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HOME ASSIGNMENT-6

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6.6 find value of z if area under a normal standard curve

a) to right of z is 0.3622

$P(Z > z) = 0.3622$

$P(Z > z) = 1 - P(Z \le z)$

$P(Z \le z) = 1 - 0.3622$

$P(Z \le z) = 0.6378$

using excel function

$z = \text{NORMSINV}(0.6378)$

$= \boxed{0.35}$

From Table A-3

b) To the left of z is 0.1131

$P(Z < z) = 0.1131$

$z = \boxed{-1.21}$ From Table A-3

c) b/w 0 & z, with z > 0, is 0.4838

$P(0 < Z < z) = 0.4838$

$P(0 < Z < z) = P(Z < z) - P(Z < 0)$

$0.4838 = P(Z < z) - P(Z < 0)$

$0.4838 + 0.5 = P(Z < z)$

$P(Z < z) = 0.9838$

using excel function

$z = \text{NORMSINV}(0.9838)$

$= \boxed{2.14}$ from Table A-3

d) b/w $-z$ & z, with $z>0$, is 0.9500

$P(-z < Z < z) = 0.9500$

$P(-z < Z < z) = P(Z < z) - P(Z < -z)$

$0.95 = P(Z<z) - (1 - P(Z<z))$

$2P(Z<z) = 1 + 0.95$

$2P(Z<z) = 1.95$

$P(Z<z) = 0.9750$

using excel function

$z = $ NORMSINV(0.9750

$= \boxed{1.96}$ from Table A.3

6.9 Given normally distributed variable X with mean 18 & Std is 2.5. Find

a) $P(X<15)$ $x = 15$

$z = \frac{x - \mu}{\sigma}$ $\mu = 18$

$\sigma = 2.5$

$= \frac{15 - 18}{2.5} = -1.2$

$P(X<15) = p(Z < -1.2)$

$= \boxed{0.1151}$ from Table A.3)

b) value of k such that $P(X<k) = 0.2236$

$P(x) = 0.2236$

$P\left(Z < \frac{k-\mu}{\sigma}\right) = 0.2236$

$P(Z < -0.76) = 0.2236$

from Table A.3

$$\frac{k-\mu}{\sigma} = -0.76$$

$$k-\mu = 0.76(\sigma)$$

$$k = 0.76(2.5)+18$$

$$\boxed{k = 16.1}$$

c) the value of k such that $P(X>k) = 0.1814$

$$P(X) = 0.1814$$

$$P\left(\frac{X-\mu}{\sigma} > \frac{k-\mu}{\sigma}\right) = 0.1814$$

$$1-P\left(z \leq \frac{k-\mu}{\sigma}\right) = 0.1814$$

$$P\left(z \leq \frac{k-\mu}{\sigma}\right) = 1-0.1814$$

$$P\left(z \leq \frac{k-\mu}{\sigma}\right) = 0.8186$$

$$P(z < 0.91) = 0.8186$$

From Table A.3

$$\frac{k-\mu}{\sigma} = 0.91$$

$$k = 0.91(2.5)+18$$

$$\boxed{k = 20.275}$$

d) $P(17 < X < 21)$ $x_1 = 17$ $x_2 = 21$

$$z_1 = \frac{x_1-\mu}{\sigma} = \frac{17-18}{2.5} = -0.4$$

$$z_2 = \frac{x_2-\mu}{\sigma} = \frac{21-18}{2.5} = 1.2$$

$P(17 < X < 21) = P(-0.4 < Z < 1.2)$
$= P(Z < 1.2) - P(Z < -0.4)$
$= 0.8849 - 0.3446$ Table A3
$= \boxed{0.5403}$

6.12 X = loaves of rye bread normally distributed
$\mu = 30cm$ $\sigma = 2cm$
a) longer than 31.7 cm
$z = \frac{x - \mu}{\sigma}$ $x = 31.7$

$= \frac{31.7 - 30}{2} = 0.85$

$P(X > 31.7) = P(Z > 0.85)$
$= 1 - P(Z < 0.85)$
$= 1 - 0.8023$ from Table A3
$= 0.1977 = \boxed{19.77\%}$
b) b/w 29.3 & 33.5 cm in length
$z_1 = \frac{x_1 - \mu}{\sigma}$ $x_1 = 29.3$
$x_2 = 33.5$
$= \frac{29.3 - 30}{2} = -0.35$

$z_2 = \frac{x_2 - \mu}{\sigma}$

$= \frac{33.5 - 30}{2} = 1.75$

$P(29.3 < X < 31.7) = P(-0.35 < Z < 1.75)$

$= P(Z < 1.25) - P(Z < -0.35)$

$= 0.9599 - 0.3632$ from Table A.3

$= 0.5967 = \boxed{59.67\%}$

c) Shorter than 25.5 cm

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{25.5 - 30}{2} = -2.25$$

$P(X < 25.5) = P(Z < -2.25)$ from Table A.3

$= 0.0122 = \boxed{1.22\%}$

6.22 If set of observations is normally distributed what percent of these differ from the mean by

a) more than 1.3σ

$P(|X - \mu| \geq 1.3\sigma) = 1 - P(|X - \mu| < 1.3\sigma)$

$= 1 - P(-1.3\sigma < X - \mu < 1.3\sigma)$

$= 1 - P\left(\frac{-1.3\sigma}{\sigma} < \frac{X - \mu}{\sigma} < \frac{1.3\sigma}{\sigma}\right)$

$= 1 - P(-1.3 < Z < 1.3)$

$= 1 - P(Z < 1.3) + P(Z < -1.3)$

$= 1 - 0.9032 + 0.0968$

$= 0.1936 = \boxed{19.36\%}$

b) less than 0.52σ

$P(|X - \mu| < 0.52\sigma) = P(-0.52\sigma < X - \mu < 0.52\sigma)$

$= P\left(\frac{-0.52\sigma}{\sigma} < \frac{X - \mu}{\sigma} < \frac{0.52\sigma}{\sigma}\right)$

$= P(-0.52 < Z < 0.52)$

$= P(Z < 0.52) - P(Z < -0.52)$

$= 0.6985 - 0.3015$ from Table A.3

$= 0.3970 = \boxed{39.70\%}$

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6.26 $n = 100$

Probability of Success in each trail $p = 10\%$

$= 0.1$

Probability of failure in each trail $q = 1 - p$

$= 1 - 0.1 = 0.9$

X = binomial distribution

Mean $(\mu) = np$

$= 100(0.1)$

$= 10$

Std $(\sigma) = \sqrt{npq}$

$= \sqrt{100(0.1)(0.9)}$

$= \sqrt{9} = 3$

a) exceeds 13

$x = 13.5$

$z = \dfrac{x - \mu}{\sigma} = \dfrac{13.5 - 10}{3}$

$= 1.166$

$P(X > 13) = P(Z > 1.17)$

$= 1 - P(Z < 1.17)$ From Table A.3

$= 1 - 0.8790$

$= \boxed{0.1210}$

b) is less than 8

$x = 7.5$

$z = \frac{x-\mu}{\sigma} = \frac{7.5-10}{3} = -0.83$

$P(X<8) = P(Z<-0.83)$ from Table A.3

$= \boxed{0.2033}$

6.34 A pair of dice is rolled 180 times. What is probability that total of 7 occurs

a) atleast 25 times

$n = 180$

No.of favourable cases ~~(when rolling)~~ to get total 7 is

(1,6) (2,5) (3,4) (4,3) (5,2) (6,1) = 6

total no.of cases when rolling pair of dice $= 6 \cdot 6 = 36$

probability of Success in each trail $p = 6/36 = 0.1667$

probability of failure in each trail $q = 1-0.1667$

$= 0.8333$

X = binomial distribution

Mean $(\mu) = np = 180 \times 0.1667$

$= 30$

Std $(\sigma) = \sqrt{npq} = 180 \times 0.1667 \times 0.8333$

$= \sqrt{25} = 5$

a) $x = 24.5$

$z = \frac{x-\mu}{\sigma}$

2T

$$\frac{24.5-30}{5} = -1.1$$

$P(X \geq 25) = P(Z \geq -1.1)$

$= 1 - P(Z < -1.1)$

$= 1 - 0.1357$ From Table A.3

$= \boxed{0.8643}$

b) b/w 33 and 41 times inclusive

$x_1 = 32.5 \qquad x_2 = 41.5$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{32.5-30}{5}$$

$= 0.5$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{41.5-30}{5}$$

$= 2.3$

$P(33 \leq X \leq 41) = P(0.50 < Z < 2.30)$

$= P(Z < 2.30) - P(Z < 0.50)$

$= 0.9893 - 0.6915$ From Table A.3

$= \boxed{0.2978}$

c) exactly 30 times

$x_1 = 29.5 \qquad x_2 = 30.5$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{29.5-30}{5}$$

$= -0.1$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{30.5-30}{5} = 0.1$$

$$P(X=30) = P(-0.10 \le Z \le 0.10)$$
$$= P(Z \le 0.10) - P(Z \le -0.10)$$
$$= 0.5398 - 0.4602 \text{ from Table A.3}$$
$$= \boxed{0.0796}$$

6.38 Probability of Success $p = 0.01$
Probability of failure $q = 1 - 0.01 = 0.99$

a) $n = 20$

X = no. of damaged letters among 20 letters in batch

X = binomial distribution

Probability mass function X is

$$P(X=x) = Bin(x; 20, 0.01)$$
$$= \binom{20}{x}(0.01)^x (0.99)^{20-x}$$

where $x = 0, 1 \cdots 20$

$$P(X>1) = 1 - P(X \le 1)$$
$$= 1 - \sum_{x=0}^{1} b(x; 20, 0.01)$$
$$= 1 - \sum_{x=0}^{1} \binom{20}{x}(0.01)^x (1-0.01)^{n-x}$$

$$= 1 - P(X=0) - P(X=1)$$
$$= 1 - \binom{20}{0}(0.01)^0 (0.99)^{20-0} - \binom{20}{1}(0.01)^1 (0.99)^{20-1}$$
$$= 1 - 0.8179 - 0.1652$$
$$= \cancel{0.6527}\ 1 - 0.9831$$
$$= \boxed{0.0169}$$

b) $n = 500$

X = no of damaged letters, among 500 letters in batch

X is binomial distribution

$\text{Mean}(\mu) = np = 500(0.01)$
$= 5$

$\text{Std}(\sigma) = \sqrt{npq}$
$= \sqrt{(500)(0.01)(0.99)}$
$= \sqrt{4.95} = 2.225$

$x = 8.5$

$$z = \frac{x-\mu}{\sigma} = \frac{8.5-5}{2.225}$$
$= 1.57$

$P(X > 8.5) = 1 - P(X \leq 8.5)$
$= 1 - P(z < 1.57)$
$= 1 - 0.9418$ from Table A.3
$= \boxed{0.0582}$

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6.56

X = power usage of particular company
$\mu = 4$, $\sigma = 2$

$P(X > 270) = P(\ln(X) > \ln(270))$

$$= P\left(\frac{\ln(x)-\mu}{\sigma} > \frac{\ln(270)-4}{2}\right)$$

$$= P\left(\frac{\ln(X)-\mu}{\sigma} > \frac{5.598-4}{2}\right)$$

$$= P\left(\frac{\ln(x)-\mu}{\sigma} > \frac{1.598}{2}\right)$$

$$= P\left(\frac{\ln(X)-\mu}{\sigma} > 0.799\right)$$

$= P(Z > 0.80)$ using Table A.3

$= 0.2119$

6.57 For ex. 6.56 what is mean power usage. what is variance

X = lognormal distribution with $\mu = 4$ $\sigma = 2$

Mean $E(X) = e^{\mu + \sigma^2/2}$

$= e^{4 + 2^2/2}$

$= e^{4+2}$

$= \boxed{e^6} = 403.4288$

Variance $V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$

$= e^{2(4)+2^2}(e^{2^2} - 1)$

$= e^{8+4}(e^4 - 1)$

$= \boxed{e^{12}(e^4 - 1)}$

6.58 X = no. of automobiles that arrives at certain intersection per minute

X = poisson distribution with mean 5

a) $P(X > 10) = 1 - P(X \leq 10)$

$= 1 - \sum_{x=0}^{10} \frac{e^{-\lambda} \lambda^x}{x!}$

$= (1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) - P(X=4) - P(X=5) - P(X=6) - P(X=7) - P(X=8) - P(X=9) - P(X=10))$

$= (1 - 0.0067 - 0.0377 - 0.0842 - 0.1404 - 0.1755 - 0.1755 - 0.1462 - 0.1044 -$

$0.0653 - 0.0363 - 0.0181)$

$= 1 - 0.9863$

$= \boxed{0.0137}$

∴ probability that more than 2 min elapse before 10 cars arrive

Let $y = \frac{x}{\beta}$

$\alpha = 10$

$\beta = \frac{x}{y} = 2/10 = 1/5$

Hence

$$P(X \le 2) = \int_0^2 \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^{\alpha} \Gamma(\alpha)} dx$$

$$P(X > 2) = 1 - P(X \le 2)$$

$$= 1 - P(Y \le 10)$$

$$= 1 - \int_0^{10} \frac{(y\beta)^{10-1} e^{-y}}{\beta^{10} \Gamma(\alpha)} \beta dy$$

$$= 1 - \int_0^{10} \frac{y^9 e^{-y}}{\Gamma(10)} dy$$

$$= 1 - \int_0^{10} \frac{y^9 e^{-y}}{9!} dy$$

$$= 1 - 0.5421$$

$$= \boxed{0.4579}$$

6.61 $n = 1000$

probability of success $p = 49\% = 0.49$

Probability of failure $q = 1 - 0.49 = 0.51$

X = no. of white-collar worker, among 1000 random selected Valium users

X = binomial distribution

$Mean(\mu) = np = 1000 \times 0.49 = 490$

$Std(\sigma) = \sqrt{npq} = \sqrt{100 \times 0.49 \times 0.51} = \sqrt{249.9} = 15.808$

$x_1 = 481.5 \quad x_2 = 510.5$

$$z_1 = \frac{x_1 - \mu}{\sigma} = \frac{481.5 - 490}{15.8} = -0.54$$

$$z_2 = \frac{x_2 - \mu}{\sigma} = \frac{510.5 - 490}{15.8} = 1.3$$

$$P(481.5 \leq X \leq 510.5) = \sum_{x=482}^{510} b(x; 1000, 0.49)$$

$$= P(-0.54 \leq Z \leq 1.3)$$

$$= P(Z \leq 1.30) - P(Z \leq -0.54)$$

$$= 0.9032 - 0.2946$$

$$= \boxed{0.6086}$$

Chapter 6 - Some Continuous Probability Distribution

Formula Sheet

① Continuous Uniform Distribution

$f_X(x;A,B) = \frac{1}{B-A}$ for $A \le x \le B$

$P(X \le x) = \frac{x-A}{B-A}$ for $A \le x \le B$

$E[X] = \frac{A+B}{2}$ $V[X] = \frac{(B-A)^2}{12}$

② Normal distribution (ND)

$n(x;\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$

$E[X] = \mu$ $V[X] = \sigma^2$

Excel function

NORMSINV $(x, \mu, \sigma, 0)$

standard normal distribution

$z = \frac{x-\mu}{\sigma}$ $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-1/2 z^2}$

$\Phi(z) = P[Z \le z]$ for $-\infty < z < \infty$

ND Probabilities

$P[a \le X \le b] = P\left[\frac{a-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{b-\mu}{\sigma}\right]$

$= P\left[\frac{a-\mu}{\sigma} \le Z \le \frac{b-\mu}{\sigma}\right]$

$= \left(\frac{b-\mu}{\sigma}\right) - \left(\frac{a-\mu}{\sigma}\right) \Rightarrow z_1$

Normal Approxi to Binomial

$z = \frac{X-np}{\sqrt{np(1-p)}}$

③ Exponential Distribution

$f_X(x;\beta) = 1/\beta \, e^{-x/\beta}$ $P[X \le x] = 1 - e^{-x/\beta}$

$E[X] = \beta$ $V[X] = \beta^2$

Gamma Distribution: $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$

$\Gamma(\alpha) = (\alpha-1)\Gamma(\alpha-1)$ | $\Gamma(1/2) = \sqrt{\pi}$

+ve integer $\Gamma(n) = (n-1)!$

$F(a;\alpha) = \int_0^a \frac{y^{\alpha-1} e^{-y}}{\Gamma(\alpha)} dy$

$\boxed{f_X(x;\alpha,\beta) = \frac{x^{\alpha-1} e^{-x/\beta}}{\beta^\alpha \Gamma(\alpha)}}$

$E[X] = \alpha\beta$ $V[X] = \alpha\beta^2$

excel function GAMMADIST $(x, \alpha, \beta, 0)$

Relationship to Poisson Process

Poisson$(\mu = \lambda t)$ | $T \sim Exp(\beta = 1/\lambda)$

Gamma$(\alpha = k, \beta = 1/\lambda)$

④ Chi-Squared Distribution

$f_X(x;v) = \frac{x^{v/2-1} e^{-x/2}}{2^{v/2}\Gamma\left(\frac{v}{2}\right)}$ for $x > 0$;

$f_X(x) = 0$

$E[X] = v$ $V[X] = 2v$

⑤ Lognormal Distribution

$f_X(x;\mu,\sigma) = \frac{e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2}}{\sigma x\sqrt{2\pi}}$ for $x > 0$;

$E[X] = e^{\mu+\sigma^2/2}$

$V[X] = (e^{2\mu+\sigma^2})(e^{\sigma^2}-1)$