BRANCH AND BOUND

INTRODUCTION TO ALGORITHMS

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INTRODUCTION

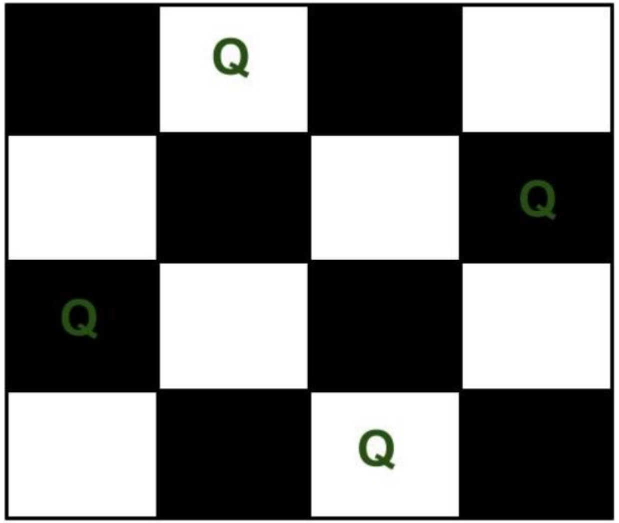
Branch-and-bound is a general technique for improving the searching process by systematically enumerating all candidate solutions and disposing of obviously impossible solutions.

Branching is the process of spawning subproblems, and *bounding* refers to ignoring partial solutions that cannot be better than the current best solution. To this end, lower and upper bounds *L* and *U* are maintained. Since global control values on the solution quality improve over time, branch-and-bound is effective in solving *optimization problems*, in which a cost-optimal assignment to the problem variables has to be found.

Problem statement

N Queen problem

The N Queen is the problem of placing N chess queens on an N×N chessboard so that no two queens attack each other. For example, following is a solution for 4 Queen problem.



Approach

In Branch and Bound solution, after building a partial solution, we figure out that there is no point going any deeper as we are going to hit a dead end. We need to figure out an efficient way of keeping track of which cells are under attack. In previous solution we kept an 8-by-8 Boolean matrix and update it each time we placed a queen, but that required linear time to update as we need to check for safe cells.

Basically, we have to ensure 4 things:

1. No two queens share a column.

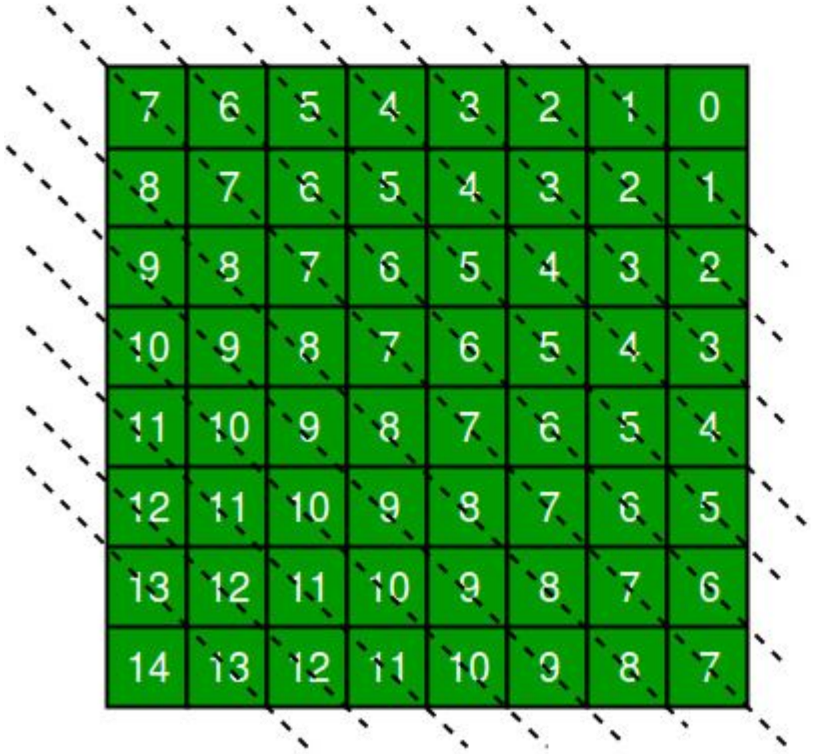
2. No two queens share a row.

3. No two queens share a top-right to left-bottom diagonal.

4. No two queens share a top-left to bottom-right diagonal.

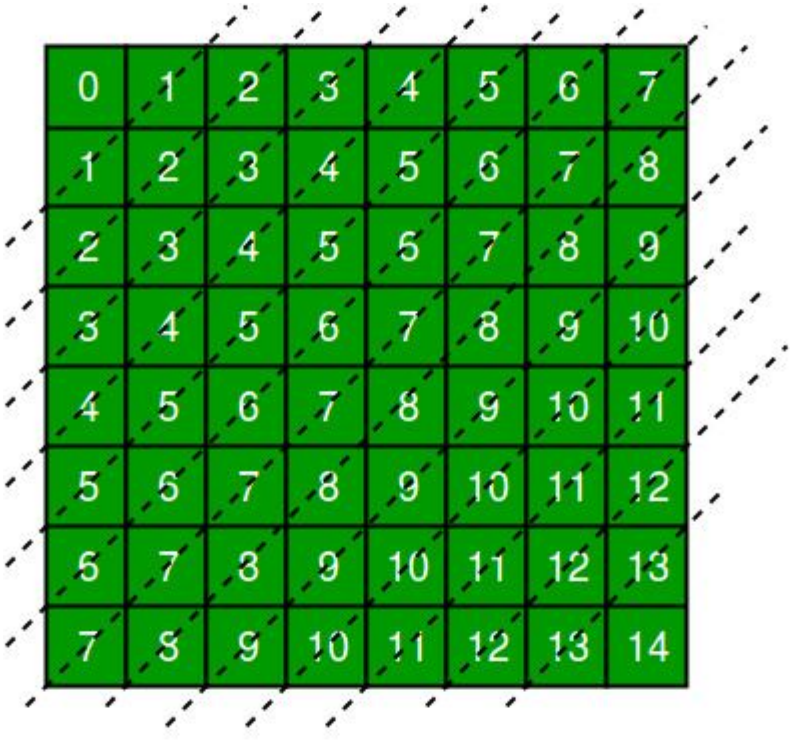
Let’s create two N x N matrix one for / diagonal and other one for \ diagonal. Let’s call them slashCode and backslashCode respectively. The trick is to fill them in such a way that two queens sharing a same /diagonal will have the same value in matrix slashCode, and if they share same \diagonal, they will have the same value in backslashCode matrix.

Using above algo we will get given matrices -



7	6	5	4	3	2	1	0
8	7	6	5	4	3	2	1
9	8	7	6	5	4	3	2
10	9	8	7	6	5	4	3
11	10	9	8	7	6	5	4
12	11	10	9	8	7	6	5
13	12	11	10	9	8	7	6
14	13	12	11	10	9	8	7

r -c + 7



0	1	2	3	4	5	6	7
1	2	3	4	5	6	7	8
2	3	4	5	6	7	8	9
3	4	5	6	7	8	9	10
4	5	6	7	8	9	10	11
5	6	7	8	9	10	11	12
6	7	8	9	10	11	12	13
7	8	9	10	11	12	13	14

r + c

The ‘N – 1’ in the backslash code is there to ensure that the codes are never negative because we will be using the codes as indices in an array.

Now before we place queen i on row j, we first check whether row j is used (use an array to store row info). Then we check whether slash code (j + i) or backslash code (j – i + 7) are used (keep two arrays that will tell us which diagonals are occupied). If yes, then we have to try a different location for queen i. If not, then we mark the row and the two diagonals as used and recurse on queen i + 1. After the recursive call returns and before we try another position for queen i, we need to reset the row, slash code and backslash code as unused again.