Chapter 1 - Energy in Thermal Physics

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Problem 1.1

The Fahrenheit temperature scale is defined so that ice melts at 32F and water boils at 212F.

(a) Derive the formulas for converting from Fahrenheit to Celsius and back.
(b) What is absolute zero on the Fahrenheit scale?

Actually derive it instead of reiterating stated facts.

GPT's Answer to Problem 1.1

The Fahrenheit temperature scale is defined so that ice melts at $32°F$ and water boils at $212°F$.

(a) Deriving Formulas for Temperature Conversion

Given:

- The melting point of ice is $32°F$ and $0°C$.

- The boiling point of water is $212°F$ and $100°C$.

Let's define two linear functions:

$$C(m) = a \cdot m + b \quad \text{(i)} \tag{1}$$

where C is the temperature in Celsius and m is the temperature in Fahrenheit, and

$$F(c) = p \cdot c + q \quad \text{(ii)} \tag{2}$$

where F is the temperature in Fahrenheit and c is the temperature in Celsius.

We can find a and b (and later p and q) by using the two given points in the Celsius and Fahrenheit scales.

1. $C(32) = 0$ leads to:

$$0 = 32a + b \quad \text{(iii)} \tag{3}$$

2. $C(212) = 100$ leads to:

$$100 = 212a + b \quad \text{(iv)} \tag{4}$$

To find the value of a we subtract equation (iii) from (iv):

$$100 - 0 = 212a - 32a$$
$$100 = 180a$$
$$a = \frac{100}{180} = \frac{5}{9}$$

Now we find b using equation (iii) and the value of a:

$$0 = 32a + b$$
$$b = -32a$$
$$b = -32 \times \frac{5}{9}$$

So,

$$C(m) = \frac{5}{9}m - \frac{160}{9} \quad \text{(v)} \tag{5}$$

Now for the Fahrenheit formula in terms of Celsius, using similar steps, we derive:

$$F(c) = \frac{9}{5}c + 32 \quad \text{(vi)} \tag{6}$$

(b) Absolute Zero on the Fahrenheit Scale

Absolute zero is the lowest possible temperature where nothing could be colder and no heat energy remains in a substance. It is defined as $-273.15°C$ in the Celsius scale.

To find absolute zero on the Fahrenheit scale, we use equation (vi):

$$F(-273.15) = \frac{9}{5} \times (-273.15) + 32$$
$$F(-273.15) = -491.67$$

So, absolute zero is $-491.67°F$ on the Fahrenheit scale.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.2

The Rankine temperature scale (abbreviated $°R$) uses the same size degrees as Fahrenheit, but measured up from absolute zero like kelvin (so Rankine is to Fahrenheit as kelvin is to Celsius). Find the conversion formula between Rankine and Fahrenheit, and also between Rankine and kelvin. What is room temperature on the Rankine scale? Actually derive it instead of reiterating stated facts.

GPT's Answer to Problem 1.1

The Rankine temperature scale (denoted as $°R$) uses the same size degrees as Fahrenheit, but measured up from absolute zero like Kelvin (so Rankine is to Fahrenheit as Kelvin is to Celsius).

Conversion between Rankine and Fahrenheit

We know that:

- Absolute zero is $-491.67°F$.

Given that both Fahrenheit and Rankine use the same interval size, the conversion formula from Fahrenheit (F) to Rankine (R) is found by adjusting the zero point from the Fahrenheit scale to absolute zero.

$$R = F + 491.67 \quad \text{(i)} \tag{7}$$

And conversely:

$$F = R - 491.67 \quad \text{(ii)} \tag{8}$$

Conversion between Rankine and Kelvin

Now let's determine the conversion between the Kelvin (K) and Rankine (R) scales.

- Absolute zero is $0K$ and also $0°R$.

- The boiling point of water is $373.15K$ and $671.67°R$ (using the conversion from Celsius to Kelvin and Fahrenheit to Rankine).

Let's define:
$$K = m \cdot R + n \quad \text{(iii)} \tag{9}$$
where K is the temperature in Kelvin and R is the temperature in Rankine, and m and n are constants to be determined.

Using the two points above, we can form two simultaneous equations:

1. $K(0) = 0$ implies:

$$0 = m \cdot 0 + n \implies n = 0 \quad \text{(iv)} \tag{10}$$

2. $K(671.67) = 373.15$ implies:

$$373.15 = 671.67 \cdot m + n \quad \text{(v)} \tag{11}$$

Solving these two equations gives:

$$m = \frac{373.15}{671.67}$$

So,

$$K = \frac{373.15}{671.67} \cdot R \quad \text{(vi)} \qquad\qquad (12)$$

Room Temperature on the Rankine Scale

Typically, room temperature is around $20°C$ or $68°F$.

Using the conversion from Fahrenheit to Rankine (equation i):

$$R = 68 + 491.67$$

$$R = 559.67°R$$

Comments

GPT4 gives the correct answers and explanations. However, it chooses unwieldy number as the slope for Rankine to Kelvin equation. Upon reiteration which asks to simplify the formula, it does give the more well known form:

$$K = \frac{5}{9}R$$

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Problem 1.3

Determine the kelvin temperature for each of the following:

(a) Human body temperature;

(b) The boiling point of water (at the standard pressure of 1 atm);

(c) The coldest day you can remember;

(d) The boiling point of liquid nitrogen $(-196°C)$;

(e) The melting point of lead $(327°C)$.

Actually derive it instead of reiterating stated facts

GPT's Answer to Problem 1.3

To convert temperatures from Celsius to Kelvin, we use the formula: $K = C + 273.15$. Applying this formula to each scenario:

(a) The average human body temperature is approximately 37°C.
So, $K = 37 + 273.15 = 310.15\,K$.

(b) Water boils at 100°C under standard conditions.
So, $K = 100 + 273.15 = 373.15\,K$.

(c) Assuming a very cold day of -30°C for this example.
So, $K = -30 + 273.15 = 243.15\,K$.

(d) Liquid nitrogen boils at -196°C.
So, $K = -196 + 273.15 = 77.15\,K$.

(e) Lead has a melting point of 327°C.
So, $K = 327 + 273.15 = 600.15\,K$.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.4

Does it ever make sense to say that one object is "twice as hot" as another? Does it matter whether one is referring to Celsius or kelvin temperatures? Explain.

GPT's Answer to Problem 1.4

The concept of one object being "twice as hot" as another is not scientifically or thermodynamically sound when using the Celsius or Fahrenheit scales, because these scales have arbitrary zero points based on specific physical milestones (e.g., the freezing and boiling points of water for the Celsius scale). However, the Kelvin scale is an absolute temperature scale with its zero point

at absolute zero, the theoretically coldest temperature and a natural zero indicating the absence of thermal energy.

To illustrate, consider two objects:

- Object A at 100°C

- Object B at 50°C

In the Celsius scale, stating that object A is "twice as hot" as object B is misleading because the scale's zero point is arbitrary, not representing a true absence of heat.

However, when converted to the Kelvin scale (using $K = C + 273.15$), we have:

- Object A: $100 + 273.15 = 373.15K$

- Object B: $50 + 273.15 = 323.15K$

Here, $373.15K$ is not double $323.15K$, showing that temperature ratios don't preserve when switching scales. This issue arises from the arbitrary zero points in the Celsius and Fahrenheit scales.

Conversely, for temperatures on the Kelvin scale:

- Object C at $100K$

- Object D at $200K$

Object D has double the Kelvin temperature of Object C, meaning it has twice the thermal energy. This comparison is valid because the Kelvin scale's zero point is absolute zero, a natural and absolute reference point indicating no thermal motion.

In conclusion, the validity of the statement that one object is "twice as hot" as another depends on the temperature scale used. It doesn't hold for Celsius or Fahrenheit, due to their arbitrary zero points, but it does for Kelvin, as this scale starts at absolute zero, allowing meaningful comparisons of thermal energy ratios.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.5

When you're sick with a fever and you take your temperature with a thermometer, approximately what is the relaxation time?

GPT's Answer to Problem 1.5

The relaxation time refers to the duration required for the thermometer to reach thermal equilibrium with the body, thus accurately indicating the body's temperature. This period is associated with the thermal time constant, which is linked to the thermometer's thermal mass, the specific heat capacity of its material, and the heat transfer coefficient between the thermometer and the body.

The heat transfer can be represented by the differential equation:

$$\frac{dT}{dt} = \frac{(T_{\text{body}} - T)}{\tau}$$

where:

- $\frac{dT}{dt}$ is the rate of temperature change of the thermometer,

- T_{body} is the body's constant temperature,

- T is the thermometer's temperature at time t,

- τ is the system's thermal time constant.

The solution to this equation is:

$$T(t) = T_{\text{body}} + (T_{\text{initial}} - T_{\text{body}})e^{-t/\tau}$$

where:

- $T(t)$ is the thermometer's temperature at time t,

- T_{initial} is the thermometer's temperature at $t = 0$,

- e is the base of the natural logarithm,

- t is time,

- τ is the thermal time constant.

The relaxation time is approximately the time for the thermometer to be within a small percentage (e.g., 1%) of the body's temperature, often around 3 to 5 time constants (3τ to 5τ). However, the exact value for τ and consequently the relaxation time cannot be calculated without specific details about the thermometer's physical properties and the heat transfer coefficient. Practically, relaxation times range from several seconds for advanced digital thermometers to a few minutes for traditional mercury or alcohol thermometers.

Comments

GPT4 gives the correct answers and explanations. However, some of the concepts were too advanced for students with minimal understanding of chemistry. Here is the detailed explanation:

Understanding Thermal Concepts

Let's simplify some scientific concepts to understand how a thermometer measures temperature:

1. **Thermal Time Constant:** This term describes how fast a system (like a thermometer) responds to temperature changes. A smaller thermal time constant means the thermometer can read your temperature quickly, while a larger one means it takes longer.

2. **Thermal Mass:** This is the thermometer's ability to absorb and store heat. A thermometer with high thermal mass needs more time to absorb heat, hence taking longer to display the correct temperature.

3. **Specific Heat Capacity:** This property tells us how much heat a material (the substance making up the thermometer) needs to change its temperature. Materials with high specific heat capacities require more heat to increase in temperature.

4. **Heat Transfer Coefficient:** This coefficient measures the efficiency of heat transfer between your body and the thermometer. Better heat transfer means a quicker and more accurate reading.

Now, let's decipher the equation that represents heat transfer:

$$\frac{dT}{dt} = \frac{(T_{\text{body}} - T)}{\tau}$$

The terms represent:

- $\frac{dT}{dt}$: The rate of the thermometer's temperature change. A higher value means a rapid change.

- T_{body}: The constant temperature of your body.

- T: The thermometer's temperature at any moment, which rises from room temperature as it absorbs your body's heat.

- τ: The thermal time constant, indicating how quickly the thermometer responds to temperature changes.

In essence, this equation says that the speed of the thermometer's temperature change depends on both the temperature difference between your body and the thermometer and on the thermometer's ability to adapt to temperature changes (represented by τ).

Problem 1.6

Give an example to illustrate why you cannot accurately judge the temperature of an object by how hot or cold it feels to the touch.

GPT's Answer to Problem 1.6

To comprehend why the human touch is an unreliable thermometer, we need to consider thermodynamics, heat transfer, and human physiology principles.

1. **Subjectivity of Human Perception:** Our nervous system perceives temperature via thermal receptors in the skin. These receptors are sensitive to heat flow rather than absolute temperature, responding more to temperature changes than to constant temperatures.

2. **Heat Transfer:** The sensation of heat or coldness comes from heat transfer between our hand and the object, not just the object's temperature. The rate of heat transfer depends on the temperature difference and the object's thermal conductivity.

   Let's consider the heat transfer q based on Newton's Law of Cooling:

   $$q = hA(T_{\text{object}} - T_{\text{skin}})$$

   where:

   - q is the heat transfer rate,

   - h is the heat transfer coefficient,

   - A is the contact area,

   - T_{object} is the object's temperature,

   - T_{skin} is the skin's temperature, approximately 34°C.

   Here, h is influenced by the material's thermal conductivity k. High thermal conductivity means rapid heat transfer, leading to intense sensations of heat or coldness, even if the object's temperature isn't extreme.

   For instance, metal feels colder than wood at the same temperature because metal's higher thermal conductivity pulls heat from your hand more quickly, which your receptors interpret as a colder sensation.

3. **Adaptation:** Our sensory receptors adapt over time, becoming less

responsive to unchanging stimuli. This is why an object doesn't feel as hot or cold after you've been holding it for some time.

These insights demonstrate that our sensory system doesn't gauge actual temperature—instead, it interprets heat flow, a perception influenced by material properties, contact area, and individual physiological differences. Hence, we cannot accurately assess an object's true temperature by how hot or cold it feels to our touch.

Comments

GPT4 gives the correct answers and explanations. However, the explanation could have been more clarified by giving more examples that explain the influence of different factors such as contact area.

Problem 1.7

When the temperature of liquid mercury increases by one degree Celsius (or one kelvin), its volume increases by one part in 5500 . The fractional increase in volume per unit change in temperature (when the pressure is held fixed) is called the thermal expansion coefficient, β :

$$\beta \equiv \frac{\Delta V/V}{\Delta T}$$

(where V is volume, T is temperature, and Δ signifies a change, which in this case should really be infinitesimal if β is to be well defined). So for mercury, $\beta = 1/5500 \text{ K}^{-1} = 1.81 \times 10^{-4} \text{ K}^{-1}$. (The exact value varies with temperature, but between 0°C and 200°C the variation is less than 1%.)

(a) Get a mercury thermometer, estimate the size of the bulb at the bottom, and then estimate what the inside diameter of the tube has to be in order for the thermometer to work as required. Assume that the thermal expansion of the glass is negligible.

(b) The thermal expansion coefficient of water varies significantly with temperature: It is $7.5 \times 10^{-4} \text{ K}^{-1}$ at 100°C, but decreases as the temperature is lowered until it becomes zero at 4°C. Below 4°C it is slightly negative, reaching a value of $-0.68 \times 10^{-4} \text{ K}^{-1}$ at 0°C. (This behavior is related to the

fact that ice is less dense than water.) With this behavior in mind, imagine the process of a lake freezing over, and discuss in some detail how this process would be different if the thermal expansion coefficient of water were always positive.

GPT's Answer to Problem 1.7

(a) The question involves determining the necessary inner diameter of the capillary tube in a mercury thermometer, given that the thermal expansion of mercury is the primary factor affecting the thermometer's readings. Assuming the bulb of a typical thermometer holds about 0.5 ml (or 0.5 cubic centimeters) of mercury — an estimated average — we can use the given thermal expansion coefficient of mercury to find out how much the volume of mercury expands per degree Celsius and then determine the necessary diameter of the tube.

For mercury, $\beta = \frac{1}{5500}$ per K (or per °C). If the volume of the mercury in the bulb is 0.5 cc, then for each degree Celsius increase in temperature, the volume of the mercury will increase by $(0.5\,\text{cc}) \times (\frac{1}{5500}) = 9.09 \times 10^{-5}\,\text{cc}$.

This additional volume of mercury must rise into the capillary tube of the thermometer. The capillary tube is essentially a cylindrical shape, and the volume of a cylinder is given by $V = \pi r^2 h$, where r is the radius and h is the height (which will correspond to the increased length that the mercury column rises per degree Celsius, which is how we read the temperature).

If we want the thermometer to register a 1°C change for every millimeter that the mercury rises (which is a typical scale), the change in height (h) will be 1 mm = 0.1 cm. Therefore, we can solve for the radius (r) of the tube using the volume formula:

$$9.09 \times 10^{-5}\,\text{cc} = \pi r^2 \times 0.1\,\text{cm}$$

Solving for r, we get approximately $r \approx 0.0054\,\text{cm}$ or $0.054\,\text{mm}$. Therefore, the inner diameter of the tube should be approximately twice this, or about $0.11\,\text{mm}$.

(b) The unique property of water is that its density increases as it cools from 100°C to 4°C, but below 4°C, it expands (its density decreases) until it freezes. This is due to the molecular structure of ice, which forms a hexagonal lattice that occupies more space than liquid water. Because of this, ice floats on water, and lakes freeze from the top down.

If the thermal expansion coefficient of water were always positive (meaning that water continued to contract upon cooling), water would become denser as it cooled, all the way until it freezes. In this scenario, colder water would always sink to the bottom, and lakes would freeze from the bottom up.

This would have profound ecological implications. Aquatic life relies on the bottom layer of the lake remaining liquid and warmer during the winter, a result of ice's insulating properties when it floats on the surface. If lakes froze from the bottom up, many aquatic species might not survive the cold temperatures.

Additionally, if ice didn't float, it would continue to accumulate at the bottom of bodies of water, potentially never melting entirely in some areas and leading to a "runaway" freezing effect, drastically lowering the water levels and making it much more difficult for temperatures to rise enough to melt the ice during warmer seasons. This phenomenon could have significantly altered the climate, geography, and biological makeup of the Earth's aquatic systems.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.8

For a solid, we also define the linear thermal expansion coefficient, α, as the fractional increase in length per degree:

$$\alpha \equiv \frac{\Delta L / L}{\Delta T}$$

(a) For steel, α is $1.1 \times 10^{-5}\,\mathrm{K}^{-1}$. Estimate the total variation in length of a 1-km steel bridge between a cold winter night and a hot summer day.

(b) The dial thermometer in Figure 1.2 uses a coiled metal strip made of two different metals laminated together. Explain how this works.

(c) Prove that the volume thermal expansion coefficient of a solid is equal to the sum of its linear expansion coefficients in the three directions: $\beta = \alpha_x + \alpha_y + \alpha_z$. (So for an isotropic solid, which expands the same in all directions, $\beta = 3\alpha$.)

GPT's Answer to Problem 1.8

(a)

Given the linear thermal expansion coefficient of steel, $\alpha = 1.1 \times 10^{-5}\,\mathrm{K^{-1}}$, we can estimate the change in length of a 1-km steel bridge due to temperature changes.

Let's assume a temperature variation between a cold winter night and a hot summer day is approximately $40\,°\mathrm{C}$ (for example, from $-20\,°\mathrm{C}$ to $20\,°\mathrm{C}$).

The change in length ΔL is given by the formula:

$$\Delta L = \alpha \cdot L \cdot \Delta T$$

where $L = 1000\,\mathrm{meters} = 1\,\mathrm{km}$ (original length of the bridge) and $\Delta T = 40\,°\mathrm{C}$ (change in temperature).

Plugging in the numbers:

$$\Delta L = (1.1 \times 10^{-5}\,\mathrm{K^{-1}}) \cdot (1000\,\mathrm{m}) \cdot (40\,°\mathrm{C}) = 0.44\,\mathrm{m}$$

So, the bridge would expand by 0.44 meters, or 44 centimeters, in the temperature range considered.

(b)

The dial thermometer works based on the principle of thermal expansion and the fact that different metals expand at different rates. The coiled metal strip in the thermometer is made of two different metals laminated together. When there is a change in temperature, one metal in the strip expands more than the other, causing the strip to coil or uncoil. The greater the temperature

change, the tighter or looser the coil becomes. This mechanical change in the coil is then translated into a rotational movement of the needle in the dial of the thermometer, providing a temperature reading.

(c)

To prove that the volume thermal expansion coefficient β of a solid is the sum of its linear expansion coefficients in the three directions $(\alpha_x, \alpha_y, \alpha_z)$, we start by considering the initial volume of the solid $V = L_x \cdot L_y \cdot L_z$, where L_x, L_y, and L_z are the initial lengths in the three directions.

When the temperature changes by ΔT, the new lengths will be $L_x + \Delta L_x, L_y + \Delta L_y$, and $L_z + \Delta L_z$, where $\Delta L_x = \alpha_x \cdot L_x \cdot \Delta T$ and similarly for ΔL_y and ΔL_z.

The new volume V' is:

$$V' = (L_x + \Delta L_x) \cdot (L_y + \Delta L_y) \cdot (L_z + \Delta L_z)$$

Expanding and neglecting the higher-order terms (ΔL^2 and ΔL^3 because they are very small compared to the first order changes):

$$V' \approx L_x \cdot L_y \cdot L_z + L_x \cdot L_y \cdot \Delta L_z + L_x \cdot \Delta L_y \cdot L_z + \Delta L_x \cdot L_y \cdot L_z$$

So, the change in volume $\Delta V = V' - V$ is:

$$\Delta V \approx L_x \cdot L_y \cdot \Delta L_z + L_x \cdot \Delta L_y \cdot L_z + \Delta L_x \cdot L_y \cdot L_z$$

Substituting $\Delta L_x = \alpha_x \cdot L_x \cdot \Delta T$ and similarly for y and z, we get:

$$\Delta V \approx L_x \cdot L_y \cdot L_z \cdot (\alpha_x + \alpha_y + \alpha_z) \cdot \Delta T = V \cdot (\alpha_x + \alpha_y + \alpha_z) \cdot \Delta T$$

The volume expansion coefficient β is then:

$$\beta = \frac{\Delta V / V}{\Delta T} = \alpha_x + \alpha_y + \alpha_z$$

For an isotropic solid, $\alpha_x = \alpha_y = \alpha_z = \alpha$, so $\beta = 3\alpha$.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.9

Problem 1.9. What is the volume of one mole of air, at room temperature and 1 atm pressure?

GPT's Answer to Problem 1.9

To find the volume of one mole of air at room temperature and 1 atm pressure, we use the ideal gas law:

$$PV = nRT$$

where:

- P is the pressure,

- V is the volume,

- n is the number of moles,

- R is the universal gas constant, and

- T is the temperature in kelvins.

Given that the pressure $P = 1\,\text{atm}$, the number of moles $n = 1$, and the universal gas constant $R = 8.31\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$, we need to convert the pressure from atmospheres to pascals (the SI unit) and the temperature from Celsius to kelvins (the SI unit). Room temperature is usually considered to be $25\,°\text{C}$, which is 298 K $(25 + 273)$. The pressure of 1 atm is equivalent to $101\,325\,\text{Pa}$.

Plugging in the values, we have:

$$(101\,325\,\text{Pa}) \cdot V = (1) \cdot (8.31\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}) \cdot (298\,\text{K})$$

Solving for V, we find:

$$V \approx 0.024\,\text{m}^3 \,\text{or}\, 24\,\text{L}$$

So, the volume of one mole of air at room temperature and 1 atm pressure is approximately 24 L.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.10

Problem 1.10. Estimate the number of air molecules in an average-sized room.

GPT's Answer to Problem 1.10

To estimate the number of air molecules in an average-sized room, we need to know the volume of the room and the conditions of the air in the room (temperature and pressure). We then use the ideal gas law to find the number of moles of air, and finally, we calculate the number of molecules.

Let's assume the room is 10 m x 10 m x 2.5 m (which gives $250\,\text{m}^3$ or $250\,000\,\text{L}$), the temperature is room temperature ($25\,^{\circ}\text{C}$ or 298 K), and the pressure is 1 atm ($101\,325\,\text{Pa}$).

The ideal gas law is:
$$PV = nRT$$

where P is the pressure, V is the volume, n is the number of moles, R is the universal gas constant ($8.31\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1}$), and T is the temperature in kelvins.

Solving for n (the number of moles) gives:

$$n = \frac{PV}{RT}$$

Substituting in the known values gives:

$$n = \frac{(101\,325\,\text{Pa})(250\,\text{m}^3)}{(8.31\,\text{J}\,\text{mol}^{-1}\,\text{K}^{-1})(298\,\text{K})}$$

$$n \approx 10\,156\,\text{mol}$$

Since each mole of air contains Avogadro's number ($N_A \approx 6.02 \times 10^{23}\,\text{mol}^{-1}$) of molecules, the total number of molecules (N) is:

$$N = n \cdot N_A$$

$$N \approx 10\,156\,\text{mol} \cdot 6.02 \times 10^{23}\,\text{mol}^{-1}$$

$$N \approx 6.11 \times 10^{28}\,\text{molecules}$$

So, there are approximately 6.11×10^{28} molecules of air in an average-sized room.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.11

Problem 1.11. Rooms A and B are the same size, and are connected by an open door. Room A, however, is warmer (perhaps because its windows face the sun). Which room contains the greater mass of air? Explain carefully.

GPT's Answer to Problem 1.11

To determine which room contains the greater mass of air, we need to consider the principles behind the ideal gas law and the behavior of gases. According to the ideal gas law,

$$PV = nRT$$

the product of pressure (P) and volume (V) is proportional to the product of the number of moles (n), the gas constant (R), and the temperature (T). However, the mass of the gas is directly related to the number of moles, as the mass (m) is the product of the number of moles and the molar mass (M): $m = nM$.

Given that Rooms A and B are connected by an open door, we can assume that the pressure in both rooms is equal due to the free movement of air between them. Also, since they are the same size, their volumes are equal.

Now, since $PV = nRT$, and P, V, and R are constants under these conditions, we have $nT = $ constant. If Room A is warmer, i.e., has a higher temperature, then it must have fewer moles of gas to maintain this equality (since nT must be constant). And since the mass of the gas is proportional to the number of moles ($m = nM$), Room A, with fewer moles of gas, contains less mass of air.

Therefore, under the assumption of constant pressure and equal volumes, Room B, which is cooler, contains the greater mass of air.

Problem 1.12

Problem 1.12. Calculate the average volume per molecule for an ideal gas at room temperature and atmospheric pressure. Then take the cube root to get an estimate of the average distance between molecules. How does this distance compare to the size of a small molecule like N2 or H2O?

GPT's Answer to Problem 1.12

The ideal gas law is given by:

$$PV = NkT$$

where P is the pressure, V is the volume, N is the number of molecules, k is the Boltzmann constant, and T is the temperature.

We're interested in finding the average volume per molecule, V_{avg}, at room temperature and atmospheric pressure. To find V_{avg}, we rearrange the ideal gas law to solve for V/N, which gives us $V_{avg} = \frac{kT}{P}$.

Given:

$$P = 1\,\text{atm} = 1.013 \times 10^5\,\text{Pa}$$
$$T = \text{room temperature} \approx 298\,\text{K}$$
$$k = 1.381 \times 10^{-23}\,\text{J/K}$$

We find:

$$V_{\text{avg}} = \frac{(1.381 \times 10^{-23}\,\text{J/K})(298\,\text{K})}{1.013 \times 10^5\,\text{Pa}} \approx 4.07 \times 10^{-26}\,\text{m}^3$$

The average distance between molecules, d, is the cube root of V_{avg}:

$$d = \sqrt[3]{V_{\text{avg}}} \approx \sqrt[3]{4.07 \times 10^{-26}\,\text{m}^3} \approx 3.44 \times 10^{-9}\,\text{m} = 3.44\,\text{nm}$$

Typical sizes of small molecules like N2 or H2O are on the order of a tenth of a nanometer (about 0.1 nm), so the average distance between gas molecules is roughly 30 times the size of a small molecule. This significant distance relative to their size is what allows us to treat them as point particles in an ideal gas, ignoring intermolecular forces and volume occupied by the gas molecules themselves.

Comments

GPT4 gives the correct answers and explanations. Although it is technically correct that small molecules like N2 or H2O are on the order of a tenth of a nanometer, to actually answer the question, the precise size of the gasses should be used.

Van Der Waals Radius and Kinetic Diameter

The discrepancy between the size of the nitrogen molecule initially referred to (0.155 nm) and the size provided (0.363 nm) arises from the different methods and definitions used to describe the "size" of a molecule.

1. **Van der Waals Radius**: The value initially provided is closer to what's known as the "van der Waals radius," which is determined based on the distance at which two non-bonded atoms or molecules exert a repulsive force on each other. This radius is typically smaller than other measures of molecular size.

2. **Kinetic Diameter**: The value provided ($0.363\,\text{nm}$) is closer to the "kinetic diameter," which is a measure often used in the context of gas molecules passing through pores, like in a filter or a membrane. It considers not just the electron clouds of the atoms in the molecule, but also the space the molecule effectively occupies as it moves, including its rotations and vibrations.

These two different measures are used in different contexts and can both be valid, depending on what specific property you're interested in. In the context of estimating the average distance between air molecules compared to the size of the molecules themselves, the kinetic diameter might be more relevant, as it gives a better sense of the "space" the molecules occupy in their thermal motion.

This example underscores the importance of specificity in scientific communication: different "sizes" can be reported for the same molecule, and each is correct within its own context. When using or comparing reported values, it's important to ensure they're defined the same way and pertain to the same property.

Problem 1.13

A mole is approximately the number of protons in a gram of protons. The mass of a neutron is about the same as the mass of a proton, while the mass of an electron is usually negligible in comparison, so if you know the total number of protons and neutrons in a molecule (i.e., its "atomic mass"), you know the approximate mass (in grams) of a mole of these molecules. * Referring to the periodic table at the back of this book, find the mass of a mole of each of the following: water, nitrogen (N_2), lead, quartz (SiO_2).

GPT's Answer to 1.13

To find the molar mass of these substances, we sum the atomic masses of the atoms in each molecule, using the atomic masses from the periodic table. One mole of a substance has a mass equal to its molecular (or atomic) mass in grams.

1. **Water** (H_2O): The molecular mass of water is $2\times$ the atomic mass of hydrogen (approximately $1\,g/mol$) plus the atomic mass of oxygen (approximately $16\,g/mol$).
   $\Rightarrow 2(1) + 16 = 18\,g/mol$

2. **Nitrogen** (N_2): The molecular mass of nitrogen gas is $2\times$ the atomic mass of nitrogen (approximately $14\,g/mol$).
   $\Rightarrow 2(14) = 28\,g/mol$

3. **Lead (Pb)**: Lead is an element, so its molar mass is the atomic mass listed on the periodic table, which is approximately $207\,g/mol$.

4. **Quartz** (SiO_2): The molecular mass of quartz is the atomic mass of silicon (approximately $28\,g/mol$) plus $2\times$ the atomic mass of oxygen (approximately $16\,g/mol$).
   $\Rightarrow 28 + 2(16) = 60\,g/mol$

Therefore, the masses of a mole of each substance are approximately $18\,g/mol$ for water, $28\,g/mol$ for nitrogen, $207\,g/mol$ for lead, and $60\,g/mol$ for quartz.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.14.

Calculate the mass of a mole of dry air, which is a mixture of N_2 (78% by volume), O_2 (21%), and argon (1%).

GPT's Answer to 1.14:

Dry air is primarily a mixture of nitrogen, oxygen, and argon. The percentages given in the problem are by volume, which are roughly equivalent to mole percentages since the gases are in the same conditions and the ideal gas law applies. Thus, we will consider these percentages as mole fractions. We calculate the molar mass of dry air by taking a weighted average of the molar masses of its components.

1. **Nitrogen** (N_2): The molar mass of nitrogen is about $28\,\text{g/mol}$. Given that it constitutes about 78% of dry air by volume, its contribution to the molar mass of dry air is 0.78×28.

2. **Oxygen** (O_2): The molar mass of oxygen is about $32\,\text{g/mol}$. Given that it constitutes about 21% of dry air by volume, its contribution to the molar mass of dry air is 0.21×32.

3. **Argon**: The molar mass of argon is about $40\,\text{g/mol}$. Given that it constitutes about 1% of dry air by volume, its contribution to the molar mass of dry air is 0.01×40.

Therefore, the molar mass of dry air is calculated as follows:

$$0.78 \times 28\,\text{g/mol} + 0.21 \times 32\,\text{g/mol} + 0.01 \times 40\,\text{g/mol} = 28.84\,\text{g/mol}$$

So, the mass of one mole of dry air is approximately 28.84 grams.

Comments

GPT4 gives the correct answers and explanations.

Problem 1.15

Estimate the average temperature of the air inside a hot-air balloon. Assume that the total mass of the unfilled balloon and payload is 500 kg. What is the mass of the air inside the balloon?

GPT's Answer to 1.15

First, let's calculate the weight of the unfilled balloon and payload:

$$W = 500 \, \text{kg} \times 9.81 \, \text{m/s}^2 = 4905 \, \text{N}$$

Now, the weight of the displaced air is also $W = 4905 \, \text{N}$. Given that the density of air at room temperature is about $1.2 \, \text{kg/m}^3$, the volume V of the displaced air (which will be roughly the volume of the hot air inside the balloon) can be given by:

$$W = \rho \times V \times g$$

$$V = \frac{W}{\rho \times g}$$

Substituting the given values:

$$V \approx \frac{4905 \, \text{N}}{1.2 \, \text{kg/m}^3 \times 9.81 \, \text{m/s}^2} = 416.7 \, \text{m}^3$$

Now, the mass of the hot air inside the balloon can be found by using the fact that its density is lesser than the density of the surrounding air. If ρ_{hot} is the density of the hot air and ρ is the density of the surrounding air, and T_{hot} is the temperature of the hot air, then:

$$\frac{\rho_{\text{hot}}}{\rho} = \frac{T}{T_{\text{hot}}}$$

From which, $\rho_{\text{hot}} = \rho \times \frac{T}{T_{\text{hot}}}$

Now, using the weight and volume relationships:

$$W = \rho_{\text{hot}} \times V \times g$$

$$W = \rho \times \frac{T}{T_{\text{hot}}} \times V \times g$$

Given that W is 4905 N and rearranging for T_{hot}:

$$T_{\text{hot}} = \rho \times T \times V \times g/W$$

Substituting the given values and T as 293 K:

$$T_{\text{hot}} \approx \frac{1.2\,\text{kg/m}^3 \times 293\,\text{K} \times 416.7\,\text{m}^3 \times 9.81\,\text{m/s}^2}{4905\,\text{N}} = 293\,\text{K} \times 1.17$$

This yields a T_{hot} value greater than 293 K, showing that the air inside the balloon is warmer than the surrounding. The exact temperature would require more specific information or more detailed assumptions about the properties of the balloon and the conditions of the environment.

Comments

GPT assumed that the weight of the air inside was also 4905 N. After prompting about this, it responded as such:

I apologize for the confusion. My assumption was based on the principle of buoyancy: when the hot air balloon is floating (or just lifting off the ground), it's essentially displacing its own weight in air. In other words, the weight of the air being displaced by the balloon is equal to the total weight of the balloon and its contents (including the hot air inside it). This is derived from Archimedes' principle.

To clarify:

Archimedes' Principle: Any object, wholly or partially immersed in a fluid (gas or liquid), is buoyed up by a force equal to the weight of the fluid that the object displaces.

For the hot air balloon to become airborne, or even just to float in equilibrium, the weight of the hot air inside the balloon and the balloon material itself (along with any payload) must be equal to the weight of the cooler air displaced by the balloon.

However, if the assumption is not clear or seems unfounded in the context of the problem statement, we can explore the question from a different angle. Let me know how you'd like to proceed.

Mistake

However, the weight of the cool air that is displaced is greater than 4905N. Let's say that the weight of the hot air is x Newtons. Then, the weight of the cool air is 4905 + x Newtons. Please redo the question with this new logic.

GPT4 gave a nonsense answer that led to it making contradictory statement like the following: 4905 + x = 4905 + x + x, and x = 4905.

Mistake 2

This is wrong. You have to start with the fact that the volume of the balloon gets to a place where the radius = 8m. From there, you calculate the mass of the displaced air. Then, calculate the mass of the hot air by subtracting the mass of the displaced air by 500kg. Then, use the mass of the hot air to calculate the temperature. All these steps can be done with PV = nRT, assuming molar mass of air is 28.97g/mol.

After being confronted with a mistake, GPT responded with this:

1. Volume of the balloon, $V = \frac{4}{3}\pi(8^3) = 2144.66 \, \text{m}^3$ (Using radius $r = 8 \, \text{m}$)

2. Weight of the unfilled balloon and payload, $W_{\text{payload}} = 500 \times 9.81 = 4905 \, \text{N}$

3. Density of air under standard conditions $\rho = 1.225 \, \text{kg/m}^3$

4. Ideal Gas Constant, $R = 8.3145 \, \text{J/mol·K}$

5. Molar mass of air $M = 28.97 \times 10^{-3} \, \text{kg/mol}$

Mass of Displaced Air:
$$m_{\text{displaced}} = \rho \times V = 1.225 \times 2144.66 = 2627.71 \, \text{kg}$$

Mass of Hot Air: Given that the upward force due to the displaced air must counteract the combined weight of the balloon and the hot air inside:
$$m_{\text{hot air}} = m_{\text{displaced}} - 500 = 2127.71 \, \text{kg}$$

To Find the Average Temperature: Using the Ideal Gas Law in the form $PV = nRT$, where $n = \frac{m_{\text{hot air}}}{M}$:

Given atmospheric pressure $P = 1\,\text{atm} = 101325\,\text{Pa}$ and rearranging for T:

$$T = \frac{PV \times M}{R \times m_{\text{hot air}}}$$

Substituting the known values:

$$T = \frac{101325 \times 2144.66 \times 28.97 \times 10^{-3}}{8.3145 \times 2127.71}$$
$$T \approx 313.12\,\text{K}$$

Thus, the average temperature of the air inside the hot-air balloon is approximately $313.12\,\text{K}$, or $40.12°C$.

This new answer is correct in terms of the logic. The final expression for Temperature is good, but GPT4 gets the actual calculation wrong. The answer is 355.8 K. The main problem that GPT4 displayed here was that when given insufficient information to solve the question, instead of saying that it cannot solve the question due to lack of information, it starts to use incorrect knowledge to force an answer. It also displayed errors in calculation.

Problem 1.16

(a) Consider a horizontal slab of air whose thickness (height) is dz. If this slab is at rest, the pressure holding it up from below must balance both the pressure from above and the weight of the slab. Use this fact to find an expression for $\frac{dP}{dz}$, the variation of pressure with altitude, in terms of the density of air.

(b) Use the ideal gas law to write the density of air in terms of pressure, temperature, and the average mass m of the air molecules. Show, then, that the pressure obeys the differential equation

$$\frac{dP}{dz} = -\frac{mg}{kT}P$$

called the barometric equation.

(c) Assuming that the temperature of the atmosphere is independent of height, solve the barometric equation to obtain the pressure as a function of height:
$$P(z) = P(0)e^{-mgz/kT}$$
Show also that the density obeys a similar equation.

(d) Estimate the pressure, in atmospheres, at the following locations.

GPT's Answer to 1.16:

(a) For the horizontal slab of air at pressure P and $P + dP$ from below and above respectively:
$$\frac{dP}{dz} = -\rho g$$

(b) Using the ideal gas law:
$$PV = nRT \implies \rho = \frac{mP}{RT}$$

Substituting this expression in (a):
$$\frac{dP}{dz} = -\frac{mgP}{RT}$$

This is the barometric equation.

(c) Solving the above differential equation:
$$\frac{dP}{P} = -\frac{mg}{RT}dz$$

Integrating both sides:
$$\ln(P) = -\frac{mgz}{RT} + C$$
Where C is an integration constant. At $z = 0$, $P = P(0)$:
$$P(z) = P(0)e^{-mgz/kT}$$

Similarly, the density follows an analogous expression.

(d) Using given constants and the derived formula: For Ogden, Utah:

$$P(1430) = e^{-28.97 \times 10^{-3} \times 9.81 \times 1430/8.31/288} \approx 0.85 \text{ atm}$$

Similarly, for Leadville: $P \approx 0.7$ atm, Mt. Whitney: $P \approx 0.61$ atm, Mt. Everest: $P \approx 0.33$ atm.

Comments

GPT4 gives the correct answers and explanations. However, the numerical values are slightly off.

Problem 1.17.

Even at low density, real gases don't quite obey the ideal gas law. A systematic way to account for deviations from ideal behavior is the virial expansion,

$$PV = nRT \left(1 + \frac{B(T)}{V/n} + \frac{C(T)}{(V/n)^2} + \cdots \right), \tag{13}$$

where the functions $B(T)$, $C(T)$, and so on are called the virial coefficients. When the density of the gas is fairly low, so that the volume per mole is large, each term in the series is much smaller than the one before.

GPT's Answer to 1.17

(a) Using the ideal gas law $PV = nRT$, the volume per mole $\frac{V}{n}$ at atmospheric pressure $P = 1$atm and $R = 0.0821$L.atm/mol.K is:

$$\frac{V}{n} = \frac{RT}{P}. \tag{14}$$

Given the table of $B(T)$ values for different temperatures, one can compute the term $\frac{B(T)}{V/n}$. If the magnitude of $\frac{B(T)}{V/n}$ is much less than 1, then the ideal gas law is a good approximation.

(b) At low temperatures, gas molecules move slower and have stronger intermolecular attractions, leading to a negative $B(T)$. At high temperatures, the

molecules move faster, resulting in dominant repulsive forces and a positive $B(T)$.

(c) To derive the virial coefficients from the van der Waals equation, expand the equation in terms of powers of $1/V$ and match the coefficients with those of the virial expansion. The binomial expansion can be applied to the term $\left[1 - \frac{nb}{V}\right]^{-1}$.

(d) To plot the graph of $B(T)$ using the van der Waals equation, one needs to adjust the constants a and b to fit the given data for nitrogen. Then, by comparing the van der Waals prediction with the actual values, the accuracy of the equation can be assessed.

Comments