Practice Test two problems

1a You state the random variable

X = temp @ noon on a march day

$X \sim N(\mu = 10 , \sigma = 20)$

$P[X > 0] = 1 - P[X < 0]$

You must standardize to solve it.

$$Z = \frac{x - \mu}{\sigma}$$

$$1 - P\left[\frac{X - \mu}{\sigma} \leq \frac{0 - 10}{20}\right]$$

$= 1 - P[Z \leq -0.5)$

$= 1 - \Phi[-0.5]$

Using table A.3

$1 - 0.3085$

$= \boxed{0.6915}$

b i X = number of success out of 16days

$X \sim Bin(n = 16, P = 0.6915)$

$$P[X \geq 9] = \sum_{x=9}^{16} \binom{n}{x} (p)^x (1-p)^{n-x}$$

$$= \sum_{x=9}^{16} \binom{16}{9} (0.6915)^9 (1 - 0.6915)^{16-9}$$

ii $E[X] = np = (16)(0.6915)$

$= 11.064$ days

$V[X] = np(1-p)$
$= (16)(0.6915)(0.3085)$
$= 3.4132$

Standard deviation $= \sqrt{3.4132}$
$= \boxed{1.8475 \text{ days}}$

2. X = Weight of the book
$X \sim$ Uniform $(a=1, b=2)$

(a) $P[X > 1.8] = 1 - P[X \leq 1.8]$ cdf.
$= 1 - F_X[1.8; 1, 2]$ cdf
$= 1 - \frac{x-A}{B-A}$
$= 1 - \frac{1.8-1}{2-1}$
$= 1 - 0.8 = \boxed{0.2}$

(b) F F F S 4 books
4th

We are looking at the success occuring on the fourth.
X = number of books to see 1st success
$X \sim \text{Geo}(p = 0.2)$
$P[X=4] = p^1 (1-p)^{x-1}$
$= (0.2)^1 (1-0.2)^{4-1}$

$$P(X=4) = (0.2)^1 \, (0.8)^3$$
$$= (0.2)(0.512)$$
$$= \boxed{0.1024}$$

3 X = number of defectives (success) in a sample of 100

$X \sim Bin(n=100, \ p=0.02)$

Poisson approximation

$\mu = np = (100)(0.02) = 2$

$P[X \leq 5] \approx P(5; \mu=2)$

Table A.2

$= \boxed{0.9834}$

4 14 balls ; 5 white, 4 blacks, 3 red, 2 green

Sample 8 balls

X_1 = number of white balls

X_2 = number of black balls

X_3 = number of red balls

X_4 = number of green balls

(a) With replacements ⇒ multinomial distribution

$n = 8, \quad P_1 = 5/14, \quad P_2 = 4/14, \quad P_3 = 3/14$
$P_4 = 2/14$

$$f(x_1, \ldots x_k; n, P_1, \ldots P_k) = \binom{n}{x_1, \ldots x_k} P_1^{x_1} \cdots P_k^{x_k}$$

where $\sum_{i=1}^{k} x_i = n$

and $\sum_{i=1}^{k} P_i = 1$

$$\binom{8}{4,2,1,1} \left(\frac{5}{14}\right)^4 \left(\frac{4}{14}\right)^2 \left(\frac{3}{14}\right)^1 \left(\frac{2}{14}\right)^1$$

b <u>without</u> replacement it follows a multivariate hypergeometric

$N = 14, \quad n = 8$

$P[X_1 = 4 \quad x_2 = 2 \quad X_3 = 1 \quad X_4 = 1]$

$f(x_1, \cdots x_k; N, n, a_1, \ldots a_k)$

$$= \frac{\binom{a_1}{x_1} \cdots \binom{a_k}{x_k}}{\binom{N}{n}}$$

where $\sum_{i=1}^{k} x_i = n$

and $\sum_{i=1}^{k} a_i = N$

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$$\frac{\binom{5}{4}\binom{4}{2}\binom{3}{1}\binom{2}{1}}{\binom{14}{8}}$$

⑤ X = annual inches of rainfall

$X \sim \log N(\mu = 3.5, \sigma = 0.1)$

$\ln(30) = 3.401 \quad \ln(40) = 3.689$

$P[30 < X < 40] = P[\ln(30) < \ln(x) < \ln(40)]$

$$P\left[\left[\frac{3.401 - 3.5}{0.1}\right] < Z < \left[\frac{3.689 - 3.5}{0.1}\right]\right]$$

$P[-0.99 < Z < 1.89]$

$= \Phi(1.89) - \Phi(-0.99)$

Using Table A.3

$= 0.9706 - 0.1611$

$= \boxed{0.8095}$

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⑥ a rate $\lambda = 90/\text{hour} = 90/60 \text{ min} = 3/2 \text{ min}$

$t = 3\text{min} \Rightarrow \mu = \lambda t$

$\frac{3}{2} \times 3 = \frac{9}{2} = 4.5$

$N_3 = N_t$ = number of Meteorites in t

$t = 3\text{min}$

$N_3 \sim \text{Poisson}\ (\mu = 4.5)$

$P[N_3 \geq 4] = 1 - P[N_3 \leq 3]$

$= 1 - P(3; \mu = 4.5)$

using Table A.2

$= 1 - 0.3423$

$= \boxed{0.6577}$

b T = time between Meteorites

$T \sim \text{Exp}(\beta = 1/\lambda = 1/3/2 = 2/3)$

$T > 20\text{sec} = \frac{20 \text{ sec}}{60 \text{ sec/min}} = 1/3 \text{ min}$

$P[T > 1/3] = 1 - P[T \leq 1/3]$

$= 1 - [1 - e^{-x/\beta})$

$= e^{-1/3 / 2/3} = \boxed{e^{-1/2}}$

(c) W_4 = time until four meteorites strike the earth

$W_4 \sim \text{Gamma}(\alpha = 4, \beta = 2/3)$

$K = \alpha = 4$ meteorites

$P[W_4 > 2] = 1 - P[W_4 \le 2]$

$1 - F_{W_4}(2; \alpha = 4, \beta = 2/3]$

$a = x/\beta = 2/(2/3) = 3 \; ; \; \alpha = 4$

Using Table A.23

$1 - 0.353$

$= \boxed{0.647}$

(d) $E[W_4] = \alpha\beta = (4)(2/3) = 8/3$

$= \boxed{8/3 \text{ mins}}$

0
00
00
00
00
00
10
30
10
00
01
37
23
39
00
00
05
86
15
95
00
00
01
13
59
43
86
00
00
02
27
57
97
17
00

⑦ X = number of success in a sample of 20

$X \sim Bin(n=20, p=0.9)$

$P[X=19] = P[X \le 19] - P[X \le 18]$

$P[X=19] = b(19; 20, 0.9)$

$= B[19; 20, 0.9] - B[18; 20, 0.9]$

Using Table A.1

$= 0.8784 - 0.6083$

$= \boxed{0.2701}$

⑧ a) mean $= \dfrac{\sum x_i}{n}$

7.3, 8.6, 10.4, 16.1, 12.2, 15.1, 14.5, 9.3

$n = 8$

$\dfrac{\sum x_i}{n} = \dfrac{7.3 + 8.6 + 10.4 + 16.1 + 12.2 + 15.1 + 14.5 + 9.3}{8}$

$= \dfrac{93.5}{8} = 11.6875$ milligrams

b) Variance $= S^2 = \dfrac{\sum x^2 - (\sum x)^2/n}{n-1}$

or $S^2 = \dfrac{n\sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}{n(n-1)}$

$$\Sigma x^2 = 53.29 + 73.96 + 108.16 + 259.21 + 148.84 + 228.01 + 210.25 + 86.49$$

$$= 1168.21$$

$$= \frac{1168.21 - (93.5)^2/8}{8-1}$$

$$= \frac{1168.21 - \frac{8742.25}{8}}{7}$$

$$= \frac{1168.21 - 1092.78}{7}$$

$$= \frac{75.42875}{7}$$

$$S^2 = 10.776.$$

If you are asked to find the Standard deviation

$$S = \sqrt{S^2}$$

$$= \sqrt{10.776}$$

Standard deviation $= 3.2826$

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9 $p = \frac{1}{1000} = 0.001$

X = number of income tax return with error

$\sim$ Poisson distribution

$\mu = np = (10000)(0.001) = 10$ so

$P[a \le X \le b] = P[X \le b] - P[X < a]$

$= P[X \le b] - P[X \le a-1]$

$= P(b; \mu) - P[a-1; \mu]$

$P[6 \le X \le 8] = P[X \le 8) - P[X \le 5]$

$\approx \sum_{x=0}^{8} p(x; 10) - \sum_{x=0}^{5} p(x; 10)$

Using Table A.2

$0.3328 - 0.0671$

$= \boxed{0.2657}$

76 Norms Approx

$\mu = np = (100)(0.1) = 10$

$\sigma^2 = np(1-p) = (100)(0.1)(0.9) = 9$

$\sigma = \sqrt{\sigma^2} = \sqrt{9} = 3$

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In Bin $P[X \geq 8]$

$= 1 - P[X < 8]$

$= 1 - P(X \leq 7)$ cdf

now you can do the approximation

$$\approx 1 - P\left[\frac{X-\mu}{\sigma} \leq \frac{7+0.5-10}{3}\right]$$

Use continuity correction because the normal is a continous distribution

$= 1 - P[Z \leq -0.83]$

$1 - \Phi(-0.83)$

Using Table A-3

$1 - 0.2033$

$= \boxed{0.7967}$

NOTE

① At least 20 $= P[X \geq 20] = 1 - P[X \leq 19]$

② No more than 20 $= P[X \leq 20]$

③ Exactly 20 $= P[X = 20] = P[X \leq 20] - P[X \leq 19]$

④ Fewer than 20 $= P[X < 20] = P[X \leq 19]$