Assignment-6

i) 6.6) a) to the right of z is 0.3622

$P(Z > z) = 0.3622$

$P(Z > z) = 1 - P(Z \leq z) = 1 - 0.3622$

$\Rightarrow P(Z \leq z) = 0.6378$

from table A.3

$Z = 0.35$

b) to the left of z is 0.1131

$P(Z < z) = 0.1131$

from table A.3

$Z = -1.21$

c) between 0 and z, with $z > 0$, is 0.4838

$P(0 < Z < z) = 0.4838$

$P(Z < z) - P(0) = 0.4838$

$P(Z < z) = 0.4838 - P(0)$

W.K.T $P(0) = 0.5$

$\Rightarrow P(Z < z) = 0.4838 + 0.5$

$= 0.9838$

from table area under normal curves

$Z = 2.14$

d) between -z and z, with $z > 0$, is 0.9500

$P(-z < Z < z) = 0.95$

$P(Z < z) - P(Z < -z) = 0.9500$

$P(Z < z) - (1 - P(Z < z)) = 0.95$

$2[P(Z < z)] = 1 + 0.95$

$P(Z < z) = \frac{1.95}{2} = 0.975$

from table

$P(Z < z) = 1.96$

2) 6.9) a) $P(X<15)$

given $\mu = 18$ $\sigma = 2.5$

$P(X<15) = P\left[\frac{x-\mu}{\sigma} < \frac{15-18}{2.5}\right]$

$P(Z<-1.2)$

$= 0.1151$ from table A.3

b) the value of k such that $P(X<k) = 0.2236$

$P(X<k) = P\left(\frac{x-\mu}{\sigma} < \frac{k-18}{2.5}\right) = 0.2236$

$P\left(Z < \frac{k-18}{2.5}\right) = 0.2236$

from table A.3

$\frac{k-18}{2.5} = -0.76$

$k-18 = -1.9$

$k = 18-1.9 = 16.1$

c) the value of k such that $P(X>k) = 0.1814$

$P\left[\frac{x-\mu}{\sigma} > \frac{k-18}{2.5}\right] = 0.1814$

$P\left[Z > \frac{k-18}{2.5}\right] = 0.1814$

$\Rightarrow 1 - P\left[Z \leq \frac{k-18}{2.5}\right] = 0.1814$

from table A.3

$\frac{k-18}{2.5} = 0.91$

$k-18 = 2.275$

$k = 18+2.275$

$k = 20.275$

3) 6.12) a) longer than ~~32 months~~ 31.7 centimeters

$P(X > 31.7) = 1 - P(X \le 31.7)$

$P(X \le 31.7) = P\left(\frac{x-\mu}{\sigma} < \frac{31.7-30}{2}\right)$

$= P(Z < 0.85)$

$\Rightarrow 1 - P(Z \le 0.85)$

from table A.3

$= 1 - 0.8023 = 0.1977$

b) between 29.3 and 33.5 centimeters in length

$P(29.3 < Z < 33.5)$

$= P\left(\frac{29.3-30}{2} < \frac{x-\mu}{\sigma} < \frac{33.5-30}{2}\right)$

$= P(-0.35 < Z < 1.75)$

$= P(Z < 1.75) - P(Z < -0.35)$

$= 0.9599 - 0.3632$

$= 0.5967$

c) shorter than 25.5

$P(X < 25.5) = P\left(\frac{x-\mu}{\sigma} < \frac{25.5-30}{2}\right)$

$= P(Z < -2.25)$

$= 0.0122$

4) 6.22) a) more than 1.3σ

$P(x > \mu + 1.3\sigma) = P\left(Z > \frac{\mu + 1.3\sigma - \mu}{\sigma}\right)$

$= P(Z > 1.3) = 1 - P(Z < 1.3)$

from table A.3 $= 1 - 0.9032$

~~from table A.3~~

$= 0.0968$

b) less than 0.52σ

$P(x < \mu - 0.52\sigma) = P(z < \frac{\mu - 0.5\sigma - \mu}{\sigma})$

from table A3 $= P(z < -0.52)$

$= 0.3015$

5) 6.26) x = no. of defections

$n = 100$

$\mu = np$

$\sigma^2 = np(1-p)$

$z = \frac{x - np}{\sqrt{np*(1-p)}}$

$p = 0.1 \quad n = 100 \quad \mu = 100(0.1) = 10$

$\sigma^2 = 100(0.1)(1-0.1)$

$= 9$

$\Rightarrow \sigma = 3$

a) exceeds 13

$P(x > 13) = 1 - P(x \leq 13)$

$= 1 - P(z \leq \frac{13 + 0.5 - 10}{3})$

$= 1 - P(z \leq 1.17)$

from table A.3

$= 1 - 0.8790$

$= 0.121$

b) less than 8

$P(x < 8) = P(z < \frac{8 - 0.5 - 10}{3})$

$= P(z < -0.83)$

from table A.3

$= 0.2033$

6) 6.34) x = variable for 7

$n = 180$

probability of getting 7,

Sample space = (1,6), (2,5), (3,4), (4,3), (5,2), (6,1)

= 6

$p = \frac{6}{36} = \frac{1}{6}$

$\mu = np = 180 \times \frac{1}{6} = 30$

$\sigma^2 = np(1-p) = 30(1-\frac{1}{6}) = 25$

$\Rightarrow \sigma = 5$

a) at least 25 times

$P(x \geq 25) = 1 - P(x \leq 25)$

$= 1 - P\left(z \leq \frac{25-0.5-30}{5}\right)$

$= 1 - P(z \leq -1.1)$

from table A.3 $= 1 - 0.1357$

$= 0.8643$

b) between 33 and 41 times inclusive

$P(33 \leq x \leq 41) = P(x \leq 41) - P(x \leq 33)$

$= P\left(z \leq \frac{41+0.5-30}{5}\right) - P\left(z \leq \frac{33+0.5-30}{5}\right)$

$= P(z \leq 2.3) - P(z \leq 0.5)$

from table A.3 $= 0.9893 - 0.695$

$= 0.2978$

c) ~~at least 35 but less than 47~~ exactly 30 times

$x_1 = 29.5$ $x_2 = 30.5$

$z_1 = \frac{x_1 + np}{\sqrt{np(1-p)}} = \cancel{20} - 0.1$

$$Z_2 = \frac{x_2 - np}{\sqrt{np(1-p)}} = 0.1$$

$P(x=30) = P(-0.1 \le Z \le 0.1)$

$= P(Z \le 0.1) - P(Z \le 0.1)$

from A-3 $= 0.5398 - 0.4609$

$= 0.0796$

7) (6.38) a) $n = 20$ $P_s = 0.01$

$P_f = 1 - 0.01 = 0.99$

x = no. of damaged letters among 20

~~P.M.F x is~~

~~$P(X=x) = Bin(x; 20, 0.01)$~~

~~$= \binom{20}{x} (0.01) (0.99)^{20-x}$~~

$\mu = np = 20(0.01) = 0.2$

$\sigma^2 = np(1-p) = 0.2(1-0.01)$

$= 0.198$

$P(x > 1) = 1 - P(x \le 1)$

$= 1 - P\left(Z \le \frac{1-0.2}{\sqrt{0.198}}\right)$

$= 1 - P(Z \le 1.79)$

from A-3 $= 1 - 0.9633$

$= 0.0367$

b) $n = 500$ $p = 0.01$

$\mu = np = 500 \times 0.01 = 5$

$\sigma^2 = np(1-p)$

$= 5(1-0.01) = 4.95$

$P(x > 8) = 1 - P(x \le 8)$

$= 1 - P\left(Z \le \frac{8-5}{\sqrt{4.95}}\right)$

from table A-3 $= 1 - 0.9099 = 0.0901$

8) 6.56)

$\mu = 4 \quad \sigma = 2$

$P[X > 270] = 1 - P[X \leq 270]$

$= 1 - P\left[Z \leq \frac{\ln(270) - 4}{2}\right]$

$= 1 - P[Z \leq 0.79]$

$= 1 - 0.7852 = 0.2148$

9) 6.57)

X = lognormal distribution with $\mu = 4$ $\sigma = 2$

Mean $E[X] = e^{\mu + \sigma^2/2}$

$= e^{4 + 2^2/2}$

$= e^{4+2} = e^6$

Variance $V(X) = e^{2\mu + \sigma^2}(e^{\sigma^2} - 1)$

$= e^{2(4)+2^2}(e^{2^2} - 1)$

$= e^{8+4}(e^4 - 1)$

$= e^{12}(e^4 - 1)$

10) 6.58)

$\mu = 5$

$N = 10 \times 5 = 50$

N_{50} = Poisson(5)

$P[N_{50} < 10] = P(10; 5)$

from table A2 $= 0.9863$

11) 6.61)

$n = 1000 \quad p = 0.49$

$\mu = np = 1000(0.49) = 490$

$\sigma^2 = np(1-p) = 490(1 - 0.49)$

$= 490(0.51) = 249.9$

$\Rightarrow \sigma = 15.8$

$P[482 \leq X \leq 510] = P(X \leq 510) - P(X \leq 482)$

$= P\left(Z \leq \frac{510 + 0.5 - 490}{15.8}\right] - P\left[Z \leq \frac{482 + 0.5 - 490}{15.8}\right]$ $= P(Z \leq 1.29) - P(Z \leq 0$

from A.3 $= 0.9015 - 0.3192 = 0.5823$

formula Sheet

1) Continous uniform distribution:

$f_x(x; A, B) = \frac{1}{B-A}$ for $A \le x \le B$

$P[x \le x] = \frac{x-A}{B-A}$ for $A \le x \le B$

$E[x] = \frac{A+B}{2}$ $\quad V[x] = \frac{(B-A)^2}{2}$

2) Normal distribution:

$n(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$ for $-\infty < x < \infty$

$E[x] = \mu$ $\quad V[x] = \sigma^2$

3) standard normal distribution:

$Z = \frac{x-\mu}{\sigma}$ $\quad \phi(z) = \frac{1}{\sqrt{2\pi}} e^{-1/2 z^2}$

$\Phi(z) = P[Z \le z)$ for $-\infty < z < \infty$

4) Normal distribution probabilities:

$P[a \le x \le b] = P\left[\frac{a-\mu}{\sigma} \le \frac{x-\mu}{\sigma} < \frac{b-\mu}{\sigma}\right]$

$= \left(\frac{b-\mu}{\sigma}\right) - \left(\frac{a-\mu}{\sigma}\right) \Rightarrow z_1$

5) Normal Approximation to binomial

$Z = \frac{x - np}{\sqrt{np(1-p)}}$

6) Exponential distribution

$f_x(x; \beta) = 1/\beta e^{x/\beta}$ $\quad P[x \le x] = 1 - e^{-x/\beta}$ $\quad E[x] = \beta$ $\quad V[x] = \beta^2$

7) Chi-squared Distribution:

$f_x(x; v) = \frac{x^{v/2} + e^{-x/2}}{2^{v/2} - \left(\frac{v}{2}\right)}$ for $x > 0$ $\quad f_x(x) = 0$

$E[x] = v$ $\quad V[x] = 2v$

8) longnormal Distribution:

$f_x(x; \mu, \sigma) = \frac{e^{-\frac{1}{2}\left(\frac{\ln(x)-\mu}{\sigma}\right)^2}}{\sigma x\sqrt{2\pi}}$ for $x > 0$;

$E[x] = e^{\mu + \sigma^2/2}$ $\quad V[x] = (e^{2\mu + \sigma^2})(e^{\sigma^2} - 1)$

~~formula~~ Formula Sheet

1) Discrete uniform distribution:

$$f(x;k) = \frac{1}{k}$$

Mean $E[x] = \frac{1}{k} \sum_{i=1}^{k} x_i$

Variance $= \frac{1}{k} \sum_{i=1}^{k} (x_i^2) - (E[x])^2$

2) Bernoulli distribution:

$$f(x;p) = \underbrace{p^x}_{E[x]} \underbrace{(1-p)^{1-x}}_{V(x)}$$

3) Binomial distribution:

$$b(x;n,p) = \binom{n}{x} p^x (1-p)^{n-x}$$

$$E[x] = np$$

$$V[x] = np(1-p)$$

$$P[X \leq x] = \sum_{t=0}^{x} b(t;n,p)$$

$$P[a \leq X \leq b] = P[X \leq b] - P[X \leq a-1)$$
$$= B(b;n,p) - B(a-1;n,p)$$

4) Multinomial distribution;

$$f(x_1 \cdots x_r; P_1 \cdots P_k) = \binom{n}{x_1 \cdots x_k} P_1^{x_1} \cdots P_k^{x_k}$$

5) Hypergeometric Distribution:

$$h(x; N, n, k) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

$$E[x] = \frac{nk}{N}$$

$$V(x) = \left(\frac{N-n}{N-1}\right) n \left(\frac{k}{N}\right)\left(1 - \frac{k}{N}\right)$$

6) Multivariate Hypergeometric:

$$f(x_1 \dots x_k; N, a_1 \dots a_k) = \frac{\binom{a_1}{x_1} \cdots \binom{a_k}{x_k}}{\binom{N}{n}}$$

7) Geometric Distribution:

$$g(x; p) = p(1-p)^{x-1}$$

$$E[x] = \frac{1}{p}$$

$$V(x) = \frac{1-p}{p^2}$$

8) Negative Binomial:

$$b^*(x; k, p) = \binom{x-1}{k-1} p^k (1-p)^{x-k}$$

9) Poisson Distribution:

$$P(x; \mu) = \frac{e^{-\mu} \mu^x}{x!}$$