“P and NP Problems”

P

P is a set of the problems that represents all the decision problems which are solvable in polynomial time.

Problems of this set can be solved in $O(n^k)$.

Example:-

If we want to color the vertices of a given connected graph in such a way that no edge is monochromatic.

Solution :- Start with any arbitrary vertex, color it red all of its neighbor vertices as blue, do it for all thee vertices and stop when you don’t have any vertices to color or there are two vertices of the same color on both side of the same edge.

NP

NP is the set of the problems in which all the containing problems have some solution and that solution can be verified in polynomial time.

In simple words we can say that it is the set of the problems which are verifiable in the polynomial time.

Example:-

If two given integers are a and b and if any other integer x exists, which is greater than 1 and less than b, such that x divides a. This NP problem is called *Integer Factorization.*

This is a decision problem and if someone claims that there exists an x which is factor of a and lies in the given interval and proofs that it is true than we can verify his/her answer in polynomial time by a simple division which takes polynomial time.

NP Complete

NP Complete is the set of problems X in NP for which it is possible to reduce any other NP problem Y to X in polynomial time.

In simple words we can say that if we have some similar problems in NP which needs same approach and if we can find solution for one problem than we can also find the solution for the similar problem using similar approach. So this is why NP complete problems are important because if we can find solution for one problem we can find solution for every NP problem which needs similar approach.

The Minesweeper Problem is the Example of NP Complete problem.

NP Hard

If any NP Complete problem X is reducible to any problem Y in polynomial time than we can say that the problem X is NP Hard problem. All NP Complete problems can be reduced to any NP hard Problem in polynomial time then if there is solution to one NP Hard problem in polynomial time then there will definitely be a solution to all NP problems in polynomial time. *The Subset Sum* Problem and *The Halting Problem* are the example of NP Hard Problem.