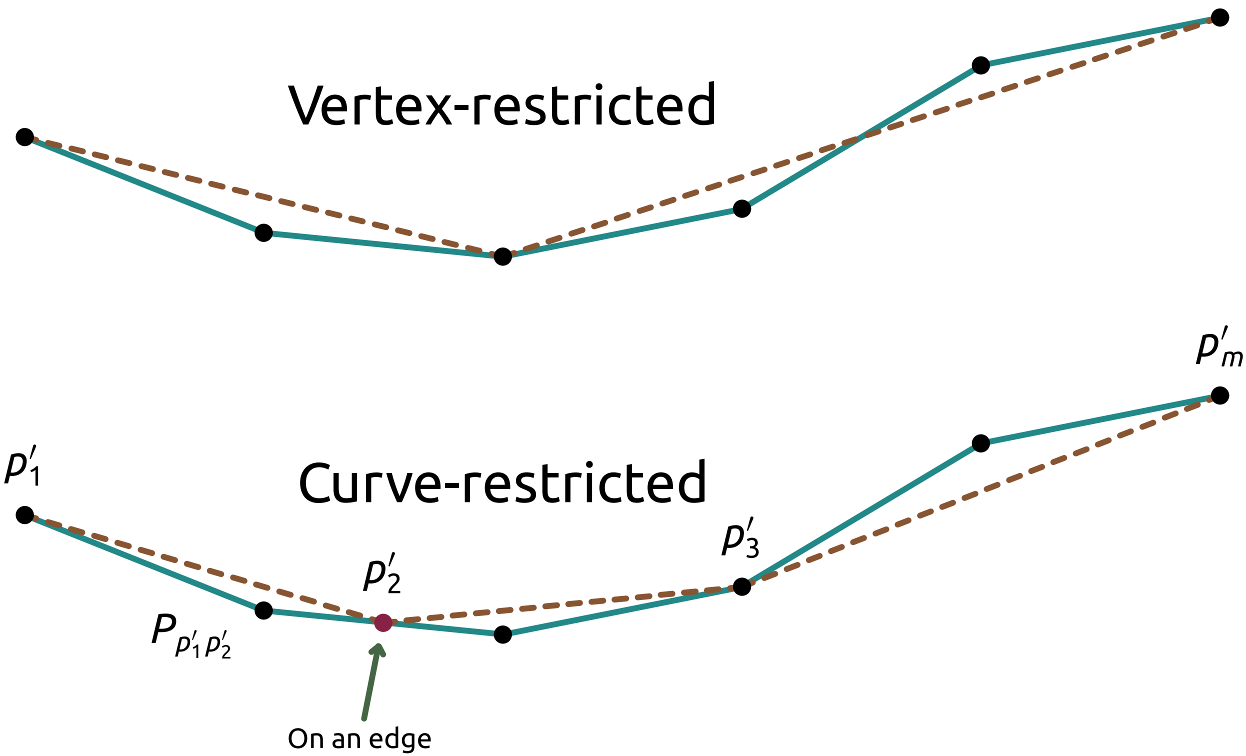
- $p_1 = p'_1$ and $p_n = p'_1$
- $m \leq n$
- $D(P, P') \leq \varepsilon$



Simplified Curve: - - - - -

Curve-Restricted Simplification

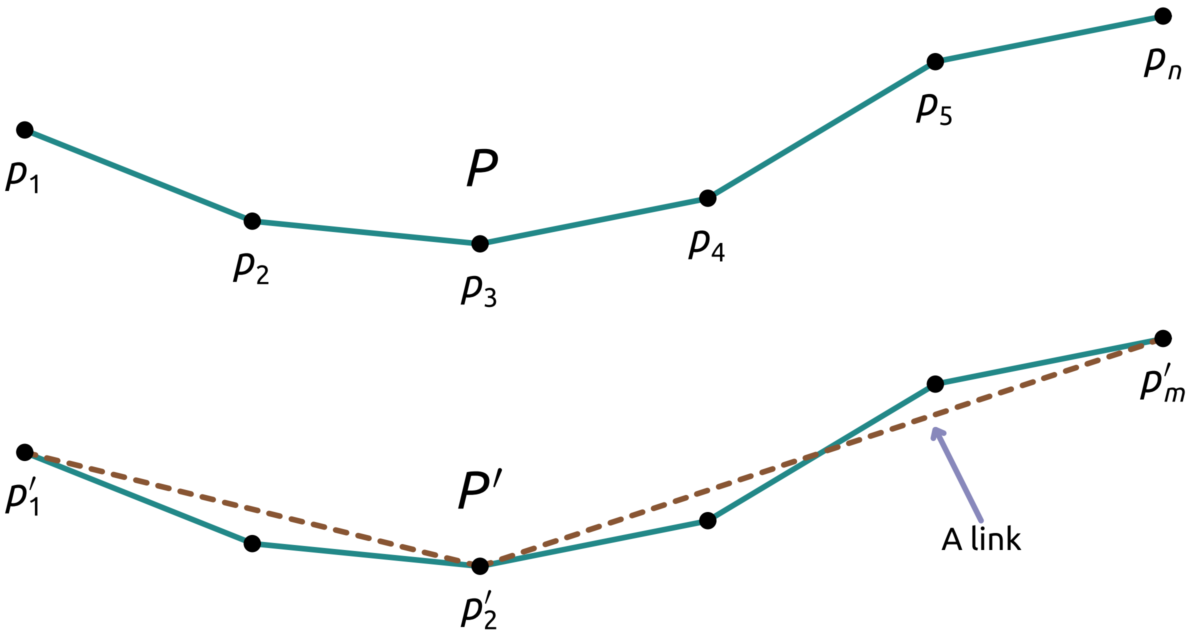
Vertex-restricted: for each i, $p'_i = p_j$ for some j.



Simplification Goals

Usually studied in two settings:

- Min-ε problem: m specified, minimize ε
- Min-# problem: ε fixed, minimize m

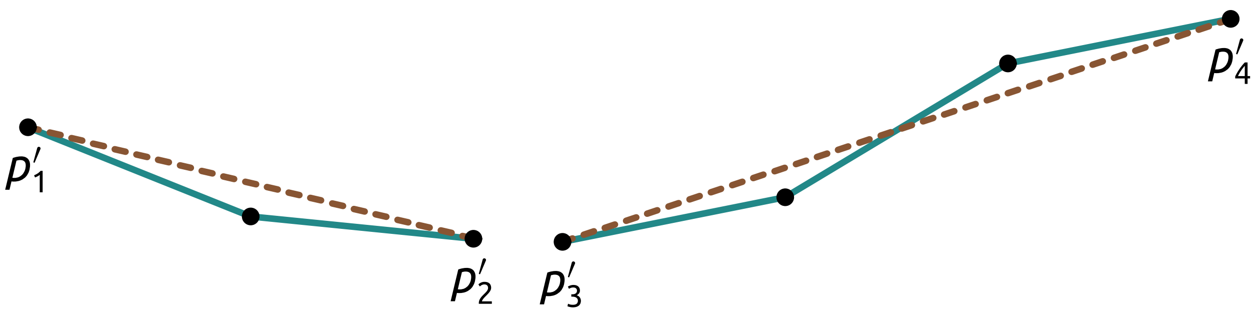


Simplified Curve: - - - - -

Local/Global Error

The error between P and P': computed globally or locally.

- global: distance between whole curves
  - $D(S, T)$: the distance between two curves
- local: max distance between corresponding sub-curves



$D_1 = D(P_{p'_1 p'_2}, p'_1 p'_2)$ $\quad$ $D_2 = D(P_{p'_3 p'_4}, p'_3 p'_4)$

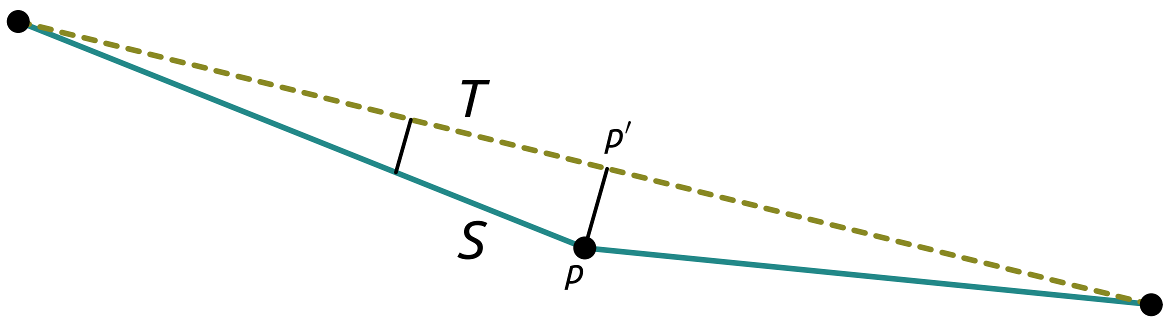
$$E(P, P') = \max(D_1, D_2)$$

Distance Measure

Distance measures: Fréchet or Hausdorff

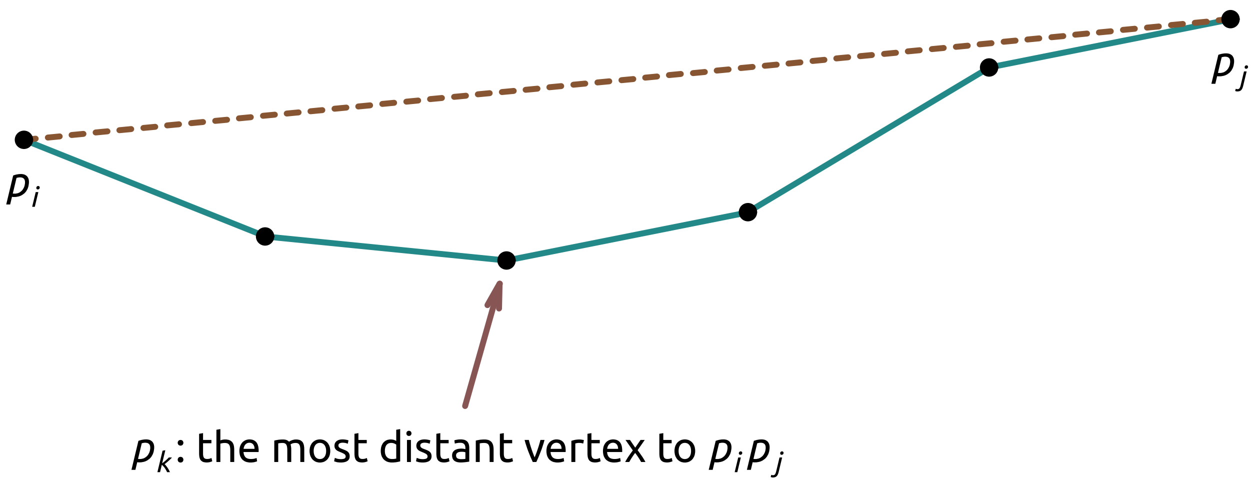
- Directed Hausdorff from S to T

$$H(S, T) = \max_{p \in S} dist(p, T)$$



Douglass and Peucker (1973)

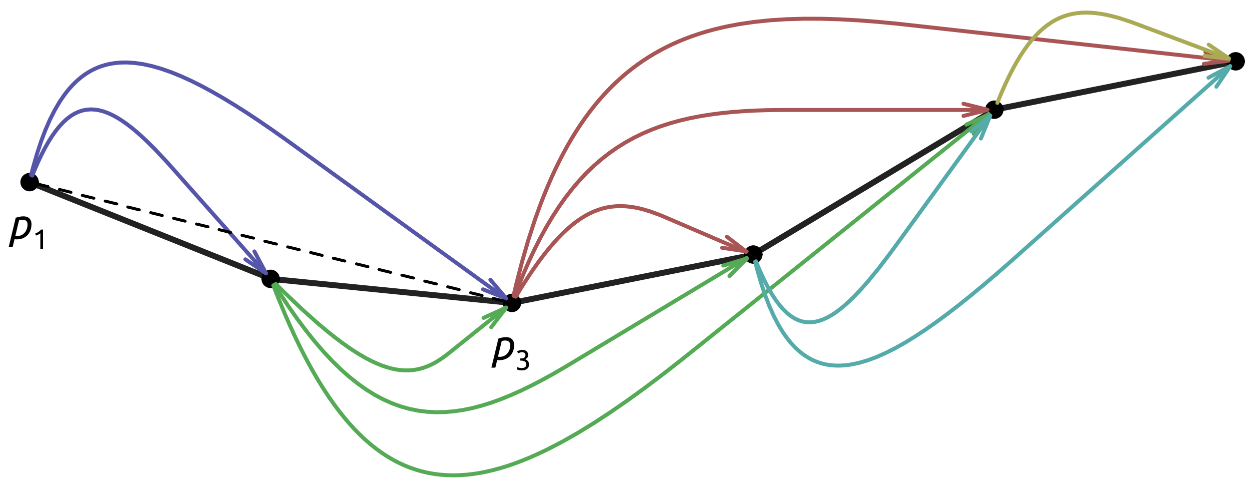
- Recursively splits at the most distant vertex if $d > \varepsilon$
- Does not minimise m
- Time complexity improved to $O(n \log n)$



Classic Algorithms

Imai and Iri (1988)

- Builds a shortcut graph
- Minimises m
- Time complexity improved to $O(n^2)$



Results on G. Vertex-Restricted Simp.

Global undirected Hausdorff distance

- Proved NP-Hard: van Kreveld et al. (2018)

Global directed Hausdorff from P' to P

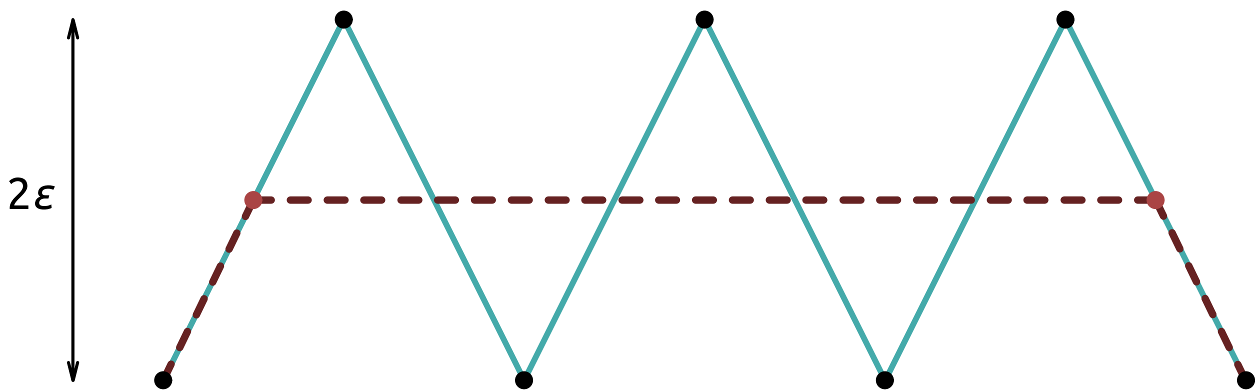
- $O(n^4)$: van Kreveld et al. (2018)
- $O(n^2 \log n)$: van de Kerkhoff et al. (preprint; 2018)

Global Fréchet distance

- $O(mn^5)$: van Kreveld et al. (2018)
- $O(n^4)$: van de Kerkhof et al. (preprint; 2018)
- $O(n^3)$: Bringmann and Chaudhury (preprint; 2018)

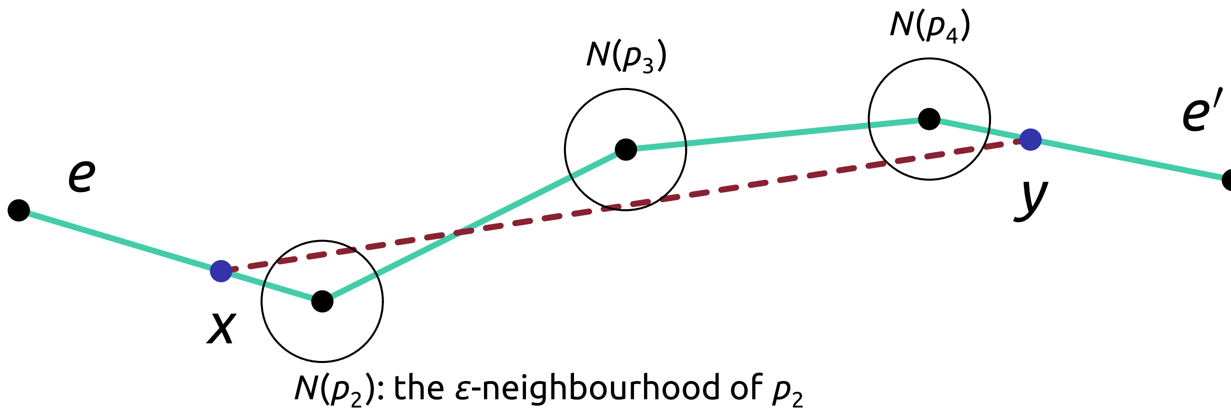
Our Goal

- Min-#
- Curve-restricted simplification
- Local directed Hausdorff from P to P'

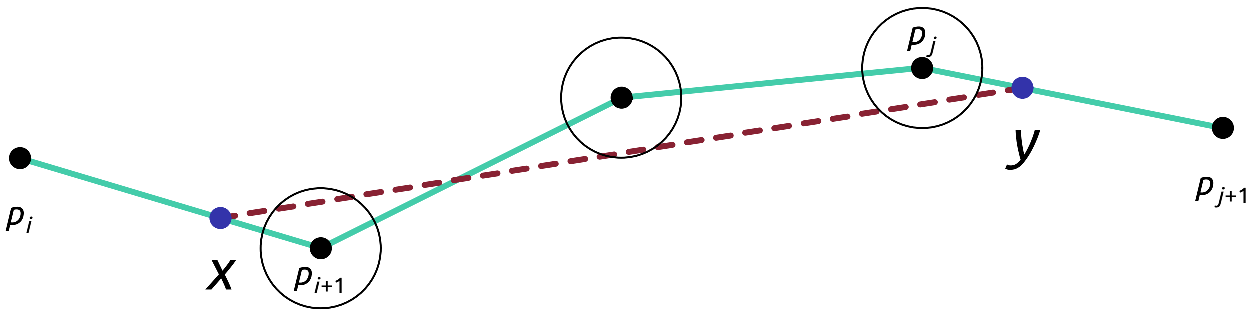


Global curve-rest. simp. for directed Hausdorff is NP-hard (Kerkhof, Kostitsyna, Löffler, Mirzanezhad, Wenk; 2018)

Lemma: intersecting the ε-neighbourhood of vertices



Lemma: If there is a link from $p_i p_{i+1}$ to $p_j p_{j+1}$, there exists another link, for which one of the following properties (next page) holds for at least two values of k for $i < k \leq j$:

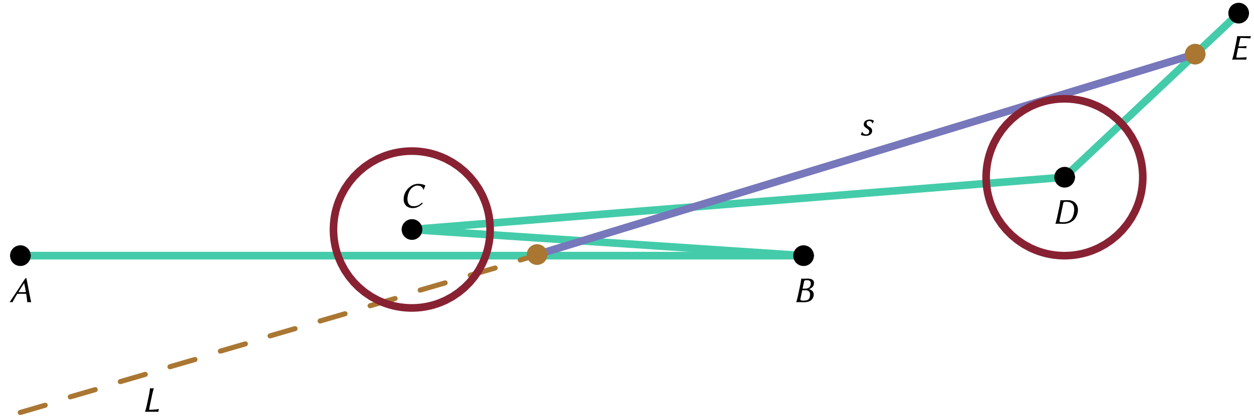


Adjusting links

Lemma: If there is a link from $p_i p_{i+1}$ to $p_j p_{j+1}$, there exists another link, for which one of the following properties holds for at least two values of k for $i < k \leq j$:

- It is a tangent to $N(p_k)$.
- It passes through the end points of $p_i p_{i+1}$ or $p_j p_{j+1}$ or their intersection with $N(p_k)$.

Adjusting links



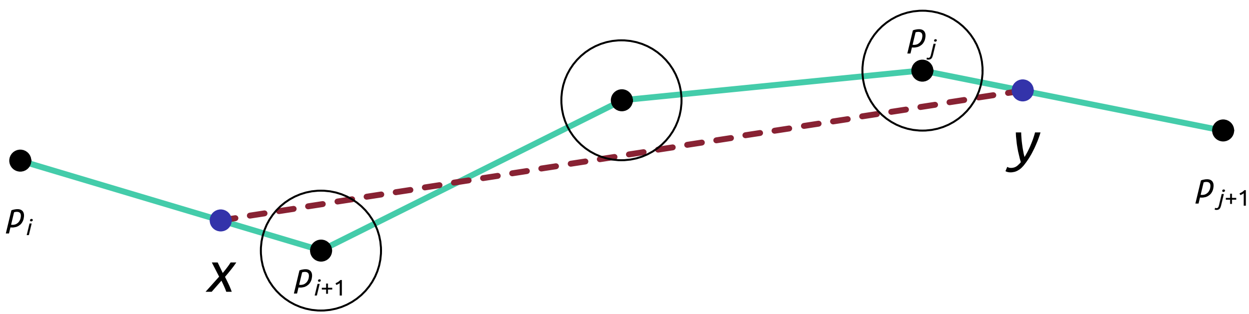
Finding the links

Algorithm for finding a link between $p_i p_{i+1}$ and $p_j p_{j+1}$:

- Try every pair of indices for k
- test if there is a link

Time complexity: $O(n^3)$.

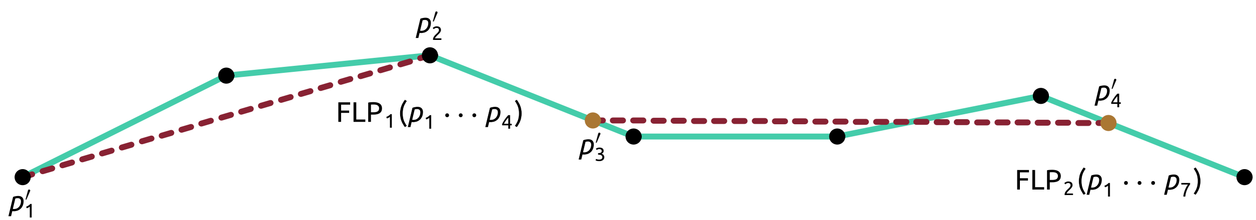
Handling tangents: two parallel lines with distance ε



Disjoint Link Chain (DLC)

Def: A disjoint link chain (DLC) is a sequence of segments D:

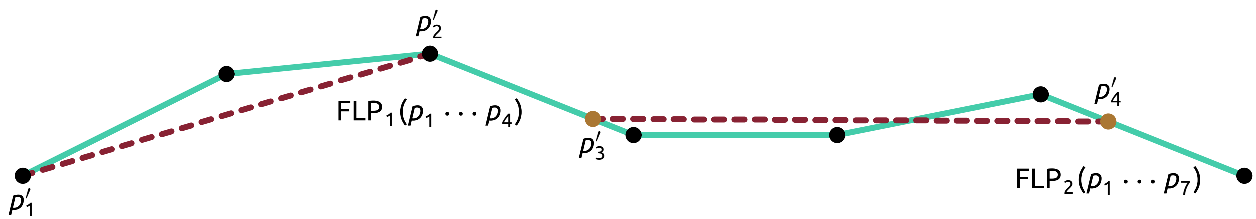
- $D = \langle p'_1 p'_2, p'_3 p'_4, \cdots, p'_{2k-1} p'_{2k} \rangle$
- $p'_{2i-1} p_{2i}$ is a valid link
- p'_{2i} and p'_{2i+1} are on the same edge of P
- The vertices appear in order on P



DLC to curve-restricted simp.

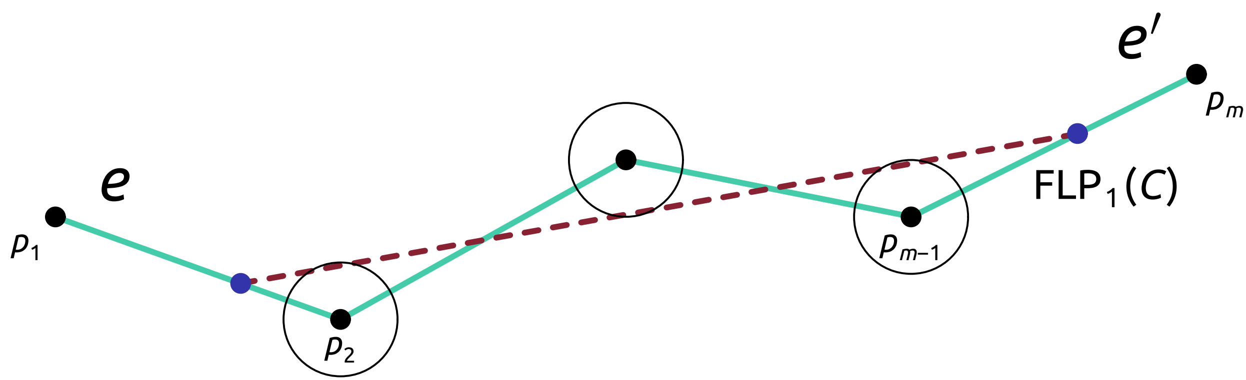
From a DLC $D = \langle p'_1 p'_2, p'_3 p'_4, \cdots, p'_{2k-1} p'_{2k} \rangle$

- A simp. with $2k$ vertices can be obtained (if $p'_1 = p_1$)



Def: For curve $C = \langle p_1, p_2, \cdots, p_m \rangle$,

- $\mathrm{FLP}_k(C)$ is the first point on $p_{m-1}p_m$
- to which there is a DLC with k links.



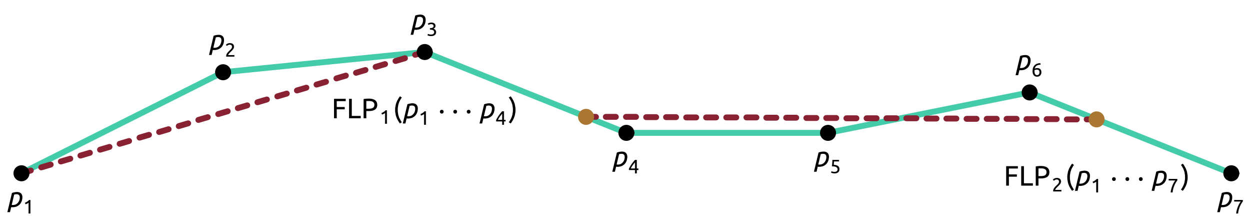
$\mathrm{FLP}_1(\langle p_1, \cdots, p_m \rangle)$ can be computed in $O(m^3)$

Finding the Smallest DLC Using DP

Dyn. Prog. for obtaining minimum link DLC:

$F[i][j]$ is $\mathrm{FLP}_j(\langle p_1, \cdots, p_i \rangle)$

- $F[i][1] = \mathrm{FLP}_1(\langle p_1, \cdots, p_i \rangle)$
- $F[i][j] = \min\limits_{1 \leq k < i} FLP_1(\langle F[k][j-1], p_k, \cdots, p_i \rangle)$



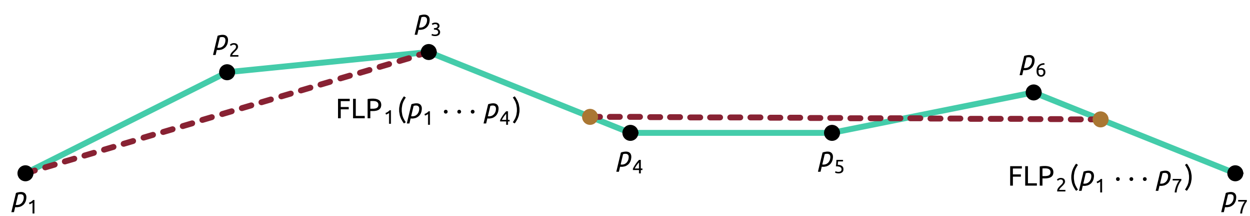
Time complexity: $O(n^6)$

Obtain a DLC using $F[n][m]$

- m is the largest index such that $F[n][m]$ is filled

Induction: $F[i][j]$ is filled iff:

- A DLC with j links exists for $\langle p_1, \cdots, p_i \rangle$



To Curve-restricted Simplification

The DLC computed using DP: D with k links

- Simp. P' from D with $2k$ links
- An optimal curve-restricted simp.
  - Is also a DLC
  - Has no less than k links

Concluding Remarks

- $O(n^6)$ is too slow
- Faster exact/approximate results
  - Introducing new vertices and using vertex-rest. algs.