ALL PAIRS SHORTEST PATHS

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Presentation Outline

LIST OF TOPICS

- What is All Pairs Shortest Path Problem?
- Floyd-Warshall Algorithm
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WHAT IS ALL PAIRS SHORTEST PATH PROBLEM?

- *The all-pairs shortest path problem is the determination of the shortest graph distances between every pair of vertices in a given graph.*
- *We have to calculate the minimum cost to find the shortest path.*

THE ALL PAIR SHORTEST PATH ALGORITHM IS ALSO KNOWN AS FLOYD - WARSHALL ALGORITHM

Floyd-Warshall Algorithm is an algorithm for finding the shortest path between all the pairs of vertices in a weighted graph.

- This algorithm follows the dynamic programming approach to find the shortest paths.
- This algorithm works for both the directed and undirected weighted graphs. But, it does not work for the graphs with negative cycles (where the sum of the edges in a cycle is negative).

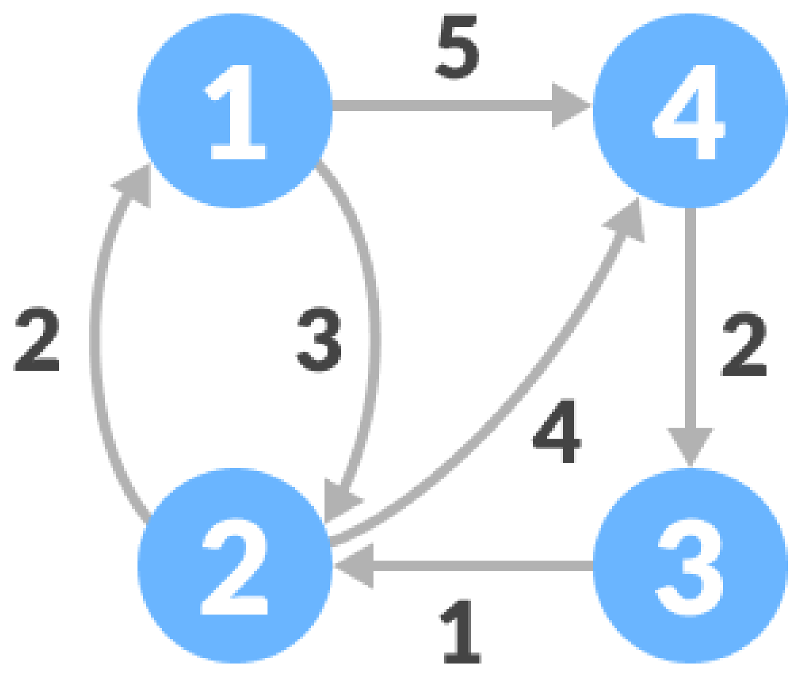
FLOYD-WARSHALL ALGORITHM

PSEUDOCODE

FLOYD - WARSHALL (W)

1. n ← rows [W].
2. D0 ← W
3. for k ← 1 to n
4. do for i ← 1 to n
5. do for j ← 1 to n
6. do dij(k) ← min (dij(k-1),
   dik(k-1)+dkj(k-1))
7. return D(n)

Let the given graph be:



Follow the steps below to find the shortest path between all the pairs of vertices.

EXAMPLE

CREATE A MATRIX A0 OF DIMENSION N*N, WHERE 'N' IS THE NO. OF VERTICES.

Each cell A[i][j] is filled with the distance from the ith vertex to the jth vertex. If there is no path from ith vertex to jth vertex, the cell is left as infinity.

$$A^0 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & \infty & 5 \\ 2 & 0 & \infty & 4 \\ \infty & 1 & 0 & \infty \\ \infty & \infty & 2 & 0 \end{bmatrix} \end{array}$$

Now, create a matrix A1 using matrix A0.

$$A^1 = \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & \infty & 5 \\ 2 & 0 & & \\ \infty & & 0 & \\ \infty & & & 0 \end{bmatrix} \end{array} \longrightarrow \begin{array}{cc} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & \infty & 5 \\ 2 & 0 & 9 & 4 \\ \infty & 1 & 0 & 8 \\ \infty & \infty & 2 & 0 \end{bmatrix} \end{array}$$

The elements in the first column and row are left as they are.

$$A^2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & & \\ 2 & 0 & 9 & 4 \\ & 1 & 0 & \\ & \infty & & 0 \end{bmatrix} \end{array} \rightarrow \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 3 & 9 & 5 \\ 2 & 0 & 9 & 4 \\ 3 & 1 & 0 & 5 \\ \infty & \infty & 2 & 0 \end{bmatrix} \end{array}$$

In a similar way, A2 is created using A3.
The elements in the second column and the second row are left as they are.

SIMILARLY, A3 AND A4 IS ALSO CREATED.

$$A^3 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & & \infty & \\ & 0 & 9 & \\ \infty & 1 & 0 & 8 \\ & & 2 & 0 \end{bmatrix} \end{array} \rightarrow \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 3 & 9 & 5 \\ 2 & 0 & 9 & 4 \\ 3 & 1 & 0 & 5 \\ 5 & 3 & 2 & 0 \end{bmatrix} \end{array}$$

$$A^4 = \begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ 1 & 0 & & & 5 \\ 2 & & 0 & & 4 \\ 3 & & & 0 & 5 \\ 4 & 5 & 3 & 2 & 0 \end{array}$$

$$\downarrow$$

$$\begin{array}{c|cccc|} & 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 7 & 5 \\ 2 & 2 & 0 & 6 & 4 \\ 3 & 3 & 1 & 0 & 5 \\ 4 & 5 & 3 & 2 & 0 \end{array}$$

4 GIVES THE SHORTEST PATH BETWEEN EACH PAIR OF VERTICES.

Time Complexity

There are three loops. Each loop has constant complexities. So, the time complexity of the Floyd-Warshall algorithm is O(n3).

Space Complexity

The space complexity of the Floyd-Warshall algorithm is O(n2).

Floyd Warshall Algorithm Applications

- *To find the shortest path is a directed graph*
- *To find the transitive closure of directed graphs*
- *To find the Inversion of real matrices*
- *For testing whether an undirected graph is bipartite*