Estimating KLD for log normal mixture model

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In this paper I'll present a method to estimate KLD for mixture model of log-normal distribution, relevant for the brain research conducted by prof. Daphna Yoel.

1 The model

We test the following hypothesis:

$$H_0 : x \sim N(\mu, \sigma^2) \tag{1}$$

$$H_1 : x \sim \frac{\delta}{\delta + \varepsilon} N(\mu + \varepsilon, \sigma^2) + \frac{\varepsilon}{\delta + \varepsilon} N(\mu - \delta, \sigma^2) \tag{2}$$

where $\varepsilon > 0, \delta > 0$. Our goal is to estimate the KLD under the alternative to find a region on the manifold of the parameters where we can reject the null hypothesis.

2 Estimate KLD

In order to estimate KLD we follow this algorithm:

1. create a grid of parameters: $\{\mu, \delta, \varepsilon, \sigma^2\}$
2. for each set on the grid we sample with the right proportions from the shifted densities
3. we compute KLD using numerical integration using the formula:

$$w(y) = log[\frac{ep}{ep + de} exp^{de*y - \frac{de^2}{2}} + \frac{de}{ep + de} exp^{-ep*y - \frac{ep^2}{2}}]$$

where:$y = \frac{x-\mu}{\sigma}$, $ep = \frac{\varepsilon}{\sigma}$, $de = \frac{\delta}{\sigma}$

$$KLD = \frac{ep}{ep+de}\int w(y)\phi(y-de)dy + \frac{de}{ep+de}\int w(y)\phi(y-ep)dy \quad (3)$$

where ϕ denotes the standard normal density.

next we plot a 3D map of values of μ, p, KLD, where $p = \frac{\delta}{\delta+\varepsilon}$