Formuler Sheet

[redacted]: *Z*-tests

Single Population: $X_1, \ldots, X_n$ i.i.d. $N(\mu, \sigma^2)$, where σ is known
Parameter of Interest: $\theta = \mu$
Point estimator: $\hat{\theta} = \overline{X}$
Standard error: $\text{s.e.}\{\hat{\theta}\} = \dfrac{\sigma}{\sqrt{n}}$

Two Independent Populations: $X_{11}, \ldots, X_{1n_1}$ i.i.d. $N(\mu_1, \sigma_1^2)$, where σ_1 is known
$X_{21}, \ldots, X_{2n_2}$ i.i.d. $N(\mu_2, \sigma_2^2)$, where σ_2 is known

Parameter of Interest: $\theta = \mu_1 - \mu_2$
Point estimator: $\hat{\theta} = \overline{X}_1 - \overline{X}_2$
Standard error: $\text{s.e.}\{\hat{\theta}\} = \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}$

Test Form (C) $H_0 : \theta = \theta_0$ vs. $H_1 : \theta > \theta_0$

Equivalent Decision Rules:

(i) Using a rejection region for $\hat{\theta}$:
Decision Rule: If $\hat{\theta} > \theta_0 + z_\alpha \, \text{s.e.}\{\hat{\theta}\}$, then reject H_0.

(ii) Using a test statistic: Define the z-statistic to be $z^* = \dfrac{\hat{\theta} - \theta_0}{\text{s.e.}\{\hat{\theta}\}}$.

Decision Rule: If $z^* > z_\alpha$, then reject H_0.

(iii) Using a p-value: The p-value of this z-test is

$$\begin{aligned} p &= P[Z > z^*] = 1 - \Phi(z^*) \\ &= \text{the probability that a } N(0,1) \text{ random variable would achieve a value} > z^*. \end{aligned}$$

If this is small, then the data **do not support** $H_0 \Rightarrow \dfrac{\hat{\theta} - \theta_0}{\text{s.e.}\{\hat{\theta}\}}$ is NOT $N(0, 1)$.

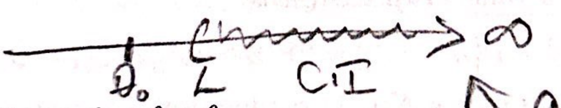
Decision Rule: If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1 - \alpha)\%$ one-sided lowerbound z-interval for θ:

$$(L, \infty) = \left(\hat{\theta} - z_\alpha \, \text{s.e.}\{\hat{\theta}\}, \infty\right).$$

$\alpha = 0.5$

Draw case with $\theta_0 < L$:

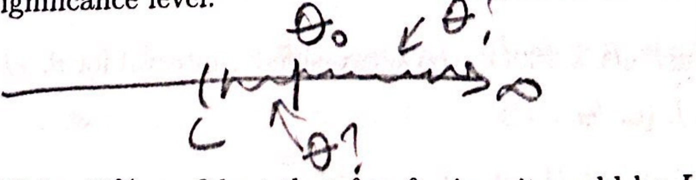


95% CI

↑ 95% confident that the true is $\theta > \theta_0$

⇒ If $\alpha = 0.05$, we are 95% confident that $\theta > \theta_0$
⇒ Reject H_0 at the 0.05 significance level.

Draw case with $\theta_0 > L$:



⇒ If $\alpha = 0.05$, we CANNOT be 95% confident that $\theta > \theta_0$ since it could be $L < \theta < \theta_0$
⇒ Fail to Reject H_0 at the 0.05 significance level.

Decision Rule: If θ_0 is NOT in the CI, then reject H_0.

Test Form (B) $H_0 : \theta = \theta_0$ vs. $H_1 : \theta < \theta_0$ [redacted]

Equivalent Decision Rules:

(i) Using a rejection region for $\hat{\theta}$:

Decision Rule: If $\hat{\theta} < \theta_0 - z_\alpha \text{ s.e.}\{\hat{\theta}\}$, then reject H_0.

(ii) Using a test statistic:

Decision Rule: If $z^* < -z_\alpha$, then reject H_0.

(iii) Using a p-value: The p-value of this z-test is

$$\begin{aligned} p &= P[Z < z^*] = \Phi(z^*) \\ &= \text{the probability that a } N(0,1) \text{ random variable would achieve a value} < z^*. \end{aligned}$$

Decision Rule: If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ one-sided upperbound z-interval for θ:

$$(-\infty, U) = \left(-\infty\, , \, \hat{\theta} + z_\alpha \text{ s.e.}\{\hat{\theta}\}\right).$$

Draw case with $\theta_0 > U$:

Draw case with $\theta_0 < U$:

Decision Rule: If θ_0 is NOT in the C.I., then reject H_0.

Test Form (A) $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$

Equivalent Decision Rules:

(i) Using a rejection region for $\hat{\theta}$:

Decision Rule: If $\hat{\theta} > \theta_0 + z_{\alpha/2} \text{ s.e.}\{\hat{\theta}\}$ or $\hat{\theta} < \theta_0 - z_{\alpha/2} \text{ s.e.}\{\hat{\theta}\}$, then reject H_0.

(ii) Using a test statistic:

Decision Rule: If $|z^*| > z_{\alpha/2}$, then reject H_0.

(iii) Using a p-value: The p-value of this z-test is

$$\begin{aligned} p &= P[Z < -|z^*|] + P[Z > |z^*|] = 2P[Z > |z^*|] = 2(1 - \Phi(|z^*|)) \\ &= \text{the probability that a } N(0,1) \text{ random variable would achieve a value} < z^*. \end{aligned}$$

Decision Rule: If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ two-sided z-interval for θ: $(L,\, U) = \hat{\theta} \pm z_{\alpha/2} \text{ s.e.}\{\hat{\theta}\}$.

Draw case with $\theta_0 < L$ (or $\theta_0 > U$):

Draw case with $L < \theta_0 < U$:

Decision Rule: If θ_0 is NOT in the C.I., then reject H_0.

t-test

[redacted]: [redacted]

Single Population: $X_1, \ldots, X_n$ i.i.d. $N(\mu, \sigma^2)$, where both μ and σ are unknown

Parameter of Interest: $\theta = \mu$; Point estimator: $\hat{\theta} = \overline{X}$

Standard error: $\text{s.e.}\{\hat{\theta}\} = \dfrac{S}{\sqrt{n}}$

t-distribution d.f.: $v = n - 1$

Two Independent Populations: $X_{11}, \ldots, X_{1n_1}$ i.i.d. $N(\mu_1, \sigma_1^2)$, both μ_1 and σ_1 are unknown
$X_{21}, \ldots, X_{2n_2}$ i.i.d. $N(\mu_2, \sigma_2^2)$, both μ_2 and σ_2 are unknown

Parameter of Interest: $\theta = \mu_1 - \mu_2$; Point estimator: $\hat{\theta} = \overline{X}_1 - \overline{X}_2$

Standard error: $(\sigma_1 = \sigma_2)$ $\text{s.e.}\{\hat{\theta}\} = S_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}$; $(\sigma_1 \neq \sigma_2)$ $\text{s.e.}\{\hat{\theta}\} = \sqrt{\dfrac{S_1^2}{n_1} + \dfrac{S_2^2}{n_2}}$

t-distribution d.f.: $(\sigma_1 = \sigma_2)$ $v = n_1 + n_2 - 2$; $(\sigma_1 \neq \sigma_2)$ $v =$ messy formula from Ch. 9

Test Form (C) $H_0 : \theta = \theta_0$ vs. $H_1 : \theta > \theta_0$

(i) Using a rejection region for $\hat{\theta}$: *Decision Rule*: If $\hat{\theta} > \theta_0 + t_{\alpha,v}\,\text{s.e.}\{\hat{\theta}\}$, then reject H_0.

(ii) Using a test statistic: Define the t-statistic to be $t^* = \dfrac{\hat{\theta} - \theta_0}{\text{s.e.}\{\hat{\theta}\}}$.

Decision Rule: If $t^* > t_{\alpha,v}$, then reject H_0.

(iii) Using a p-value: (Software) $p = P[T > t^*]$, where $T \sim t(v)$. If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ one-sided lowerbound t-interval for θ:

$$(L, \infty) = \left(\hat{\theta} - t_{\alpha,v}\,\text{s.e.}\{\hat{\theta}\}\,,\, \infty\right).$$

Decision Rule: If θ_0 is NOT in the CI, then reject H_0.

Test Form (B) $H_0 : \theta = \theta_0$ vs. $H_1 : \theta < \theta_0$

(i) Using a rejection region for $\hat{\theta}$: *Decision Rule*: If $\hat{\theta} < \theta_0 - t_{\alpha,v}\,\text{s.e.}\{\hat{\theta}\}$, then reject H_0.

(ii) Using a test statistic: *Decision Rule*: If $t^* < -t_{\alpha,v}$, then reject H_0.

(iii) Using a p-value: (Software) $p = P[T < t^*]$, where $T \sim t(v)$. If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ one-sided upperbound t-interval for θ:

$$(-\infty, U) = \left(-\infty\,,\, \hat{\theta} + t_{\alpha,v}\,\text{s.e.}\{\hat{\theta}\}\right).$$

Decision Rule: If θ_0 is NOT in the C.I., then reject H_0.

Test Form (A) $H_0 : \theta = \theta_0$ vs. $H_1 : \theta \neq \theta_0$

(i) Using a rejection region for $\hat{\theta}$:

Decision Rule: If $\hat{\theta} > \theta_0 + t_{\alpha/2,v}\,\text{s.e.}\{\hat{\theta}\}$ or $\hat{\theta} < \theta_0 - t_{\alpha/2,v}\,\text{s.e.}\{\hat{\theta}\}$, then reject H_0.

(ii) Using a test statistic: *Decision Rule*: If $|t^*| > t_{\alpha/2,v}$, then reject H_0.

(iii) Using a p-value: (Software) $p = 2P[T > |t^*|]$, where $T \sim t(v)$. If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ two-sided t-interval for θ: $(L, U) = \hat{\theta} \pm t_{\alpha/2,v}\,\text{s.e.}\{\hat{\theta}\}$.

Decision Rule: If θ_0 is NOT in the C.I., then reject H_0.

χ^2-tests

[redacted] χ^2-[redacted]

Single Population: $X_1, \ldots, X_n$ i.i.d. $N(\mu, \sigma^2)$, where both μ and σ are unknown
Parameter of Interest: σ^2 ; Point estimator: S^2 ; χ^2-distribution d.f.: $n-1$

Test Form (C) $H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma^2 > \sigma_0^2$

(i) Using a rejection region for S^2: *Decision Rule*: If $S^2 > \dfrac{\sigma_0^2}{n-1}\chi^2_{\alpha,\,n-1}$, then reject H_0.

(ii) Using a test statistic: Define the χ^2-statistic to be $\chi^2_* = \dfrac{(n-1)s^2}{\sigma_0^2}$.

Decision Rule: If $\chi^2_* > \chi^2_{\alpha,\,n-1}$, then reject H_0.

(iii) Using a p-value: (Software) $p = P[Y > \chi^2_*]$, where $Y \sim \chi^2(n-1)$. If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ one-sided lowerbound χ^2-interval for σ^2:

$$(L,\,\infty) = \left(\frac{(n-1)s^2}{\chi^2_{\alpha,\,n-1}},\,\infty\right).$$

Decision Rule: If σ_0^2 is NOT in the CI, then reject H_0.

Test Form (B) $H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma^2 < \sigma_0^2$

(i) Using a rejection region for S^2: *Decision Rule*: If $S^2 < \dfrac{\sigma_0^2}{n-1}\chi^2_{1-\alpha,\,n-1}$, then reject H_0.

(ii) Using a test statistic: *Decision Rule*: If $\chi^2_* < \chi^2_{1-\alpha,\,n-1}$, then reject H_0.

(iii) Using a p-value: (Software) $p = P[Y < \chi^2_*]$, where $Y \sim \chi^2(n-1)$. If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ one-sided upperbound χ^2-interval for σ^2:

$$(0,\,U) = \left(0\,,\,\frac{(n-1)s^2}{\chi^2_{1-\alpha,\,n-1}}\right).$$

Decision Rule: If σ_0^2 is NOT in the C.I., then reject H_0.

Test Form (A) $H_0 : \sigma^2 = \sigma_0^2$ vs. $H_1 : \sigma^2 \neq \sigma_0^2$

(i) Using a rejection region for S^2:

Decision Rule: If $S^2 > \dfrac{\sigma_0^2}{n-1}\chi^2_{\alpha/2,\,n-1}$ or $S^2 < \dfrac{\sigma_0^2}{n-1}\chi^2_{1-\alpha/2,\,n-1}$, then reject H_0.

(ii) Using a test statistic: *Decision Rule*: If $\chi^2_* > \chi^2_{\alpha/2,\,n-1}$ or $\chi^2_* < \chi^2_{1-\alpha/2,\,n-1}$, then reject H_0.

(iii) Using a p-value: (Software) If $s^2 \geq \sigma_0^2$, then $p = 2P[Y > \chi^2_*]$, otherwise $p = 2P[Y < \chi^2_*]$, where $Y \sim \chi^2(n-1)$. If $p < \alpha$, then reject H_0.

(iv) Using a C.I.: Construct a $100(1-\alpha)\%$ two-sided χ^2-interval for σ^2:

$$(L,\,U) = \left(\frac{(n-1)s^2}{\chi^2_{\alpha/2,\,n-1}},\,\frac{(n-1)s^2}{\chi^2_{1-\alpha/2,\,n-1}}\right).$$

Decision Rule: If σ_0^2 is NOT in the C.I., then reject H_0.

[redacted] F-tests

Two Independent Populations: $X_{11}, \ldots, X_{1n_1}$ i.i.d. $N(\mu_1, \sigma_1^2)$, both μ_1 and σ_1 are unknown
$X_{21}, \ldots, X_{2n_2}$ i.i.d. $N(\mu_2, \sigma_2^2)$, both μ_2 and σ_2 are unknown

Parameter of Interest: $\frac{\sigma_1^2}{\sigma_2^2}$; Point estimator: $\frac{S_1^2}{S_2^2}$

F-distribution d.f.: (numerator) $v_1 = n_1 - 1$; (denominator) $v_2 = n_2 - 1$

Test Form (C) $H_0 : \sigma_1^2 = \sigma_2^2 \Rightarrow \sigma_1^2/\sigma_2^2 = 1$

vs. $H_1 : \sigma_1^2 > \sigma_2^2$ $\sigma_1^2/\sigma_2^2 > 1$

(i) Using a test statistic: Define the F-statistic to be $F_* = \frac{s_1^2}{s_2^2}$.

Decision Rule: If $F_* > f_{\alpha, n_1-1, n_2-1}$, then reject H_0.

(ii) Using a p-value: (Software) $p = P[F > F_*]$, where $F \sim F(v_1, v_2)$. If $p < \alpha$, then reject H_0.

(iii) Using a C.I.: Construct a $100(1-\alpha)\%$ one-sided lowerbound F-interval for $\frac{\sigma_1^2}{\sigma_2^2}$:

$$(L, \infty) = \left(\frac{s_1^2}{s_2^2} f_{1-\alpha, n_2-1, n_1-1}\,,\ \infty \right).$$

Decision Rule: If 1 is NOT in the CI, then reject H_0.

Test Form (B) $H_0 : \sigma_1^2 = \sigma_2^2$ vs. $H_1 : \sigma_1^2 < \sigma_2^2$

(i) Using a test statistic:

Decision Rule: If $F_* < f_{1-\alpha, n_1-1, n_2-1}$, then reject H_0.

(ii) Using a p-value: (Software) $p = P[F < F_*]$, where $F \sim F(v_1, v_2)$. If $p < \alpha$, then reject H_0.

(iii) Using a C.I.: Construct a $100(1-\alpha)\%$ one-sided upperbound F-interval for $\frac{\sigma_1^2}{\sigma_2^2}$:

$$(0, U) = \left(0\,,\ \frac{s_1^2}{s_2^2} f_{\alpha, n_2-1, n_1-1} \right).$$

Decision Rule: If 1 is NOT in the C.I., then reject H_0.

Test Form (A) $H_0 : \sigma_1^2 = \sigma_2^2$ vs. $H_1 : \sigma_1^2 \neq \sigma_2^2$

(i) Using a test statistic:

Decision Rule: If $F_* > f_{\alpha/2, n_1-1, n_2-1}$ or $F_* < f_{1-\alpha/2, n_1-1, n_2-1}$, then reject H_0.

(ii) Using a p-value: (Software) If $s_1^2 \geq s_2^2$, then $p = 2P[F > F_*]$, otherwise $p = 2P[F < F_*]$, where $F \sim F(v_1, v_2)$. If $p < \alpha$, then reject H_0.

(iii) Using a C.I.: Construct a $100(1-\alpha)\%$ two-sided F-interval for $\frac{\sigma_1^2}{\sigma_2^2}$:

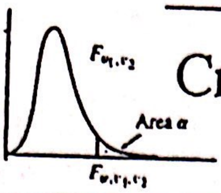
$$(L, U) = \left(\frac{s_1^2}{s_2^2} f_{1-\alpha/2, n_2-1, n_1-1}\,,\ \frac{s_1^2}{s_2^2} f_{\alpha/2, n_2-1, n_1-1} \right).$$

Decision Rule: If 1 is NOT in the C.I., then reject H_0.

ADDITIONAL F-TABLE

TABLE IV • Critical Points of the F-distribution

[redacted]



$F_{.10,\nu_1,\nu_2}$ α = 0.10

Degrees of freedom for the numerator (ν_1); Degrees of freedom for the denominator (ν_2)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	59.86	60.19	60.71	61.22	61.74	62.00	62.26	62.53	62.79	63.06	63.33
2	8.53	9.00	9.16	9.24	9.29	9.33	9.35	9.37	9.38	9.39	9.41	9.42	9.44	9.45	9.46	9.47	9.47	9.48	9.49
3	5.54	5.46	5.39	5.34	5.31	5.28	5.27	5.25	5.24	5.23	5.22	5.20	5.18	5.18	5.17	5.16	5.15	5.14	5.13
4	4.54	4.32	4.19	4.11	4.05	4.01	3.98	3.95	3.94	3.92	3.90	3.87	3.84	3.83	3.82	3.80	3.79	3.78	3.76
5	4.06	3.78	3.62	3.52	3.45	3.40	3.37	3.34	3.32	3.30	3.27	3.24	3.21	3.19	3.17	3.16	3.14	3.12	3.10
6	3.78	3.46	3.29	3.18	3.11	3.05	3.01	2.98	2.96	2.94	2.90	2.87	2.84	2.82	2.80	2.78	2.76	2.74	2.72
7	3.59	3.26	3.07	2.96	2.88	2.83	2.78	2.75	2.72	2.70	2.67	2.63	2.59	2.58	2.56	2.54	2.51	2.49	2.47
8	3.46	3.11	2.92	2.81	2.73	2.67	2.62	2.59	2.56	2.54	2.50	2.46	2.42	2.40	2.38	2.36	2.34	2.32	2.29
9	3.36	3.01	2.81	2.69	2.61	2.55	2.51	2.47	2.44	2.42	2.38	2.34	2.30	2.28	2.25	2.23	2.21	2.18	2.16
10	3.29	2.92	2.73	2.61	2.52	2.46	2.41	2.38	2.35	2.32	2.28	2.24	2.20	2.18	2.16	2.13	2.11	2.08	2.06
11	3.23	2.86	2.66	2.54	2.45	2.39	2.34	2.30	2.27	2.25	2.21	2.17	2.12	2.10	2.08	2.05	2.03	2.00	1.97
12	3.18	2.81	2.61	2.48	2.39	2.33	2.28	2.24	2.21	2.19	2.15	2.10	2.06	2.04	2.01	1.99	1.96	1.93	1.90
13	3.14	2.76	2.56	2.43	2.35	2.28	2.23	2.20	2.16	2.14	2.10	2.05	2.01	1.98	1.96	1.93	1.90	1.88	1.85
14	3.10	2.73	2.52	2.39	2.31	2.24	2.19	2.15	2.12	2.10	2.05	2.01	1.96	1.94	1.91	1.89	1.86	1.83	1.80
15	3.07	2.70	2.49	2.36	2.27	2.21	2.16	2.12	2.09	2.06	2.02	1.97	1.92	1.90	1.87	1.85	1.82	1.79	1.76
16	3.05	2.67	2.46	2.33	2.24	2.18	2.13	2.09	2.06	2.03	1.99	1.94	1.89	1.87	1.84	1.81	1.78	1.75	1.72
17	3.03	2.64	2.44	2.31	2.22	2.15	2.10	2.06	2.03	2.00	1.96	1.91	1.86	1.84	1.81	1.78	1.75	1.72	1.69
18	3.01	2.62	2.42	2.29	2.20	2.13	2.08	2.04	2.00	1.98	1.93	1.89	1.84	1.81	1.78	1.75	1.72	1.69	1.66
19	2.99	2.61	2.40	2.27	2.18	2.11	2.06	2.02	1.98	1.96	1.91	1.86	1.81	1.79	1.76	1.73	1.70	1.67	1.63
20	2.97	2.59	2.38	2.25	2.16	2.09	2.04	2.00	1.96	1.94	1.89	1.84	1.79	1.77	1.74	1.71	1.68	1.64	1.61
21	2.96	2.57	2.36	2.23	2.14	2.08	2.02	1.98	1.95	1.92	1.87	1.83	1.78	1.75	1.72	1.69	1.66	1.62	1.59
22	2.95	2.56	2.35	2.22	2.13	2.06	2.01	1.97	1.93	1.90	1.86	1.81	1.76	1.73	1.70	1.67	1.64	1.60	1.57
23	2.94	2.55	2.34	2.21	2.11	2.05	1.99	1.95	1.92	1.89	1.84	1.80	1.74	1.72	1.69	1.66	1.62	1.59	1.55
24	2.93	2.54	2.33	2.19	2.10	2.04	1.98	1.94	1.91	1.88	1.83	1.78	1.73	1.70	1.67	1.64	1.61	1.57	1.53
25	2.92	2.53	2.32	2.18	2.09	2.02	1.97	1.93	1.89	1.87	1.82	1.77	1.72	1.69	1.66	1.63	1.59	1.56	1.52
26	2.91	2.52	2.31	2.17	2.08	2.01	1.96	1.92	1.88	1.86	1.81	1.76	1.71	1.68	1.65	1.61	1.58	1.54	1.50
27	2.90	2.51	2.30	2.17	2.07	2.00	1.95	1.91	1.87	1.85	1.80	1.75	1.70	1.67	1.64	1.60	1.57	1.53	1.49
28	2.89	2.50	2.29	2.16	2.06	2.00	1.94	1.90	1.87	1.84	1.79	1.74	1.69	1.66	1.63	1.59	1.56	1.52	1.48
29	2.89	2.50	2.28	2.15	2.06	1.99	1.93	1.89	1.86	1.83	1.78	1.73	1.68	1.65	1.62	1.58	1.55	1.51	1.47
30	2.88	2.49	2.28	2.14	2.03	1.98	1.93	1.88	1.85	1.82	1.77	1.72	1.67	1.64	1.61	1.57	1.54	1.50	1.46
40	2.84	2.44	2.23	2.09	2.00	1.93	1.87	1.83	1.79	1.76	1.71	1.66	1.61	1.57	1.54	1.51	1.47	1.42	1.38
60	2.79	2.39	2.18	2.04	1.95	1.87	1.82	1.77	1.74	1.71	1.66	1.60	1.54	1.51	1.48	1.44	1.40	1.35	1.29
120	2.75	2.35	2.13	1.99	1.90	1.82	1.77	1.72	1.68	1.65	1.60	1.55	1.48	1.45	1.41	1.37	1.32	1.26	1.19
∞	2.71	2.30	2.08	1.94	1.85	1.77	1.72	1.67	1.63	1.60	1.55	1.49	1.42	1.38	1.34	1.30	1.24	1.17	1.00

$F_{.025,\nu_1,\nu_2}$ α = 0.025

Degrees of freedom for the numerator (ν_1); Degrees of freedom for the denominator (ν_2)

ν_2 \ ν_1	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40	60	120	∞
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.7	963.3	968.6	976.7	984.9	993.1	997.2	1001	1006	1010	1014	1018
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47	39.48	39.49	39.50
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04	13.99	13.95	13.90
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41	8.36	8.31	8.26
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18	6.12	6.07	6.02
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01	4.96	4.90	4.85
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.42	4.36	4.31	4.25	4.20	4.14
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84	3.78	3.73	3.67
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51	3.45	3.39	3.33
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26	3.20	3.14	3.08
11	6.72	5.26	4.63	4.28	4.04	3.88	3.75	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06	3.00	2.94	2.88
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91	2.85	2.79	2.72
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78	2.72	2.66	2.60
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67	2.61	2.55	2.49
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59	2.52	2.46	2.40
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51	2.45	2.38	2.32
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44	2.38	2.32	2.25
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38	2.32	2.26	2.19
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33	2.27	2.20	2.13
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29	2.22	2.16	2.09
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25	2.18	2.11	2.04
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21	2.14	2.08	2.00
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18	2.11	2.04	1.97
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15	2.08	2.01	1.94
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12	2.05	1.98	1.91
26	5.66	4.27	3.67	3.33	3.10	2.94	2.82	2.73	2.65	2.59	2.49	2.39	2.28	2.22	2.16	2.09	2.03	1.95	1.88
27	5.63	4.24	3.65	3.31	3.08	2.92	2.80	2.71	2.63	2.57	2.47	2.36	2.25	2.19	2.13	2.07	2.00	1.93	1.85
28	5.61	4.22	3.63	3.29	3.06	2.90	2.78	2.69	2.61	2.55	2.45	2.34	2.23	2.17	2.11	2.05	1.98	1.91	1.83
29	5.59	4.20	3.61	3.27	3.04	2.88	2.76	2.67	2.59	2.53	2.43	2.32	2.21	2.15	2.09	2.03	1.96	1.89	1.81
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01	1.94	1.87	1.79
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88	1.80	1.72	1.64
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74	1.67	1.58	1.48
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61	1.53	1.43	1.31
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48	1.39	1.27	1.00