CHAPTER NINE

A random sample of 12 graduates of a certain secretarial school typed an average of 79.3 words per minute with a standard deviation of 7.8 words per minute. Assuming a normal distribution for the number of words typed per minute.

a. Find a 95% confidence interval for the average number of words typed by all graduates of this school.
b. Interpret your answer.

(a) $n = 12$ $\bar{x} = 79.3$ $S = 7.8$ and $1-\alpha = 95\%$ $\alpha = 0.05$ $t_{\alpha/2, 11} = t_{0.025} = 2.201$

$V = n-1 = 11$ $\alpha/2 = 0.025$

Table A.4 =

A 95% confidence Interval for the population mean is

$$\bar{x} - t_{\alpha/2, n-1} \, s/\sqrt{n} < \mu < \bar{x} + t_{\alpha/2, n-1} \, s/\sqrt{n}$$

$$79.3 - (2.201)(7.8/\sqrt{12}) < \mu < 79.3 + (2.201)\left(\frac{7.8}{\sqrt{12}}\right)$$

$$= 79.3 - (2.201)(7.8/3.4641) < \mu < 79.3 + (2.201)\left(\frac{7.8}{3.464}\right)$$

$$= 79.3 - (2.201)(2.2517) < \mu < 79.3 + (2.201)(2.2517)$$

$$79.3 - 4.9559 < \mu < 79.3 + 4.9559$$

$$74.34 < \mu < 84.26$$

C.I (74.34, 84.26)

(b) We are 95% Confident that the average numbertyped by all graduates of this School lies between 74.34 and 84.26.

The heights of a random sample of 50 college students showed a mean of 174.5 centimeters and a standard deviation of 6.9 centimeters.

(a) Construct a 98% confidence interval for the mean height of all college students.

(b) Interpret your answer.

$n = 50 \quad \bar{x} = 174.5 \quad \sigma = 6.9$

98% CI $= 0.98$ and $\alpha = 0.02$ since it is a two-sided test $Z_{\alpha/2} = Z_{0.01} = 2.33$

$\bar{x} - Z_{0.01}(\sigma/\sqrt{n}) < \mu < \bar{x} + Z_{0.01}(\sigma/\sqrt{n})$

$174.5 - (2.33)(6.9/\sqrt{50}) < \mu < 174.5 + (2.33)(6.9/\sqrt{50})$

$= 174.5 - (2.33)(6.9/7.0711) < \mu < 174.5 + (2.33)(6.9/7.0711)$

$174.5 - (2.33)(0.9758) < \mu < 174.5 + (2.33)(0.9758)$

$174.5 - 2.2736 < \mu < 174.5 + 2.2736$

$172.23 < \mu < 176.77$

CI $= (172.23, 176.77)$

(b) We are 98% confident that the mean height of all college students lies between 172.23 and 176.77

CHAPTER 10

A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years?

Use a 0.05 level of significance.

Use the Hypothesis test steps
This is a one sided test and it is a test form C

$Z_{0.05} = 1.645$

① $H_0 : \mu = \mu_0 \Rightarrow \mu = 70$ years

② $H_1 : \mu > \mu_0 \Rightarrow \mu > 70$ years

③ $\alpha = 0.05$, $n = 100$, $\bar{x} = 71.8$, $\sigma = 8.9$, $\mu_0 = 70$

④ Equivalent Decision Rules.

ⓐ Using a rejection region for $\hat{\mu}$
Decision rule: If $\bar{x} > \mu_0 + Z_\alpha \, se\{\bar{x}\}$ then reject H_0

ⓑ Using a test statistic: Define the z-statistic to be
$Z^* = \dfrac{\hat{\mu} - \mu_0}{se\{\hat{\mu}\}}$ or $\dfrac{\bar{x} - \mu_0}{se(\hat{\mu})}$. Decision Rule: If $Z^* > Z_\alpha$ reject H_0

ⓒ Using a P-value: The P-value of this Z-test is
$P = P[Z > Z^*] = 1 - \Phi(Z^*)$ Decision Rule: If $P < \alpha$, then reject H_0

ⓓ Using a C.I: Construct a $100(1-\alpha)\%$ one sided lower bound Z-interval for μ $(L, \infty) = (\hat{\mu} - Z_\alpha \, s.e\{\mu\}, \infty)$

$Z_\alpha = Z_{0.05} = 1.645$.

$s.e\{\mu\} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{8.9}{100} = 0.89$, Where $Z^* = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$

⑤ Computations: $\bar{x} = 71.8$ years, $\sigma = 8.9$ years and hence

$$Z^* = \frac{71.8 - 70}{8.9/\sqrt{100}} = \frac{1.8}{0.89} = 2.02$$

$Z^* > Z_\alpha$; $2.02 > 1.645$

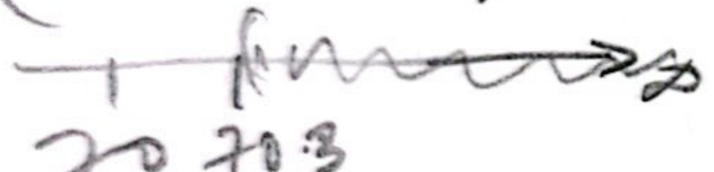
⑥ Decision: Reject H_0 and conclude that the mean life span today is greater than 70 years; $\hat{\mu} = 71.8$

$\hat{\mu} > 70 + (1.645)(0.89)$

$\hat{\mu} > 70 + 1.464$; $71.8 > 71.46$

using a rejection region, we reject H_0. The P-value corresponding to $Z = 2.02$.
using Table A-3 we have $P = P[Z > 2.02] = 1 - \Phi(2.02)$
$= 1 - 0.9783 = 0.0217$; $P < \alpha$; $0.0217 < 0.05$
As a result, the evidence in favor of H_1 is even stronger than the suggested by a 0.05 level of significance.

using a C.I $(\hat{\mu} - Z_\alpha \, se\{\hat{\mu}\}, \infty)$
$(71.8 - (1.645)(0.89), \infty) \Rightarrow (70.3, \infty)$



Since $\mu_0 = 70$ is not in the C.I then we reject H_0.

CHAPTER ELEVEN

The grades of a class of 10 students on a midterm report (x) and on the final examination (y) are as follows:

S/N	final examination (y)	midterm report (x)	xy	x^2		
1	68	86	5848	7396		
2	50	66	3300	4356		
3	71	78	5538	6084		
4	72	34	2448	1156		
5	81	47	3807	2209		
6	94	85	7990	7225		
7	96	99	9504	9801		
8	99	99	9801	9801		
9	67	68	4556	4624		
10	77	82	6314	6724		
Totals	775	744	59106	59376		

$n=10, \ \Sigma y_i = 775 \quad \Sigma x_i = 744 \quad \Sigma x_i^2 = 59376 \quad \Sigma x_i y_i = 59106$

(a) Estimate the linear regression line.

(b) Estimate the final examination grade of a student who received a grade of 85 on the midterm report.

(a) $\hat{y} = b_0 + b_1 x$

$$b_1 = \frac{\Sigma x_i y_i - \frac{(\Sigma x_i)(\Sigma y_i)}{n}}{\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}}$$

$$b_1 = \frac{59106 - \frac{(744)(775)}{10}}{59376 - \frac{(744)^2}{10}}$$

$$b_1 = \frac{59106 - \frac{576600}{10}}{59376 - \frac{553536}{10}}$$

$$b_1 = \frac{59106 - 57660}{59376 - 55353.6} = \frac{1446}{4022.4} = \boxed{0.359}$$

$$b_0 = \frac{1}{n}\left(\Sigma y_i - b_1 \Sigma x_i\right)$$

$$= \frac{1}{10}\left(775 - (0.359)(744)\right)$$

$$= \frac{1}{10}\left(775 - 267.096\right)$$

$$= \frac{1}{10}\left(507.904\right)$$

$$b_0 = \boxed{50.79}$$

fitted values: $\hat{y} = b_0 + b_1 x_i$

$$\hat{y} = 50.79 + 0.359(x_i)$$

(b) For $x = 85$

$$\hat{y} = 50.79 + (0.359)(85)$$

$$= 50.97 + 30.515$$

$$= 81.305$$

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