1 Convlution

i typedef complex<double> cd;

1.1 FFT

```
const double PI = acos(-1);
  void FFT(vector<cd> &a, bool inv){
      int n = a.size();
      for (int i=1, j=0; i<n; i++){</pre>
          int bit = (n>>1);
          for ( ; j&bit ; bit>>=1){
              j ^= bit;
           i ^= bit:
          if (i<j){</pre>
               swap(a[i], a[j]);
      for (int len=2 ; len<=n ; len<<=1){</pre>
          cd wlen = polar(1.0, (inv ? 2 : -2)*PI/len);
          for (int i=0 ; i<n ; i+=len){</pre>
              cd w(1);
               for (int j=0 ; j<len/2 ; j++){</pre>
                   cd u = a[i+j];
                   cd v = a[i+j+len/2]*w;
                   a[i+j] = u+v;
                   a[i+j+len/2] = u-v;
                   w *= wlen;
      if (inv){
           for (auto &x : a){
              x /= n;
      }
43 vector<cd> polyMul(vector<cd> a, vector<cd> b){
      int sa = a.size(), sb = b.size(), n = 1;
      while (n<sa+sb-1) n *= 2;</pre>
      a.resize(n);
      b.resize(n);
      vector<cd> c(n);
      FFT(a, 0);
      FFT(b, 0);
      for (int i=0 ; i<n ; i++) c[i] = a[i]*b[i];</pre>
      FFT(c, 1);
      c.resize(sa+sb-1);
      return c:
```

1.2 FFT-2

```
1 typedef complex < double > cd;
 void FFT(vector<cd> &a) {
      int n = a.size(), L = 31-__builtin_clz(n);
      vector<complex<long double>> R(2, 1);
      vector<cd> rt(2, 1);
      for (int k=2 ; k<n ; k*=2){</pre>
          R.resize(n);
          rt.resize(n);
          auto x = polar(1.0L, acos(-1.0L) / k);
          for (int i=k ; i<2*k ; i++){</pre>
               rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
      vector<int> rev(n);
      for (int i=0 ; i<n ; i++){</pre>
          rev[i] = (rev[i/2] | (i&1) << L)/2;
      for (int i=0 ; i<n ; i++){</pre>
          if (i<rev[i]) swap(a[i], a[rev[i]]);</pre>
      for (int k=1; k<n; k*=2){</pre>
          for (int i=0; i < n; i+=2*k){
               for (int j=0 ; j<k ; j++){</pre>
                   auto x = (double *)&rt[j+k];
auto y = (double *)&a[i+j+k];
                   cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*
                        y[0]);
                   a[i+j+k] = a[i+j]-z;
                   a[i+j] += z;
          }
      return;
  vector<double> PolyMul(const vector<double> a, const vector<</pre>
      if (a.empty() || b.empty()) return {};
      vector<double> res(a.size()+b.size()-1);
      int L = 32 - __builtin_clz(res.size()), n = 1 << L;</pre>
      vector<cd> in(n), out(n);
      copy(a.begin(), a.end(), begin(in));
      for (int i=0 ; i<b.size() ; i++){</pre>
          in[i].imag(b[i]);
      FFT(in);
      for (cd& x : in) x *= x;
      for (int i=0 ; i<n ; i++){</pre>
          out[i] = in[-i & (n - 1)] - conj(in[i]);
      FFT(out);
      for (int i=0 ; i<res.size() ; i++){</pre>
          res[i] = imag(out[i]) / (4 * n);
      return res;
```

1.3 NTT-998244353

```
| \text{const} \text{ int } \text{MOD} = (119 \iff 23) + 1, \text{ ROOT} = 62; // = 998244353
2 // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
  // and 483 << 21 (same root). The last two are > 10^9.
  // 9cd58a
  void NTT(vector<int> &a) {
      int n = a.size();
      int L = 31-__builtin_clz(n);
      vector<int> rt(2, 1);
      for (int k=2, s=2; k<n; k*=2, s++){
          rt.resize(n);
          int z[] = {1, qp(ROOT, MOD>>s)};
          for (int i=k; i<2*k; i++){
               rt[i] = rt[i/2]*z[i&1]%MOD;
      vector<int> rev(n);
      for (int i=0 ; i<n ; i++){</pre>
           rev[i] = (rev[i/2]|(i&1)<<L)/2;
      for (int i=0 ; i<n ; i++){</pre>
          if (i<rev[i]){</pre>
               swap(a[i], a[rev[i]]);
      for (int k=1; k<n; k*=2){</pre>
           for (int i=0; i<n; i+=2*k){
               for (int j=0 ; j<k ; j++){</pre>
                   int z = rt[j+k]*a[i+j+k]%MOD, &ai = a[i+j];
                   a[i+j+k] = ai-z+(z>ai ? MOD : 0);
                   ai += (ai+z)=MOD ? z-MOD : z);
  vector<int> polyMul(vector<int> &a, vector<int> &b){
      if (a.empty() || b.empty()) return {};
      int s = a.size()+b.size()-1, B = 32-__builtin_clz(s), n =
      int inv = qp(n, MOD-2);
      vector<int> L(a), R(b), out(n);
      L.resize(n), R.resize(n);
      NTT(L), NTT(R);
      for (int i=0 ; i<n ; i++){</pre>
          out[-i&(n-1)] = L[i]*R[i]%MOD*inv%MOD;
      NTT(out);
53
      out.resize(s);
      return out;
```

1.4 FFT-mod

```
2| 修改 const int MOD = 998244353 更改要取餘的數字
3 POLVMul(a, b) 回傳多項式乘法的結果 (c_k = \sum_{i+j} a_i+b_j)
       mod MOD )
5| 大約可以支援 5e5 · a i. b i 皆在 MOD 以下的非負整數
  const int MOD = 998244353;
  typedef complex<double> cd;
void FFT(vector<cd> &a) {
      int n = a.size(), L = 31- builtin clz(n);
      vector<complex<long double>> R(2, 1);
      vector<cd> rt(2, 1);
      for (int k=2; k<n; k*=2){</pre>
          R.resize(n);
          rt.resize(n);
          auto x = polar(1.0L, acos(-1.0L) / k);
          for (int i=k ; i<2*k ; i++){</pre>
              rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
      vector<int> rev(n);
      for (int i=0 ; i<n ; i++){</pre>
          rev[i] = (rev[i/2] | (i&1) << L)/2;
      for (int i=0 ; i<n ; i++){</pre>
          if (i<rev[i]) swap(a[i], a[rev[i]]);</pre>
      for (int k=1; k<n; k*=2){
          for (int i=0 ; i<n ; i+=2*k){</pre>
              for (int j=0 ; j<k ; j++){</pre>
                  auto x = (double *)&rt[j+k];
                  auto y = (double *)&a[i+j+k];
                  cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*
                       y[0]);
                  a[i+j+k] = a[i+j]-z;
                  a[i+j] += z;
      return;
46 vector<int> PolyMul(vector<int> a, vector<int> b){
      if (a.empty() || b.empty()) return {};
      vector<int> res(a.size()+b.size()-1);
      int B = 32-__builtin_clz(res.size()), n = (1<<B), cut =</pre>
           int(sqrt(MOD));
      vector<cd> L(n), R(n), outs(n), outl(n);
      for (int i=0 ; i<a.size() ; i++){</pre>
          L[i] = cd((int) a[i]/cut, (int)a[i]%cut);
      for (int i=0 ; i<b.size() ; i++){</pre>
          R[i] = cd((int) b[i]/cut, (int)b[i]%cut);
      FFT(L);
      FFT(R);
      for (int i=0 ; i<n ; i++){</pre>
          int j = -i&(n-1);
```

```
outl[j] = (L[i]+conj(L[j])) * R[i]/(2.0*n);
    outs[j] = (L[i]-conj(L[j])) * R[i]/(2.0*n)/1i;
}
FFT(outl);
FFT(outs);
for (int i=0; i<res.size(); i++){
    int av = (int)(real(outl[i])+0.5), cv = (int)(imag(
        outs[i])+0.5);
    int bv = (int)(imag(outl[i])+0.5) + (int)(real(outs[i])+0.5);
    res[i] = ((av%MoD*cut+bv) % MoD*cut+cv) % MOD;
}
return res;
}</pre>
```

2 Data-Structure

2.1 GP-Hash-Table

```
| #include <ext/pb_ds/assoc_container.hpp>
  using namespace gnu pbds;
  typedef tree<int,null_type,less<int>,rb_tree_tag,
       tree order statistics node update> order set;
  struct custom hash {
      static uint64_t splitmix64(uint64_t x) {
          // http://xorshift.di.unimi.it/splitmix64.c
          x += 0x9e3779b97f4a7c15;
          x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
          x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
          return x ^ (x >> 31);
      size_t operator()(uint64_t x) const {
          static const uint64 t FIXED RANDOM = chrono::
               steady clock::now().time since epoch().count();
          return splitmix64(x + FIXED RANDOM);
  };
19 gp hash table (int, int, custom hash) ss;
```

2.2 Sparse-Table

```
vector<vector<int>> st;
void build(vector<int>> v){
    int h = __lg(v.size());
    st.resize(h+1);
    st[0] = v;

for (int i=1; i<=h; i++){
        int gap = (1<<(i-1));
        for (int j=0; j+gap<st[i-1].size(); j++){
            st[i].push_back(min(st[i-1][j], st[i-1][j+gap]));
    }
}
}
}
</pre>
```

2.3 Order-Set

15 // 回傳 [LL, rr) 的最小值

int RMQ(int 11, int rr){
 int h = lg(rr-11);

return min(st[h][l1], st[h][rr-(1<<h)]);</pre>

2.4 BIT

```
vector<int> BIT(MAX_SIZE);
  void update(int pos, int val){
      for (int i=pos ; i<MAX_SIZE ; i+=i&-i){</pre>
          BIT[i]+=val;
  int query(int pos){
      int ret=0;
      for (int i=pos ; i>0 ; i-=i&-i){
          ret+=BIT[i]:
      return ret;
16 // const int MAX N = (1<<20)
17 // const int LOG_N = 20;
18 | int k th(int k){ // 回傳 BIT 中第 k 小的元素(based-1)
      int target = k-1, now = 0;
      for (int i=LOG N-1; i>=0; i--){
          if (BIT[now+(1<<i)]<=k){</pre>
               k -= BIT[now+(1<<i)];</pre>
               now += 1<<i;
25
      return now+1;
```

2.5 Treap

```
struct Treap{
Treap *1, *r;
int pri, val, sz;
```

```
Treap(int val){
          1 = nullptr;
          r = nullptr:
          pri = rand();
          val = val;
          sz = 1;
  } *root;
14 int size(Treap *a){
      return a ? a->sz : 0;
18 void pull(Treap *t){
      t\rightarrow sz = size(t\rightarrow l) + size(t\rightarrow r) + 1;
22 Treap *merge(Treap *a, Treap *b){
      // 如果一個為空,就回傳另一個
      if (!a || !b) return a ? a : b;
      if (a->pri>b->pri){
          a \rightarrow r = merge(a \rightarrow r, b);
          pull(a);
          return a;
      }else{
          b \rightarrow 1 = merge(a, b \rightarrow 1);
          pull(b);
          return b;
void split(Treap *&t, int k, Treap *&a, Treap *&b){
      // 如果樹為空就直接返回
      if (!t) a = b = nullptr;
      else if (size(t->l)+1<=k){ // 用 k 分割 treap
          // 如果以左子節點為根 + 目前節點合法:
          split(t->r, k-size(t->l)-1, a->r, b);
          pull(a);
      }else{
          b = t;
          split(t->1, k, a, b->1);
          pull(b);
54 ostream & operator << (ostream &os, Treap *t){
      if (t==0) return os:
      os << t->1;
      os << (char)t->val;
      os << t->r;
      return os;
62 void print(Treap *t){
      if (t->1!=0) print(t->1);
      cout << (char)t->val;
      if (t->r!=0) print(t->r);
```

2.6 Persistent-Segment-Tree

全部都是 0-based

```
Persistent Segment Tree st(n+q);
  st.build(v, 0);
  update_version(pos, val, ver): 對版本 ver 的 pos 位置改成 val 73
  query version(ql, qr, ver): 對版本 ver 查詢 [ql, qr) 的區間和 74
11 clone version(ver): 複製版本 ver 到最新的版本
  struct Persistent_Segment_Tree{
      int node cnt = 0;
      struct Node{
          int lc = -1;
          int rc = -1;
          int val = 0;
      vector<Node> arr;
      vector<int> version;
      Persistent Segment Tree(int sz){
          arr.resize(32*sz);
          version.push back(node cnt++);
          return;
      void pull(Node &c, Node a, Node b){
          c.val = a.val+b.val;
      void build(vector<int> &v, int idx, int ll = 0, int rr =
          auto &now = arr[idx];
                                                                99
          if (rr-ll==1){
                                                                100
              now.val = v[11];
                                                                101
              return;
          int mid = (11+rr)/2;
          now.lc = node cnt++:
          now.rc = node cnt++;
          build(v, now.lc, ll, mid);
          build(v, now.rc, mid, rr);
          pull(now, arr[now.lc], arr[now.rc]);
          return:
      void update(int pos, int val, int idx, int ll = 0, int rr
          auto &now = arr[idx];
          if (rr-ll==1){
              now.val = val;
              return;
          int mid = (11+rr)/2;
          if (pos<mid){</pre>
```

```
arr[node_cnt] = arr[now.lc];
        now.lc = node cnt;
        node cnt++:
        update(pos, val, now.lc, ll, mid);
        arr[node cnt] = arr[now.rc];
        now.rc = node cnt;
        node cnt++:
        update(pos, val, now.rc, mid, rr);
    pull(now, arr[now.lc], arr[now.rc]);
    return;
}
void update_version(int pos, int val, int ver){
    update(pos, val, version[ver]);
Node query(int ql, int qr, int idx, int ll = 0, int rr =
    auto &now = arr[idx];
    if (q1<=11 && rr<=qr) return now;</pre>
    if (rr<=ql || qr<=ll) return Node();</pre>
    int mid = (11+rr)/2;
    pull(ret, query(ql, qr, now.lc, ll, mid), query(ql,
         qr, now.rc, mid, rr));
    return ret;
}
Node query_version(int ql, int qr, int ver){
    return query(ql, qr, version[ver]);
void clone_version(int ver){
    version.push_back(node_cnt);
    arr[node_cnt] = arr[version[ver]];
    node cnt++;
```

2.7 Trie

```
struct Trie{
    struct Data{
        int nxt[2]={0, 0};
};

int sz=0;
vector<Data> arr;

void init(int n){
    arr.resize(n);
}

void insert(int n){
    int now=0;
    for (int i=N; i>=0; i--){
        int v=(n>>i)&1;
        if (!arr[now].nxt[v]){
```

```
arr[now].nxt[v]=++sz;
              now=arr[now].nxt[v];
      }
      int query(int n){
          int now=0, ret=0;
          for (int i=N ; i>=0 ; i--){
              int v=(n>>i)&1;
              if (arr[now].nxt[1-v]){
                  ret+=(1<<i);
                  now=arr[now].nxt[1-v];
              }else if (arr[now].nxt[v]){
                  now=arr[now].nxt[v];
              }else{
                  return ret;
          return ret;
40 } tr;
```

2.8 LC-Segment-Tree

```
2 全部都是 0-based
5 LC_Segment_Tree st(n);
7 函式:
s|update(val):將一個 pair <a, b> 代表插入一條 y=ax+b 的直線
9 | query(x): 查詢所有直線在位置 x 的最小值
struct LC Segment Tree{
     struct Node{ // y = ax+b
         int a = 0;
         int b = INF;
         int y(int x){
            return a*x+b;
     vector<Node> arr;
     LC Segment Tree(int n = 0){
         arr.resize(4*n);
26
     void update(Node val, int idx = 0, int ll = 0, int rr =
          MAX V){
         if (rr-ll==1){
            if (val.y(ll) < arr[idx].y(ll)){</pre>
                arr[idx] = val;
            return;
```

```
int mid = (11+rr)/2;
          if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
37
              的線斜率要比較小
          if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
              update(val, idx*2+1, ll, mid);
          }else{ // 交點在右邊
              swap(arr[idx], val); // 在左子樹中,新線比舊線還
              update(val, idx*2+2, mid, rr);
43
          return;
      }
      int query(int x, int idx = 0, int ll = 0, int rr = MAX V)
          if (rr-ll==1){
              return arr[idx].y(11);
          int mid = (11+rr)/2;
          if (x<mid){</pre>
              return min(arr[idx].y(x), query(x, idx*2+1, ll,
              return min(arr[idx].y(x), query(x, idx*2+2, mid,
59 };
```

2.9 Persistent-Disjoint-Set

```
1 struct Persistent Disjoint Set{
       Persistent_Segment_Tree arr, sz;
       void init(int n){
           arr.init(n);
           vector<int> v1;
           for (int i=0 ; i<n ; i++){</pre>
               v1.push_back(i);
           arr.build(v1, 0);
           sz.init(n):
           vector<int> v2;
           for (int i=0 ; i<n ; i++){</pre>
               v2.push back(1);
           sz.build(v2, 0);
       int find(int a){
           int res = arr.query_version(a, a+1, arr.version.size
                ()-1).val;
           if (res==a) return a;
           return find(res);
       bool unite(int a, int b){
           a = find(a):
27
           b = find(b);
```

2.10 Add-Set-Segment-Tree

42

43

```
1 // [ll, rr), based-0
2 // 使用前記得 init(陣列大小), build(陣列名稱)
3 // add(LL, rr): 區間修改
4 // set(ll, rr): 區間賦值
5 // query(ll, rr): 區間求和 / 求最大值
  struct SegmentTree{
      struct node{
          int add_tag = 0;
          int set_tag = 0;
          int sum = 0;
          int ma = 0;
      };
      vector<node> arr;
      SegmentTree(int n){
          arr.resize(n<<2);</pre>
17
20
      node pull(node A, node B){
          node C:
          C.sum = A.sum+B.sum;
          C.ma = max(A.ma, B.ma);
          return C;
       // cce0c8
      void push(int idx, int ll, int rr){
          if (arr[idx].set tag!=0){
29
30
              arr[idx].sum = (rr-ll)*arr[idx].set_tag;
              arr[idx].ma = arr[idx].set_tag;
31
              if (rr-ll>1){
32
                  arr[idx*2+1].add_tag = 0;
33
                  arr[idx*2+1].set_tag = arr[idx].set_tag;
34
35
                  arr[idx*2+2].add tag = 0;
36
                  arr[idx*2+2].set_tag = arr[idx].set_tag;
```

```
arr[idx].set tag = 0;
    if (arr[idx].add_tag!=0){
        arr[idx].sum += (rr-ll)*arr[idx].add tag;
        arr[idx].ma += arr[idx].add tag;
        if (rr-ll>1){
            arr[idx*2+1].add tag += arr[idx].add tag;
            arr[idx*2+2].add tag += arr[idx].add tag;
        arr[idx].add tag = 0;
    }
}
void build(vector<int> &v, int idx = 0, int ll = 0, int
     rr = n){
    if (rr-ll==1){
        arr[idx].sum = v[11];
        arr[idx].ma = v[ll]:
        int mid = (11+rr)/2;
        build(v, idx*2+1, ll, mid);
        build(v, idx*2+2, mid, rr);
        arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
}
void add(int ql, int qr, int val, int idx = 0, int ll =
     0, int rr =n){
    push(idx, ll, rr);
    if (rr<=ql || qr<=ll) return;</pre>
    if (q1<=11 && rr<=qr){</pre>
        arr[idx].add tag += val;
        push(idx, ll, rr);
        return:
    int mid = (11+rr)/2;
    add(ql, qr, val, idx*2+1, ll, mid);
    add(ql, qr, val, idx*2+2, mid, rr);
    arr[idx]=pull(arr[idx*2+1], arr[idx*2+2]);
void set(int ql, int qr, int val, int idx=0, int ll=0,
     int rr=n){
    push(idx, ll, rr);
    if (rr<=ql || qr<=ll) return;</pre>
    if (q1<=11 && rr<=qr){</pre>
        arr[idx].add tag = 0;
        arr[idx].set tag = val;
        push(idx, ll, rr);
        return:
    int mid = (11+rr)/2;
    set(ql, qr, val, idx*2+1, ll, mid);
    set(ql, qr, val, idx*2+2, mid, rr);
    arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
node query(int ql, int qr, int idx = 0, int ll = 0, int
     rr = n){
    push(idx, 11, rr);
    if (rr<=ql || qr<=ll) return node();</pre>
    if (ql<=ll && rr<=qr) return arr[idx];</pre>
    int mid = (11+rr)/2:
```

```
return pull(query(ql, qr, idx*2+1, ll, mid), query(ql 31
                , gr, idx*2+2, mid, rr));
100 } ST;
```

3 Dynamic-Programming

3.1 SOS-DP

```
1 // 總時間複雜度為 O(n 2^n)
2 // 計算 dp[i] = i 所有 bit mask 子集的和
 for (int i=0 ; i<n ; i++){</pre>
      for (int mask=0; mask<(1<<n); mask++){</pre>
          if ((mask>>i)&1){
              dp[mask] += dp[mask^(1<<i)];</pre>
```

3.2 Digit-DP

27

```
| #include <bits/stdc++.h>
 using namespace std;
 long long l, r;
s| long long dp[20][10][2][2]; // dp[pos][pre][limit] = 後 pos
     位·pos 前一位是 pre·(是/否)有上界·(是/否)有前綴零
     的答案數量
 long long memorize search(string &s, int pos, int pre, bool
     limit, bool lead){
     // 已經被找過了,直接回傳值
     if (dp[pos][pre][limit][lead]!=-1) return dp[pos][pre][
         limit][lead];
     // 已經搜尋完畢,紀錄答案並回傳
     if (pos==(int)s.size()){
        return dp[pos][pre][limit][lead] = 1;
     // 枚舉目前的位數數字是多少
     long long ans = 0:
     for (int now=0; now<=(limit ? s[pos]-'0': 9); now++){
        if (now==pre){
            // 1~9 絕對不能連續出現
            if (pre!=0) continue;
            // 如果已經不在前綴零的範圍內 · a 不能連續出現
            if (lead==false) continue;
        ans += memorize search(s, pos+1, now, limit&(now==(s[
            pos]-'0')), lead&(now==0));
```

```
// 已經搜尋完畢,紀錄答案並回傳
33
     return dp[pos][pre][limit][lead] = ans;
34
  // 回傳 [0. n] 有多少數字符合條件
  long long find answer(long long n){
      memset(dp, -1, sizeof(dp));
      string tmp = to string(n);
      return memorize_search(tmp, 0, 0, true, true);
42
  int main(){
      // input
     cin >> 1 >> r;
     // output - 計算 [L, r] 有多少數字任意兩個位數都不相同
     cout << find answer(r)-find answer(l-1) << "\n";</pre>
      return 0:
```

3.3 整數拆分

```
dp[i][x] = 要將整數 x 拆成 i 堆的「組合數」
dp[i+1][x+1]+=dp[i][x] ( 創造新的一堆 )
dp[i][x+i]+=dp[i][x] (把每一堆都增加1)
```

Geometry

4.1 Point-Struct

```
const int EPS = 1e-6;
typedef int pt;
struct point{
    pt x, y;
    point(pt _x = 0, pt _y = 0){
       x = _x;
        y = y;
    point operator * (pt a){return {a*x, a*y};};
    point operator / (pt a){return {a/x, a/y};};
    point operator + (point a){return {x+a.x, y*a.y};};
    point operator - (point a){return {x-a.x, y-a.y};};
    // 內積、外積
    double operator * (point a){return x*a.x+y*a.y;};
    double operator ^ (point a){return x*a.y-y*a.x;};
```

4.2 Line-Intersection

4.3 Pick's-Theorem

給定頂點坐標均是整點的簡單多邊形,面積 = 內部格點數 + 邊上格點數/2 - 1

4.4 Point-In-Polygon

```
for (int i=0; i<n; i++){
    if (in(v[i], v[i+1], a)) return 0;
    if (banana(a, b, v[i], v[i+1])) cnt++;
}
return cnt%2 ? 1 : -1;</pre>
```

4.5 Convex-Hull

```
// e0a719
vector<point> convex_hull(vector<point> v){
    sort(v.begin(), v.end());
    v.resize(unique(v.begin(), v.end())-v.begin());
    vector<point> hull;
    for (int =0; _<2; _++){
        int sz=hull.size();
        for (int i=0; i<v.size(); i++){
            while (hull.size()>=sz+2 && ori(hull[hull.size() -2], hull[hull.size()-1], v[i])<0){
            hull.pop_back();
            }
            hull.pop_back(v[i]);
    }
    hull.pop_back();
    reverse(v.begin(), v.end());
}
return hull;
}</pre>
```

5 Graph

5.1 Find-Bridge

```
vector<int> dep(MAX N), low(MAX N);
vector<pair<int, int>> bridge;
bitset<MAX N> vis;
void dfs(int now, int pre){
    vis[now] = 1;
    low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
    for (auto x : G[now]){
        if (x==pre){
            continue;
        }else if (vis[x]==0){
            // 沒有走過的節點
            dfs(x, now);
            low[now] = min(low[now], low[x]);
        }else if (vis[x]==1){
            low[now] = min(low[now], dep[x]);
    if (now!=1 && low[now]==dep[now]){
        bridge.push back({now, pre});
```

5.2 Find-AP

return;

25 }

```
1 vector<int> dep(MAX_N), low(MAX_N), AP;
  bitset<MAX N> vis;
  void dfs(int now, int pre){
      int cnt = 0;
      bool ap = 0;
      vis[now] = 1;
      low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
      for (auto x : G[now]){
          if (x==pre){
              continue;
          }else if (vis[x]==0){
              cnt++;
               dfs(x, now);
              low[now] = min(low[now], low[x]);
              if (low[x]>=dep[now]) ap=1;
              low[now] = min(low[now], dep[x]);
22
      if ((now==pre && cnt>=2) || (now!=pre && ap)){
          AP.push back(now);
25
26 }
```

5.3 MCMF

```
1 // frostray 會用,所以他超強
  struct Flow {
      struct Edge {
          int u, rc, k, rv;
      };
      vector<vector<Edge>> G;
      vector<int> par, par_eid;
      Flow(int n): G(n + \overline{1}), par(n + 1), par eid(n + 1) {}
      // v->u / 流量:c, cost: k
      void add(int v, int u, int c, int k) {
          G[v].push_back({u, c, k, (int)G[u].size()});
          G[u].push_back({v, 0, -k, (int)G[v].size() - 1});
      int spfa(int s, int t) {
          fill(par.begin(), par.end(), -1);
          vector<int> dis(par.size(), LONG_LONG_MAX);
          vector<bool> in_q(par.size(), false);
          queue<int> 0;
          dis[s] = 0; in_q[s] = true;
21
          Q.push(s);
          while (! Q.empty()) {
23
              int v = Q.front(); Q.pop();
              in q[v] = false;
              for (int i = 0; i < (int)G[v].size(); i++) {</pre>
```

```
auto [u, rc, k, rv] = G[v][i];
                  if (rc > 0 \&\& dis[v] + k < dis[u]) {
                      dis[u] = dis[v] + k:
                      par[u] = v;
                      par_eid[u] = i;
                      if (! in q[u]) Q.push(u);
                      in_q[u] = true;
                  }
             }
          return dis[t];
      // 〈最大流,最小費用〉
     pair<int, int> flow(int s, int t) {
          int fl = 0, cost = 0, d;
          while ((d = spfa(s, t)) < LONG_LONG_MAX) {</pre>
              int cur = LONG LONG MAX;
              for (int v = t; v != s; v = par[v])
                  cur = min(cur, G[par[v]][par_eid[v]].rc);
              cost += d * cur;
              for (int v = t; v != s; v = par[v]) {
                  G[par[v]][par eid[v]].rc -= cur;
                  G[v][G[par[v]][par_eid[v]].rv].rc += cur;
          return {fl, cost};
55 };
```

5.4 HLD

```
| #include <bits/stdc++.h>
2 #define int long long
 using namespace std;
 const int N = 100005;
 vector <int> G[N];
 struct HLD {
     vector<int> pa, sz, depth, mxson, topf, id;
     int n, idcnt = 0;
     HLD(int _n) : n(_n), pa(_n + 1), sz(_n + 1), depth(_n +
          1), mxson(_n + 1), topf(_n + 1), id(_n + 1) {}
     void dfs1(int v = 1, int p = -1) {
         pa[v] = p; sz[v] = 1; mxson[v] = 0;
         depth[v] = (p == -1 ? 0 : depth[p] + 1);
         for (int u : G[v]) {
             if (u == p) continue;
             dfs1(u, v);
             sz[v] += sz[u];
             if (sz[u] > sz[mxson[v]]) mxson[v] = u;
     void dfs2(int v = 1, int top = 1) {
         id[v] = ++idcnt;
         topf[v] = top;
         if (mxson[v]) dfs2(mxson[v], top);
         for (int u : G[v]) {
             if (u == mxson[v] || u == pa[v]) continue;
             dfs2(u, u);
```

```
// query 為區間資料結構
int path query(int a, int b) {
   int res = 0:
   while (topf[a] != topf[b]) { /// 若不在同一條鍊上
       if (depth[topf[a]] < depth[topf[b]]) swap(a, b);</pre>
       res = max(res, 011); // query : l = id[topf[a]],
            r = id[a]
       a = pa[topf[a]];
   /// 此時已在同一條鍊上
   if (depth[a] < depth[b]) swap(a, b);</pre>
   res = \max(\text{res}, 011); // query : l = id[b], r = id[a]
   return res;
```

5.5 Tree-Isomorphism

```
| #include <bits/stdc++.h>
  #pragma GCC optimize("03,unroll-loops")
  #define fastio ios::sync_with_stdio(0), cin.tie(0), cout.tie
  #define dbg(x) cerr << #x << " = " << x << endl
  #define int long long
  using namespace std;
  // declare
  const int MAX SIZE = 2e5+5;
  const int INF = 9e18;
  const int MOD = 1e9+7;
  const double EPS = 1e-6;
  typedef vector<vector<int>> Graph;
  typedef map<vector<int>, int> Hash;
  int n, a, b;
  int id1, id2;
  pair<int, int> c1, c2;
  vector<int> sz1(MAX_SIZE), sz2(MAX_SIZE);
  vector<int> we1(MAX SIZE), we2(MAX SIZE);
  Graph g1(MAX_SIZE), g2(MAX_SIZE);
  Hash m1, m2;
23 int testcase=0:
  void centroid(Graph &g, vector<int> &s, vector<int> &w, pair< 90</pre>
       int, int> &rec, int now, int pre){
      s[now]=1;
      w[now]=0:
      for (auto x : g[now]){
          if (x!=pre){
              centroid(g, s, w, rec, x, now);
              s[now]+=s[x];
              w[now]=max(w[now], s[x]);
      }
      w[now]=max(w[now], n-s[now]);
      if (w[now]<=n/2){</pre>
          if (rec.first==0) rec.first=now;
          else rec.second=now;
```

```
vector<int> v;
    for (auto x : g[now]){
        if (x!=pre){
            int add=dfs(g, m, id, x, now);
            v.push back(add);
    sort(v.begin(), v.end());
    if (m.find(v)!=m.end()){
        return m[v];
    }else{
        m[v]=++id;
        return id;
void solve1(){
    // init
    id1=0;
    id2=0:
    c1={0, 0};
    c2={0, 0};
    fill(sz1.begin(), sz1.begin()+n+1, 0);
    fill(sz2.begin(), sz2.begin()+n+1, 0);
    fill(we1.begin(), we1.begin()+n+1, 0);
    fill(we2.begin(), we2.begin()+n+1, 0);
    for (int i=1 ; i<=n ; i++){</pre>
        g1[i].clear();
        g2[i].clear();
    m1.clear();
    m2.clear();
    // input
    cin >> n;
    for (int i=0 ; i<n-1 ; i++){</pre>
        cin >> a >> b;
        g1[a].push_back(b);
        g1[b].push_back(a);
    for (int i=0 ; i<n-1 ; i++){</pre>
        cin >> a >> b;
        g2[a].push back(b);
        g2[b].push_back(a);
    // get tree centroid
    centroid(g1, sz1, we1, c1, 1, 0);
    centroid(g2, sz2, we2, c2, 1, 0);
    // process
    int res1=0, res2=0, res3=0;
    if (c2.second!=0){
        res1=dfs(g1, m1, id1, c1.first, 0);
        m2=m1;
        id2=id1:
        res2=dfs(g2, m1, id1, c2.first, 0);
        res3=dfs(g2, m2, id2, c2.second, 0);
    }else if (c1.second!=0){
        res1=dfs(g2, m1, id1, c2.first, 0);
        m2=m1:
        id2=id1;
```

106

107

43 int dfs(Graph &g, Hash &m, int &id, int now, int pre){

```
res2=dfs(g1, m1, id1, c1.first, 0);
                                                                  36 }
                                                                                                                                     15 #define deb1 cerr << "deb1" << endl;
110
           res3=dfs(g1, m2, id2, c1.second, 0);
                                                                                                                                     #define deb2 cerr << "deb2" << endl;</pre>
111
       }else{
                                                                                                                                       #define deb3 cerr << "deb3" << endl;</pre>
           res1=dfs(g1, m1, id1, c1.first, 0);
                                                                                                                                       #define deb4 cerr << "deb4" << endl;</pre>
112
           res2=dfs(g2, m1, id1, c2.first, 0);
                                                                     5.7 Cut BCC
                                                                                                                                       #define deb5 cerr << "deb5" << endl;</pre>
113
                                                                                                                                       #define bye exit(0);
114
115
                                                                                                                                       using namespace std;
116
                                                                     #include <bits/stdc++.h>
       cout << (res1==res2 || res1==res3 ? "YES" : "NO") << endl</pre>
                                                                                                                                       const int mxn = (int)(2e5) + 10;
117
                                                                     using namespace std;
                                                                                                                                       const int mxlg = 17;
118
                                                                                                                                       int last special node = (int)(1e5) + 1;
                                                                     const int N = 200005;
119
       return:
                                                                                                                                       vector<int> E[mxn], F[mxn];
                                                                     vector <int> G[N];
120 }
                                                                     int low[N], depth[N];
121
                                                                                                                                       struct edg{
                                                                     bool vis[N];
   signed main(void){
122
                                                                                                                                           int fr, to;
                                                                     vector <vector <int>> bcc;
                                                                                                                                           edg(int _fr, int _to){
123
       fastio;
                                                                     stack <int> stk;
                                                                                                                                               fr = _fr;
124
125
       int t=1;
                                                                                                                                    32
                                                                                                                                               to = _to;
                                                                     void dfs(int v, int p) {
126
       cin >> t;
                                                                                                                                    33
                                                                         stk.push(v);
       while (t--){
                                                                                                                                       };
127
                                                                                                                                    34
                                                                         vis[v] = true;
           solve1();
                                                                                                                                       ostream& operator<<(ostream& os, edg x){os << x.fr << "--" <<
128
                                                                         low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
129
                                                                         for (int u : G[v]) {
                                                                                                                                       vector<edg> EV;
130
       return 0;
                                                                             if (u == p) continue;
131 }
                                                                             if (!vis[u]) {
                                                                                                                                       void tarjan(int v, int par, stack<int>& S){
                                                                                 /// (v, u) 是樹邊
                                                                                                                                           static vector<int> dfn(mxn), low(mxn);
                                                                                 dfs(u, v);
                                                                                                                                           static vector<bool> to add(mxn);
                                                                                 low[v] = min(low[v], low[u]);
                                                                                                                                           static int nowT = 0;
  5.6 Bridge BCC
                                                                                 /// u 無法在不經過父邊的情況走到 v 的祖先
                                                                                 if (low[u] >= depth[v]) {
                                                                                                                                           int childs = 0;
                                                                                                                                    43
                                                                                     bcc.emplace_back();
                                                                                                                                           nowT += 1;
 i #include <bits/stdc++.h>
                                                                                     while (stk.top() != u) {
                                                                                                                                           dfn[v] = low[v] = nowT;
   using namespace std;
                                                                                         bcc.back().push back(stk.top());
                                                                                                                                           for(auto &ne:E[v]){
                                                                                         stk.pop();
                                                                                                                                               int i = EV[ne].to;
   const int N = 200005;
                                                                                                                                               if(i == par) continue;
   vector <int> G[N];
                                                                                     bcc.back().push_back(stk.top());
                                                                                                                                               if(!dfn[i]){
   int low[N], depth[N];
                                                                                     stk.pop();
                                                                                                                                                   S.push(ne);
   bool vis[N];
                                                                                     bcc.back().push back(v);
                                                                                                                                                   tarjan(i, v, S);
   vector <vector <int>> bcc;
                                                                                                                                    52
                                                                                                                                                   childs += 1;
                                                                             } else {
   stack <int> stk:
                                                                                                                                                   low[v] = min(low[v], low[i]);
                                                                                 /// (v, u) 是回邊
                                                                                                                                    54
   void dfs(int v, int p) {
                                                                                 low[v] = min(low[v], depth[u]);
                                                                                                                                    55
                                                                                                                                                   if(par >= 0 && low[i] >= dfn[v]){
      stk.push(v);
                                                                                                                                                       vector<int> bcc;
       vis[v] = true;
                                                                                                                                                       int tmp;
                                                                                                                                    57
       low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
       for (int u : G[v]) {
                                                                                                                                                            tmp = S.top(); S.pop();
           if (u == p) continue;
                                                                                                                                                           if(!to add[EV[tmp].fr]){
           if (!vis[u]) {
                                                                                                                                                               to add[EV[tmp].fr] = true;
                                                                           圓方樹
               /// (v, u) 是樹邊
                                                                                                                                                               bcc.pb(EV[tmp].fr);
               dfs(u, v);
               low[v] = min(low[v], low[u]);
                                                                                                                                                           if(!to_add[EV[tmp].to]){
           } else {
                                                                                                                                                               to add[EV[tmp].to] = true;
                                                                     #include <bits/stdc++.h>
               /// (v, u) 是回邊
                                                                     #define lp(i,a,b) for(int i=(a);i<(b);i++)
                                                                                                                                                               bcc.pb(EV[tmp].to);
               low[v] = min(low[v], depth[u]);
                                                                     #define pii pair<int,int>
                                                                                                                                                       }while(tmp != ne);
                                                                     #define pb push back
                                                                     #define ins insert
                                                                                                                                                       for(auto &j:bcc){
       /// v 在不依靠父邊的情況下永遠沒辦法走到它的祖先
                                                                     #define ff first
                                                                                                                                                            to add[j] = false;
       if (low[v] == depth[v]) {
                                                                     #define ss second
                                                                                                                                                           F[last_special_node].pb(j);
                                                                     #define opa(x) cerr << #x << " = " << x << ", ";
                                                                                                                                                           F[j].pb(last_special_node);
           bcc.emplace back();
                                                                                                                                    72
                                                                     #define op(x) cerr << #x << " = " << x << endl;
           while (stk.top() != v) {
                                                                                                                                    73
                                                                                                                                                       last_special_node += 1;
               bcc.back().push_back(stk.top());
                                                                   10 #define ops(x) cerr << x;</pre>
                                                                   #define etr cerr << endl;</pre>
               stk.pop();
                                                                  12 #define spc cerr << ' ';
```

#define BAE(x) (x).begin(), (x).end()

qwe << ' '; cerr << endl;</pre>

14 #define STL(x) cerr << #x << " : "; for(auto &qwe:x) cerr << 78

else{

low[v] = min(low[v], dfn[i]);

bcc.back().push_back(stk.top());

stk.pop();

int fr, to, relay; cin >> fr >> to >> relay;

dep[relay] >= dep[lca(fr, to)]){

if((can_reach(fr, relay) || can_reach(to, relay)) &&

if(fr == relay || to == relay){

cout << "NO\n";

cout << "NO\n";

continue;

continue:

cout << "YES\n";</pre>

E[u].pb(EV.size());

E[v].pb(EV.size());

EV.pb(edg(u, v));

EV.pb(edg(v, u));

E[0].pb(EV.size());

EV.pb(edg(0, 1)); stack<int> S;

tarjan(0, -1, S);

lp(queries,0,q){

build lca();

147

148

149

150

151

152

153

155

156

157

158

160 161

162

163

164

165

166

167 }

```
if(dfn[i] < dfn[v]){ // edge i--v will be visited 144</pre>
                      twice at here, but we only need one.
                    S.push(ne);
       }
s6 int dep[mxn], jmp[mxn][mxlg];
   void dfs_lca(int v, int par, int depth){
       dep[v] = depth;
       for(auto &i:F[v]){
           if(i == par) continue;
           jmp[i][0] = v;
           dfs_lca(i, v, depth + 1);
   inline void build lca(){
       jmp[1][0] = 1;
       dfs_lca(1, -1, 1);
       lp(j,1,mxlg){
           lp(i,1,mxn){
               jmp[i][j] = jmp[jmp[i][j-1]][j-1];
102
103
104
105
   inline int lca(int x, int y){
       if(dep[x] < dep[y]){ swap(x, y); }</pre>
108
109
       int diff = dep[x] - dep[y];
110
       lp(j,0,mxlg){
111
           if((diff >> j) & 1){
112
               x = jmp[x][j];
113
114
       if(x == y) return x;
115
116
117
       for(int j = mxlg - 1; j >= 0; j--){
           if(jmp[x][j] != jmp[y][j]){
118
               x = jmp[x][j];
               y = jmp[y][j];
120
121
122
123
       return jmp[x][0];
124
125
   inline bool can reach(int fr, int to){
       if(dep[to] > dep[fr]) return false;
128
       int diff = dep[fr] - dep[to];
129
130
           if((diff >> j) & 1){
131
132
               fr = jmp[fr][j];
133
134
135
       return fr == to;
136
137
       ios::sync with stdio(false); cin.tie(0);
        freopen("test_input.txt", "r", stdin);
       int n, m, q; cin >> n >> m >> q;
141
       lp(i,0,m){
```

int u, v; cin >> u >> v;

5.9 Dijkstra

```
1 // 可以在 O(E Log E) 的時間複雜度解決在無負權有向圖單點源最短
  const int INF = 2e18; // 要確保 INF 開的足夠大
  vector<vector<pair<int, int>>> G(n); // G[i] = <節點, 權重>
  vector<int> dis(n, INF);
  priority_queue<pair<int, int>, vector<pair<int, int>>,
      greater<pair<int, int>>> pq;
  dis[s] = 0:
  pq.push({0, s});
  while (pq.size()){
      int now_dis = pq.top().first;
      int now node = pq.top().second;
      pq.pop();
      if (now dis>dis[now node]) continue;
      for (auto x : G[now node]){
          if (now dis+x.second<dis[x.first]){</pre>
              dis[x.first] = now_dis+x.second;
              pq.push({x.first, dis[x.first]});
21
22
```

5.10 SCC 與縮點

```
2 給定一個有向圖, 迴回傳縮點後的圖、SCC 的資訊
3 所有點都以 based-0 編號
```

63

64

```
5 函式:
6 SCC compress G(n): 宣告一個有 n 個點的圖
7 .add edge(u, v): 加上一條邊 u -> v
8 .compress: O(n log n) 計算 G3、SCC、SCC_id 的資訊,並把縮點後
       的結果存在 result 裡
10 | SCC[i] = 某個 SCC 中的所有點
|| SCC id[i] = 第 i 個點在第幾個 SCC
12 */
13 // c8b146
  struct SCC compress{
      int n = 0, m = 0;
      vector<vector<int>>> G, inv_G, result;
      vector<pair<int, int>> edges;
      vector<bool> vis;
      vector<int> order;
      vector<vector<int>> SCC;
      vector<int> SCC_id;
      SCC_compress(int _n){
          n = n;
          G.resize(n);
          inv G.resize(n);
          result.resize(n):
          vis.resize(n);
          SCC id.resize(n);
      void add edge(int u, int v){
          G[u].push_back(v);
          inv G[v].push back(u);
          edges.push back({u, v});
          m++;
39
      void dfs1(vector<vector<int>> &G, int now){
          vis[now] = 1;
          for (auto x : G[now]){
              if (vis[x]==0){
                 dfs1(G, x);
          order.push back(now);
          return;
48
      void dfs2(vector<vector<int>> &G, int now){
          SCC id[now] = SCC.size()-1;
          SCC.back().push_back(now);
          vis[now] = 1;
          for (auto x : G[now]){
              if (vis[x]==0){
                 dfs2(G, x);
60
61
          return;
      }
62
      void compress(){
          fill(vis.begin(), vis.end(), 0);
          for (int i=0 ; i<n ; i++){</pre>
```

```
if (vis[i]==0){
                                                                  int flow(int s, int t){
        dfs1(G, i);
                                                                      int ans = 0;
                                                                      while (true){
                                                                          fill(dis.begin(), dis.end(), INF);
                                                                          queue<int> q;
fill(vis.begin(), vis.end(), 0);
                                                                          q.push(s);
reverse(order.begin(), order.end());
                                                                          dis[s] = 0;
for (int i=0 ; i<n ; i++){</pre>
    if (vis[order[i]]==0){
                                                                          while (q.size()){
        SCC.push_back(vector<int>());
                                                                               int u = q.front(); q.pop();
                                                                              for (auto [v, rc, rid] : G[u]){
   if (rc<=0 || dis[v]<INF) continue;</pre>
        dfs2(inv G, order[i]);
    }
}
                                                                                   dis[v] = dis[u]+1;
                                                                                   q.push(v);
for (int i=0 ; i<m ; i++){</pre>
    if (SCC id[edges[i].first]!=SCC id[edges[i].
         second1){
                                                                          if (dis[t]==INF) break;
        result[SCC_id[edges[i].first]].push_back(
             SCC_id[edges[i].second]);
                                                                          fill(it.begin(), it.end(), 0);
                                                                          while (true){
                                                                               int df = dfs(s, t, INF);
for (int i=0 ; i<SCC.size() ; i++){</pre>
                                                                              if (df<=0) break;</pre>
    sort(result[i].begin(), result[i].end());
                                                                              ans += df;
    result[i].resize(unique(result[i].begin(), result
         [i].end())-result[i].begin());
                                                                      return ans;
                                                           60 };
```

5.11 Dinic

```
1 // 時間複雜度: O(V^2E)
2 struct Flow{
     struct Edge{
         int v, rc, rid;
     vector<vector<Edge>> G;
     void add(int u, int v, int c){
         G[u].push_back({v, c, G[v].size()});
         G[v].push_back({u, 0, G[u].size()-1});
     vector<int> dis, it;
     Flow(int n){
         G.resize(n):
         dis.resize(n);
         it.resize(n);
     int dfs(int u, int t, int f){
         if (u==t || f==0) return f;
         for (int &i=it[u] ; i<G[u].size() ; i++){</pre>
             auto &[v, rc, rid] = G[u][i];
             if (dis[v]!=dis[u]+1) continue;
             int df = dfs(v, t, min(f, rc));
             if (df<=0) continue;</pre>
             rc -= df;
             G[v][rid].rc += df;
             return df;
         return 0;
```

6 Math

6.1 Burnside's-Lemma

$$\sum_{k=1}^{n} \frac{c(k)}{n}$$

- n:有多少種置換方式(例如:旋轉方式)
- c(k): 所有可能中,經過 k 次旋轉後,仍不會和別人相同的方式的數量

6.2 線性篩

```
}
```

6.3 Lucas's-Theorem

```
1 // 對於很大的 C^n_{m} 對質數 p 取模·只要 p 不大就可以用。
2 int Lucas(int n, int m, int p){
3     if (m==0) return 1;
     return (C(n%p, m%p, p)*Lucas(n/p, m/p, p)%p);
}
```

6.4 Matrix

```
| struct Matrix{
       int n, m;
       vector<vector<int>> arr;
       Matrix(int n, int m){
           n = _n;
           m = _m;
           arr.resize(n, vector<int>(m));
       Matrix operator * (const Matrix B){
           Matrix ret(n, B.m);
           for (int i=0 ; i<n ; i++){</pre>
                for (int j=0; j<B.m; j++){
                    for (int k=0; k<m; k++){</pre>
                        ret.arr[i][j] += arr[i][k]*B.arr[k][j];
ret.arr[i][j] %= MOD;
20
22
23
           return ret;
24
25 };
```

6.5 最大質因數

```
void max_fac(int n, int &ret){
    if (n<=ret || n<2) return;
    if (isprime(n)){
        ret = max(ret, n);
        return;
}

int p = Pollard_Rho(n);
max_fac(p, ret), max_fac(n/p, ret);
</pre>
```

6.6 中國剩餘定理(m 互質)

```
1 vector<int> a, m;
  int extgcd(int a, int b, int &x, int &y){
      if (b==0){
          x=1, y=0;
          return a;
      int ret=extgcd(b, a%b, y, x);
      v-=a/b*x:
      return ret;
14 // n = 有幾個式子·求解 x \equiv a i \bmod m i
int CRT(int n, vector<int> &a, vector<int> &m){
      int p=1, ans=0;
      vector<int> M(n), inv_M(n);
      for (int i=0 ; i<n ; i++) p*=m[i];</pre>
      for (int i=0 ; i<n ; i++){</pre>
          M[i]=p/m[i];
          extgcd(M[i], m[i], inv_M[i], tmp);
          ans+=a[i]*inv_M[i]*M[i];
          ans%=p:
      }
      return (ans%p+p)%p;
```

6.7 中國剩餘定理(m 不互質)

```
i int extgcd(int a, int b, int &x, int &y){
     if (b==0){
         x=1, y=0;
          return a;
     int ret=extgcd(b, a%b, y, x);
     y-=a/b*x;
      return ret;
12 // 對於方程組的式子兩兩求解
13 // {是否有解, {a, m}}
14 pair < bool, pair < int, int >> CRT(int a1, int m1, int a2, int m2
      int g=__gcd(m1, m2);
     if ((a2-a1)%g!=0) return {0, {-1, -1}};
     extgcd(m1, m2, x, y);
     x=(a2-a1)*x/g; // 兩者不能相反
     a1=x*m1+a1;
     m1=m1*m2/g;
     a1=(a1%m1+m1)%m1;
     return {1, {a1, m1}};
```

6.8 歐拉公式

```
1 / / phi(n) = 小於 n 並與 n 互質的正整數數量。
2 // O(sqrt(n)) · 回傳 phi(n)
3 int phi(int n){
     int ret = n;
     for (int i=2; i*i<=n; i++){</pre>
         if (n%i==0){
              while (n%i==0) n /= i;
              ret = ret*(i-1)/i;
     if (n>1) ret = ret*(n-1)/n;
      return ret;
 // O(n log n) · 回傳 1~n 的 phi 值
 vector<int> phi 1 to n(int n){
     vector<int> phi(n+1);
      phi[0]=0;
      phi[1]=1;
      for (int i=2 ; i<=n ; i++){</pre>
         phi[i]=i-1;
      for (int i=2 ; i<=n ; i++){</pre>
         for (int j=2*i; j<=n; j+=i){ // 枚舉所有倍數
             phi[j]-=phi[i];
     }
      return phi;
```

6.9 卡特蘭數

任意括號序列: $C_n = \frac{1}{n+1} {2n \choose n}$

6.10 歐拉定理

```
若 a,m 互質 \cdot 則: a^n \bmod m = a^{n \bmod \varphi(m)} \bmod m 若 a,m 可能是任何數 \cdot 則: a^{\varphi(m)+[n \bmod \varphi(m)]} \bmod m
```

6.11 Fraction

```
#include <bits/stdc++.h>
using namespace std;
#include <br/>
```

```
4 /// Fraction template starts ///
  #define fraction template bonus check
  const long long ll overflow warning value = (long long)(3e9);
  long long gcd(long long a, long long b){
      if(a == 0) return 0:
      if(b == 0) return a;
      if(a < b) return gcd(b,a);</pre>
      return gcd(b, a%b);
13 }
14 struct frac{
      long long a, b;
      frac(long long _a = 0, long long _b = 1){
          a = a; b = b;
          if(b == 0){
               cerr << "Error: division by zero\n";</pre>
               cerr << "Called : Constructor(" << a << ", " <<</pre>
                    _b << ")\n";
              return:
          if(a == 0){b = 1; return;}
          if(b < 0){a = -a; b = -b;}
          long long gcd_ab = gcd(std::abs(a), b);
          if(gcd_ab != 1){a /= gcd_ab; b /= gcd_ab;}
          #ifdef fraction_template_bonus_check
          if(std::abs(a) > 11 overflow warning value || b >
               11_overflow_warning_value){
               cerr << "Overflow warning: " << a << "/" << b <<
                     " \setminus n"; 
          #endif // fraction_template_bonus_check
      frac operator+(frac const &B){
33
          return frac(a*(B.b)+(B.a)*b, b*(B.b));}
      frac operator-(frac const &B){
          return frac(a*(B.b)-(B.a)*b, b*(B.b));}
      frac operator*(frac const &B){
          return frac(a*(B.a), b*(B.b));}
      frac operator/(frac const &B){
          return frac(a*(B.b), b*(B.a));}
      frac operator+=(frac const &B){
          *this = frac(a*(B.b)+(B.a)*b, b*(B.b));}
      frac operator -= (frac const &B){
          *this = frac(a*(B.b)-(B.a)*b, b*(B.b));}
      frac operator*=(frac const &B){
          *this = frac(a*(B.a), b*(B.b));}
      frac operator/=(frac const &B){
          *this = frac(a*(B.b), b*(B.a));}
      frac abs(){
          a = std::abs(a);
53
          return *this;
      bool operator<(frac const &B){</pre>
          return a*B.b < B.a*b;}</pre>
      bool operator <= (frac const &B){</pre>
          return a*B.b <= B.a*b;}</pre>
      bool operator>(frac const &B){
          return a*B.b > B.a*b;}
      bool operator>=(frac const &B){
          return a*B.b >= B.a*b:}
      bool operator == (frac const &B){
          return a * B.b == B.a * b:}
      bool operator!=(frac const &B){
```

```
return a * B.b != B.a * b;}
68 };
69 ostream& operator<<(ostream &os, const frac& A){
        os << A.a << "/" << A.b;
        return os;
73 /// Fraction template ends ///
75 void test(frac A, frac B){
        cout << "A = " << A << endl;
        cout << "B = " << B << endl;
        cout << endl;</pre>
        cout \langle\langle "A + B = " \langle\langle A + B \rangle\langle\langle endl;
        cout \langle\langle "A - B = " \langle\langle A - B \langle\langle endl;
        cout \langle\langle "A * B = " \langle\langle A * B \langle\langle endl;
        cout << "A / B = " << A / B << endl;
        cout << endl:</pre>
        cout \langle\langle "(A \langle B) = " \langle\langle (A \langle B) \langle\langle endl;
        cout \langle\langle "(A \langle = B) = " \langle\langle (A \langle = B) \rangle\langle\langle endl;
        cout \langle\langle "(A > B)' = " \langle\langle (A > B) \langle\langle endl;
        cout \langle\langle "(A \rangle = B) = " \langle\langle (A \rangle = B) \langle\langle endl;
        cout << "(A == B) =" << (A == B) << endl;
        cout \langle\langle "(A != B) = " \langle\langle (A != B) \rangle\langle\langle endl;
        cout << "----\n":
        return;
   int main(){
        frac tmp1(-7, 2);
        frac tmp2(5, 3);
        test(tmp1, tmp2);
        frac tmp3(-7);
        frac tmp4(0);
        test(tmp3, tmp4);
        return 0;
```

6.12 錯排公式

錯排公式: (n 個人中,每個人皆不再原來位置的組合數)

$$dp_i = \begin{cases} 1 & i = 0\\ 0 & i = 1\\ (i-1)(dp_{i-1} + dp_{i-2}) & \text{otherwise} \end{cases}$$

6.13 Quick-Pow

```
int qp(int b, int p, int m = MOD){
   int ret = 1;
   for (; p; p>>=1){
       if (p&1) ret = ret*b%m;
       b = b*b%m;
   }
   return ret;
   }
}
```

6.14 二元一次方程式

```
\begin{cases} ax+by=c\\ dx+ey=f\\ \end{cases} = \begin{cases} x=\frac{ed-bf}{ad-bc}\\ y=\frac{af-ec}{ad-bc}\\ \end{cases} 若 x=\frac{0}{0} 且 y=\frac{0}{0} · 則代表無限多組解。若 x=\frac{*}{0} 且 y=\frac{*}{0} · 則代表無 24 \end{cases} 25
```

6.15 Josephus

6.16 數論分塊

6.17 Miller-Rabin

```
20 | vector<int> 11sprp = {2, 325, 9375, 28178, 450775, 9780504,
       1795265022};
  bool isprime(int n, vector(int) sprp = llsprp){
      if (n==2) return 1;
      if (n<2 || n%2==0) return 0;
      int t = 0;
      int u = n-1:
      for ( ; u%2==0 ; t++) u>>=1;
      for (int i=0 ; i<sprp.size() ; i++){</pre>
          int a = sprp[i]%n;
          if (a==0 || a==1 || a==n-1) continue;
          int x = qp(a, u, n);
          if (x==1 || x==n-1) continue;
          for (int j=0 ; j<t ; j++){</pre>
              x = modmul(x, x, n);
               if (x==1) return 0;
               if (x==n-1) break;
          if (x==n-1) continue;
          return 0;
41
      }
42
43
      return 1;
```

6.18 Pollard-Rho

```
nt19937 seed(chrono::steady clock::now().time since epoch().
       count());
  int rnd(int 1, int r){
      return uniform int distribution<int>(1, r)(seed);
6 // O(n^{1/4}) 回傳 1 或自己的因數、記得先判斷 n 是不是質數
       (用 Miller-Rabin)
  // c1670c
  int Pollard_Rho(int n){
      int s = 0, t = 0;
      int c = rnd(1, n-1);
      int step = 0, goal = 1;
      int val = 1:
      for (goal=1 ; ; goal<<=1, s=t, val=1){</pre>
          for (step=1 ; step<=goal ; step++){</pre>
              t = ((\_int128)t*t+c)%n;
              val = (__int128)val*abs(t-s)%n;
              if ((step % 127) == 0){
                  int d = __gcd(val, n);
                  if (d>1) return d;
23
24
          int d = __gcd(val, n);
27
          if (d>1) return d;
29
30 }
```

7 Misc

7.1 Xor-Basis

```
vector<int> basis:
void add vector(int x){
     for (auto v : basis){
         x=min(x, x^v);
     if (x) basis.push_back(x);
9 | // 給一數字集合 S · 求能不能 XOR 出 x
10 bool check(int x){
     for (auto v : basis){
         x=min(x, x^v);
     return x;
17 // 給一數字集合 S, 求能 XOR 出多少數字
18 // 答案等於 2^{basis 的大小}
20 | // 給一數字集合 S · 求 XOR 出最大的數字
21 int get_max(){
     int ans=0;
     for (auto v : basis){
         ans=max(ans, ans^v);
     return ans;
```

7.2 Default-Code

```
i #include <bits/stdc++.h>
 #define int long long
  #define fastio ios::sync with stdio(0), cin.tie(0);
  using namespace std;
 #ifdef LOCAL
  #define cout cout << "\033[0;32m"
  #define cerr cerr << "\033[0;31m"
 #define endl "\n" << "\033[0m"
#pragma GCC optimize("03,unroll-loops")
12 #pragma GCC target("avx,avx2,sse,sse2,sse3,sse4,popcnt")
13 #define endl "\n"
14 #endif
 const int MAX N = 5e5+10;
  const int INF = 2e18;
19 void solve1(){
      return:
  signed main(){
      fastio;
```

```
27

28

29

30

31

32

33

return 0;
```

7.3 Radix-Sort

```
1 // 值域限制:0~1073741823(2^30-1)
  inline void radix_sort(vector<int> &a, int n){
      static int cnt[32768] = {0};
       vector<int> tmpa(n);
       for(int i = 0; i < n; ++i)</pre>
          ++cnt[a[i] & 32767];
       for(int i = 1; i < 32768; ++i)</pre>
          cnt[i] += cnt[i-1];
       static int temp;
       for(int i = n-1; i >= 0; --i){
          temp = a[i] & 32767;
           --cnt[temp];
           tmpa[cnt[temp]] = a[i];
       static int cnt2[32768] = {0};
       for(int i = 0; i < n; ++i)
           ++cnt2[(tmpa[i]>>15)];
       for(int i = 1; i < 32768; ++i)
          cnt2[i] += cnt2[i-1];
       for(int i = n-1; i >= 0; --i){
          temp = (tmpa[i]>>15);
           --cnt2[temp];
          a[cnt2[temp]] = tmpa[i];
27
       return;
```

7.4 Set-Pq-Sort

```
1  // priority_queue
2  struct cmp{
      bool operator () (Data a, Data b){
          return a.x<b.x;
      }
6  };
7  priority_queue<Data, vector<Data>, cmp> pq;
8
9  // set
10  struct Data{
      int x;
12
13  bool operator < (const Data &b){
          return x<b.x;
      }
16  };
</pre>
```

7.5 2-SAT

```
| #include <bits/stdc++.h>
  using namespace std;
  struct TWO_SAT {
      int n, N;
      vector<vector<int>> G, rev_G;
      deque<bool> used;
      vector<int> order, comp;
      deque<bool> assignment;
      void init(int n) {
          n = _n;
          N = n * 2;
          G.resize(N + 5);
          rev_G.resize(N + 5);
      void dfs1(int v) {
          used[v] = true;
          for (int u : G[v]) {
              if (!used[u])
                  dfs1(u);
          order.push_back(v);
23
24
      void dfs2(int v, int cl) {
          comp[v] = cl;
          for (int u : rev G[v]) {
              if (comp[u] == -1)
                  dfs2(u, c1);
      bool solve() {
          order.clear();
          used.assign(N, false);
          for (int i = 0; i < N; ++i) {
               if (!used[i])
                  dfs1(i);
          comp.assign(N, -1);
          for (int i = 0, j = 0; i < N; ++i) {
               int v = order[N - i - 1];
               if (comp[v] == -1)
                  dfs2(v, j++);
          assignment.assign(n, false);
          for (int i = 0; i < N; i += 2) {
               if (comp[i] == comp[i + 1])
                  return false;
               assignment[i / 2] = (comp[i] > comp[i + 1]);
50
          return true;
51
52
      void add_disjunction(int a, bool na, int b, bool nb) { //
          // na means whether a is negative or not
          // nb means whether b is negative or not
          a = 2 * a ^ na;
          b = 2 * b ^ nb;
          int neg_a = a ^ 1;
          int neg_b = b ^ 1;
58
          G[neg_a].push_back(b);
59
          G[neg_b].push_back(a);
60
          rev G[b].push back(neg a);
          rev_G[a].push_back(neg_b);
```

```
return;
       void get result(vector<int>& res) {
           res.clear();
           for (int i = 0; i < n; i++)
               res.push_back(assignment[i]);
71 /* CSES Giant Pizza
72 3 5
|73| + 1 + 2
|74| - 1 + 3
   - + + + -
   */
78 int main() {
       int n, m;
       cin >> n >> m;
       TWO SAT E;
       E.init(m);
       char c1, c2;
       int inp1, inp2;
       for (int i = 0; i < n; i++) {</pre>
           cin >> c1 >> inp1;
           cin >> c2 >> inp2;
           E.add disjunction(inp1 - 1, c1 == '-', inp2 - 1, c2
                == '-');
       bool able = E.solve();
       if (able) {
           vector <int> ans;
           E.get_result(ans);
           for (int i : ans)
               cout << (i == true ? '+' : '-') << ' ';
           cout << ' \ n';
       } else {
           cout << "IMPOSSIBLE\n";</pre>
101
102
103
       return 0;
```

7.6 Enumerate-Subset

```
ı | // 時間複雜度 0(3^n)
2 1 / / 枚舉每個 mask 的子集
3 for (int mask=0; mask<(1<<n); mask++){</pre>
     for (int s=mask ; s>=0 ; s=(s-1)&m){
         // s 是 mask 的子集
         if (s==0) break;
```

7.7 Fast-Input

```
ı // fast IO
2 // 6f8879
```

```
3 inline char readchar(){
      static char buffer[BUFSIZ], * now = buffer + BUFSIZ, *
           end = buffer + BUFSIZ:
      if (now == end)
                                                                   19
                                                                   20 }
          if (end < buffer + BUFSIZ)</pre>
              return EOF;
                                                                   22 int hash value(int 1, int r){ // 取得 s[l..r] 的數值
          end = (buffer + fread(buffer, 1, BUFSIZ, stdin));
          now = buffer;
                                                                   25
      return *now++;
 inline int nextint(){
      int x = 0, c = readchar(), neg = false;
      while (('0' > c | c > '9') \&\& c! = '-' \&\& c! = EOF) c =
           readchar():
      if(c == '-') neg = true, c = readchar();
      while ('0' \le c \&\& c \le '9') x = (x << 3) + (x << 1) + (c^{0})
           , c = readchar();
      if(neg) x = -x;
      return x; // returns 0 if EOF
```

7.8 setup

```
se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
:inoremap " ""<Esc>i
:inoremap {<CR> {<CR>}<Esc>ko
:inoremap {{ {}}<ESC>i
map <F9> :w<bar>!g++ "%" -o %:r -std=c++17 -Wall -Wextra -
    Wshadow -O2 -DLOCAL -g -fsanitize=undefined,address<CR>
map <F8> :!./%:r<CR>
ca hash w !cpp -dD -P -fpreprocessed \| tr -d "[:space:]" \|
    md5sum \| cut -c-6
```

String

8.1 Hash

```
i mt19937 rnd(chrono::steady clock::now().time since epoch().
      count());
 int A = rnd(), B = 1000000007;
 vector<int> myPow, myPre;
 void hash init(string s){
     myPow.resize(s.size());
     myPre.resize(s.size());
     for (int i=0 ; i<s.size() ; i++){</pre>
          if (i==0){
              myPow[i] = 1;
              myPre[i] = s[i];
          }else{
              myPow[i] = myPow[i-1]*A%B;
              myPre[i] = (myPre[i-1]*A+s[i])%B;
```

8.2 Manacher

if (l==0) return myPre[r];

return ((myPre[r]-myPre[l-1]*myPow[r-l+1])%B+B)%B;

return;

```
i string Manacher(string str) {
      string tmp = "$#";
      for(char i : str) {
          tmp += i;
          tmp += '#';
      vector<int> p(tmp.size(), 0);
      int mx = 0, id = 0, len = 0, center = 0;
      for(int i=1; i<(int)tmp.size(); i++) {</pre>
          p[i] = mx > i ? min(p[id*2-i], mx-i) : 1;
12
          while(tmp[i+p[i]] == tmp[i-p[i]]) p[i]++;
13
          if(mx < i+p[i]) mx = i+p[i], id = i;
          if(len<p[i]) len = p[i], center = i;</pre>
      return str.substr((center-len)/2, len-1);
```

8.3 KMP

```
ı | // 給一個字串 S·定義函數 \pi(i) = k 代表 S[1 ... k] = S[i-k
      +1 ... i]
  // 61e853
 vector<int> KMP(string &s){
      int n = s.size();
      vector<int> ret(n);
      for (int i=1 ; i<n ; i++){</pre>
          int j = ret[i-1];
          while (j>0 && s[i]!=s[j]){
              j = ret[i-1];
          if (s[i]==s[j]) j++;
          ret[i] = j;
14
      return ret;
```

8.4 Z-Function

```
lcp.erase(lcp.begin(), lcp.begin()+2);
                                                               32
                                                               33 };
1 \mid // 定義一個長度為 n 的文本為 T · 則陣列 Z 的 Z[i] 代表 T[0:n]
        和 T[i:n] 最長共同前綴
2 // bcfbd6
vector<int> z_function(string s){
      vector<int> ret(s.size());
      int 11 = 0, rr = 0;
      for (int i=1; i<s.size(); i++){</pre>
          int j = 0;
          if (i<rr) j = min(ret[i-l1], rr-i);</pre>
          while (s[j]==s[i+j]) j++;
          ret[i] = j;
          if (i+j>rr){
             11 = i;
              rr = i+j;
      }
      ret[0] = s.size();
      return ret;
22 }
```

8.5 Suffix-Array

```
1 struct SuffixArray {
     vector<int> sa, lcp;
     SuffixArray(string& s, int lim = 256) {
          // 64418e
          int n = s.size()+1, k = 0, a, b;
          vector<int> x(s.begin(), s.end()), y(n), ws(max(n,
               lim)), rank(n);
          x.push back(0);
          sa = 1cp = y;
          iota(sa.begin(), sa.end(), 0);
          for (int j=0, p=0 ; p<n ; j=max(1LL, j*2), lim=p) {</pre>
              p = j;
              iota(y.begin(), y.end(), n-j);
              for (int i=0; i<n; i++) if (sa[i] >= j) y[p++]
              = sa[i] - j;
fill(ws.begin(), ws.end(), 0);
              for (int i=0 ; i<n ; i++) ws[x[i]]++;</pre>
              for (int i=1; i<lim; i++) ws[i] += ws[i - 1];</pre>
              for (int i = n; i--;) sa[--ws[x[y[i]]]] = y[i];
              swap(x, y), p = 1, x[sa[0]] = 0;
for (int i=1; i<n; i++){
                  a = sa[i - 1];
                  b = sa[i];
                  x[b] = (y[a] == y[b] && y[a + j] == y[b + j])
                         ? p - 1 : p++;
              }
          // b8debe
          for (int i=1; i<n; i++) rank[sa[i]] = i;</pre>
          for (int i=0, j ; i<n-1 ; lcp[rank[i++]]=k)</pre>
              for (k && k--, j=sa[rank[i]-1]; s[i+k]==s[j+k];
                    k++);
          sa.erase(sa.begin());
```

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