Day - 3

Introduction to algorithms

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Single pair shortest path or dijkstra's algorithm

Below are the detailed steps used in Dijkstra's algorithm to find the shortest path from a single source vertex to all other vertices in the given graph.

Algorithm

1) Create a set *sptSet* (shortest path tree set) that keeps track of vertices included in shortest path tree, i.e., whose minimum distance from source is calculated and finalized. Initially, this set is empty.

2) Assign a distance value to all vertices in the input graph. Initialize all distance values as INFINITE. Assign distance value as 0 for the source vertex so that it is picked first.

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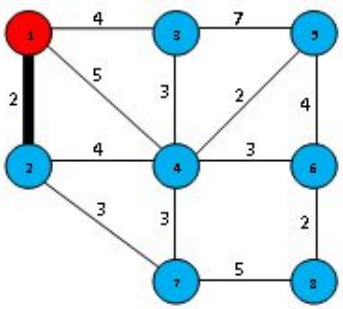
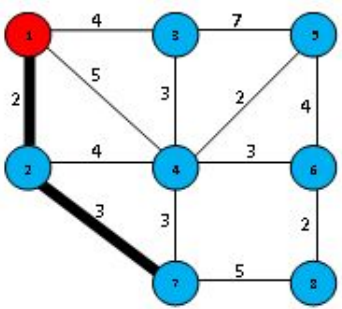
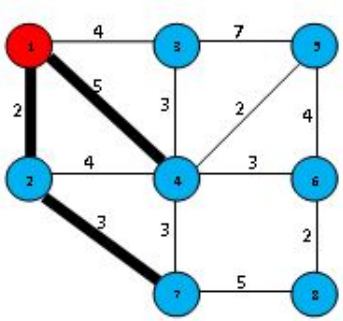
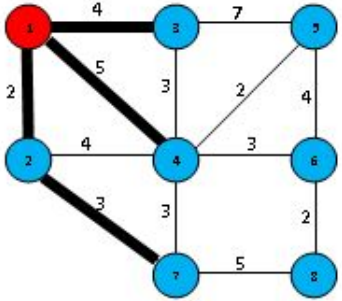
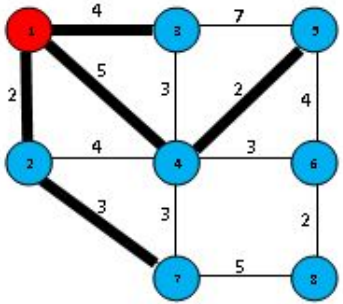
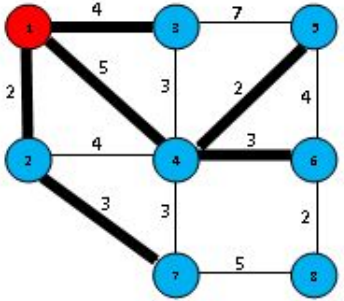
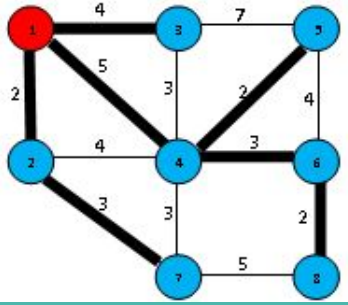
3) While *sptSet* doesn't include all vertices

....**a)** Pick a vertex u which is not there in *sptSet* and has minimum distance value.

....**b)** Include u to *sptSet*.

….**c)** Update distance value of all adjacent vertices of u. To update the distance values, iterate through all adjacent vertices. For every adjacent vertex v, if sum of distance value of u (from source) and weight of edge u-v, is less than the distance value of v, then update the distance value of v

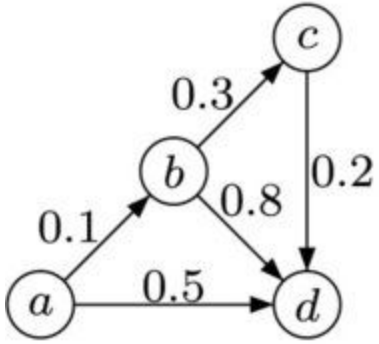








All shortest path or Warshall's algorithm

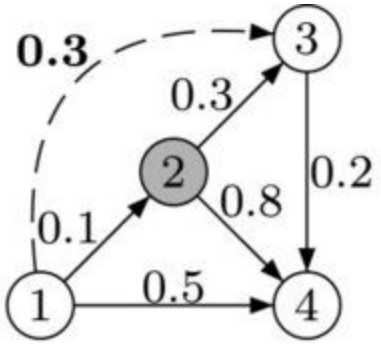
We initialize the solution matrix same as the input graph matrix as a first step. Then we update the solution matrix by considering all vertices as an intermediate vertex. The idea is to one by one pick all vertices and updates all shortest paths which include the picked vertex as an intermediate vertex in the shortest path. When we pick vertex number k as an intermediate vertex, we already have considered vertices {0, 1, 2, .. k-1} as intermediate vertices. For every pair (i, j) of the source and destination vertices respectively, there are two possible cases.

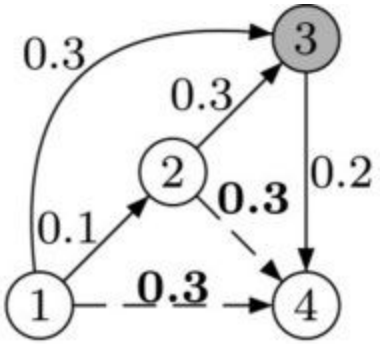
1) k is not an intermediate vertex in shortest path from i to j. We keep the value of dist[i][j] as it is.

2) k is an intermediate vertex in shortest path from i to j. We update the value of dist[i][j] as dist[i][k] + dist[k][j] if dist[i][j] > dist[i][k] + dist[k][j]

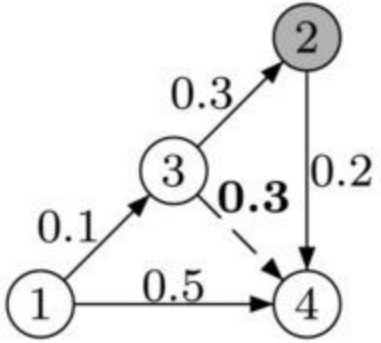


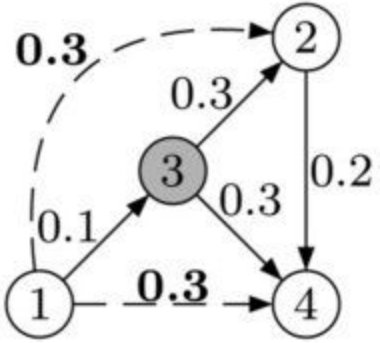
(a) Input graph



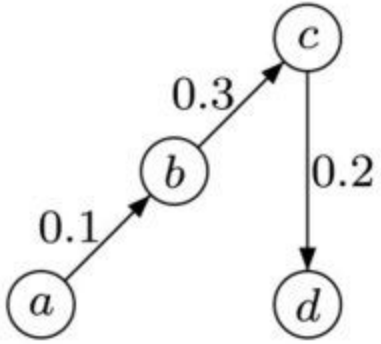


(b) Order I: $1 = a, 2 = b, 3 = c, 4 = d$.





(c) Order II: $1 = a, 2 = c, 3 = b, 4 = d$.



(d) Reduced graph