Assignment - 6

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6.6 Find value of z if area under a normal standard curve

a) To right of z is 0.3622

$P(Z > z) = 0.3622$

$P(Z > z) = 1 - P(Z \le z)$

$P(Z \le z) = 1 - 0.3622$

$P(Z \le z) = 0.6378$

Using excel funⁿ

$z = NORMSINV(0.6378)$

$= 0.35$

From Table A.3

b) To the left of z is 0.1131

$P(Z < z) = 0.1131$

$\boxed{z = -1.21}$

From Table A.3

c) b/w 0 & z, with z>0, is 0.4838

$P(0 < Z < z) = 0.4838$

$P(0 < Z < z) = P(Z < z) - P(Z < 0)$

$0.4838 = P(Z < z) - P(Z < 0)$

$0.4838 + 0.5 = P(Z < z$

$P(Z < z) = 0.9838$

Using excel funⁿ

$z = NORMSINV(0.9838)$

from Table A3

d) b/w $-z$ & z with $z>0$ is 0.9500

$P(-z<Z<z) = 0.9500$

$P(-z<Z<z) = P(Z<z) - P(Z<-z)$

$0.95 = P(Z<z) - [1-P(Z<z)]$

$2P(Z<z) = 1+0.95$

$2P(Z<z) = 1.95$

$P(Z<z) = 0.9750$

Using exel fun^n z: NORMSINV (0.9750)

$= 1.96$ From Table A3

6.9) Given normally distributed variable x with mean 18 & std is 2.5

a) $P(x<15)$ $x=15$

$z = \frac{x-\mu}{\sigma}$ $\mu = 18$

$\sigma = 2.5$

$= \frac{15-18}{2.5} = -1.2$

b) Value of k such that $P(x<k) = 0.2236$

$P(x) = 0.2236$

$P\left(z<\frac{k-\mu}{\sigma}\right) = 0.2236$

$P(z<-0.76) = 0.2236$ From Table A3

$\frac{k-\mu}{\sigma} = 0.76$

$k-\mu = 0.76(\sigma)$

$k = 0.76(2.5)+18$

$\boxed{k = 16.1}$

c) Thee value of k such that $P(x > k) = 0.1814$

$$P(x) = 0.1814$$

$$P\left(\frac{x-\mu}{\sigma} > \frac{k-\mu}{\sigma}\right) = 0.1814$$

$$1 - P\left(z \leq \frac{k-\mu}{\sigma}\right) = 0.1814$$

$$P\left(z \leq \frac{k-\mu}{\sigma}\right) = 1 - 0.1814$$

$$P\left(z \leq \frac{k-\mu}{\sigma}\right) = 0.8186$$

$$P(z < 0.91) = 0.8186$$

from Table A3

$$\frac{k-\mu}{\sigma} = 0.91$$

$$k = 0.91(2.5) + 18$$

$$\boxed{k = 20.275}$$

d) $P(17 < x < 21)$ $\quad x_1 = 17 \quad x_2 = 21$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{17-18}{2.5} = -0.4$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{21-18}{2.5} = 1.2$$

$$P(17 < x < 21) = P(-0.4 < z < 1.2)$$

$$= P(z < 1.2) - P(z < -0.4)$$

$$= 0.8849 - 0.3446$$

$$= 0.5403$$

From Table A_3

6.12 X: loaves of rye bread normally distributed

$\mu: 30cm \quad \sigma = 2cm$

a) longer than 31.7cm

$Z = \frac{x-\mu}{\sigma} \quad x = 31.7$

$= \frac{31.7-30}{2} = 0.85$

$P(x > 31.7) = P(z > 0.85)$

$= 1 - P(z < 0.85)$

$= 1 - 0.8023$

$= 0.1977 = 19.77\%$ from Table A.3

b) b/w 29.3 & 33.5cm in length

$Z_1 = \frac{x_1-\mu}{\sigma} \quad x_1 = 29.3$

$x_2 = 33.5$

$= \frac{29.3-30}{2} = -0.35$

$Z_2 = \frac{x_2-\mu}{\sigma}$

$= \frac{33.5-30}{2} = 1.75$

$P(29.3 < x < 31.7) = P(-0.35 < z < 1.75)$

$= P(z < 1.75) - P(z < -0.35)$

$= 0.9599 - 0.3632$

$= 0.5967$ From Table A.3

$= 59.67\%$

c) shorter than 25.5cm $Z = \frac{x-\mu}{\sigma} = \frac{25.5-30}{2} = -2.25$

$P(x < 25.5) = P(z < -2.25)$ From Table A.3

$= 0.0122 \Rightarrow \boxed{1.22\%}$

6.22) If set of observations is normally distributed what percent of these differ from the mean by

a) more than 1.3σ

$P(|x-\mu| \geq 1.3\sigma) = 1 - P(|x-\mu| < 1.3\sigma)$

$= 1 - P(-1.3\sigma < x-\mu < 1.3\sigma)$

$= 1 - P\left(\frac{-1.3\sigma}{\sigma} < \frac{x-\mu}{\sigma} < \frac{1.3\sigma}{\sigma}\right)$

$= 1 - P(-1.3 < z < 1.3)$

$= 1 - P(z < 1.3) + P(z < -1.3)$

$= 1 - 0.9032 + 0.0968$

$= 0.1936 = \boxed{19.36\%}$

b) Less than 0.52σ

$P(|x-\mu| < 0.52\sigma) = P(-0.52\sigma < x-\mu < 0.52\sigma)$

$= P\left(\frac{-0.52\sigma}{\sigma} < \frac{x-\mu}{\sigma} < \frac{0.52}{\sigma}\sigma\right)$

$= P(-0.52 < z < 0.52)$

$= P(z < 0.52) - P(z < 0.52)$

$= 0.6985 - 0.3015$ from Table A_3

$= 0.3970 \Rightarrow \boxed{39.70\%}$

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6.36) $n = 100$

Probability of success in each trail $p = 10\%$
$= 0.1$

Probability of failure in each trail $q = 1-p$
$= 1 - 0.1 = 0.9$

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X: binomial distribution

Mean $(\mu) = np$

$= 100(0.1)$

$= 10$

Std $(\sigma) = \sqrt{npq}$

$= \sqrt{(100)(0.1)(0.9)}$

$= \sqrt{9} \boxed{= 3}$

a) exceeds 13

$x = 13.5$

$Z = \frac{x - \mu}{\sigma} = \frac{13.5 - 10}{3}$

$= 1.166$

$P(X > 13) = P(Z > 1.17)$

$= 1 - P(Z < 1.17)$

$= 1 - 0.8790$

$\boxed{= 0.1210}$

b) is less than 8

$x = 7.5$

$Z = \frac{x - \mu}{\sigma} = \frac{7.5 - 10}{3} = -0.83$

$P(X < 8) = P(Z < 0.83)$ from Table A3

$\boxed{= 0.2033}$

6.34) A pair of dice is rolled 180 times. what is probability that total of 7 occurs

a) atleast 25 times

$n = 180$

No. of favourable cases to get total 7 is

(1,6) (2,5) (3,4) (4,3) (5,2) (6,1) = 6

total no of cases when rolling $= 6 \times 6 = 36$
pair of dice

Probability of success in each trail $p = 6/36 = 0.1667$

Probability of failure in each trail $q = 1 - 0.1667$

$= 0.83333$

X = bionomial distribution

mean $(\mu) = np = 180 \times 0.1667$

$= 30$

Std$(\sigma) = \sqrt{npq} = 180 \times 0.1667 \times 0.8333$

$= \sqrt{25} = 5$

a) $x = 24.5$

$$z = \frac{x - \mu}{\sigma} = \frac{24.5 - 30}{5} = -1.1$$

$P(X \geq 25) = P(z \geq -1.1)$

$= 1 - P(z < -1.1)$

$= 1 - 0.1357$ from Table A_3

$= 0.8643$

b) b/w 33 and 41 times inclusive

$x_1 = 32.5 \quad x_2 = 41.5$

$$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{32.5 - 30}{5} = 0.5$$

$$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{41.5 - 30}{5} = 2.3$$

$P(33 < X < 41) = P(0.50 < Z < 2.30)$

$= P(Z < 2.30) - P(Z < 0.50)$

$= 0.9893 - 0.6915$ from Table A_3

$= 0.2978$

c) exactly 30 times

$x_1 = 29.5$ $\quad x_2 = 30.5$

$$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{29.5 - 30}{5} = -0.1$$

$$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{30.5 - 30}{5} = 0.1$$

$P(X = 30) = P(-0.10 \leq Z \leq 0.10)$

$= P(Z \leq 0.10) - P(Z \leq -0.10)$

$= 0.5398 - 0.4602$ from Table A_3

$= 0.0796$

6.38) Probability of success $p = 0.01$

Probability of failure $q = 1 - 0.01 = 0.99$

a) $n = 20$

X: no of damaged letters among 20 letters in batch

X: binomial distribution

Probability mass function X is

$P(X = x) = Bin(x; 20, 0.01)$

$$= \binom{20}{x} (0.01)^x (0.99)^{20-x}$$

where $x = 0, 1, \ldots 20$

$$p(x>1) = 1 - p(x \leq 1)$$

$$= 1 - \sum_{x=0}^{1} b(x: 20, 0.01)$$

$$= 1 - \sum_{x=0}^{1} \binom{20}{x} (0.01)^x (1-0.01)^{n-x}$$

$$= 1 - p(x=0) - p(x=1)$$

$$= 1 - \binom{20}{0} (0.01)^0 (0.99)^{20-0} - \binom{20}{1} (0.01)^1 (0.99)^{20-1}$$

$$= 1 - 0.8179 - 0.1652$$

$$= 1 - 0.9831$$

$$= 0.0169$$

b) $n = 500$

X: no. of damaged kettles among 500 kettles in batch X is binomial distribution

$$\text{Mean } (\mu) = np = 500 (0.01)$$

$$= 5$$

$$\text{Std } (\sigma) = \sqrt{npq}$$

$$= \sqrt{(500)(0.01)(0.99)}$$

$$= \sqrt{4.95} = 2.225$$

$x = 8.5$

$$z = \frac{x-\mu}{\sigma} = \frac{8.5-5}{2.225} = 1.57$$

$P(X > 8.5) = 1 - P(X \le 8.5)$

$= 1 - P(z < 1.57)$

$= 1 - 0.9418$ From Table A3

$= 0.0582$

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6.56) X = Power usage of particular company.

$\mu = 4 \quad \sigma = 2$

$P(X > 270) = P(\ln(x) > \ln(270))$

$= P\left(\frac{\ln(x) - \mu}{\sigma} > \frac{\ln(270) - 4}{2}\right)$

$= P\left(\frac{\ln(x) - \mu}{\sigma} > \frac{5.598 - 4}{2}\right)$

$= P\left(\frac{\ln(x) - \mu}{\sigma} > \frac{1.598}{2}\right)$

$= P\left(\frac{\ln(x) - \mu}{\sigma} > 0.799\right)$

$= P(z > 0.80)$ using Table A3

$= 0.2119$

6.57) For ex:- 656 what s mean power usage. what is Variance

X = log normal distribution with $\mu = 4 \quad \sigma = 2$

Mean $E(x) = e^{\mu + \sigma^2/2}$

$= e^{4 + 2^2/2}$

$= e^{4+2}$

$\boxed{= e^6} = 403.4288$

$\text{Variance}(x) = e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$

$= e^{2(4)+2^2}\left(e^{2^2}-1\right)$

$= e^{8+4}\left(e^{4}-1\right)$

$= e^{12}\left(e^{4}-1\right)$

6.58)

x : no of automobiles that arrives at certain intersection/min

x : poisson distribution with mean 5

a) $P(x>10) = 1-P(x\leq 10)$

$= 1-\sum_{x=0}^{10}\frac{e^{-\lambda}\lambda^{x}}{x!}$

$= (1-P(x=0)) - P(x=1) - P(x=2) - P(x=3) - P(x=4) - P(x=5)$
$- P(x=6) - P(x=7) - P(x=8) - P(x=9) - P(x=10)$

$= (1-0.0067-0.0337-0.0842-0.1404-0.1755-0.1755$
$-0.1462-0.1044-0.0653-0.0363-0.0181)$

$= 1-0.9863$

$= 0.0137$

Probality that more than 2min elapse befn 10 cars arrive

Let $y = x/\beta$; $\alpha = 10$; $\beta = x/y = 2/10 = 1/5$

Hence

$P(x>2) = 1-P(x\leq 2)$

$= 1-P(y\leq 10)$

$= 1-\int_0^{10}\frac{(y\beta)^{10-1}}{\beta\Gamma(\alpha)}e^{-y}\,\beta dy$

$= 1-\int_0^{10}\frac{y^{9}e^{-y}}{9!}\,dy$

$= 1 - 0.5421$

$\boxed{= 0.4579}$

6.61) $n = 1000$

Probability of success $p = 49\% = 0.49$

Probability of failure $q = 1 - 0.49 = 0.51$

X = no. of white-collar worker among 1000 random selects valiumors

X = binomial distribution

$\text{mean}(\mu) = np = 1000 \times 0.49 = 490$

$\text{Std}(\sigma) = \sqrt{npq} = \sqrt{100 \times 0.49 \times 0.51} = \sqrt{24.99} = 15.808$

$x_1 = 481.5 \qquad x_2 = 510.5$

$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{481.5 - 490}{15.8} = -0.54$

$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{510.5 - 490}{15.8} = 1.3$

$P(481.5 \le X \le 510.5) = \sum_{x=482}^{510} b(x; 1000, 0.49)$

$= P(-0.54 \le z \le 1.3)$

$= P(z \le 1.30) - P(z \le -0.54)$

$= 0.9032 - 0.2946$

$\boxed{= 0.6086}$

Chapter 6 - Some Continuous Probability Distribution

Formulae sheet

1) Continuous Uniform distribution

$f(x; A, B) = B - A$ for $A \le x \le B$

$P(X \le x) = \dfrac{x - A}{B - A}$ for $A \le x \le B$

$E[X] = \dfrac{A+B}{2}$ $V[X] = \dfrac{(B-A)^2}{2}$

2) Normal distribution

$n(x; \mu \sigma) = \dfrac{1}{\sigma \sqrt{2\pi}} e^{-1/2} \dfrac{(x-\mu)^2}{\sigma}$

for $-\infty < x < \infty$

$E[X] = \mu$ $V[X] = \sigma^2$

Excel funn

NORMSINV $(x, \mu, \sigma, 0)$

standard normal distribution

$z = \dfrac{x - \mu}{\sigma}$ $\phi(z) = \dfrac{1}{\sqrt{2\pi}} e^{-1/2 z^2}$

$\phi(z) = P(Z \le z)$ for $-\infty < z < \infty$

ND Probability :

$P(a \le X \le b) = P\left[\dfrac{a-\mu}{\sigma} \le \dfrac{X-\mu}{\sigma} \le \dfrac{b-\mu}{\sigma}\right]$

$= P\left[\dfrac{a-\mu}{\sigma} \le Z \le \dfrac{b-\mu}{\sigma}\right]$

$= \left(\dfrac{a-\mu}{\sigma}\right) - \left(\dfrac{a-\mu}{\sigma}\right) \Rightarrow z_1$

Normal Approxi to Binomial :

$z = \dfrac{\cancel{X} - np}{\sqrt{np(1-p)}}$

③ Exponential Distribution

$f_X(x;\beta) = 1/\beta \; e^{-x/\beta}$ $\quad P[X \le x] = 1 - e^{-x/\beta}$

$E[X] = \beta \quad V[X] = \beta^2$

Gamma Distribution : $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$

$\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1) \;|\; \Gamma(1/2) = \sqrt{\pi}$

+ve integer $\Gamma(n) = (n-1)$

$F(a;\alpha) = \int_0^a \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)} dy$

$E[X] = \alpha\beta \quad V[X] = \alpha\beta^2$

Excel func:- GAMMADIST $(x, \alpha, \beta, 0)$

Relationship to poission Process:

Poisson $(\mu = \lambda)$ $\quad (T \sim \text{Exp}(\beta = 1/\lambda))$

Gamma $(\alpha = k, \; \beta = 1/\lambda)$

④ chi- squared distibution:-

$f_X(x;v) = \frac{x^{v/2 - 1} e^{-x/2}}{2^{v/2} \Gamma(v/2)}$ for $x > 0$

$f_X(x) = 0$; $E[X] = v$; $V[X] = 2v$

⑤ Lognormal Distribution:

$f_X(x;\mu,\sigma) = e^{-1/2} \left(\frac{\ln(x) - \mu}{\sigma}\right)^2$ for $x > 0$;

$E[X] = e^{\mu + \sigma^2/2}$

$V[X] = (e^{2\mu} - \sigma^2)(e^{\sigma^2} - 1)$