

# P vs NP PROBLEMS

AYUSH MAHAJAN

181210014

CSE 2<sup>ND</sup> YEAR (G1)

# P – COMPLEXITY CLASS

- P IS A COMPLEXITY CLASS THAT REPRESENTS THE SET OF ALL DECISION PROBLEMS THAT CAN BE SOLVED IN POLYNOMIAL TIME.
- THAT IS, GIVEN AN INSTANCE OF THE PROBLEM, THE ANSWER YES OR NO CAN BE DECIDED IN POLYNOMIAL TIME.
- **EXAMPLE**
- GIVEN A CONNECTED GRAPH G, CAN ITS VERTICES BE COLOURED USING TWO COLOURS SO THAT NO EDGE IS MONOCHROMATIC?
- ALGORITHM: START WITH AN ARBITRARY VERTEX, COLOR IT RED AND ALL OF ITS NEIGHBOURS BLUE AND CONTINUE. STOP WHEN YOU RUN OUT OF VERTICES OR YOU ARE FORCED TO MAKE AN EDGE HAVE BOTH OF ITS ENDPOINTS BE THE SAME COLOR.

# NP – COMPLEXITY CLASS

- NP IS A COMPLEXITY CLASS THAT REPRESENTS THE SET OF ALL DECISION PROBLEMS FOR WHICH THE INSTANCES WHERE THE ANSWER IS "YES" HAVE PROOFS THAT CAN BE VERIFIED IN POLYNOMIAL TIME.
- THIS MEANS THAT IF SOMEONE GIVES US AN INSTANCE OF THE PROBLEM AND A CERTIFICATE (SOMETIMES CALLED A WITNESS) TO THE ANSWER BEING YES, WE CAN CHECK THAT IT IS CORRECT IN POLYNOMIAL TIME.
- **EXAMPLE**
- INTEGER FACTORISATION IS IN NP. THIS IS THE PROBLEM THAT GIVEN INTEGERS N AND M, IS THERE AN INTEGER F WITH  $1 < F < M$ , SUCH THAT F DIVIDES N (F IS A SMALL FACTOR OF N)?
- THIS IS A DECISION PROBLEM BECAUSE THE ANSWERS ARE YES OR NO. IF SOMEONE HANDS US AN INSTANCE OF THE PROBLEM (SO THEY HAND US INTEGERS N AND M) AND AN INTEGER F WITH  $1 < F < M$ , AND CLAIM THAT F IS A FACTOR OF N (THE CERTIFICATE), WE CAN CHECK THE ANSWER IN POLYNOMIAL TIME BY PERFORMING THE DIVISION  $N / F$ .

# REAL LIFE EXAMPLE

- AN EXAMPLE WOULD BE BASIC MULTIPLICATION. YOU CAN CHECK THAT A NUMBER IS THE PRODUCT OF TWO OTHER NUMBERS BY SIMPLY MULTIPLYING THOSE NUMBERS (IN POLYNOMIAL TIME) AND CHECKING THAT THEIR PRODUCT IS INDEED THE PRODUCT YOU WERE CHECKING.
- AN EXAMPLE IN WHICH CHECKING THE ANSWER IS FASTER THAN FINDING THE ANSWER WOULD BE FINDING THE SOLUTION TO A MULTIVARIABLE SYSTEM OF EQUATIONS.
- IN THIS CASE, ONE NEED ONLY SUBSTITUTE THE SOLUTION VALUES IN FOR THE VARIABLES IN THE EQUATIONS AND CHECK THAT ALL THE EQUATIONS ARE INDEED SATISFIED. TYPICALLY, SOLVING SUCH A SYSTEM IS VERY DIFFICULT, IF IT IS EVEN EXACTLY SOLVABLE AT ALL.