PRACTICE EXAM THREE SOLUTIONS

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① Requirement : $\mu \geq 175$ ← requirement satisfied

$H_0 : \mu \geq 175$ $\quad H_1 : \mu < 175$

Type I error = Reject H_0 when H_0 is true = Conclude not satified when it really is satisfied.

Type II error = Accept H_0 when H_0 is false = Conclude is satisfied when it really it is not

Type II error is the more serious error because the company would proceed to use/sell shipment not knowing the the requirement is not satisfied. Because it is type II error that is more serious we need to swap H_0, H_1 to assign type I error as the more serious. What does it mean to swap them

$\mu_0 = 175$

② $H_0 : \mu \leq 175$

$H_1 : \mu > 175$ ← requirement is not satisfied

$\alpha = 0.05$ Test form C use a lower bound C.I to do the test. This will be a t-test. $\bar{x} = 178$, $n = 20$, $\nu = 19$

$t_{0.05,19} = 1.729$

$(\bar{x} - t_{0.05,19}\,(s.e(\bar{x})), \infty)$

$$S^2 = \frac{\sum_{i=1}^{20} (x_i - \bar{x})^2}{n-1} = \frac{304}{20-1} = 16$$ Page 2

$S = \sqrt{16} = 4$

$$s.e(\bar{x}) = \frac{S}{\sqrt{n}} = \frac{4}{\sqrt{20}} = \frac{4}{4.4721} = 0.8944$$

$(178 - (1.729)(0.8944), \infty)$

$(178 - 1.5464, \infty) = (176.45, \infty)$

95% confident that μ is in the C.I

Since 175 is not in C.I $\Rightarrow$ Reject H_0

Conclude that requirement is satisfied

c Largest μ_0 for which we can reject H_0 has to be outside C.I

d $\sigma = 4.1$, use a p-value to test your hypotheses in part (a)

So you do a Z-test

$$Se(\bar{x}) = \frac{\sigma}{\sqrt{n}} = \frac{4.1}{\sqrt{20}} = \frac{4.1}{4.4721} = 0.9168$$

$$Z^* = \frac{\bar{x} - \mu_0}{se} = \frac{178-175}{0.9168} = \frac{3}{0.9168} = 3.27$$

Test form C

$P = 1 - \Phi(z^*) = 1 - \Phi(3.27)$

$P = 1 - 0.99946 = 0.00054$

$\alpha = 0.05$ so $p < \alpha$

Reject H_0.

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Bushing i	1	2	3	4	5	6	7	8
Student (x)	10.2	9.8	6.5	7.6	8.2	9.9	8.6	9.1
Student (y)	10.0	10.1	6.9	7.5	8.2	9.7	8.3	8.8
d	0.2	-0.3	-0.4	0.1	0	0.2	0.3	0.3

Paired case → same spanner bushing for both students.

$H_0: \mu_x = \mu_y \Rightarrow \mu_x - \mu_y = 0$

$H_1: \mu_x \neq \mu_y \quad \mu_x - \mu_y \neq 0$

add the difference between x and y

$$d = \frac{0.2 - 0.3 - 0.4 + 0.1 + 0 + 0.2 + 0.3 + 0.3}{8}$$

$$\bar{d} = \frac{0.4}{8} = 0.05$$

$$\Sigma d_i^2 = 0.04 + 0.09 + 0.16 + 0.01 + 0 + 0.04 + 0.09 + 0.09$$
$$= 0.52$$

$$S_D^2 = \frac{(\Sigma d_i^2) - \frac{(\Sigma d)^2}{n}}{n-1} = \frac{0.52 - \frac{(0.4)^2}{8}}{7}$$

$$= \frac{0.52 - \frac{0.16}{8}}{7} = \frac{0.52 - 0.02}{7}$$

$$S_D^2 = \frac{0.5}{7} = 0.0714$$

$$S_D = \sqrt{S_D^2} = 0.2673$$

$s.e\{\Delta\} = \frac{S_D}{\sqrt{n}} = \frac{0.2673}{\sqrt{8}} = \frac{0.2673}{2.8284}$

$= 0.0945$

$t^* = \frac{\bar{d} - 0}{se} = \frac{0.05}{0.0945} = 0.529$

$\alpha = 0.10 \Rightarrow \frac{\alpha}{2} = 0.05 \Rightarrow t_{0.05,7} = 1.895$

$|t^*| = 0.529 < 1.895$

Accept H_0 or Fail to Reject H_0

It means that you cannot conclude that μ_x is different from μ_y. It could be the same.

b 95% C.I, $\alpha = 0.05$, $\alpha/2 = 0.025$

$t_{0.025,7} = 2.365$

$\bar{d} \pm (2.365)(s.e(\bar{d}))$

$0.05 \pm (2.365)(0.0945)$

0.05 ± 0.2235

$(-0.1735, 0.273)$

Lower Limit, Upper Limit

③ μ_1 = before μ_2 = after

Two means independent samples

~~σ_1 σ_2 known~~

$\sigma_1 = \sigma_2$ unknown → Pooled

This is case 2 because the variances are the same.

Test $\mu_1 > \mu_2$

$H_0: \mu_1 \leq \mu_2$; $H_0: \mu_1 - \mu_2 \leq 0$

$H_1: \mu_1 > \mu_2$ $H_1: \mu_1 - \mu_2 > 0$

This is form C lower bound C.I

Assumed pooled $\sigma_1 = \sigma_2 = \sigma$ - common

estimate the $S_p = 0.3$

$\alpha = 0.01$ degree of freedom $= n_1 + n_2 - 2$

$= 22 + 20 - 2 = 40$

$t_{0.01, 40} = 2.423$

$$\left((\bar{X}_1 - \bar{X}_2) - t_{0.01,40}\,(se\,(\bar{x}_1 - \bar{x}_2), \infty\right)$$

$$S.e = S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}} = 0.3 \times \sqrt{\frac{1}{22} + \frac{1}{20}}$$

$$= 0.0927$$

$$\left((9.83 - 9.67) - (2.423)(0.0927), \infty\right)$$

$$\left((0.16) - 0.2246\right), \infty)$$

$$(-0.0646, \infty)$$

0 is inside C.I so we fail to reject H_0 FTR H_0

④ A: $n_1 = 10$ $\bar{x}_1 = 30.1$ $s_1^2 = 0.17$

B: $n_2 = 11$ $\bar{x}_2 = 69.8$ $s_2^2 = 0.19$

95% C.I $\frac{\sigma_1^2}{\sigma_2^2}$ $\alpha = 0.05$

$\alpha/2 = 0.025$

$$\left(\frac{S_1^2}{S_2^2} f_{1-0.025, n_2-1, n_1-1}, \frac{S_1^2}{S_2^2} f_{0.025, n_2-1, n_1-1} \right)$$

$$\left(\frac{S_1^2}{S_2^2} f_{0.975, 10, 9}, \frac{S_1^2}{S_2^2} f_{0.025, 10, 9} \right)$$

$$\left(\frac{S_1^2}{S_2^2} \frac{1}{f_{0.025, 9, 10}}, \frac{S_1^2}{S_2^2} f_{0.025, 10, 9} \right)$$

$($

$(0.2367, \quad 3.543)$

Testing $H_0: \sigma_1^2 = \sigma_2^2 \Rightarrow \sigma_1^2/\sigma_2^2 = 1$

$H_1: \sigma_1^2 \neq \sigma_2^2 \Rightarrow \frac{\sigma_1^2}{\sigma_2} \neq 1$

Since 1 is inside C.I we fail to reject H_0. So we cannot conclude different

(5) $n = 15, \quad S^2 = 0.08$

Claim : $\sigma^2 \leq 0.05$

$H_0 : \sigma^2 \leq 0.05$

$H_1 : \sigma^2 > 0.05$

Test form C, use CI lowerbound χ^2

$$\left(\frac{(n-1)S^2}{\chi^2_{\alpha, n-1}}, \infty\right) = \left(\frac{(n-1)S^2}{\chi^2_{0.01, 15-1}}, \infty\right)$$

$$\left(\frac{(15-1)(0.08)}{29.141}, \infty\right)$$

$$(0.0384, \infty)$$

We can clearly see that 0.05 is inside CI Fail to reject H_0

We cannot reject the claim.

b) $\sigma_0^2 = 0.0384$

The producer is to claim that $\sigma^2 \leq 0.0384$ still inside the CI

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6 We need to know Sxx, Syy, Sxy

$$b_1 = \frac{\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{n}}{\sum x_i^2 - \frac{(\sum x_i)^2}{n}}$$

$$= \frac{S_{xy}}{S_{xx}}$$

$$\frac{871.77 - \frac{(92.25)(135)}{15}}{589.1875 - \frac{(92.25)^2}{15}}$$

$$= \frac{871.77 - \frac{12453.75}{15}}{589.1875 - \frac{8510.0625}{15}}$$

$$\frac{871.77 - 830.25}{589.1875 - 567.3375} = \frac{41.52}{21.85}$$

Sxx

$$b_1 = 1.9$$

$$b_0 = \frac{1}{n}\left(\sum y_i - b_1 \sum x_i\right)$$

$$= \frac{1}{15}\left(135 - (1.9)(92.25)\right)$$

$$= \frac{1}{15}\left(135 - 175.275\right)$$

$$= \frac{1}{15}\left(-40.275\right)$$

$$b_0 = -2.685$$

$$\hat{y} = b_0 + b_1 x$$

$$\hat{y} = -2.685 + 1.9x$$

b. When you have this type of question it is asking R^2 coefficient of determination, which is the explained

$$R^2 = \frac{SSR}{SST} \text{ or } 1 - \frac{SSE}{SST}$$

$$SSR = b_1^2 S_{xx} \text{ or } b_1 S_{xy}$$

$$= (1.9)^2 (21.85)$$

$$= (3.61)(21.85)$$

$$= 78.8785$$

$$SST = S_{yy} = \sum y^2 - (\sum y_i)^2 / n$$

$$= 1312.4 - \frac{(135)^2}{15}$$

$$1312.4 - \frac{18225}{15}$$

$$1312.4 - 1215 = 97.4$$

$$R^2 = \frac{78.87785}{97.4} = 0.8098$$

c $H_0 : \beta_1 = 0$
$H_1 : \beta_1 \neq 0$ ← Slope is Significant

Using test Statistic

$$t^* = \frac{b_1 - 0}{se}$$

$$se\{b_1\} = \sqrt{\frac{mse}{S_{xx}}}$$

$$SSE = SST - SSR = 97.4 - 78.87785$$
$$= 18.5215$$

$$MSE = \frac{SSE}{n-2} = \frac{18.5215}{15-2} = 1.4247$$

$$se\{b_1\} = \sqrt{\frac{1.4247}{21.85}} = 0.255$$

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$$t^* = \frac{b_1 - 0}{se} = \frac{1.9}{0.255} = 7.44$$

I need to compare against the $t_{0.025,13} = 2.160$

$|7.44| > 2.160 \Rightarrow$ Reject H_0

Conclude slope is significant

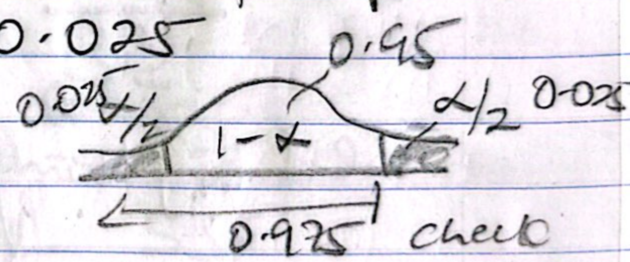
7 p = true proportion immunized

$$\hat{p} = \frac{1125}{1500} = 0.75$$

two-sided 95% CI for p

$\alpha = 0.05 \rightarrow \frac{\alpha}{2} = 0.025$

$Z_{0.025} = 1.96$



check Table A.3 for the 0.975 inside the Z-table.

$$\hat{p} \pm Z_{0.025}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.75 \pm (1.96)\sqrt{\frac{(0.75)(1-0.75)}{1500}}$$

$$0.75 \pm (1.96)\sqrt{\frac{0.1875}{1500}}$$

$0.75 \pm (1.96)\sqrt{(0.000125)}$
$0.75 \pm (1.96)(0.01118)$
0.75 ± 0.02191
$(0.728, 0.772)$
C.I length $\leq 0.03 \Rightarrow e = \frac{0.03}{2}$
$= 0.015$

$n \geq \left(\frac{Z_{0.025}}{e}\right)^2 \hat{p}(1-\hat{p})$
$= \left(\frac{1.96}{0.015}\right)^2 (0.75)(0.25)$
$= (130.66)^2 (0.75)(0.25)$
$= (17073.78)(0.75)(0.25)$
$= 3201.33$
round up to 3202.

For Clevelend we have no data
we will use $\hat{p} = 0.5$
$n \geq \left(\frac{1.96}{0.015}\right)^2 (0.5)(0.5)$
$\simeq (17073.78)(0.5)^2$
$= 4268.45$
$\simeq 4269$

b p_B = Baltimore $\hat{p}_B = 0.75$
p_C = Clevelend $\hat{p}_C = \frac{960}{1200} = 0.8$

Test $p_B < p_C$

$H_0: p_B \geq p_C$
$H_1: p_B < p_C$ ← Baltimore < Cleveland

use $\hat{p} = \frac{1125 + 960}{1500 + 1200} = 0.772\overline{2}$

$\alpha = 0.01 \rightarrow Z_{0.01} = 2.326$
using a test statistic so we need Z^*

$$Z^* = \frac{\hat{p}_B - \hat{p}_C}{Se}$$

$$Se\{\hat{p}_B - \hat{p}_C\} = \sqrt{\hat{p}(1-\hat{p})(1/n_1 + 1/n_2)}$$

$$= \sqrt{(0.7722)(0.2278)\left(\frac{1}{1500} + \frac{1}{1200}\right)}$$

$$= 0.01624$$

$$Z^* = \frac{\hat{p}_B - \hat{p}_C}{Se} = \frac{0.75 - 0.8}{0.01624}$$

$$= -3.08$$

Reject H_0 if $Z^* < -Z_{0.01}$

$-3.08 < -2.326 \Rightarrow$ Reject H_0

We Conclude that $P_B < P_C$

ii What is the p-value of your test?

$P = \Phi(z^*) = \Phi(-3.08)$

$= \boxed{0.00103}$