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# Misc

## 1.1 Note

開始寫題目之前，請做下面的事：

- 在「開始寫任何題目之前」，應該要先自己看過「所有」範例測資的邏輯和演算法是否有跟範例輸出對上
- 如果你覺得別人的某段程式碼有錯誤，就應該直接講出來
- +2~+3 後就開始生測資跟對拍（根據寫 generator 跟 checker 的時間決定）

寫程式請遵照以下原則：

- 準確使用註解分段程式碼
  - declare
  - init
  - input
  - process / queries
  - output
- 陣列若可以開到最大，則使用常數宣告大小

上傳之前，請依序檢查以下資訊：

- 是否開啟 IO 優化
- 是否有 t 筆輸入但忘了輸入
- 是否有初始化容器
- 題目範圍有沒有開到最大
- 跑過所有範例測資，並嚴格確認是否正確

## 1.2 Default Code [d9f980]

```
1 #include <bits/stdc++.h>
2 using namespace std;
3 #define int long long
4
5 void solve(){
6
7 }
8
9 signed main(){
10     ios::sync_with_stdio(0), cin.tie(0);
11
12     int t = 1;
13     while (t--){
14         solve();
15     }
16
17     return 0;
18 }
```

## 1.3 Run

```
1 from os import *
2
3 f = "pA"
4
5 while 1:
6     i = input("input: ")
7     system("clear")
8     p = listdir(".")
9     if i != "":
10         f = i
11         print(f"file = {f}")
12         if system(f"g++ {f}.cpp -std=c++17 -Wall -Wextra -Wshadow
13             -O2 -D LOCAL -g -fsanitize=undefined,address -o {f}
14             "):
15             print("CE")
16             continue
17
18         for x in sorted(p):
19             if f in x and ".in" in x:
20                 print(x)
21                 if system(f"./{f} < {x}"):
22                     print("RE")
23                 print()
```

## 1.4 Custom Set PQ Sort [2892de]

```
1 // priority_queue · 務必檢查相等的 case · 給所有元素一個排序的
2 // 依據
3 struct cmp{
4     bool operator () (Data a, Data b){
5         return a.x<b.x;
6     }
7 };
8 priority_queue<Data, vector<Data>, cmp> pq;
9
10 // set · 務必檢查相等的 case · 給所有元素一個排序的依據
11 auto cmp = [](int a, int b) {
12     return a > b;
13 };
14 set<int, decltype(cmp)> s = {1, 2, 3, 4, 5};
15 cout << *s.begin() << '\n';
```

## 1.5 Dynamic Bitset [c78aa8]

```
1 const int MAXN = 2e5 + 5;
2 template <int len = 1>
3 void solve(int n) {
4     if (n > len) {
5         solve<min(len*2, MAXN)>(n);
6         return;
7     }
8     bitset<len> a;
9 }
```

## 1.6 Enumerate Subset [a13e46]

```
1 // 時間複雜度  $O(3^n)$ 
2 // 枚舉每個 mask 的子集
3 for (int mask=0; mask<(1<<n); mask++){
4     for (int s=mask; s>=0; s=(s-1)&m){
5         // s 是 mask 的子集
6         if (s==0) break;
7     }
8 }
```

## 1.7 Fast Input [6f8879]

```
1 // fast IO
2 // 6f8879
3 inline char readchar(){
4     static char buffer[BUFSIZ], *now = buffer + BUFSIZ, *
5     end = buffer + BUFSIZ;
6     if (now == end)
7     {
8         if (end < buffer + BUFSIZ)
9             return EOF;
10        end = (buffer + fread(buffer, 1, BUFSIZ, stdin));
11        now = buffer;
12    }
13    return *now++;
14 }
15 inline int nextint(){
16     int x = 0, c = readchar(), neg = false;
17     while(('0' > c || c > '9') && c!='-' && c!=EOF) c =
18         readchar();
19     if(c == '-') neg = true, c = readchar();
20     while('0' <= c && c <= '9') x = (x<<3) + (x<<1) + (c^'0')
21         , c = readchar();
22     if(neg) x = -x;
23     return x; // returns 0 if EOF
```

## 1.8 OEIS [f915c2]

```
1 // 若一個線性遞迴有 k 項，給他恰好 2*k 個項可以求出線性遞迴
2 // f915c2
3 template <typename T>
4 vector<T> BerlekampMassey(vector<T> a) {
5     auto scalarProduct = [](vector<T> v, T c) {
6         for (T &x: v) x *= c;
7         return v;
8     };
9     vector<T> s, best;
10     int bestPos = 0;
11     for (size_t i = 0; i < a.size(); i++) {
12         T error = a[i];
13         for (size_t j = 0; j < s.size(); j++) error -= s[j] *
14             a[i-1-j];
15         if (error == 0) continue;
16         if (s.empty()) {
17             s.resize(i + 1);
18             bestPos = i;
19         }
```

```

18         best.push_back(1 / error);
19         continue;
20     }
21     vector<T> fix = scalarProduct(best, error);
22     fix.insert(fix.begin(), i - bestPos - 1, 0);
23     if (fix.size() >= s.size()) {
24         best = scalarProduct(s, - 1 / error);
25         best.insert(best.begin(), 1 / error);
26         bestPos = i;
27         s.resize(fix.size());
28     }
29     for (size_t j = 0; j < fix.size(); j++)
30         s[j] += fix[j];
31 }
32 return s;
33 }

```

## 1.9 Pragma [09d13e]

```

1 #pragma GCC optimize("O3,unroll-loops")
2 #pragma GCC target("avx,avx2,sse,sse2,sse3,sse4,popcnt")

```

## 1.10 Xor Basis [840136]

```

1 vector<int> basis;
2 void add_vector(int x){
3     for (auto v : basis){
4         x=min(x, x^v);
5     }
6     if (x) basis.push_back(x);
7 }
8
9 // 給一數字集合 S · 求能不能 XOR 出 x
10 bool check(int x){
11     for (auto v : basis){
12         x=min(x, x^v);
13     }
14     return 0;
15 }
16
17 // 給一數字集合 S · 求能 XOR 出多少數字
18 // 答案等於 2^{basis 的大小}
19
20 // 給一數字集合 S · 求 XOR 出最大的數字
21 int get_max(){
22     int ans=0;
23     for (auto v : basis){
24         ans=max(ans, ans^v);
25     }
26     return ans;
27 }

```

## 1.11 random int [9cc603]

```

1 mt19937 seed(chrono::steady_clock::now().time_since_epoch().
2     count());
3 int rng(int l, int r){
4     return uniform_int_distribution<int>(l, r)(seed);
5 }

```

## 1.12 Python

```

1 # system setting
2 sys.setrecursionlimit(100000)
3 sys.set_int_max_str_digits(10000)
4
5 # turtle
6 from turtle import *
7
8 N = 3000000010
9 setworldcoordinates(-N, -N, N, N)
10 hideturtle()
11 speed(100)
12
13 def draw_line(a, b, c, d):
14     teleport(a, b)
15     goto(c, d)
16
17 def write_dot(x, y, text, diff=1): # diff = 文字的偏移
18     teleport(x, y)
19     dot(5, "red")
20
21     teleport(x+N/100*diff, y+N/100*diff)
22     write(text, font=("Arial", 5, "bold"))
23
24 # usage
25 draw_line(*a[i], *(a[i-1]))
26 write_dot(*a[i], str(a[i]))

```

## 1.13 diff

```

1 set -e
2 g++ ac.cpp -o ac
3 g++ wa.cpp -o wa
4 for ((i=0;;i++))
5 do
6     echo "$i"
7     python3 gen.py > input
8     ./ac < input > ac.out
9     ./wa < input > wa.out
10    diff ac.out wa.out || break
11 done

```

## 1.14 hash command

```

1 cat file.cpp | cpp -dD -P -fpreprocessed | tr -d "[:space:]"
2 | md5sum | cut -c-6

```

## 1.15 setup

```

1 se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
2
3 :inoremap " ""<Esc>i
4 :inoremap {<CR> {<CR><Esc>ko
5 :inoremap {{ {}<ESC>i
6
7 function! F(...)
8     execute '!./%:r < ./' . a:1
9 endfunction
10 command! -nargs=* R call F(<f-args>)
11
12 map <F7> :w<bar>!g++ "%" -o %:r -std=c++17 -Wall -Wextra -
13     Wshadow -O2 -DLOCAL -g -fsanitize=undefined,address<CR>
14 map <F8> :!./%:r<CR>
15 map <F9> :!./%:r < ./%:r.in<CR>
16
17 ca hash w !cpp -dD -P -fpreprocessed \ | tr -d "[:space:]" \ |
18     md5sum \ | cut -c-6
19
20 " i+<esc>25A---+<esc>
21 " o|<esc>25A |<esc>
22 " "ggVGyG35pGdd

```

## 2 Convolution

### 2.1 FFT any mod [234f9e]

```

1 /*
2 修改 const int MOD = 998244353 更改要取餘的數字
3 PolyMul(a, b) 回傳多項式乘法的結果 (c_k = \sum_{i+j=k} a_i b_j
4     mod MOD)
5
6 大約可以支援 5e5 · a_i, b_i 皆在 MOD 以下的非負整數
7 */
8 const int MOD = 998244353;
9 typedef complex<double> cd;
10
11 // b9c90a
12 void FFT(vector<cd> &a) {
13     int n = a.size(), L = 31-__builtin_clz(n);
14     vector<complex<long double>> R(2, 1);
15     vector<cd> rt(2, 1);
16     for (int k=2; k<n; k*=2){
17         R.resize(n);
18         rt.resize(n);
19         auto x = polar(1.0L, acos(-1.0L) / k);
20         for (int i=k; i<2*k; i++){
21             rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
22         }
23     }
24
25     vector<int> rev(n);
26     for (int i=0; i<n; i++){
27         rev[i] = (rev[i/2] | (i&1)<<L)/2;
28     }
29     for (int i=0; i<n; i++){
30         if (i<rev[i]) swap(a[i], a[rev[i]]);
31     }
32 }

```

```

30 }
31 for (int k=1 ; k<n ; k*=2){
32     for (int i=0 ; i<n ; i+=2*k){
33         for (int j=0 ; j<k ; j++){
34             auto x = (double *)&rt[j+k];
35             auto y = (double *)&a[i+j+k];
36             cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*
37                 y[0]);
38             a[i+j+k] = a[i+j]-z;
39             a[i+j] += z;
40         }
41     }
42     return;
43 }
44 // d3c65e
45 vector<int> PolyMul(vector<int> a, vector<int> b){
46     if (a.empty() || b.empty()) return {};
47     vector<int> res(a.size()+b.size()-1);
48     int B = 32-__builtin_clz(res.size()), n = (1<<B), cut =
49         int(sqrt(MOD));
50     vector<cd> L(n), R(n), outs(n), outl(n);
51     for (int i=0 ; i<a.size() ; i++){
52         L[i] = cd((int) a[i]/cut, (int)a[i]%cut);
53     }
54     for (int i=0 ; i<b.size() ; i++){
55         R[i] = cd((int) b[i]/cut, (int)b[i]%cut);
56     }
57     FFT(L);
58     FFT(R);
59     for (int i=0 ; i<n ; i++){
60         int j = -i&(n-1);
61         outl[j] = (L[i]+conj(L[j])) * R[i]/(2.0*n);
62         outs[j] = (L[i]-conj(L[j])) * R[i]/(2.0*n)/1i;
63     }
64     FFT(outl);
65     FFT(outs);
66     for (int i=0 ; i<res.size() ; i++){
67         int av = (int)(real(outl[i])+0.5), cv = (int)(imag(
68             outs[i])+0.5);
69         int bv = (int)(imag(outl[i])+0.5) + (int)(real(outs[i]
70             )+0.5);
71         res[i] = ((av%MOD*cut+bv) % MOD*cut+cv) % MOD;
72     }
73     return res;
74 }
75 }

```

## 2.2 FFT new [c95bb8]

```

1 typedef complex<double> cd;
2
3 // b9c90a
4 void FFT(vector<cd> &a) {
5     int n = a.size(), L = 31-__builtin_clz(n);
6     vector<complex<long double>> R(2, 1);
7     vector<cd> rt(2, 1);
8     for (int k=2 ; k<n ; k*=2){
9         R.resize(n);
10        rt.resize(n);

```

```

11        auto x = polar(1.0L, acos(-1.0L) / k);
12        for (int i=k ; i<2*k ; i++){
13            rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
14        }
15    }
16    vector<int> rev(n);
17    for (int i=0 ; i<n ; i++){
18        rev[i] = (rev[i/2] | (i&1)<<L)/2;
19    }
20    for (int i=0 ; i<n ; i++){
21        if (i<rev[i]) swap(a[i], a[rev[i]]);
22    }
23    for (int k=1 ; k<n ; k*=2){
24        for (int i=0 ; i<n ; i+=2*k){
25            for (int j=0 ; j<k ; j++){
26                auto x = (double *)&rt[j+k];
27                auto y = (double *)&a[i+j+k];
28                cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*
29                    y[0]);
30                a[i+j+k] = a[i+j]-z;
31                a[i+j] += z;
32            }
33        }
34    }
35    return;
36 }
37 // 39029d
38 vector<double> PolyMul(const vector<double> a, const vector<
39     double> b){
40     if (a.empty() || b.empty()) return {};
41     vector<double> res(a.size()+b.size()-1);
42     int L = 32 - __builtin_clz(res.size()), n = 1 << L;
43     vector<cd> in(n), out(n);
44     copy(a.begin(), a.end(), begin(in));
45     for (int i=0 ; i<b.size() ; i++){
46         in[i].imag(b[i]);
47     }
48     FFT(in);
49     for (cd& x : in) x *= x;
50     for (int i=0 ; i<n ; i++){
51         out[i] = in[-i & (n - 1)] - conj(in[i]);
52     }
53     FFT(out);
54     for (int i=0 ; i<res.size() ; i++){
55         res[i] = imag(out[i]) / (4 * n);
56     }
57     return res;
58 }
59
60
61 }

```

## 2.3 FFT short [70c01a]

```

1 #define int long long
2
3 using Cplx = complex<double>;
4 const double pi = acos(-1);
5 const int mod = 998244353, g = 3;
6 int power(int a, int b) {
7     int res = 1;

```

```

8     while (b) {
9         if (b & 1) res = res * a % mod;
10        a = a * a % mod;
11        b >>= 1;
12    }
13    return res;
14 }
15 int inv(int x) { return power(x, mod - 2); }
16 // FFT use Cplx, NTT use LL
17 void FFT(vector<int> &a, int n, int op) {
18     // n must be 2^k
19     vector<int> R(n);
20     FOR (i, 0, n - 1)
21         R[i] = R[i/2]/2 + (i&1)*(n/2);
22     FOR (i, 0, n - 1)
23         if (i < R[i]) swap(a[i], a[R[i]]);
24     for (int m = 2; m <= n; m *= 2) {
25         // Cplx w1(cos(2*pi/m), sin(2*pi/m)*op);
26         int w1 = power(g, (mod-1)/m * op + mod-1);
27         for (int i = 0; i < n; i += m) {
28             // Cplx wk(1, 0);
29             int wk = 1;
30             FOR (k, 0, m / 2 - 1) {
31                 auto x = a[i+k], y = a[i+k+m/2] * wk % mod;
32                 a[i+k] = (x+y) % mod;
33                 a[i+k+m/2] = (x-y+mod) % mod;
34                 wk = wk * w1 % mod;
35             }
36         }
37     }
38     if (op == -1)
39         FOR (i, 0, n - 1) {
40             // a[i] = a[i] / n;
41             a[i] = a[i] * inv(n) % mod;
42         }
43 }

```

## 2.4 FWT [832aa5]

```

1 // 已經把 mint 刪掉 · 需要增加註解
2 vector<int> xor_convolution(vector<int> a, vector<int> b, int
3     k) {
4     if (k == 0) {
5         return vector<int>{a[0] * b[0]};
6     }
7     vector<int> aa(1 << (k - 1)), bb(1 << (k - 1));
8     FOR (i, 0, (1 << (k - 1)) - 1) {
9         aa[i] = a[i] + a[i + (1 << (k - 1))];
10        bb[i] = b[i] + b[i + (1 << (k - 1))];
11    }
12    vector<int> X = xor_convolution(aa, bb, k - 1);
13    FOR (i, 0, (1 << (k - 1)) - 1) {
14        aa[i] = a[i] - a[i + (1 << (k - 1))];
15        bb[i] = b[i] - b[i + (1 << (k - 1))];
16    }
17    vector<int> Y = xor_convolution(aa, bb, k - 1);
18    vector<int> c(1 << k);
19    FOR (i, 0, (1 << (k - 1)) - 1) {
20        c[i] = (X[i] + Y[i]) / 2;
21        c[i + (1 << (k - 1))] = (X[i] - Y[i]) / 2;
22    }
23    return c;

```

## 2.5 Min Convolution Concave Concave [ffb28d]

```

1 // 需要增加註解
2 // min convolution
3 vector<int> mkk(vector<int> a, vector<int> b) {
4     vector<int> slope;
5     FOR (i, 1, ssize(a) - 1) slope.pb(a[i] - a[i - 1]);
6     FOR (i, 1, ssize(b) - 1) slope.pb(b[i] - b[i - 1]);
7     sort(all(slope));
8     slope.insert(begin(slope), a[0] + b[0]);
9     partial_sum(all(slope), begin(slope));
10    return slope;
11 }

```

## 2.6 NTT mod 998244353 [5c6335]

```

1 const int MOD = (119 << 23) + 1, ROOT = 62; // = 998244353
2 // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
3 // and 483 << 21 (same root). The last two are > 10^9.
4 // 9cd58a
5 void NTT(vector<int> &a) {
6     int n = a.size();
7     int L = 31 - __builtin_clz(n);
8     vector<int> rt(2, 1);
9     for (int k=2, s=2; k<n; k*=2, s++){
10        rt.resize(n);
11        int z[] = {1, qp(ROOT, MOD>>s)};
12        for (int i=k; i<2*k; i++){
13            rt[i] = rt[i/2]*z[i&1]%MOD;
14        }
15    }
16 }
17 vector<int> rev(n);
18 for (int i=0; i<n; i++){
19     rev[i] = (rev[i/2] | (i&1)<<L)/2;
20 }
21 for (int i=0; i<n; i++){
22     if (i<rev[i]){
23         swap(a[i], a[rev[i]]);
24     }
25 }
26 }
27 for (int k=1; k<n; k*=2){
28     for (int i=0; i<n; i+=2*k){
29         for (int j=0; j<k; j++){
30             int z = rt[j+k]*a[i+j+k]%MOD, &ai = a[i+j];
31             a[i+j+k] = ai-z+(z>ai ? MOD : 0);
32             ai += (ai+z>MOD ? z-MOD : z);
33         }
34     }
35 }
36 }
37 }
38 // 0b0e99
39 vector<int> polyMul(vector<int> &a, vector<int> &b){
40     if (a.empty() || b.empty()) return {};
41     int s = a.size()+b.size()-1, B = 32 - __builtin_clz(s), n =
42         1<<B;
43     int inv = qp(n, MOD-2);
44 }

```

```

45 vector<int> L(a), R(b), out(n);
46 L.resize(n), R.resize(n);
47 NTT(L), NTT(R);
48 for (int i=0; i<n; i++){
49     out[-i&(n-1)] = L[i]*R[i]%MOD*inv%MOD;
50 }
51 NTT(out);
52 out.resize(s);
53 return out;
54 }
55 }

```

## 3 Data-Structure

### 3.1 BIT [7ef3a9]

```

1 vector<int> BIT(MAX_SIZE);
2
3 // const int MAX_N = (1<<20)
4 int k_th(int k){ // 回傳 BIT 中第 k 小的元素 (based-1)
5     int res = 0;
6     for (int i=MAX_N>>1; i>=1; i>=1){
7         if (BIT[res+i]<k)
8             k -= BIT[res+i];
9         return res+1;
10    }

```

### 3.2 Disjoint Set Persistent [447002]

```

1 struct Persistent_Disjoint_Set{
2     Persistent_Segment_Tree arr, sz;
3
4     void init(int n){
5         arr.init(n);
6         vector<int> v1;
7         for (int i=0; i<n; i++){
8             v1.push_back(i);
9         }
10        arr.build(v1, 0);
11
12        sz.init(n);
13        vector<int> v2;
14        for (int i=0; i<n; i++){
15            v2.push_back(1);
16        }
17        sz.build(v2, 0);
18    }
19
20    int find(int a){
21        int res = arr.query_version(a, a+1, arr.version.size()
22            (-1).val;
23        if (res==a) return a;
24        return find(res);
25    }
26
27    bool unite(int a, int b){
28        a = find(a);
29        b = find(b);

```

```

29    if (a!=b){
30
31        int sz1 = sz.query_version(a, a+1, arr.version.
32            size()-1).val;
33        int sz2 = sz.query_version(b, b+1, arr.version.
34            size()-1).val;
35
36        if (sz1<sz2){
37            arr.update_version(a, b, arr.version.size()
38                -1);
39            sz.update_version(b, sz1+sz2, arr.version.
40                size()-1);
41        }else{
42            arr.update_version(b, a, arr.version.size()
43                -1);
44            sz.update_version(a, sz1+sz2, arr.version.
45                size()-1);
46        }
47    }
48    return true;
49 }
50 return false;
51 }
52 }
53 }
54 }
55 }

```

### 3.3 PBDS GP Hash Table [866cf6]

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 typedef tree<int, null_type, less<int>, rb_tree_tag,
4     tree_order_statistics_node_update> order_set;
5 struct custom_hash {
6     static uint64_t splitmix64(uint64_t x) {
7         // http://xorshift.di.unimi.it/splitmix64.c
8         x += 0x9e3779b97f4a7c15;
9         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
10        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
11        return x ^ (x >> 31);
12    }
13
14    size_t operator()(uint64_t x) const {
15        static const uint64_t FIXED_RANDOM = chrono::
16            steady_clock::now().time_since_epoch().count();
17        return splitmix64(x + FIXED_RANDOM);
18    }
19 };
20 gp_hash_table<int, int, custom_hash> ss;

```

### 3.4 PBDS Order Set [231774]

```

1 /*
2 .find_by_order(k) 回傳第 k 小的值 (based-0)
3 .order_of_key(k) 回傳有多少元素比 k 小
4 不能在 #define int long long 後 #include 檔案
5 */
6
7 #include <ext/pb_ds/assoc_container.hpp>
8 #include <ext/pb_ds/tree_policy.hpp>

```

```

9 using namespace __gnu_pbds;
10 typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> order_set;

```

### 3.5 Segment Tree Add Set [bb1898]

```

1 // [ll, rr), based-0
2 // 使用前記得 init(陣列大小), build(陣列名稱)
3 // add(ll, rr): 區間修改
4 // set(ll, rr): 區間賦值
5 // query(ll, rr): 區間求和 / 求最大值
6 struct SegmentTree{
7     struct node{
8         int add_tag = 0;
9         int set_tag = 0;
10        int sum = 0;
11        int ma = 0;
12    };
13
14    vector<node> arr;
15
16    SegmentTree(int n){
17        arr.resize(n<<2);
18    }
19
20    node pull(node A, node B){
21        node C;
22        C.sum = A.sum+B.sum;
23        C.ma = max(A.ma, B.ma);
24        return C;
25    }
26
27    // cce0c8
28    void push(int idx, int ll, int rr){
29        if (arr[idx].set_tag!=0){
30            arr[idx].sum = (rr-ll)*arr[idx].set_tag;
31            arr[idx].ma = arr[idx].set_tag;
32            if (rr-ll>1){
33                arr[idx*2+1].add_tag = 0;
34                arr[idx*2+1].set_tag = arr[idx].set_tag;
35                arr[idx*2+2].add_tag = 0;
36                arr[idx*2+2].set_tag = arr[idx].set_tag;
37            }
38            arr[idx].set_tag = 0;
39        }
40        if (arr[idx].add_tag!=0){
41            arr[idx].sum += (rr-ll)*arr[idx].add_tag;
42            arr[idx].ma += arr[idx].add_tag;
43            if (rr-ll>1){
44                arr[idx*2+1].add_tag += arr[idx].add_tag;
45                arr[idx*2+2].add_tag += arr[idx].add_tag;
46            }
47            arr[idx].add_tag = 0;
48        }
49    }
50
51    void build(vector<int> &v, int idx = 0, int ll = 0, int
        rr = n){
52        if (rr-ll==1){
53            arr[idx].sum = v[ll];
54            arr[idx].ma = v[ll];
55        }else{

```

```

56        int mid = (ll+rr)/2;
57        build(v, idx*2+1, ll, mid);
58        build(v, idx*2+2, mid, rr);
59        arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
60    }
61
62    void add(int ql, int qr, int val, int idx = 0, int ll =
        0, int rr = n){
63        push(idx, ll, rr);
64        if (rr<=ql || qr<=ll) return;
65        if (ql<=ll && rr<=qr){
66            arr[idx].add_tag += val;
67            push(idx, ll, rr);
68            return;
69        }
70        int mid = (ll+rr)/2;
71        add(ql, qr, val, idx*2+1, ll, mid);
72        add(ql, qr, val, idx*2+2, mid, rr);
73        arr[idx]=pull(arr[idx*2+1], arr[idx*2+2]);
74    }
75
76    void set(int ql, int qr, int val, int idx=0, int ll=0,
        int rr=n){
77        push(idx, ll, rr);
78        if (rr<=ql || qr<=ll) return;
79        if (ql<=ll && rr<=qr){
80            arr[idx].add_tag = 0;
81            arr[idx].set_tag = val;
82            push(idx, ll, rr);
83            return;
84        }
85        int mid = (ll+rr)/2;
86        set(ql, qr, val, idx*2+1, ll, mid);
87        set(ql, qr, val, idx*2+2, mid, rr);
88        arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
89    }
90
91    node query(int ql, int qr, int idx = 0, int ll = 0, int
        rr = n){
92        push(idx, ll, rr);
93        if (rr<=ql || qr<=ll) return node();
94        if (ql<=ll && rr<=qr) return arr[idx];
95
96        int mid = (ll+rr)/2;
97        return pull(query(ql, qr, idx*2+1, ll, mid), query(ql
            , qr, idx*2+2, mid, rr));
98    }
99 }
100 } ST;

```

### 3.6 Segment Tree Li Chao Line [45b8ba]

```

1 /*
2 全部都是 0-based
3
4 宣告
5 LC_Segment_Tree st(n);
6
7 函式：
8 update({a, b})：插入一條  $y=ax+b$  的全域直線
9 query(x)：查詢所有直線在位置  $x$  的最小值
10 */

```

```

11 const int MAX_V = 1e6+10; // 值域最大值
12
13 struct LC_Segment_Tree{
14     struct Node{ //  $y = ax+b$ 
15         int a = 0;
16         int b = INF;
17
18         int y(int x){
19             return a*x+b;
20         }
21     };
22     vector<Node> arr;
23
24     LC_Segment_Tree(int n = 0){
25         arr.resize(4*n);
26     }
27
28     void update(Node val, int idx = 0, int ll = 0, int rr =
        MAX_V){
29         if (rr-ll==0) return;
30         if (rr-ll==1){
31             if (val.y(ll)<arr[idx].y(ll)){
32                 arr[idx] = val;
33             }
34             return;
35         }
36
37         int mid = (ll+rr)/2;
38         if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
            的線斜率要比較小
39         if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
40             update(val, idx*2+1, ll, mid);
41         }else{ // 交點在右邊
42             swap(arr[idx], val); // 在左子樹中，新線比舊線還
                要好
43             update(val, idx*2+2, mid, rr);
44         }
45         return;
46     }
47
48     int query(int x, int idx = 0, int ll = 0, int rr = MAX_V)
        {
49         if (rr-ll==0) return INF;
50         if (rr-ll==1){
51             return arr[idx].y(ll);
52         }
53
54         int mid = (ll+rr)/2;
55         if (x<mid){
56             return min(arr[idx].y(x), query(x, idx*2+1, ll,
                mid));
57         }else{
58             return min(arr[idx].y(x), query(x, idx*2+2, mid,
                rr));
59         }
60     }
61 };

```

### 3.7 Segment Tree Li Chao Segment [2cb0a4]

```

1 /*
2 全部都是 0-based

```

```

3
4 宣告
5 LC_Segment_Tree st(n);
6
7 函式：
8 update_segment({a, b}, ql, qr)：在 [ql, qr) 插入一條  $y=ax+b$ 
  的線段
9 query(x)：查詢所有直線在位置 x 的最小值
10 */
11 const int MAX_V = 1e6+10; // 值域最大值
12
13 struct LC_Segment_Tree{
14     struct Node{ //  $y = ax+b$ 
15         int a = 0;
16         int b = INF;
17
18         int y(int x){
19             return a*x+b;
20         }
21     };
22     vector<Node> arr;
23
24     LC_Segment_Tree(int n = 0){
25         arr.resize(4*n);
26     }
27
28     void update(Node val, int idx = 0, int ll = 0, int rr =
      MAX_V){
29         if (rr-ll==0) return;
30         if (rr-ll<=1){
31             if (val.y(ll)<arr[idx].y(ll)){
32                 arr[idx] = val;
33             }
34             return;
35         }
36         int mid = (ll+rr)/2;
37         if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
          的線斜率要比較小
38         if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
39             update(val, idx*2+1, ll, mid);
40         }else{ // 交點在右邊
41             swap(arr[idx], val); // 在左子樹中，新線比舊線還
              要好
42             update(val, idx*2+2, mid, rr);
43         }
44         return;
45     }
46 }
47
48 // 在 [ql, qr) 加上一條 val 的線段
49 void update_segment(Node val, int ql, int qr, int idx =
      0, int ll = 0, int rr = MAX_V){
50     if (rr-ll==0) return;
51     if (rr<=ql || qr<=ll) return;
52     if (ql<=ll && rr<=qr){
53         update(val, idx, ll, rr);
54         return;
55     }
56     int mid = (ll+rr)/2;
57     update_segment(val, ql, qr, idx*2+1, ll, mid);
58     update_segment(val, ql, qr, idx*2+2, mid, rr);
59     return;
60 }
61

```

```

62
63     int query(int x, int idx = 0, int ll = 0, int rr = MAX_V)
        {
64         if (rr-ll==0) return INF;
65         if (rr-ll==1){
66             return arr[idx].y(ll);
67         }
68         int mid = (ll+rr)/2;
69         if (x<mid){
70             return min(arr[idx].y(x), query(x, idx*2+1, ll,
              mid));
71         }else{
72             return min(arr[idx].y(x), query(x, idx*2+2, mid,
              rr));
73         }
74     }
75 }
76 };

```

### 3.8 Segment Tree Persistent [3b5aa9]

```

1 /*
2 全部都是 0-based
3
4 宣告
5 Persistent_Segment_Tree st(n+q);
6 st.build(v, 0);
7
8 函式：
9 update_version(pos, val, ver)：對版本 ver 的 pos 位置改成 val
10 query_version(ql, qr, ver)：對版本 ver 查詢 [ql, qr) 的區間和
11 clone_version(ver)：複製版本 ver 到最新的版本
12 */
13 struct Persistent_Segment_Tree{
14     int node_cnt = 0;
15     struct Node{
16         int lc = -1;
17         int rc = -1;
18         int val = 0;
19     };
20     vector<Node> arr;
21     vector<int> version;
22
23     Persistent_Segment_Tree(int sz){
24         arr.resize(32*sz);
25         version.push_back(node_cnt++);
26         return;
27     }
28
29     void pull(Node &c, Node a, Node b){
30         c.val = a.val+b.val;
31         return;
32     }
33
34     void build(vector<int> &v, int idx, int ll = 0, int rr =
      n){
35         auto &now = arr[idx];
36
37         if (rr-ll==1){
38             now.val = v[ll];
39             return;
40         }

```

```

41         int mid = (ll+rr)/2;
42         now.lc = node_cnt++;
43         now.rc = node_cnt++;
44         build(v, now.lc, ll, mid);
45         build(v, now.rc, mid, rr);
46         pull(now, arr[now.lc], arr[now.rc]);
47         return;
48     }
49
50     void update(int pos, int val, int idx, int ll = 0, int rr
      = n){
51         auto &now = arr[idx];
52
53         if (rr-ll==1){
54             now.val = val;
55             return;
56         }
57         int mid = (ll+rr)/2;
58         if (pos<mid){
59             arr[node_cnt] = arr[now.lc];
60             now.lc = node_cnt;
61             node_cnt++;
62             update(pos, val, now.lc, ll, mid);
63         }else{
64             arr[node_cnt] = arr[now.rc];
65             now.rc = node_cnt;
66             node_cnt++;
67             update(pos, val, now.rc, mid, rr);
68         }
69         pull(now, arr[now.lc], arr[now.rc]);
70         return;
71     }
72
73     void update_version(int pos, int val, int ver){
74         update(pos, val, version[ver]);
75     }
76
77     Node query(int ql, int qr, int idx, int ll = 0, int rr =
      n){
78         auto &now = arr[idx];
79
80         if (ql<=ll && rr<=qr) return now;
81         if (rr<=ql || qr<=ll) return Node();
82
83         int mid = (ll+rr)/2;
84
85         Node ret;
86         pull(ret, query(ql, qr, now.lc, ll, mid), query(ql,
              qr, now.rc, mid, rr));
87         return ret;
88     }
89
90     Node query_version(int ql, int qr, int ver){
91         return query(ql, qr, version[ver]);
92     }
93
94     void clone_version(int ver){
95         version.push_back(node_cnt);
96         arr[node_cnt] = arr[version[ver]];
97         node_cnt++;
98     }
99
100 };
101

```



### 3.9 Sparse Table [31f22a]

```

1 struct SparseTable{
2     vector<vector<int>>> st;
3     void build(vector<int> v){
4         int h = __lg(v.size());
5         st.resize(h+1);
6         st[0] = v;
7
8         for (int i=1 ; i<=h ; i++){
9             int gap = (1<<(i-1));
10            for (int j=0 ; j+gap<st[i-1].size() ; j++){
11                st[i].push_back(min(st[i-1][j], st[i-1][j+gap]));
12            }
13        }
14    }
15
16    // 回傳 [ll, rr) 的最小值
17    int query(int ll, int rr){
18        int h = __lg(rr-ll);
19        return min(st[h][ll], st[h][rr-(1<<h)]);
20    }
21 };

```

### 3.10 Treap [5851f5]

```

1 struct Treap{
2     Treap *l = nullptr, *r = nullptr;
3     int pri = rand(), val = 0, sz = 1;
4
5     Treap(int _val){
6         val = _val;
7     }
8 };
9
10 int size(Treap *t){return t ? t->sz : 0;}
11 void pull(Treap *t){
12     t->sz = size(t->l)+size(t->r)+1;
13 }
14
15 Treap* merge(Treap *a, Treap *b){
16     if (!a || !b) return a ? a : b;
17
18     if (a->pri>b->pri){
19         a->r = merge(a->r, b);
20         pull(a);
21         return a;
22     }else{
23         b->l = merge(a, b->l);
24         pull(b);
25         return b;
26     }
27 }
28
29 pair<Treap*, Treap*> split(Treap *t, int k){ // 1-based <前
30     k 個元素, 其他元素>
31     if (!t) return {};
32     if (size(t->l)>=k){
33         auto pa = split(t->l, k);

```

```

34         t->l = pa.second;
35         pull(t);
36         return {pa.first, t};
37     }else{
38         auto pa = split(t->r, k-size(t->l)-1);
39         t->r = pa.first;
40         pull(t);
41         return {t, pa.second};
42     }
43 }
44
45 // functions
46 Treap* build(vector<int> v){
47     Treap* ret = nullptr;
48     for (int i=0 ; i<v.size() ; i++){
49         ret = merge(ret, new Treap(v[i]));
50     }
51     return ret;
52 }
53
54 array<Treap*, 3> cut(Treap *t, int l, int r){ // 1-based <前
55     1~l-1 個元素, l~r 個元素, r+1 個元素>
56     array<Treap*, 3> ret;
57     tie(ret[1], ret[2]) = split(t, r);
58     tie(ret[0], ret[1]) = split(ret[1], l-1);
59     return ret;
60 }
61
62 void print(Treap *t, bool flag = true){
63     if (t->l!=0) print(t->l, false);
64     cout << t->val;
65     if (t->r!=0) print(t->r, false);
66     if (flag) cout << endl;

```

### 3.11 Treap2 [1bf328]

```

1 // 1-based · 請注意 MAX_N 是否足夠大
2 int root = 0;
3 int lc[MAX_N], rc[MAX_N];
4 int pri[MAX_N], val[MAX_N];
5 int sz[MAX_N], tag[MAX_N], fa[MAX_N];
6 int new_node(int v){
7     static int nodeCnt = 0;
8     nodeCnt++;
9     val[nodeCnt] = v;
10    sz[nodeCnt] = 1;
11    pri[nodeCnt] = rand();
12    return nodeCnt;
13 }
14
15 void push(int x){
16     if (tag[x]){
17         if (lc[x]) tag[lc[x]] ^= 1;
18         if (rc[x]) tag[rc[x]] ^= 1;
19     }
20     tag[x] = 0;
21 }
22
23 int pull(int x){
24     if (x){
25         fa[x] = 0;
26         sz[x] = 1+sz[lc[x]]+sz[rc[x]];

```

```

26         if (lc[x]) fa[lc[x]] = x;
27         if (rc[x]) fa[rc[x]] = x;
28     }
29     return x;
30 }
31
32 int merge(int a, int b){
33     if (!a || !b) return a||b;
34     push(a), push(b);
35
36     if (pri[a]>pri[b]){
37         rc[a] = merge(rc[a], b);
38         return pull(a);
39     }else{
40         lc[b] = merge(a, lc[b]);
41         return pull(b);
42     }
43 }
44
45 // [1, k] [k+1, n]
46 void split(int x, int k, int &a, int &b) {
47     if (!x) return a = b = 0, void();
48     push(x);
49     if (sz[lc[x]] >= k) {
50         split(lc[x], k, a, lc[x]);
51         b = x;
52         pull(a); pull(b);
53     }else{
54         split(rc[x], k - sz[lc[x]] - 1, rc[x], b);
55         a = x;
56         pull(a); pull(b);
57     }
58 }
59
60 // functions
61 // 回傳 x 在 Treap 中的位置
62 int get_pos(int x){
63     vector<int> sta;
64     while (fa[x]){
65         sta.push_back(x);
66         x = fa[x];
67     }
68     while (sta.size()){
69         push(x);
70         x = sta.back();
71         sta.pop_back();
72     }
73     push(x);
74
75     int res = sz[x] - sz[rc[x]];
76     while (fa[x]){
77         if (rc[fa[x]]==x){
78             res += sz[fa[x]]-sz[x];
79         }
80         x = fa[x];
81     }
82     return res;
83 }
84
85 // 1-based <前 [1, l-1] 個元素, [l, r] 個元素, [r+1, n] 個元素>
86 array<int, 3> cut(int x, int l, int r){
87     array<int, 3> ret;
88     split(x, r, ret[1], ret[2]);
89     split(ret[1], l-1, ret[0], ret[1]);

```



```

90     return ret;
91 }
92
93 void print(int x){
94     push(x);
95     if (lc[x]) print(lc[x]);
96     cerr << val[x] << " ";
97     if (rc[x]) print(rc[x]);
98 }

```

### 3.12 Trie [b6475c]

```

1 struct Trie{
2     struct Data{
3         int nxt[2]={0, 0};
4     };
5
6     int sz=0;
7     vector<Data> arr;
8
9     void init(int n){
10         arr.resize(n);
11     }
12
13     void insert(int n){
14         int now=0;
15         for (int i=N ; i>=0 ; i--){
16             int v=(n>>i)&1;
17             if (!arr[now].nxt[v]){
18                 arr[now].nxt[v]=++sz;
19             }
20             now=arr[now].nxt[v];
21         }
22     }
23
24     int query(int n){
25         int now=0, ret=0;
26         for (int i=N ; i>=0 ; i--){
27             int v=(n>>i)&1;
28             if (arr[now].nxt[1-v]){
29                 ret+=(1<<i);
30                 now=arr[now].nxt[1-v];
31             }else if (arr[now].nxt[v]){
32                 now=arr[now].nxt[v];
33             }else{
34                 return ret;
35             }
36         }
37         return ret;
38     }
39 } tr;

```

## 4 Dynamic-Programming

### 4.1 Digit DP [133f00]

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 long long l, r;
5 long long dp[20][10][2][2]; // dp[pos][pre][limit] = 後 pos
6 // 位 · pos 前一位是 pre · (是/否) 有上界 · (是/否) 有前綴零
7 // 的答案數量
8
9 long long memorize_search(string &s, int pos, int pre, bool
10 limit, bool lead){
11
12     // 已經被找過了 · 直接回傳值
13     if (dp[pos][pre][limit][lead]!=-1) return dp[pos][pre][
14 limit][lead];
15
16     // 已經搜尋完畢 · 紀錄答案並回傳
17     if (pos==(int)s.size()){
18         return dp[pos][pre][limit][lead] = 1;
19     }
20
21     // 枚舉目前的位數數字是多少
22     long long ans = 0;
23     for (int now=0 ; now<=(limit ? s[pos]-'0' : 9) ; now++){
24         if (now==pre){
25
26             // 1~9 絕對不能連續出現
27             if (pre!=0) continue;
28
29             // 如果已經不在前綴零的範圍內 · 0 不能連續出現
30             if (lead==false) continue;
31         }
32
33         ans += memorize_search(s, pos+1, now, limit&(now==(s[
34 pos]-'0')), lead&(now==0));
35     }
36
37     // 已經搜尋完畢 · 紀錄答案並回傳
38     return dp[pos][pre][limit][lead] = ans;
39 }
40
41 // 回傳 [0, n] 有多少數字符合條件
42 long long find_answer(long long n){
43     memset(dp, -1, sizeof(dp));
44     string tmp = to_string(n);
45
46     return memorize_search(tmp, 0, 0, true, true);
47 }
48
49 int main(){
50
51     // input
52     cin >> l >> r;
53
54     // output - 計算 [l, r] 有多少數字任意兩個位數都不相同
55     cout << find_answer(r)-find_answer(l-1) << "\n";
56
57     return 0;
58 }

```

### 4.2 Knapsack On Tree [df69b1]

```

1 // 需要重構、需要增加註解
2 #include <bits/stdc++.h>
3 #define F first
4 #define S second
5 #define all(x) begin(x), end(x)
6 using namespace std;
7
8 #define chmax(a, b) (a) = (a) < (b) ? (b) : (a)
9 #define chmin(a, b) (a) = (a) < (b) ? (a) : (b)
10
11 #define ll long long
12
13 #define FOR(i, a, b) for (int i = a; i <= b; i++)
14
15 int N, W, cur;
16 vector<int> w, v, sz;
17 vector<vector<int>> adj, dp;
18
19 void dfs(int x) {
20     sz[x] = 1;
21     for (int i : adj[x]) dfs(i), sz[x] += sz[i];
22     cur++;
23     // choose x
24     for (int i=w[x] ; i<=W ; i++){
25         dp[cur][i] = dp[cur - 1][i - w[x]] + v[x];
26     }
27     // not choose x
28     for (int i=0 ; i<=W ; i++){
29         chmax(dp[cur][i], dp[cur - sz[x]][i]);
30     }
31 }
32
33 signed main() {
34     cin >> N >> W;
35     adj.resize(N + 1);
36     w.assign(N + 1, 0);
37     v.assign(N + 1, 0);
38     sz.assign(N + 1, 0);
39     dp.assign(N + 2, vector<int>(W + 1, 0));
40     for (int i=1 ; i<=N ; i++){
41         int p; cin >> p;
42         adj[p].push_back(i);
43     }
44
45     for (int i=1 ; i<=N ; i++) cin >> w[i];
46     for (int i=1 ; i<=N ; i++) cin >> v[i];
47     dfs(0);
48     cout << dp[N + 1][W] << "\n";
49 }

```

### 4.3 SOS DP [8dfa8b]

```

1 // 總時間複雜度為  $O(n \cdot 2^n)$ 
2 // 計算  $dp[i] = i$  所有 bit mask 子集的和
3 for (int i=0 ; i<n ; i++){
4     for (int mask=0 ; mask<(1<<n) ; mask++){
5         if ((mask>>i)&1){
6             dp[mask] += dp[mask^(1<<i)];
7         }
8     }
9 }

```

## 4.4 Integer Partition

$dp[i][x]$  = 要將整數  $x$  拆成  $i$  堆的「組合數」

$dp[i+1][x+1] += dp[i][x]$  (創造新的一堆)  
 $dp[i][x+i] += dp[i][x]$  (把每一堆都增加 1)

## 5 Geometry

### 5.1 Geometry Struct [31a5e0]

```

1 using ld = double;
2
3 // 判斷數值正負：{1:正數,0:零,-1:負數}
4 int sign(long long x) {return (x >= 0) ? ((bool)x) : -1; }
5 int sign(ld x) {return (abs(x) < 1e-9) ? 0 : (x>0 ? 1 : -1); }
6
7 template<typename T>
8 struct point {
9     T x, y;
10     point() {}
11     point(const T &x, const T &y) : x(x), y(y) {}
12     explicit operator point<ld>() {return point<ld>(x, y); }
13
14     point operator+(point b) {return {x+b.x, y+b.y}; }
15     point operator-(point b) {return {x-b.x, y-b.y}; }
16     point operator*(T b) {return {x*b, y*b}; }
17     point operator/(T b) {return {x/b, y/b}; }
18     bool operator==(point b) {return x==b.x && y==b.y; }
19
20     T operator*(point b) {return x * b.x + y * b.y; }
21     T operator^(point b) {return x * b.y - y * b.x; }
22
23     // 逆時針極角排序
24     bool side() { return (y == 0) ? (x > 0) : (y < 0); }
25     bool operator<(point &b) {
26         return side() == b.side() ?
27             (x*b.y > b.x*y) : side() < b.side();
28     }
29     friend ostream& operator<<(ostream& os, point p) {
30         return os << "(" << p.x << ", " << p.y << ")";
31     }
32     // 判斷 ab 到 ac 的方向：{1:逆時鐘,0:重疊,-1:順時鐘}
33     friend int ori(point a, point b, point c) {
34         return sign((b-a)^(c-a));
35     }
36     friend bool btw(point a, point b, point c) {
37         return ori(a, b, c) == 0 && sign((a-c)*(b-c)) <= 0;
38     }
39     // 判斷線段 ab, cd 是否相交
40     friend bool banana(point a, point b, point c, point d) {
41         if (btw(a, b, c) || btw(a, b, d)
42             || btw(c, d, a) || btw(c, d, b)) return true;
43         int u = ori(a, b, c) * ori(a, b, d);
44         int v = ori(c, d, a) * ori(c, d, b);
45         return u < 0 && v < 0;
46     }
47     // 旋轉 Arg(b) 的角度 (小心溢位)
48     point rotate(point b){return {x*b.x-y*b.y, x*b.y+y*b.x};}
49     // 回傳極座標角度・值域：[-π, +π]
50
51     friend ld Arg(point b) {
52         return (b.x != 0 || b.y != 0) ? atan2(b.y, b.x) : 0;
53     }
54     friend T abs2(point b) {return b * b; }
55 };
56
57 template<typename T>
58 struct line {
59     point<T> p1, p2;
60     // ax + by + c = 0
61     T a, b, c; // |a|, |b| ≤ 2C, |c| ≤ 8C²
62     line() {}
63     line(const point<T> &x, const point<T> &y) : p1(x), p2(y){
64         build();
65     }
66     void build() {
67         a = p1.y - p2.y;
68         b = p2.x - p1.x;
69         c = (-a*p1.x)-b*p1.y;
70     }
71     // 判斷點和有向直線的關係：{1:左邊,0:在線上,-1:右邊}
72     int ori(point<T> &p) {
73         return sign((p2-p1) ^ (p-p1));
74     }
75     // 判斷直線斜率是否相同
76     bool parallel(line &l) {
77         return ((p1-p2) ^ (l.p1-l.p2)) == 0;
78     }
79     // 兩直線交點
80     point<ld> line_intersection(line &l) {
81         using P = point<ld>;
82         point<T> u = p2-p1, v = l.p2-l.p1, s = l.p1-p1;
83         return P(p1) + P(u) * ((ld(s^v)) / (u^v));
84     }
85 };
86
87 template<typename T>
88 struct polygon {
89     vector<point<T>> v;
90     polygon() {}
91     polygon(const vector<point<T>> &u) : v(u) {}
92     // simple 為 true 的時候會回傳任意三點不共線的凸包
93     void make_convex_hull(int simple) {
94         auto cmp = [&](point<T> &p, point<T> &q) {
95             return (p.x == q.x) ? (p.y < q.y) : (p.x < q.x);
96         };
97         simple = (bool)simple;
98         sort(v.begin(), v.end(), cmp);
99         v.resize(unique(v.begin(), v.end()) - v.begin());
100         vector<point<T>> hull;
101         for (int t = 0; t < 2; ++t){
102             int sz = hull.size();
103             for (auto &i:v) {
104                 while (hull.size() >= sz+2 && ori(hull[sz],
105                     hull[sz-2], hull.back(), i) < simple) {
106                     hull.pop_back();
107                 }
108                 hull.push_back(i);
109             }
110             hull.pop_back();
111             reverse(v.begin(), v.end());
112         }
113         swap(hull, v);
114     }
115 };
116
117 // 可以在有 n 個點的簡單多邊形內・用 O(n) 判斷一個點：
118
119 // {1 : 在多邊形內, 0 : 在多邊形上, -1 : 在多邊形外}
120 int in_polygon(point<T> a){
121     const T MAX_POS = 1e9 + 5; // [記得修改] 座標的最大值
122     point<T> pre = v.back(), b(MAX_POS, a.y + 1);
123     int cnt = 0;
124
125     for (auto &i:v) {
126         if (btw(pre, i, a)) return 0;
127         if (banana(a, b, pre, i)) cnt++;
128         pre = i;
129     }
130
131     return cnt%2 ? 1 : -1;
132 }
133
134 /// 警告：以下所有凸包專用的函式都只接受逆時針排序且任三點不
135 /// 共線的凸包 ///
136 // 可以在有 n 個點的凸包內・用 O(Log n) 判斷一個點：
137 // {1 : 在凸包內, 0 : 在凸包邊上, -1 : 在凸包外}
138 int in_convex(point<T> p) {
139     int n = v.size();
140     int a = ori(v[0], v[1], p), b = ori(v[0], v[n-1], p);
141     if (a < 0 || b > 0) return -1;
142     if (btw(v[0], v[1], p)) return 0;
143     if (btw(v[0], v[n-1], p)) return 0;
144     int l = 1, r = n - 1, mid;
145     while (l + 1 < r) {
146         mid = (l + r) >> 1;
147         if (ori(v[0], v[mid], p) >= 0) l = mid;
148         else r = mid;
149     }
150     int k = ori(v[l], v[r], p);
151     if (k <= 0) return k;
152     return 1;
153 }
154
155 // 凸包專用的環狀二分搜・回傳 0-based index
156 int cycle_search(auto &f) {
157     int n = v.size(), l = 0, r = n;
158     bool rv = f(1, 0);
159     while (r - l > 1) {
160         int m = (l + r) / 2;
161         if (f(0, m) ? rv : f(m, (m + 1) % n)) r = m;
162         else l = m;
163     }
164     return f(1, r % n) ? l : r % n;
165 }
166
167 // 可以在有 n 個點的凸包內・用 O(Log n) 判斷一條直線：
168 // {1 : 穿過凸包, 0 : 剛好切過凸包, -1 : 沒碰到凸包}
169 int line_cut_convex(line<T> L) {
170     L.build();
171     point<T> p(L.a, L.b);
172     auto gt = [&](int neg) {
173         auto f = [&](int x, int y) {
174             return sign((v[x] - v[y]) * p) == neg;
175         };
176         return -(v[cycle_search(f)] * p);
177     };
178     T x = gt(1), y = gt(-1);
179     if (L.c < x || y < L.c) return -1;
180     return not (L.c == x || L.c == y);
181 }
182
183 // 可以在有 n 個點的凸包內・用 O(Log n) 判斷一個線段：
184 // {1 : 存在一個凸包上的邊可以把這個線段切成兩半,
185 // 0 : 有碰到凸包但沒有任何凸包上的邊可以把它切成兩半,
186 // -1 : 沒碰到凸包}

```

```

177 /// 除非線段兩端點都不在凸包邊上，否則此函數回傳 0 的時候不一
178 定表示線段沒有通過凸包內部 ///
179 int segment_across_convex(line<T> L) {
180     L.build();
181     point<T> p(L.a, L.b);
182     auto gt = [&](int neg) {
183         auto f = [&](int x, int y) {
184             return sign((v[x] - v[y]) * p) == neg;
185         };
186         return cycle_search(f);
187     };
188     int i = gt(1), j = gt(-1), n = v.size();
189     T x = -(v[i] * p), y = -(v[j] * p);
190     if (L.c < x || y < L.c) return -1;
191     if (L.c == x || L.c == y) return 0;
192
193     if (i > j) swap(i, j);
194     auto g = [&](int x, int lim) {
195         int now = 0, nxt;
196         for (int i = 1 << __lg(lim); i > 0; i /= 2) {
197             if (now + i > lim) continue;
198             nxt = (x + i) % n;
199             if (L.ori(v[x]) * L.ori(v[nxt]) >= 0) {
200                 x = nxt;
201                 now += i;
202             } // ↓ BE CAREFUL
203             return -(ori(v[x], v[(x + 1) % n], L.p1) * ori(v[
204                 x], v[(x + 1) % n], L.p2));
205         };
206         return max(g(i, j - i), g(j, n - (j - i)));
207     };
208     /// 可以在有 n 個點的凸包內，用 O(Log n) 判斷一個線段：
209     /// {1 : 線段上存在某一點位於凸包內部（邊上不算），
210     /// 0 : 線段上存在某一點碰到凸包的邊但線段上任一點均不在凸包
211     內部，
212     -1 : 線段完全在凸包外面}
213     int segment_pass_convex_interior(line<T> L) {
214         if (in_convex(L.p1) == 1 || in_convex(L.p2) == 1)
215             return 1;
216         L.build();
217         point<T> p(L.a, L.b);
218         auto gt = [&](int neg) {
219             auto f = [&](int x, int y) {
220                 return sign((v[x] - v[y]) * p) == neg;
221             };
222             return cycle_search(f);
223         };
224         int i = gt(1), j = gt(-1), n = v.size();
225         T x = -(v[i] * p), y = -(v[j] * p);
226         if (L.c < x || y < L.c) return -1;
227         if (L.c == x || L.c == y) return 0;
228
229         if (i > j) swap(i, j);
230         auto g = [&](int x, int lim) {
231             int now = 0, nxt;
232             for (int i = 1 << __lg(lim); i > 0; i /= 2) {
233                 if (now + i > lim) continue;
234                 nxt = (x + i) % n;
235                 if (L.ori(v[x]) * L.ori(v[nxt]) > 0) {
236                     x = nxt;
237                     now += i;
238                 } // ↓ BE CAREFUL
239             }
240         };
241         return max(g(i, j - i), g(j, n - (j - i)));
242     };
243
244     return -(ori(v[x], v[(x + 1) % n], L.p1) * ori(v[
245         x], v[(x + 1) % n], L.p2));
246     };
247
248     return max(g(i, j - i), g(j, n - (j - i)));
249 }
250
251 friend int halfplane_intersection(vector<line<T>> &s,
252     polygon<T> &P) {
253     auto angle_cmp = [&](line<T> &A, line<T> &B) {
254         point<T> a = A.p2-A.p1, b = B.p2-B.p1;
255         return (a < b);
256     };
257     sort(s.begin(), s.end(), angle_cmp); // 線段左側為該
258     線段半平面
259     int L, R, n = s.size();
260     vector<point<T>> px(n);
261     vector<line<T>> q(n);
262     q[L = R = 0] = s[0];
263     for (int i = 1; i < n; ++i) {
264         while (L < R && s[i].ori(px[R-1]) <= 0) --R;
265         while (L < R && s[i].ori(px[L]) <= 0) ++L;
266         q[++R] = s[i];
267         if (q[R].parallel(q[R-1])) {
268             --R;
269             if (q[R].ori(s[i].p1) > 0) q[R] = s[i];
270         }
271         if (L < R) px[R-1] = q[R-1].line_intersection(q[R]);
272     }
273     while (L < R && q[L].ori(px[R-1]) <= 0) --R;
274     P.v.clear();
275     if (R - L <= 1) return 0;
276     px[R] = q[R].line_intersection(q[L]);
277     for (int i = L; i <= R; ++i) P.v.push_back(px[i]);
278     return R - L + 1;
279 }
280
281 }
282
283 }
284
285 }
286
287 }
288
289 }
290
291 }
292
293 }
294
295 }

```

```

296     return -(ori(v[x], v[(x + 1) % n], L.p1) * ori(v[
297         x], v[(x + 1) % n], L.p2));
298     };
299
300     return max(g(i, j - i), g(j, n - (j - i)));
301 }
302
303 pair<int,int> convex_tangent_point(point<T> p) {
304     int n = v.size(), z = -1, edg = -1;
305     auto gt = [&](int neg) {
306         auto check = [&](int x) {
307             if (v[x] == p) z = x;
308             if (btw(v[x], v[(x + 1) % n], p)) edg = x;
309             if (btw(v[(x + n - 1) % n], v[x], p)) edg = (
310                 x + n - 1) % n;
311         };
312         auto f = [&](int x, int y) {
313             check(x); check(y);
314             return ori(p, v[x], v[y]) == neg;
315         };
316         return cycle_search(f);
317     };
318     int x = gt(1), y = gt(-1);
319     if (z != -1) {
320         return {(z + n - 1) % n, (z + 1) % n};
321     }
322     else if (edg != -1) {
323         return {edg, (edg + 1) % n};
324     }
325     else {
326         return {x, y};
327     }
328 }
329
330 friend int halfplane_intersection(vector<line<T>> &s,
331     polygon<T> &P) {
332     auto angle_cmp = [&](line<T> &A, line<T> &B) {
333         point<T> a = A.p2-A.p1, b = B.p2-B.p1;
334         return (a < b);
335     };
336     sort(s.begin(), s.end(), angle_cmp); // 線段左側為該
337     線段半平面
338     int L, R, n = s.size();
339     vector<point<T>> px(n);
340     vector<line<T>> q(n);
341     q[L = R = 0] = s[0];
342     for (int i = 1; i < n; ++i) {
343         while (L < R && s[i].ori(px[R-1]) <= 0) --R;
344         while (L < R && s[i].ori(px[L]) <= 0) ++L;
345         q[++R] = s[i];
346         if (q[R].parallel(q[R-1])) {
347             --R;
348             if (q[R].ori(s[i].p1) > 0) q[R] = s[i];
349         }
350         if (L < R) px[R-1] = q[R-1].line_intersection(q[R]);
351     }
352     while (L < R && q[L].ori(px[R-1]) <= 0) --R;
353     P.v.clear();
354     if (R - L <= 1) return 0;
355     px[R] = q[R].line_intersection(q[L]);
356     for (int i = L; i <= R; ++i) P.v.push_back(px[i]);
357     return R - L + 1;
358 }
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```

## 5.2 Pick's Theorem

給定頂點坐標均是整點的簡單多邊形，面積 = 內部格點數 + 邊上格點數/2 - 1

## 6 Graph

### 6.1 2-SAT [5a6317]

```

1 struct TWO_SAT {
2     int n, N;
3     vector<vector<int>> G, rev_G;
4     deque<bool> used;
5     vector<int> order, comp;
6     deque<bool> assignment;
7     void init(int _n) {
8         n = _n;
9         N = _n * 2;
10        G.resize(N + 5);
11        rev_G.resize(N + 5);
12    }
13    void dfs1(int v) {
14        used[v] = true;
15        for (int u : G[v]) {
16            if (!used[u])
17                dfs1(u);
18        }
19        order.push_back(v);
20    }
21    void dfs2(int v, int c1) {
22        comp[v] = c1;
23        for (int u : rev_G[v]) {
24            if (comp[u] == -1)
25                dfs2(u, c1);
26        }
27    }
28    bool solve() {
29        order.clear();
30        used.assign(N, false);
31        for (int i = 0; i < N; ++i) {
32            if (!used[i])
33                dfs1(i);
34        }
35        comp.assign(N, -1);
36        for (int i = 0, j = 0; i < N; ++i) {
37            int v = order[N - i - 1];
38            if (comp[v] == -1)
39                dfs2(v, j++);
40        }
41        assignment.assign(n, false);
42        for (int i = 0; i < N; i += 2) {
43            if (comp[i] == comp[i + 1])
44                return false;
45            assignment[i / 2] = (comp[i] > comp[i + 1]);
46        }
47        return true;
48    }
49    // A or B 都是 0-based
50    void add_disjunction(int a, bool na, int b, bool nb) {
51        // na is true => ~a, na is false => a
52        // nb is true => ~b, nb is false => b
53        a = 2 * a ^ na;
54    }
55 }

```

```

54     b = 2 * b ^ nb;
55     int neg_a = a ^ 1;
56     int neg_b = b ^ 1;
57     G[neg_a].push_back(b);
58     G[neg_b].push_back(a);
59     rev_G[b].push_back(neg_a);
60     rev_G[a].push_back(neg_b);
61     return;
62 }
63 void get_result(vector<int>& res) {
64     res.clear();
65     for (int i = 0; i < n; i++)
66         res.push_back(assignment[i]);
67 }
68 };

```

## 6.2 Augment Path [f8a5dd]

```

1 struct AugmentPath{
2     int n, m;
3     vector<vector<int>> G;
4     vector<int> mx, my;
5     vector<int> visx, visy;
6     int stamp;
7
8     AugmentPath(int _n, int _m) : n(_n), m(_m), G(n), mx(n,
9         -1), my(m, -1), visx(n), visy(m){
10         stamp = 0;
11     }
12
13     void add(int x, int y){
14         G[x].push_back(y);
15     }
16
17     // bb03e2
18     bool dfs1(int now){
19         visx[now] = stamp;
20
21         for (auto x : G[now]){
22             if (my[x]==-1){
23                 mx[now] = x;
24                 my[x] = now;
25                 return true;
26             }
27         }
28         for (auto x : G[now]){
29             if (visx[my[x]]!=stamp && dfs1(my[x])){
30                 mx[now] = x;
31                 my[x] = now;
32                 return true;
33             }
34         }
35         return false;
36     }
37
38     vector<pair<int, int>> find_max_matching(){
39         vector<pair<int, int>> ret;
40
41         while (true){
42             stamp++;
43             int tmp = 0;
44             for (int i=0 ; i<n ; i++){
45                 if (mx[i]==-1 && dfs1(i)) tmp++;

```

```

45     }
46     if (tmp==0) break;
47 }
48
49 for (int i=0 ; i<n ; i++){
50     if (mx[i]!=-1){
51         ret.push_back({i, mx[i]});
52     }
53 }
54 return ret;
55 }
56
57 // 645577
58 void dfs2(int now){
59     visx[now] = true;
60
61     for (auto x : G[now]){
62         if (my[x]!=-1 && visy[x]==false){
63             visy[x] = true;
64             dfs2(my[x]);
65         }
66     }
67 }
68
69 // 要先執行 find_max_matching 一次
70 vector<pair<int, int>> find_min_vertex_cover(){
71     fill(visx.begin(), visx.end(), false);
72     fill(visy.begin(), visy.end(), false);
73
74     vector<pair<int, int>> ret;
75     for (int i=0 ; i<n ; i++){
76         if (mx[i]==-1) dfs2(i);
77     }
78
79     for (int i=0 ; i<n ; i++){
80         if (visx[i]==false) ret.push_back({1, i});
81     }
82     for (int i=0 ; i<m ; i++){
83         if (visy[i]==true) ret.push_back({2, i});
84     }
85
86     return ret;
87 }
88 };

```

## 6.3 C3C4 [d00465]

```

1 // 0-based
2 void C3C4(vector<int> deg, vector<array<int, 2>> edges){
3     int N = deg.size();
4     int M = edges.size();
5
6     vector<int> ord(N), rk(N);
7     iota(ord.begin(), ord.end(), 0);
8     sort(ord.begin(), ord.end(), [&](int x, int y) { return
9         deg[x] > deg[y]; });
10    for (int i=0 ; i<N ; i++) rk[ord[i]] = i;
11
12    vector<vector<int>> D(N), adj(N);
13    for (auto [u, v] : edges) {
14        if (rk[u] > rk[v]) swap(u, v);
15        D[u].emplace_back(v);
16        adj[u].emplace_back(v);

```

```

16     adj[v].emplace_back(u);
17 }
18
19 vector<int> vis(N);
20
21 int c3 = 0, c4 = 0;
22 for (int x : ord) { // c3
23     for (int y : D[x]) vis[y] = 1;
24     for (int y : D[x]) for (int z : D[y]){
25         c3 += vis[z]; // xyz is C3
26     }
27     for (int y : D[x]) vis[y] = 0;
28 }
29 for (int x : ord) { // c4
30     for (int y : D[x]) for (int z : adj[y])
31         if (rk[z] > rk[x]) c4 += vis[z]++;
32     for (int y : D[x]) for (int z : adj[y])
33         if (rk[z] > rk[x]) --vis[z];
34 } // both are O(M*sqrt(M)), test @ 2022 CCPC guangzhou
35 cout << c4 << "\n";
36 }

```

## 6.4 Cut BCC [2af809]

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 200005;
5 vector<int> G[N];
6 int low[N], depth[N];
7 bool vis[N];
8 vector<vector<int>> bcc;
9 stack<int> stk;
10
11 void dfs(int v, int p) {
12     stk.push(v);
13     vis[v] = true;
14     low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
15     for (int u : G[v]) {
16         if (u == p) continue;
17         if (!vis[u]) {
18             /// (v, u) 是樹邊
19             dfs(u, v);
20             low[v] = min(low[v], low[u]);
21             /// u 無法在不經過父邊的情況走到 v 的祖先
22             if (low[u] >= depth[v]) {
23                 bcc.emplace_back();
24                 while (stk.top() != u) {
25                     bcc.back().push_back(stk.top());
26                     stk.pop();
27                 }
28                 bcc.back().push_back(stk.top());
29                 stk.pop();
30                 bcc.back().push_back(v);
31             }
32         } else {
33             /// (v, u) 是回邊
34             low[v] = min(low[v], depth[u]);
35         }
36     }
37 }

```

## 6.5 Dinic [961b34]

```

1 // 一般圖:  $O(EV^2)$ 
2 // 二分圖:  $O(EV)$ 
3 struct Flow{
4     using T = int; // 可以換成別的类型
5     struct Edge{
6         int v; T rc; int rid;
7     };
8     vector<vector<Edge>> G;
9     void add(int u, int v, T c){
10         G[u].push_back({v, c, G[v].size()});
11         G[v].push_back({u, 0, G[u].size()-1});
12     }
13     vector<int> dis, it;
14
15     Flow(int n){
16         G.resize(n);
17         dis.resize(n);
18         it.resize(n);
19     }
20
21     // ce56d6
22     T dfs(int u, int t, T f){
23         if (u == t || f == 0) return f;
24         for (int &i=it[u] ; i<G[u].size() ; i++){
25             auto &[v, rc, rid] = G[u][i];
26             if (dis[v]!=dis[u]+1) continue;
27             T df = dfs(v, t, min(f, rc));
28             if (df <= 0) continue;
29             rc -= df;
30             G[v][rid].rc += df;
31             return df;
32         }
33         return 0;
34     }
35
36     // e22e39
37     T flow(int s, int t){
38         T ans = 0;
39         while (true){
40             fill(dis.begin(), dis.end(), INF);
41             queue<int> q;
42             q.push(s);
43             dis[s] = 0;
44
45             while (q.size()){
46                 int u = q.front(); q.pop();
47                 for (auto [v, rc, rid] : G[u]){
48                     if (rc <= 0 || dis[v] < INF) continue;
49                     dis[v] = dis[u] + 1;
50                     q.push(v);
51                 }
52             }
53             if (dis[t]==INF) break;
54
55             fill(it.begin(), it.end(), 0);
56             while (true){
57                 T df = dfs(s, t, INF);
58                 if (df <= 0) break;
59                 ans += df;
60             }
61         }
62         return ans;
63     }

```

```

64
65 // the code below constructs minimum cut
66 void dfs_mincut(int now, vector<bool> &vis){
67     vis[now] = true;
68     for (auto &[v, rc, rid] : G[now]){
69         if (vis[v] == false && rc > 0){
70             dfs_mincut(v, vis);
71         }
72     }
73 }
74
75 vector<pair<int, int>> construct(int n, int s, vector<
76     pair<int, int>> &E){
77     // E is G without capacity
78     vector<bool> vis(n);
79     dfs_mincut(s, vis);
80     vector<pair<int, int>> ret;
81     for (auto &[u, v] : E){
82         if (vis[u] == true && vis[v] == false){
83             ret.emplace_back(u, v);
84         }
85     }
86     return ret;
87 };

```

## 6.6 Dominator Tree [52b249]

```

1 /*
2 全部都是  $\theta$ -based
3 G 要是向有無權圖
4 一開始要初始化  $G(N, root)$  · 代表有  $N$  個節點 · 根是  $root$ 
5 用完之後要 build
6  $G[i] = i$  的 idom · 也就是從  $root$  走到  $i$  時 · 一定要走到的點且離
7  $i$  最近
8 */
9 struct DominatorTree{
10     int N;
11     vector<vector<int>> G;
12     vector<vector<int>> buckets, rg;
13     // dfn[x] = the DFS order of x
14     // rev[x] = the vertex with DFS order x
15     // par[x] = the parent of x
16     vector<int> dfn, rev, par;
17     vector<int> sdom, dom, idom;
18     vector<int> fa, val;
19     int stamp;
20     int root;
21
22     int operator [] (int x){
23         return idom[x];
24     }
25
26     DominatorTree(int _N, int _root) :
27         N(_N),
28         G(N), buckets(N), rg(N),
29         dfn(N, -1), rev(N, -1), par(N, -1),
30         sdom(N, -1), dom(N, -1), idom(N, -1),
31         fa(N, -1), val(N, -1)
32     {
33         stamp = 0;
34         root = _root;
35     }

```

```

34 }
35
36 void add_edge(int u, int v){
37     G[u].push_back(v);
38 }
39
40 void dfs(int x){
41     rev[dfn[x] = stamp] = x;
42     fa[stamp] = sdom[stamp] = val[stamp] = stamp;
43     stamp++;
44
45     for (int u : G[x]){
46         if (dfn[u]==-1){
47             dfs(u);
48             par[dfn[u]] = dfn[x];
49             rg[dfn[u]].push_back(dfn[x]);
50         }
51     }
52 }
53
54 int eval(int x, bool first){
55     if (fa[x]==x) return !first ? -1 : x;
56     int p = eval(fa[x], false);
57
58     if (p==-1) return x;
59     if (sdom[val[x]]>sdom[val[fa[x]]]) val[x] = val[fa[x]];
60     fa[x] = p;
61
62     return !first ? p : val[x];
63 }
64
65 void link(int x, int y){
66     fa[x] = y;
67 }
68
69 void build(){
70     dfs(root);
71
72     for (int x=stamp-1 ; x>=0 ; x--){
73         for (int y : rg[x]){
74             sdom[x] = min(sdom[x], sdom[eval(y, true)]);
75         }
76         if (x>0) buckets[sdom[x]].push_back(x);
77         for (int u : buckets[x]){
78             int p = eval(u, true);
79             if (sdom[p]==x) dom[u] = x;
80             else dom[u] = p;
81         }
82         if (x>0) link(x, par[x]);
83     }
84
85     idom[root] = root;
86     for (int x=1 ; x<stamp ; x++){
87         if (sdom[x]!=dom[x]) dom[x] = dom[dom[x]];
88     }
89     for (int i=1 ; i<stamp ; i++) idom[rev[i]] = rev[dom[i]];
90 }
91 };

```

## 6.7 EdgeBCC [d09eb1]

```

1 // d09eb1
2 // 0-based · 支援重邊
3 struct EdgeBCC{
4     int n, m, dep, sz;
5     vector<vector<pair<int, int>>> G;
6     vector<vector<int>> bcc;
7     vector<int> dfn, low, stk, isBridge, bccId;
8     vector<pair<int, int>> edge, bridge;
9
10    EdgeBCC(int _n) : n(_n), m(0), sz(0), dfn(n), low(n), G(n)
11        ), bcc(n), bccId(n) {}
12
13    void add_edge(int u, int v) {
14        edge.push_back({u, v});
15        G[u].push_back({v, m});
16        G[v].push_back({u, m++});
17    }
18
19    void dfs(int now, int pre) {
20        dfn[now] = low[now] = ++dep;
21        stk.push_back(now);
22
23        for (auto [x, id] : G[now]){
24            if (!dfn[x]){
25                dfs(x, id);
26                low[now] = min(low[now], low[x]);
27            } else if (id != pre){
28                low[now] = min(low[now], dfn[x]);
29            }
30        }
31
32        if (low[now] == dfn[now]){
33            if (pre != -1) isBridge[pre] = true;
34            int u;
35            do{
36                u = stk.back();
37                stk.pop_back();
38                bcc[sz].push_back(u);
39                bccId[u] = sz;
40            } while (u != now);
41            sz++;
42        }
43    }
44
45    void get_bcc() {
46        isBridge.assign(m, 0);
47        dep = 0;
48        for (int i=0; i<n; i++){
49            if (!dfn[i]) dfs(i, -1);
50        }
51
52        for (int i=0; i<m; i++){
53            if (isBridge[i]){
54                bridge.push_back({edge[i].first, edge[i].second});
55            }
56        }
57    }
58
59    };

```

## 6.8 EnumeratePlanarFace [e70ee1]

```
1 // 0-based
```

```

2 struct PlanarGraph{
3     int n, m, id;
4     vector<point<int>> v;
5     vector<vector<pair<int, int>>> G;
6     vector<int> conv, nxt, vis;
7
8     PlanarGraph(int n, int m, vector<point<int>> _v) :
9         n(n), m(m), id(0),
10        v(_v), G(n),
11        conv(2*m), nxt(2*m), vis(2*m) {}
12
13    void add_edge(int x, int y){
14        G[x].push_back({y, 2*id});
15        G[y].push_back({x, 2*id+1});
16        conv[2*id] = x;
17        conv[2*id+1] = y;
18        id++;
19    }
20
21    vector<int> enumerate_face(){
22        for (int i=0; i<n; i++){
23            sort(G[i].begin(), G[i].end(), [&](pair<int, int>
24                a, pair<int, int> b){
25                return (v[a.first]-v[i])<(v[b.first]-v[i]);
26            });
27
28            int sz = G[i].size(), pre = sz-1;
29            for (int j=0; j<sz; j++){
30                nxt[G[i][pre].second] = G[i][j].second^1;
31                pre = j;
32            }
33        }
34
35        vector<int> ret;
36        for (int i=0; i<2*m; i++){
37            if (vis[i]==false){
38                int area = 0, now = i;
39                vector<int> pt;
40
41                while (!vis[now]){
42                    vis[now] = true;
43                    pt.push_back(conv[now]);
44                    now = nxt[now];
45                }
46
47                pt.push_back(pt.front());
48                for (int i=0; i+1<pt.size(); i++){
49                    area += (v[pt[i]]^v[pt[i+1]]);
50                }
51
52                // pt = face boundary
53                if (area>0){
54                    ret.push_back(area);
55                } else{
56                    // pt is outer face
57                }
58            }
59        }
60
61        return ret;
62    }
63
64    };

```

## 6.9 HLD [f57ec6]

```

1 #include <bits/stdc++.h>
2 #define int long long
3 using namespace std;
4
5 const int N = 100005;
6 vector<int> G[N];
7 struct HLD {
8     vector<int> pa, sz, depth, mxson, topf, id;
9     int n, idcnt = 0;
10    HLD(int _n) : n(_n), pa(_n+1), sz(_n+1), depth(_n+1),
11        mxson(_n+1), topf(_n+1), id(_n+1) {}
12    void dfs1(int v = 1, int p = -1) {
13        pa[v] = p; sz[v] = 1; mxson[v] = 0;
14        depth[v] = (p == -1 ? 0 : depth[p] + 1);
15        for (int u : G[v]) {
16            if (u == p) continue;
17            dfs1(u, v);
18            sz[v] += sz[u];
19            if (sz[u] > sz[mxson[v]]) mxson[v] = u;
20        }
21    }
22    void dfs2(int v = 1, int top = 1) {
23        id[v] = ++idcnt;
24        topf[v] = top;
25        if (mxson[v]) dfs2(mxson[v], top);
26        for (int u : G[v]) {
27            if (u == mxson[v] || u == pa[v]) continue;
28            dfs2(u, u);
29        }
30    }
31    // query 為區間資料結構
32    int path_query(int a, int b) {
33        int res = 0;
34        while (topf[a] != topf[b]) { /// 若不在同一條鍊上
35            if (depth[topf[a]] < depth[topf[b]]) swap(a, b);
36            res = max(res, 0ll); // query : l = id[topf[a]],
37            a = pa[topf[a]];
38        }
39        /// 此時已在同一條鍊上
40        if (depth[a] < depth[b]) swap(a, b);
41        res = max(res, 0ll); // query : l = id[b], r = id[a]
42        return res;
43    }
44 };

```

## 6.10 Kosaraju [c7d5aa]

```

1 /* c7d5aa
2 給定一個有向圖，迴傳傳縮點後的圖、SCC 的資訊
3 所有點都以 based-0 編號
4
5 函式：
6 SCC_compress G(n): 宣告一個有 n 個點的圖
7 .add_edge(u, v): 加上一條邊 u -> v
8 .compress: O(n log n) 計算 G3、SCC、SCC_id 的資訊，並把縮點後
9 的結果存在 result 裡

```



```

10 SCC[i] = 某個 SCC 中的所有點
11 SCC_id[i] = 第 i 個點在第幾個 SCC
12 */
13 struct SCC_compress{
14     int N, M, sz;
15     vector<vector<int>> G, inv_G, result;
16     vector<pair<int, int>> edges;
17     vector<bool> vis;
18     vector<int> order;
19
20     vector<vector<int>> SCC;
21     vector<int> SCC_id;
22
23     SCC_compress(int _N) :
24         N(_N), M(0), sz(0),
25         G(N), inv_G(N),
26         vis(N), SCC_id(N)
27     {}
28
29     vector<int> operator [] (int x){
30         return result[x];
31     }
32
33     void add_edge(int u, int v){
34         G[u].push_back(v);
35         inv_G[v].push_back(u);
36         edges.push_back({u, v});
37         M++;
38     }
39
40     void dfs1(vector<vector<int>> &G, int now){
41         vis[now] = 1;
42         for (auto x : G[now]) if (!vis[x]) dfs1(G, x);
43         order.push_back(now);
44     }
45
46     void dfs2(vector<vector<int>> &G, int now){
47         SCC_id[now] = SCC.size()-1;
48         SCC.back().push_back(now);
49         vis[now] = 1;
50         for (auto x : G[now]) if (!vis[x]) dfs2(G, x);
51     }
52
53     void compress(){
54         fill(vis.begin(), vis.end(), 0);
55         for (int i=0 ; i<N ; i++) if (!vis[i]) dfs1(G, i);
56
57         fill(vis.begin(), vis.end(), 0);
58         reverse(order.begin(), order.end());
59         for (int i=0 ; i<N ; i++){
60             if (!vis[order[i]]){
61                 SCC.push_back(vector<int>());
62                 dfs2(inv_G, order[i]);
63             }
64         }
65
66         result.resize(SCC.size());
67         sz = SCC.size();
68         for (auto [u, v] : edges){
69             if (SCC_id[u]!=SCC_id[v]) result[SCC_id[u]].
                push_back(SCC_id[v]);
70         }
71         for (int i=0 ; i<SCC.size() ; i++){
72             sort(result[i].begin(), result[i].end());

```

```

73         result[i].resize(unique(result[i].begin(), result
74             [i].end())-result[i].begin());
75     }
76 };

```

## 6.11 Kuhn Munkres [e66c35]

```

1 // O(n^3) 找到最大權匹配
2 struct KuhnMunkres{
3     int n; // max(n, m)
4     vector<vector<int>> G;
5     vector<int> match, lx, ly, visx, visy;
6     vector<int> slack;
7     int stamp = 0;
8
9     KuhnMunkres(int n) : n(n), G(n, vector<int>(n)), lx(n),
        ly(n), slack(n), match(n), visx(n), visy(n) {}
10
11     void add(int x, int y, int w){
12         G[x][y] = max(G[x][y], w);
13     }
14
15     bool dfs(int i, bool aug){ // aug = true 表示要更新 match
16         if (visx[i]==stamp) return false;
17         visx[i] = stamp;
18
19         for (int j=0 ; j<n ; j++){
20             if (visy[j]==stamp) continue;
21             int d = lx[i]+ly[j]-G[i][j];
22
23             if (d==0){
24                 visy[j] = stamp;
25                 if (match[j]==-1 || dfs(match[j], aug)){
26                     if (aug){
27                         match[j] = i;
28                     }
29                     return true;
30                 }
31             }else{
32                 slack[j] = min(slack[j], d);
33             }
34         }
35         return false;
36     }
37
38     bool augment(){
39         for (int j=0 ; j<n ; j++){
40             if (visy[j]!=stamp && slack[j]==0){
41                 visy[j] = stamp;
42                 if (match[j]==-1 || dfs(match[j], false)){
43                     return true;
44                 }
45             }
46         }
47         return false;
48     }
49
50     void relabel(){
51         int delta = INF;
52         for (int j=0 ; j<n ; j++){
53             if (visy[j]!=stamp) delta = min(delta, slack[j]);
54         }

```

```

55         for (int i=0 ; i<n ; i++){
56             if (visx[i]==stamp) lx[i] -= delta;
57         }
58         for (int j=0 ; j<n ; j++){
59             if (visy[j]==stamp) ly[j] += delta;
60             else slack[j] -= delta;
61         }
62     }
63
64     int solve(){
65
66         for (int i=0 ; i<n ; i++){
67             lx[i] = 0;
68             for (int j=0 ; j<n ; j++){
69                 lx[i] = max(lx[i], G[i][j]);
70             }
71         }
72
73         fill(ly.begin(), ly.end(), 0);
74         fill(match.begin(), match.end(), -1);
75
76         for(int i = 0; i < n; i++) {
77             fill(slack.begin(), slack.end(), INF);
78             stamp++;
79             if(dfs(i, true)) continue;
80
81             while(augment()==false) relabel();
82             stamp++;
83             dfs(i, true);
84         }
85
86         int ans = 0;
87         for (int j=0 ; j<n ; j++){
88             if (match[j]!=-1){
89                 ans += G[match[j]][j];
90             }
91         }
92         return ans;
93     }
94 };

```

## 6.12 LCA [4e91da]

```

1 struct Tree{
2     int N, M = 0, H;
3     vector<vector<int>> G;
4     vector<vector<int>> LCA;
5     vector<int> parent;
6     vector<int> dep;
7
8     Tree(int _N) : N(_N), H(__lg(_N)+1){
9         G.resize(N);
10        parent.resize(N, -1);
11        dep.resize(N, 0);
12        LCA.resize(H, vector<int>(N, 0));
13    }
14
15    void add_edge(int u, int v){
16        M++;
17        G[u].push_back(v);
18        G[v].push_back(u);
19    }
20

```



```

21 void dfs(int now, int pre){ // root 的 pre 是自己
22     dep[now] = dep[pre]+1;
23     parent[now] = pre;
24     for (auto x : G[now]){
25         if (x==pre) continue;
26         dfs(x, now);
27     }
28 }
29
30 void build_LCA(int root = 0){
31     dfs(root, root);
32     for (int i=0 ; i<N ; i++) LCA[0][i] = parent[i];
33     for (int i=1 ; i<H ; i++){
34         for (int j=0 ; j<N ; j++){
35             LCA[i][j] = LCA[i-1][LCA[i-1][j]];
36         }
37     }
38 }
39
40 int jump(int u, int step){
41     for (int i=0 ; i<H ; i++){
42         if (step&(1<<i)) u = LCA[i][u];
43     }
44     return u;
45 }
46
47 int get_LCA(int u, int v){
48     if (dep[u]<dep[v]) swap(u, v);
49     u = jump(u, dep[u]-dep[v]);
50     if (u==v) return u;
51     for (int i=H-1 ; i>=0 ; i--){
52         if (LCA[i][u]!=LCA[i][v]){
53             u = LCA[i][u];
54             v = LCA[i][v];
55         }
56     }
57     return parent[u];
58 }
59 };

```

### 6.13 MCMF [1e5239]

```

1 struct Flow {
2     struct Edge {
3         int u, rc, k, rv;
4     };
5
6     vector<vector<Edge>> G;
7     vector<int> par, par_eid;
8     Flow(int n) : G(n+1), par(n+1), par_eid(n+1) {}
9
10    // v->u, capacity: c, cost: k
11    void add(int v, int u, int c, int k){
12        G[v].push_back({u, c, k, SZ(G[u])});
13        G[u].push_back({v, 0, -k, SZ(G[v])-1});
14    }
15
16    // 3701d6
17    int spfa(int s, int t){
18        fill(ALL(par), -1);
19        vector<int> dis(SZ(par), INF);
20        vector<bool> in_q(SZ(par), false);
21        queue<int> Q;

```

```

22     dis[s] = 0;
23     in_q[s] = true;
24     Q.push(s);
25
26     while (!Q.empty()){
27         int v = Q.front();
28         Q.pop();
29         in_q[v] = false;
30
31         for (int i=0 ; i<SZ(G[v]) ; i++){
32             auto [u, rc, k, rv] = G[v][i];
33             if (rc>0 && dis[v]+k<dis[u]){
34                 dis[u] = dis[v]+k;
35                 par[u] = v;
36                 par_eid[u] = i;
37                 if (!in_q[u]) Q.push(u);
38                 in_q[u] = true;
39             }
40         }
41     }
42     return dis[t];
43 }
44
45 // return <max flow, min cost>, 150093
46 pair<int, int> flow(int s, int t){
47     int fl = 0, cost = 0, d;
48     while ((d = spfa(s, t))<INF){
49         int cur = INF;
50         for (int v=t ; v!=s ; v=par[v]){
51             cur = min(cur, G[par[v]][par_eid[v]].rc);
52             fl += cur;
53             cost += d*cur;
54             for (int v=t ; v!=s ; v=par[v]){
55                 G[par[v]][par_eid[v]].rc -= cur;
56                 G[v][G[par[v]][par_eid[v]].rv].rc += cur;
57             }
58         }
59         return {fl, cost};
60     }
61 }
62
63 vector<pair<int, int>> construct(){
64     vector<pair<int, int>> ret;
65     for (int i=0 ; i<n ; i++){
66         for (auto x : G[i]){
67             if (x.rc==0){
68                 ret.push_back({i+1, x.u-n+1});
69                 break;
70             }
71         }
72     }
73     return ret;
74 }
75 };

```

### 6.14 Tarjan [8b2350]

```

1 struct tarjan_SCC {
2     int now_T, now_SCCs;
3     vector<int> dfn, low, SCC;
4     stack<int> S;
5     vector<vector<int>> E;
6     vector<bool> vis, in_stack;

```

```

7
8     tarjan_SCC(int n) {
9         init(n);
10    }
11    void init(int n) {
12        now_T = now_SCCs = 0;
13        dfn = low = SCC = vector<int>(n);
14        E = vector<vector<int>>(n);
15        S = stack<int>();
16        vis = in_stack = vector<bool>(n);
17    }
18    void add(int u, int v) {
19        E[u].push_back(v);
20    }
21    void build() {
22        for (int i = 0; i < dfn.size(); ++i) {
23            if (!dfn[i]) dfs(i);
24        }
25    }
26    void dfs(int v) {
27        now_T++;
28        vis[v] = in_stack[v] = true;
29        dfn[v] = low[v] = now_T;
30        S.push(v);
31        for (auto &i:E[v]) {
32            if (!vis[i]) {
33                vis[i] = true;
34                dfs(i);
35                low[v] = min(low[v], low[i]);
36            }
37            else if (in_stack[i]) {
38                low[v] = min(low[v], dfn[i]);
39            }
40        }
41        if (low[v] == dfn[v]) {
42            int tmp;
43            do {
44                tmp = S.top();
45                S.pop();
46                SCC[tmp] = now_SCCs;
47                in_stack[tmp] = false;
48            } while (tmp != v);
49            now_SCCs += 1;
50        }
51    }
52 };

```

### 6.15 Tarjan Find AP [1daed6]

```

1 vector<int> dep(MAX_N), low(MAX_N), AP;
2 bitset<MAX_N> vis;
3
4 void dfs(int now, int pre){
5     int cnt = 0;
6     bool ap = 0;
7     vis[now] = 1;
8     low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
9
10    for (auto x : G[now]){
11        if (x==pre){
12            continue;
13        }else if (vis[x]==0){
14            cnt++;

```

```

15     dfs(x, now);
16     low[now] = min(low[now], low[x]);
17     if (low[x] >= dep[now]) ap=1;
18 }else{
19     low[now] = min(low[now], dep[x]);
20 }
21 }
22
23 if ((now==pre && cnt>=2) || (now!=pre && ap)){
24     AP.push_back(now);
25 }
26 }

```

## 6.16 Tree Isomorphism [cd2bbc]

```

1 #include <bits/stdc++.h>
2 #pragma GCC optimize("O3,unroll-loops")
3 #define fastio ios::sync_with_stdio(0), cin.tie(0), cout.tie
4 (0)
5 #define dbg(x) cerr << #x << " = " << x << endl
6 #define int long long
7 using namespace std;
8
9 // declare
10 const int MAX_SIZE = 2e5+5;
11 const int INF = 9e18;
12 const int MOD = 1e9+7;
13 const double EPS = 1e-6;
14 typedef vector<vector<int>> Graph;
15 typedef map<vector<int>, int> Hash;
16
17 int n, a, b;
18 int id1, id2;
19 pair<int, int> c1, c2;
20 vector<int> sz1(MAX_SIZE), sz2(MAX_SIZE);
21 vector<int> we1(MAX_SIZE), we2(MAX_SIZE);
22 Graph g1(MAX_SIZE), g2(MAX_SIZE);
23 Hash m1, m2;
24 int testcase=0;
25
26 void centroid(Graph &g, vector<int> &s, vector<int> &w, pair<
27 int, int> &rec, int now, int pre){
28     s[now]=1;
29     w[now]=0;
30     for (auto x : g[now]){
31         if (x!=pre){
32             centroid(g, s, w, rec, x, now);
33             s[now]+=s[x];
34             w[now]=max(w[now], s[x]);
35         }
36     }
37     w[now]=max(w[now], n-s[now]);
38     if (w[now]<=n/2){
39         if (rec.first==0) rec.first=now;
40         else rec.second=now;
41     }
42 }
43
44 int dfs(Graph &g, Hash &m, int &id, int now, int pre){
45     vector<int> v;
46     for (auto x : g[now]){
47         if (x!=pre){

```

```

47         int add=dfs(g, m, id, x, now);
48         v.push_back(add);
49     }
50 }
51 sort(v.begin(), v.end());
52
53 if (m.find(v)!=m.end()){
54     return m[v];
55 }else{
56     m[v]=++id;
57     return id;
58 }
59 }
60
61 void solve1(){
62
63     // init
64     id1=0;
65     id2=0;
66     c1={0, 0};
67     c2={0, 0};
68     fill(sz1.begin(), sz1.begin()+n+1, 0);
69     fill(sz2.begin(), sz2.begin()+n+1, 0);
70     fill(we1.begin(), we1.begin()+n+1, 0);
71     fill(we2.begin(), we2.begin()+n+1, 0);
72     for (int i=1; i<=n; i++){
73         g1[i].clear();
74         g2[i].clear();
75     }
76     m1.clear();
77     m2.clear();
78
79     // input
80     cin >> n;
81     for (int i=0; i<n-1; i++){
82         cin >> a >> b;
83         g1[a].push_back(b);
84         g1[b].push_back(a);
85     }
86     for (int i=0; i<n-1; i++){
87         cin >> a >> b;
88         g2[a].push_back(b);
89         g2[b].push_back(a);
90     }
91
92     // get tree centroid
93     centroid(g1, sz1, we1, c1, 1, 0);
94     centroid(g2, sz2, we2, c2, 1, 0);
95
96     // process
97     int res1=0, res2=0, res3=0;
98     if (c2.second!=0){
99         res1=dfs(g1, m1, id1, c1.first, 0);
100         m2=m1;
101         id2=id1;
102         res2=dfs(g2, m1, id1, c2.first, 0);
103         res3=dfs(g2, m2, id2, c2.second, 0);
104     }else if (c1.second!=0){
105         res1=dfs(g2, m1, id1, c2.first, 0);
106         m2=m1;
107         id2=id1;
108         res2=dfs(g1, m1, id1, c1.first, 0);
109         res3=dfs(g1, m2, id2, c1.second, 0);
110     }else{
111         res1=dfs(g1, m1, id1, c1.first, 0);

```

```

113         res2=dfs(g2, m1, id1, c2.first, 0);
114     }
115
116     // output
117     cout << (res1==res2 || res1==res3 ? "YES" : "NO") << endl;
118
119     return;
120 }
121
122 signed main(void){
123     fastio;
124
125     int t=1;
126     cin >> t;
127     while (t--){
128         solve1();
129     }
130     return 0;
131 }

```

## 6.17 圓方樹 [675aec]

```

1 #include <bits/stdc++.h>
2 #define lp(i,a,b) for(int i=(a);i<(b);i++)
3 #define pii pair<int,int>
4 #define pb push_back
5 #define ins insert
6 #define ff first
7 #define ss second
8 #define opa(x) cerr << #x << " = " << x << ", ";
9 #define op(x) cerr << #x << " = " << x << endl;
10 #define ops(x) cerr << x;
11 #define etr cerr << endl;
12 #define spc cerr << ' ';
13 #define BAE(x) (x).begin(), (x).end()
14 #define STL(x) cerr << #x << " : "; for(auto &qwe:x) cerr <<
15 qwe << ' '; cerr << endl;
16 #define deb1 cerr << "deb1" << endl;
17 #define deb2 cerr << "deb2" << endl;
18 #define deb3 cerr << "deb3" << endl;
19 #define deb4 cerr << "deb4" << endl;
20 #define deb5 cerr << "deb5" << endl;
21 #define bye exit(0);
22 using namespace std;
23
24 const int mxn = (int)(2e5) + 10;
25 const int mxlg = 17;
26 int last_special_node = (int)(1e5) + 1;
27 vector<int> E[mxn], F[mxn];
28
29 struct edg{
30     int fr, to;
31     edg(int _fr, int _to){
32         fr = _fr;
33         to = _to;
34     }
35 };
36 ostream& operator<<(ostream& os, edg x){os << x.fr << "--" <<
37 x.to;}
38 vector<edg> EV;
39
40 void tarjan(int v, int par, stack<int>& S){

```

```

39 static vector<int> dfn(mxn), low(mxn);
40 static vector<bool> to_add(mxn);
41 static int nowT = 0;
42
43 int child = 0;
44 nowT += 1;
45 dfn[v] = low[v] = nowT;
46 for(auto &ne:E[v]){
47     int i = EV[ne].to;
48     if(i == par) continue;
49     if(!dfn[i]){
50         S.push(ne);
51         tarjan(i, v, S);
52         child += 1;
53         low[v] = min(low[v], low[i]);
54
55         if(par >= 0 && low[i] >= dfn[v]){
56             vector<int> bcc;
57             int tmp;
58             do{
59                 tmp = S.top(); S.pop();
60                 if(!to_add[EV[tmp].fr]){
61                     to_add[EV[tmp].fr] = true;
62                     bcc.pb(EV[tmp].fr);
63                 }
64                 if(!to_add[EV[tmp].to]){
65                     to_add[EV[tmp].to] = true;
66                     bcc.pb(EV[tmp].to);
67                 }
68             }while(tmp != ne);
69             for(auto &j:bcc){
70                 to_add[j] = false;
71                 F[last_special_node].pb(j);
72                 F[j].pb(last_special_node);
73             }
74             last_special_node += 1;
75         }
76     }
77     else{
78         low[v] = min(low[v], dfn[i]);
79         if(dfn[i] < dfn[v]){ // edge i--v will be visited
80             // twice at here, but we only need one.
81             S.push(ne);
82         }
83     }
84 }
85
86 int dep[mxn], jmp[mxn][mxlg];
87 void dfs_lca(int v, int par, int depth){
88     dep[v] = depth;
89     for(auto &i:F[v]){
90         if(i == par) continue;
91         jmp[i][0] = v;
92         dfs_lca(i, v, depth + 1);
93     }
94 }
95
96 inline void build_lca(){
97     jmp[1][0] = 1;
98     dfs_lca(1, -1, 1);
99     lp(j, 1, mxlg){
100         lp(i, 1, mxn){
101             jmp[i][j] = jmp[jmp[i][j-1]][j-1];
102         }
103     }
104 }
105
106 inline int lca(int x, int y){
107     if(dep[x] < dep[y]){ swap(x, y); }
108
109     int diff = dep[x] - dep[y];
110     lp(j, 0, mxlg){
111         if((diff >> j) & 1){
112             x = jmp[x][j];
113         }
114     }
115     if(x == y) return x;
116
117     for(int j = mxlg - 1; j >= 0; j--){
118         if(jmp[x][j] != jmp[y][j]){
119             x = jmp[x][j];
120             y = jmp[y][j];
121         }
122     }
123     return jmp[x][0];
124 }
125
126 inline bool can_reach(int fr, int to){
127     if(dep[to] > dep[fr]) return false;
128
129     int diff = dep[fr] - dep[to];
130     lp(j, 0, mxlg){
131         if((diff >> j) & 1){
132             fr = jmp[fr][j];
133         }
134     }
135     return fr == to;
136 }
137
138 int main(){
139     ios::sync_with_stdio(false); cin.tie(0);
140     // freopen("test_input.txt", "r", stdin);
141     int n, m, q; cin >> n >> m >> q;
142     lp(i, 0, m){
143         int u, v; cin >> u >> v;
144         E[u].pb(EV.size());
145         EV.pb(edg(u, v));
146         E[v].pb(EV.size());
147         EV.pb(edg(v, u));
148     }
149     E[0].pb(EV.size());
150     EV.pb(edg(0, 1));
151     stack<int> S;
152     tarjan(0, -1, S);
153     build_lca();
154
155     lp(queries, 0, q){
156         int fr, to, relay; cin >> fr >> to >> relay;
157         if(fr == relay || to == relay){
158             cout << "NO\n";
159             continue;
160         }
161         if((can_reach(fr, relay) || can_reach(to, relay)) &&
162            dep[relay] >= dep[lca(fr, to)]){
163             cout << "NO\n";
164             continue;
165         }
166         cout << "YES\n";
167     }
168 }

```

## 6.18 最大權閉合圖 [6ca663]

```

1 /*
2 邊  $u \rightarrow v$  表示選  $u$  就要選  $v$  ( $\theta$ -based)
3 保證回傳值非負
4 構造：從  $S$  開始  $dfs$ ，不走最小割的邊。
5 所有經過的點就是要選的那些點。
6 一般圖： $O(n^2m)$  / 二分圖： $O(m\sqrt{n})$ 
7 */
8 template<typename U>
9 U maximum_closure(vector<U> w, vector<pair<int,int>> EV) {
10     int n = w.size(), S = n + 1, T = n + 2;
11     Flow G(T + 5); // Graph/Dinic.cpp
12     U sum = 0;
13     for (int i = 0; i < n; ++i) {
14         if (w[i] > 0) {
15             G.add(S, i, w[i]);
16             sum += w[i];
17         }
18         else if (w[i] < 0) {
19             G.add(i, T, abs(w[i]));
20         }
21     }
22     for (auto &[u, v] : EV) { // 請務必確保  $INF > \sum w_i$ 
23         G.add(u, v, INF);
24     }
25     U cut = G.flow(S, T);
26     return sum - cut;
27 }

```

## 6.19 Theorem

### • 任意圖

- 最大匹配 + 最小邊覆蓋 =  $n$  (不能有孤點)
- 點覆蓋的補集是獨立集。最小點覆蓋 + 最大獨立集 =  $n$
- $w$ (最小權點覆蓋) +  $w$ (最大權獨立集) =  $\sum w_v$
- (帶點權的二分圖可以用最小割解，構造請參考 Augment Path.cpp)

### • 二分圖

- 最小點覆蓋 = 最大匹配 =  $n$  - 最大獨立集

### • 只有邊帶權的二分圖

- $w$ -vertex-cover (帶權點覆蓋)：每條邊的兩個連接點被選中的次數總和至少要是  $w_e$ 。
- $w$ -weight matching (帶權匹配)
- minimum vertex count of  $w$ -vertex-cover = maximum weight count of  $w$ -weight matching (一個點可以被選很多次，但邊不行)

### • 點、邊都帶權的二分圖的定理

- $b$ -matching：假設  $v$  的點權是  $b_v$ ，那所有  $v$  的匹配邊  $e$  的權重都要滿足  $\sum w_e \leq b_v$ 。
- The maximum  $w$ -weight of a  $b$ -matching equals the minimum  $b$ -weight of vertices in a  $w$ -vertex-cover.

## 7 Math

### 7.1 CRT [8d7c58]

```

1 // ax + by = c
2 int extgcd(int a, int b, int c, int &x, int &y) {
3     if (b == 0) {
4         if (c % a) return INF;
5         x = c / a, y = 0;
6         return abs(a);
7     }
8     int x1, y1;
9     int g = extgcd(b, a % b, c, x1, y1);
10    x = y1;
11    y = x1 - a / b * y1;
12    return g;
13 }
14
15 // 有 n 個式子 · 求解  $x \equiv a_i \pmod{m_i}$ 
16 int CRT_m_coprime(int n, vector<int> &a, vector<int> &m) {
17     int p = 1, ans = 0;
18     vector<int> M(n), inv_M(n);
19
20     for (int i = 0; i < n; i++) p *= m[i];
21     for (int i = 0; i < n; i++) {
22         M[i] = p / m[i];
23         int tmp;
24         extgcd(M[i], m[i], inv_M[i], tmp);
25         ans += a[i] * inv_M[i] * M[i];
26         ans %= p;
27     }
28     return (ans % p + p) % p;
29 }
30
31 // 對於方程組的式子兩兩求解
32 // 回傳: {是否有解, {a, m}}
33 pair<bool, pair<int, int>> CRT_m_NOT_coprime(int a1, int m1,
34     int a2, int m2) {
35     int g = __gcd(m1, m2);
36     if ((a2 - a1) % g != 0) return {0, {-1, -1}};
37
38     int x, y; extgcd(m1, m2, x, y);
39
40     x = (a2 - a1) * x / g; // 兩者不能相反
41     a1 = x * m1 + a1;
42     m1 = m1 * m2 / g;
43     a1 = (a1 % m1 + m1) % m1;
44     return {1, {a1, m1}};
45 }
46
47 // ans = a / b (mod m)
48 // ans = ret.F + k * ret.S, k is integer
49 pair<int, int> div(int a, int b, int m) {
50     int flag = 1;
51     if (a < 0) { a = -a; flag *= -1; }
52     if (b < 0) { b = -b; flag *= -1; }
53     int t = -1, k = -1;
54     int res = extgcd(b, m, a, t, k);
55     if (res == INF) return {INF, INF};
56     m = abs(m / res);
57     t = t * flag;
58     t = (t % m + m) % m;
59     return {t, m};

```

59 | }

### 7.2 Josephus Problem [e0ed50]

```

1 // 有 n 個人 · 第偶數個報數的人被刪掉 · 問第 k 個被踢掉的是誰
2 int solve(int n, int k){
3     if (n==1) return 1;
4     if (k<=(n+1)/2){
5         if (2*k>n) return 2*k%n;
6         else return 2*k;
7     }else{
8         int res=solve(n/2, k-(n+1)/2);
9         if (n&1) return 2*res+1;
10        else return 2*res-1;
11    }
12 }

```

### 7.3 Lagrange any x [1f2c26]

```

1 // init: (x1, y1), (x2, y2) in a vector
2 struct Lagrange{
3     int n;
4     vector<pair<int, int>> v;
5
6     Lagrange(vector<pair<int, int>> &_v){
7         n = _v.size();
8         v = _v;
9     }
10
11     // O(n^2 Log MAX_A)
12     int solve(int x){
13         int ret = 0;
14         for (int i=0 ; i<n ; i++){
15             int now = v[i].second;
16             for (int j=0 ; j<n ; j++){
17                 if (i==j) continue;
18                 now *= ((x-v[j].first)+MOD)%MOD;
19                 now %= MOD;
20                 now *= (qp((v[i].first-v[j].first+MOD)%MOD,
21                     MOD-2)+MOD)%MOD;
22                 now %= MOD;
23             }
24             ret = (ret+now)%MOD;
25         }
26         return ret;
27     }
28 };

```

### 7.4 Lagrange continuous x [57536a]

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAX_N = 5e5 + 10;
5 const int mod = 1e9 + 7;

```

```

6
7 long long inv_fac[MAX_N];
8
9 inline int fp(long long x, int y) {
10     int ret = 1;
11     for (; y >= 1) {
12         ret = (y & 1) ? (ret * x % mod) : ret;
13         x = x * x % mod;
14     }
15     return ret;
16 }
17
18 // TO USE THIS TEMPLATE, YOU MUST MAKE SURE THAT THE MOD
19 // NUMBER IS A PRIME.
20 struct Lagrange {
21     /*
22      * Initialize a polynomial with f(x_0), f(x_0 + 1), ..., f(
23      * x_0 + n).
24      * This determines a polynomial f(x) whose degree is at most
25      * n.
26      * Then you can call sample(x) and you get the value of f(x)
27      * .
28      * Complexity of init() and sample() are both O(n).
29      */
30     int m, shift; // m = n + 1
31     vector<int> v, mul;
32     // You can use this function if you don't have inv_fac array
33     // already.
34     void construct_inv_fac() {
35         long long fac = 1;
36         for (int i = 2; i < MAX_N; ++i) {
37             fac = fac * i % mod;
38         }
39         inv_fac[MAX_N - 1] = fp(fac, mod - 2);
40         for (int i = MAX_N - 1; i >= 1; --i) {
41             inv_fac[i - 1] = inv_fac[i] * i % mod;
42         }
43     }
44     // You call init() many times without having a second
45     // instance of this struct.
46     void init(int X_0, vector<int> &u) {
47         v = u;
48         shift = ((1 - X_0) % mod + mod) % mod;
49         if (v.size() == 1) v.push_back(v[0]);
50         m = v.size();
51         mul.resize(m);
52     }
53     // You can use sample(x) instead of sample(x % mod).
54     int sample(int x) {
55         x = ((long long)x + shift) % mod;
56         x = (x < 0) ? (x + mod) : x;
57         long long now = 1;
58         for (int i = m; i >= 1; --i) {
59             mul[i - 1] = now;
60             now = now * (x - i) % mod;
61         }
62         int ret = 0;
63         bool neg = (m - 1) & 1;
64         now = 1;
65         for (int i = 1; i <= m; ++i) {
66             int up = now * mul[i - 1] % mod;
67             int down = inv_fac[m - i] * inv_fac[i - 1] % mod;
68             int tmp = ((long long)v[i - 1] * up % mod) * down
69                 % mod;
70             ret += (neg && tmp) ? (mod - tmp) : (tmp);
71             ret = (ret >= mod) ? (ret - mod) : ret;
72         }
73     }
74 };

```

```

65         now = now * (x - i) % mod;
66         neg ^= 1;
67     }
68     return ret;
69 }
70 };
71
72 int main() {
73     int n; cin >> n;
74     vector<int> v(n);
75     for (int i = 0; i < n; ++i) {
76         cin >> v[i];
77     }
78     Lagrange L;
79     L.construct_inv_fac();
80     L.init(0, v);
81     int x; cin >> x;
82     cout << L.sample(x);
83 }

```

## 7.5 Lucas's Theorem [b37dcf]

```

1 // 對於很大的  $C_n^m$  對質數  $p$  取模。只要  $p$  不大就可以用。
2 int Lucas(int n, int m, int p){
3     if (m==0) return 1;
4     return (C(n%p, m%p, p)*Lucas(n/p, m/p, p)%p);
5 }

```

## 7.6 Matrix [8d1a23]

```

1 struct Matrix{
2     int n, m;
3     vector<vector<int>> arr;
4
5     Matrix(int _n, int _m){
6         n = _n;
7         m = _m;
8         arr.assign(n, vector<int>(m));
9     }
10
11     vector<int> & operator [] (int i){
12         return arr[i];
13     }
14
15     Matrix operator * (Matrix b){
16         Matrix ret(n, b.m);
17         for (int i=0 ; i<n ; i++){
18             for (int j=0 ; j<b.m ; j++){
19                 for (int k=0 ; k<m ; k++){
20                     ret.arr[i][j] += arr[i][k]*b.arr[k][j]%
21                         MOD;
22                     ret.arr[i][j] %= MOD;
23                 }
24             }
25         }
26         return ret;
27     }
28
29     Matrix pow(int p){
30         Matrix ret(n, n), mul = *this;

```

```

30     for (int i=0 ; i<n ; i++){
31         ret.arr[i][i] = 1;
32     }
33
34     for ( ; p ; p>>=1){
35         if (p&1) ret = ret*mul;
36         mul = mul*mul;
37     }
38
39     return ret;
40 }
41
42 int det(){
43     vector<vector<int>> arr = this->arr;
44     bool flag = false;
45     for (int i=0 ; i<n ; i++){
46         int target = -1;
47         for (int j=i ; j<n ; j++){
48             if (arr[j][i]){
49                 target = j;
50                 break;
51             }
52         }
53         if (target== -1) return 0;
54         if (i!=target){
55             swap(arr[i], arr[target]);
56             flag = !flag;
57         }
58     }
59
60     for (int j=i+1 ; j<n ; j++){
61         if (!arr[j][i]) continue;
62         int freq = arr[j][i]*qp(arr[i][i], MOD-2)%MOD
63             ;
64         for (int k=i ; k<n ; k++){
65             arr[j][k] -= freq*arr[i][k];
66             arr[j][k] = (arr[j][k]%MOD+MOD)%MOD;
67         }
68     }
69
70     int ret = !flag ? 1 : MOD-1;
71     for (int i=0 ; i<n ; i++){
72         ret *= arr[i][i];
73         ret %= MOD;
74     }
75     return ret;
76 }
77 };

```

## 7.7 Matrix 01 [8d542a]

```

1 const int MAX_N = (1LL<<12);
2 struct Matrix{
3     int n, m;
4     vector<bitset<MAX_N>> arr;
5
6     Matrix(int _n, int _m){
7         n = _n;
8         m = _m;
9         arr.resize(n);
10     }
11

```

```

12     Matrix operator * (Matrix b){
13         Matrix b_t(b.m, b.n);
14         for (int i=0 ; i<b.n ; i++){
15             for (int j=0 ; j<b.m ; j++){
16                 b_t.arr[j][i] = b.arr[i][j];
17             }
18         }
19
20         Matrix ret(n, b.m);
21         for (int i=0 ; i<n ; i++){
22             for (int j=0 ; j<b.m ; j++){
23                 ret.arr[i][j] = ((arr[i]&b_t.arr[j]).count()
24                     &1);
25             }
26         }
27         return ret;
28 };

```

## 7.8 Miller Rabin [2748d2]

```

1 // O(Log n)
2 typedef Uint unsigned long long
3 Uint modmul(Uint a, Uint b, Uint m) {
4     int ret = a*b - m*(Uint)((long double)a*b/m);
5     return ret + m*(ret < 0) - m*(ret >= (int)m);
6 }
7
8 int qp(int b, int p, int m){
9     int ret = 1;
10    for ( ; p ; p>>=1){
11        if (p&1){
12            ret = modmul(ret, b, m);
13        }
14        b = modmul(b, b, m);
15    }
16    return ret;
17 }
18
19 // ed23aa
20 vector<int> llsprp = {2, 325, 9375, 28178, 450775, 9780504,
21     1795265022};
22 bool isprime(int n, vector<int> sprp = llsprp){
23     if (n==2) return 1;
24     if (n<2 || n%2==0) return 0;
25
26     int t = 0;
27     int u = n-1;
28     for ( ; u%2==0 ; t++) u>>=1;
29
30     for (int i=0 ; i<sprp.size() ; i++){
31         int a = sprp[i]%n;
32         if (a==0 || a==1 || a==n-1) continue;
33         int x = qp(a, u, n);
34         if (x==1 || x==n-1) continue;
35         for (int j=0 ; j<t ; j++){
36             x = modmul(x, x, n);
37             if (x==1) return 0;
38             if (x==n-1) break;
39         }
40         if (x==n-1) continue;
41         return 0;

```

```

42     }
43
44     return 1;
45 }

```

## 7.9 Pollard Rho [a5daef]

```

1 mt19937 seed(chrono::steady_clock::now().time_since_epoch().
2   count());
3 int rnd(int l, int r){
4   return uniform_int_distribution<int>(l, r)(seed);
5 }
6 //  $O(n^{1/4})$  回傳 1 或自己的因數、記得先判斷  $n$  是不是質數
7 // (用 Miller-Rabin)
8 // c1670c
9 int Pollard_Rho(int n){
10    int s = 0, t = 0;
11    int c = rnd(1, n-1);
12
13    int step = 0, goal = 1;
14    int val = 1;
15
16    for (goal=1 ; ; goal<=1, s=t, val=1){
17        for (step=1 ; step<=goal ; step++){
18            t = ((__int128)t*t+c)%n;
19            val = ((__int128)val*abs(t-s)%n;
20
21            if ((step % 127) == 0){
22                int d = __gcd(val, n);
23                if (d>1) return d;
24            }
25        }
26
27        int d = __gcd(val, n);
28        if (d>1) return d;
29    }
30 }

```

## 7.10 Polynomial [51ca3b]

```

1 struct Poly {
2     int len, deg;
3     int *a;
4     // len = 2^k >= the original length
5     Poly(): len(0), deg(0), a(nullptr) {}
6     Poly(int _n) {
7         len = 1;
8         deg = _n - 1;
9         while (len < _n) len <= 1;
10        a = (ll*) calloc(len, sizeof(ll));
11    }
12    Poly(int l, int d, int *b) {
13        len = l;
14        deg = d;
15        a = b;
16    }
17    void resize(int _n) {

```

```

18    int len1 = 1;
19    while (len1 < _n) len1 <= 1;
20    int *res = (ll*) calloc(len1, sizeof(ll));
21    for (int i = 0; i < min(len, _n); i++) {
22        res[i] = a[i];
23    }
24    len = len1;
25    deg = _n - 1;
26    free(a);
27    a = res;
28 }
29 Poly& operator=(const Poly rhs) {
30     this->len = rhs.len;
31     this->deg = rhs.deg;
32     this->a = (ll*)realloc(this->a, sizeof(ll) * len);
33     copy(rhs.a, rhs.a + len, this->a);
34     return *this;
35 }
36 Poly operator*(Poly rhs) {
37     int l1 = this->len, l2 = rhs.len;
38     int d1 = this->deg, d2 = rhs.deg;
39     while (l1 > 0 and this->a[l1 - 1] == 0) l1--;
40     while (l2 > 0 and rhs.a[l2 - 1] == 0) l2--;
41     int l = 1;
42     while (l < max(l1 + l2 - 1, d1 + d2 + 1)) l <= 1;
43     int *x, *y, *res;
44     x = (ll*) calloc(l, sizeof(ll));
45     y = (ll*) calloc(l, sizeof(ll));
46     res = (ll*) calloc(l, sizeof(ll));
47     copy(this->a, this->a + l1, x);
48     copy(rhs.a, rhs.a + l2, y);
49     ntt.tran(l, x); ntt.tran(l, y);
50     FOR (i, 0, l - 1)
51         res[i] = x[i] * y[i] % mod;
52     ntt.tran(l, res, true);
53     free(x); free(y);
54     return Poly(l, d1 + d2, res);
55 }
56 Poly operator+(Poly rhs) {
57     int l1 = this->len, l2 = rhs.len;
58     int l = max(l1, l2);
59     Poly res;
60     res.len = l;
61     res.deg = max(this->deg, rhs.deg);
62     res.a = (ll*) calloc(l, sizeof(ll));
63     FOR (i, 0, l1 - 1) {
64         res.a[i] += this->a[i];
65         if (res.a[i] >= mod) res.a[i] -= mod;
66     }
67     FOR (i, 0, l2 - 1) {
68         res.a[i] += rhs.a[i];
69         if (res.a[i] >= mod) res.a[i] -= mod;
70     }
71     return res;
72 }
73 Poly operator-(Poly rhs) {
74     int l1 = this->len, l2 = rhs.len;
75     int l = max(l1, l2);
76     Poly res;
77     res.len = l;
78     res.deg = max(this->deg, rhs.deg);
79     res.a = (ll*) calloc(l, sizeof(ll));
80     FOR (i, 0, l1 - 1) {
81         res.a[i] += this->a[i];
82         if (res.a[i] >= mod) res.a[i] -= mod;
83     }

```

```

84     FOR (i, 0, l2 - 1) {
85         res.a[i] -= rhs.a[i];
86         if (res.a[i] < 0) res.a[i] += mod;
87     }
88     return res;
89 }
90 Poly operator*(const int rhs) {
91     Poly res;
92     res = *this;
93     FOR (i, 0, res.len - 1) {
94         res.a[i] = res.a[i] * rhs % mod;
95         if (res.a[i] < 0) res.a[i] += mod;
96     }
97     return res;
98 }
99 Poly(vector<int> f) {
100    int _n = f.size();
101    len = 1;
102    deg = _n - 1;
103    while (len < _n) len <= 1;
104    a = (ll*) calloc(len, sizeof(ll));
105    FOR (i, 0, deg) a[i] = f[i];
106 }
107 Poly derivative() {
108     Poly g(this->deg);
109     FOR (i, 1, this->deg) {
110         g.a[i - 1] = this->a[i] * i % mod;
111     }
112     return g;
113 }
114 Poly integral() {
115     Poly g(this->deg + 2);
116     FOR (i, 0, this->deg) {
117         g.a[i + 1] = this->a[i] * ::inv(i + 1) % mod;
118     }
119     return g;
120 }
121 Poly inv(int len1 = -1) {
122     if (len1 == -1) len1 = this->len;
123     Poly g(1); g.a[0] = ::inv(a[0]);
124     for (int l = 1; l < len1; l <= 1) {
125         Poly t; t = *this;
126         t.resize(l < 1);
127         t = g * t;
128         t.resize(l < 1);
129         Poly g1 = g * 2 - t;
130         swap(g, g1);
131     }
132     return g;
133 }
134 Poly ln(int len1 = -1) {
135     if (len1 == -1) len1 = this->len;
136     auto g = *this;
137     auto x = g.derivative() * g.inv(len1);
138     x.resize(len1);
139     x = x.integral();
140     x.resize(len1);
141     return x;
142 }
143 Poly exp() {
144     Poly g(1);
145     g.a[0] = 1;
146     for (int l = 1; l < len; l <= 1) {
147         Poly t, g1; t = *this;
148         t.resize(l < 1); t.a[0]++;
149         g1 = (t - g.ln(l < 1)) * g;

```



```

150         g1.resize(1 << 1);
151         swap(g, g1);
152     }
153     return g;
154 }
155 Poly pow(11 n) {
156     Poly &a = *this;
157     int i = 0;
158     while (i <= a.deg and a.a[i] == 0) i++;
159     if (i and (n > a.deg or n * i > a.deg)) return Poly(a
160         .deg + 1);
161     if (i == a.deg + 1) {
162         Poly res(a.deg + 1);
163         res.a[0] = 1;
164         return res;
165     }
166     Poly b(a.deg - i + 1);
167     int inv1 = ::inv(a.a[i]);
168     FOR (j, 0, b.deg)
169         b.a[j] = a.a[j + i] * inv1 % mod;
170     Poly res1 = (b.ln() * (n % mod)).exp() * (::power(a.a
171         [i], n));
172     Poly res2(a.deg + 1);
173     FOR (j, 0, min((11)(res1.deg), (11)(a.deg - n * i)))
174         res2.a[j + n * i] = res1.a[j];
175     return res2;
176 }
177 };

```

## 7.11 josephus [0be067]

```

1 // n 個人，每 k 個人就刪除的約瑟夫遊戲
2 int josephus(int n, int k) {
3     if (n == 1)
4         return 0;
5     if (k == 1)
6         return n-1;
7     if (k > n)
8         return (josephus(n-1, k) + k) % n;
9     int cnt = n / k;
10    int res = josephus(n - cnt, k);
11    res -= n % k;
12    if (res < 0)
13        res += n;
14    else
15        res += res / (k - 1);
16    return res;
17 }

```

## 7.12 數論分塊 [8ccab5]

```

1 /*
2 時間複雜度為 O(sqrt(n))
3 區間為 [l, r]
4 */
5 for(int i=1 ; i<=n ; i++){
6     int l = i, r = n/(n/i);
7     i = r;
8     ans.push_back(r);
9 }

```

## 7.13 最大質因數 [ca5e52]

```

1 void max_fac(int n, int &ret){
2     if (n<=ret || n<2) return;
3     if (isprime(n)){
4         ret = max(ret, n);
5         return;
6     }
7
8     int p = Pollard_Rho(n);
9     max_fac(p, ret), max_fac(n/p, ret);
10 }

```

## 7.14 歐拉公式 [85f3b1]

```

1 // phi(n) = 小於 n 並與 n 互質的正整數數量。
2 // O(sqrt(n)) · 回傳 phi(n)
3 int phi(int n){
4     int ret = n;
5
6     for (int i=2 ; i*i<=n ; i++){
7         if (n%i==0){
8             while (n%i==0) n /= i;
9             ret = ret*(i-1)/i;
10        }
11    }
12    if (n>1) ret = ret*(n-1)/n;
13
14    return ret;
15 }
16
17 // O(n Log n) · 回傳 1~n 的 phi 值
18 vector<int> phi_1_to_n(int n){
19     vector<int> phi(n+1);
20     phi[0]=0;
21     phi[1]=1;
22
23     for (int i=2 ; i<=n ; i++){
24         phi[i]=i-1;
25     }
26
27     for (int i=2 ; i<=n ; i++){
28         for (int j=2*i ; j<=n ; j+=i){ // 枚舉所有倍數
29             phi[j]-=phi[i];
30         }
31     }
32
33     return phi;
34 }

```

## 7.15 Burnside's Lemma

$$\sum_{k=1}^n \frac{c(k)}{n}$$

- $n$ : 有多少種置換方式 (例如: 旋轉方式)
- $c(k)$ : 所有可能中, 經過  $k$  次旋轉後, 仍不會和別人相同的方式的數量

## 7.16 Catalan Number

任意括號序列:  $C_n = \frac{1}{n+1} \binom{2n}{n}$

## 7.17 Matrix Tree Theorem

目標: 給定一張無向圖, 問他的生成樹數量。  
方法: 先把所有自環刪掉, 定義  $Q$  為以下矩陣

$$Q_{i,j} = \begin{cases} \deg(v_i) & \text{if } i = j \\ -(\text{邊 } v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉  $Q$  的第一個 row 跟 column, 它的 determinant 就是答案。  
目標: 給定一張有向圖, 問他的以  $r$  為根, 可以走到所有點生成樹數量。

方法: 先把所有自環刪掉, 定義  $Q$  為以下矩陣

$$Q_{i,j} = \begin{cases} \deg_{in}(v_i) & \text{if } i = j \\ -(\text{邊 } v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉  $Q$  的第  $r$  個 row 跟 column, 它的 determinant 就是答案。

## 7.18 Stirling's formula

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

## 7.19 Theorem

- $1 \sim x$  質數的數量  $\approx \frac{x}{\ln x}$
- $x$  的因數的數量  $\approx x^{\frac{1}{3}}$
- $x$  的質因數的數量  $\approx \log \log x$
- $p$  is a prime number  $\Leftrightarrow (p-1)! \equiv -1 \pmod{p}$
- 每個正整數都可以表示成四個整數的平方和
- 任何大於 2 的整數都可以表示成兩個質數的和
- $n^{k-2} \cdot \prod_{i=1}^k s_i$  個點、 $k$  的連通塊, 加上  $k-1$  條邊使得變成一個連通圖的方法數, 其中每個連通塊有  $s_i$  個點

## 7.20 二元一次方程式

$$\begin{cases} ax + by = e \\ cx + dy = f \end{cases} = \begin{cases} x = \frac{ed-bf}{ad-bc} \\ y = \frac{af-ec}{ad-bc} \end{cases}$$
  
若  $x = \frac{0}{0}$  且  $y = \frac{0}{0}$ , 則代表無限多組解。若  $x = \frac{*}{0}$  且  $y = \frac{*}{0}$ , 則代表無解。



## 7.21 歐拉定理

若  $a, m$  互質，則：

$$a^n \equiv a^{n \bmod \varphi(m)} \pmod{m}$$

若  $a, m$  不互質，則：

$$a^n \equiv a^{\varphi(m) + [n \bmod \varphi(m)]} \pmod{m}$$

## 7.22 錯排公式

錯排公式：( $n$  個人中，每個人皆不再原來位置的組合數)

$$dp_i = \begin{cases} 1 & i = 0 \\ 0 & i = 1 \\ (i-1)(dp_{i-1} + dp_{i-2}) & \text{otherwise} \end{cases}$$

# 8 String

## 8.1 AC automation [6ece7f]

```
1 const int MAXN = 5e5 + 5;
2 struct ac_automation {
3
4     int go[MAXN][26], fail[MAXN], is_end[MAXN];
5     int sz;
6
7     void add(string s) {
8         int now = 0;
9         for (char c : s) {
10             if (!go[now][c - 'a'])
11                 go[now][c - 'a'] = ++sz;
12             now = go[now][c - 'a'];
13         }
14         is_end[now]++;
15     }
16     vector<int> que;
17     void build() {
18         que.pb(0);
19         for (int i = 0; i < ssize(que); i++) {
20             auto u = que[i];
21             FOR (c, 0, 25) {
22                 if (go[u][c]) {
23                     int v = go[u][c];
24                     fail[v] = !u ? 0 : go[fail[u]][c];
25                     is_end[v] += is_end[fail[v]];
26                     que.pb(v);
27                 }
28                 else {
29                     go[u][c] = go[fail[u]][c];
30                 }
31             }
32         }
33     }
34 } AC;
```

## 8.2 Hash [942f42]

```
1 mt19937 seed(chrono::steady_clock::now().time_since_epoch().
2     count());
3 int rng(int l, int r){
4     return uniform_int_distribution<int>(l, r)(seed);
5 }
6 int A = rng(1e5, 8e8);
7 const int B = 1e9+7;
8 // 2f6192
9 struct RollingHash{
10     vector<int> Pow, Pre;
11     RollingHash(string s = ""){
12         Pow.resize(s.size());
13         Pre.resize(s.size());
14
15         for (int i=0 ; i<s.size() ; i++){
16             if (i==0){
17                 Pow[i] = 1;
18                 Pre[i] = s[i];
19             }else{
20                 Pow[i] = Pow[i-1]*A%B;
21                 Pre[i] = (Pre[i-1]*A+s[i])%B;
22             }
23         }
24
25         return;
26     }
27
28     int get(int l, int r){ // 取得 [l, r] 的數值
29         if (l==0) return Pre[r];
30         int res = (Pre[r]-Pre[l-1]*Pow[r-l+1])%B;
31         if (res<0) res += B;
32         return res;
33     }
34 };
```

## 8.3 KMP [e5b7ce]

```
1 // 給一個字串 S，定義函數  $\pi(i) = k$  代表  $S[1 \dots k] = S[i-k$ 
2      $+1 \dots i]$  (最長真前後綴)
3 // e5b7ce
4 vector<int> KMP(string &s){
5     int n = s.size();
6     vector<int> ret(n);
7     for (int i=1; i<n; i++){
8         int j = ret[i-1];
9         while (j>0 && s[i]!=s[j]) j = ret[j-1];
10        j += (s[i]==s[j]);
11        ret[i] = j;
12    }
13    return ret;
14 }
```

## 8.4 Manacher [9a4b4d]

```
1 string Manacher(string str) {
2     string tmp = "$#";
3     for(char i : str) {
4         tmp += i;
5         tmp += '#';
6     }
7
8     vector<int> p(tmp.size(), 0);
9     int mx = 0, id = 0, len = 0, center = 0;
10    for(int i=1; i<(int)tmp.size(); i++) {
11        p[i] = mx > i ? min(p[id*2-i], mx-i) : 1;
12
13        while(tmp[i+p[i]] == tmp[i-p[i]]) p[i]++;
14        if(mx<i+p[i]) mx = i+p[i], id = i;
15        if(len<p[i]) len = p[i], center = i;
16    }
17    return str.substr((center-len)/2, len-1);
18 }
```

## 8.5 Min Rotation [9d296f]

```
1 // 9d296f
2 int minRotation(string s) {
3     int a=0, N=sz(s); s += s;
4     for (int b=0; b<N; b++){
5         for (int k=0; k<N; k++){
6             if (a+k == b || s[a+k] < s[b+k]) {b += max(0LL, k
7                 -1); break;}
8             if (s[a+k] > s[b+k]) {a = b; break;}
9         }
10    }
11    return a;
```

## 8.6 Suffix Array [6352b3]

```
1 // 注意，當  $|s|=1$  時，lcp 不會有值，務必測試  $|s|=1$  的 case
2 struct SuffixArray {
3     string s;
4     vector<int> sa, lcp;
5
6     // 69ced9
7     SuffixArray(string _s, int lim = 256) {
8         s = _s;
9         int n = s.size()+1, k = 0, a, b;
10        vector<int> x(s.begin(), s.end()), y(n, ws(max(n,
11            lim)), rank(n));
12        x.push_back(0);
13        sa = lcp = y;
14        iota(sa.begin(), sa.end(), 0);
15        for (int j=0, p=0; p<n; j=max(1LL, j*2), lim=p) {
16            p = j;
17            iota(y.begin(), y.end(), n-j);
18            for (int i=0; i<n; i++) if (sa[i] >= j) y[p++]
19                = sa[i] - j;
20            fill(ws.begin(), ws.end(), 0);
21            for (int i=0; i<n; i++) ws[x[i]]++;
22            for (int i=1; i<lim; i++) ws[i] += ws[i-1];
23            for (int i = n; i--;) sa[--ws[x[i]]] = y[i];
24            swap(x, y), p = 1, x[sa[0]] = 0;
```

```

23     for (int i=1 ; i<n ; i++){
24         a = sa[i - 1];
25         b = sa[i];
26         x[b] = (y[a] == y[b] && y[a + j] == y[b + j])
                ? p - 1 : p++;
27     }
28 }
29
30 for (int i=1 ; i<n ; i++) rank[sa[i]] = i;
31 for (int i=0, j ; i<n-1 ; lcp[rank[i+1]]=k)
32     for (k && k--, j=sa[rank[i]-1] ; i+k<s.size() &&
            j+k<s.size() && s[i+k]==s[j+k] ; k++);
33 sa.erase(sa.begin());
34 lcp.erase(lcp.begin(), lcp.begin()+2);
35 }
36
37 // f49583
38 vector<int> pos; // pos[i] = i 這個值在 pos 的哪個地方
39 SparseTable st;
40 void init_lcp(){
41     pos.resize(sa.size());
42     for (int i=0 ; i<sa.size() ; i++){
43         pos[sa[i]] = i;
44     }
45     if (lcp.size()){
46         st.build(lcp);
47     }
48 }
49
50 // 用之前記得 init
51 // 回傳 [l1, r1] 跟 [l2, r2] 的 lcp · 0-based
52 int get_lcp(int l1, int r1, int l2, int r2){
53     int pos_1 = pos[l1], len_1 = r1-l1+1;
54     int pos_2 = pos[l2], len_2 = r2-l2+1;
55     if (pos_1>pos_2){
56         swap(pos_1, pos_2);
57         swap(len_1, len_2);
58     }
59
60     if (l1==l2){
61         return min(len_1, len_2);
62     }else{
63         return min({st.query(pos_1, pos_2), len_1, len_2
64             });
65     }
66 }
67
68 // 檢查 [l1, r1] 跟 [l2, r2] 的大小關係 · 0-based
69 // 如果前者小於後者 · 就回傳 <0 · 相等就回傳 =0 · 否則回傳
70 // >0
71 // 5b8db0
72 int substring_cmp(int l1, int r1, int l2, int r2){
73     int len_1 = r1-l1+1;
74     int len_2 = r2-l2+1;
75     int res = get_lcp(l1, r1, l2, r2);
76
77     if (res<len_1 && res<len_2){
78         return s[l1+res]-s[l2+res];
79     }else if (len_1==res && len_2==res){
80         // 如果不需要以 index 作為次要排序參數 · 這裡要回
81         // 傳 0
82         return l1-l2;
83     }else{
84         return len_1==res ? -1 : 1;
85     }
86 }

```

```

83 }
84
85 // 對於位置在 <=p 的後綴 · 找離他左邊/右邊最接近位置 >p 的
86 // 後綴的 lcp · 0-based
87 // pre[i] = s[i] 離他左邊最接近位置 >p 的後綴的 lcp · 0-
88 // based
89 // suf[i] = s[i] 離他右邊最接近位置 >p 的後綴的 lcp · 0-
90 // based
91 // da12fa
92 pair<vector<int>, vector<int>> get_left_and_right_lcp(int
93     p){
94     vector<int> pre(p+1);
95     vector<int> suf(p+1);
96
97     { // build pre
98         int now = 0;
99         for (int i=0 ; i<s.size() ; i++){
100             if (sa[i]<=p){
101                 pre[sa[i]] = now;
102                 if (i<lcp.size()) now = min(now, lcp[i]);
103             }else{
104                 if (i<lcp.size()) now = lcp[i];
105             }
106         }
107     }
108     { // build suf
109         int now = 0;
110         for (int i=s.size()-1 ; i>=0 ; i--){
111             if (sa[i]<=p){
112                 suf[sa[i]] = now;
113                 if (i-1>=0) now = min(now, lcp[i-1]);
114             }else{
115                 if (i-1>=0) now = lcp[i-1];
116             }
117         }
118     }
119
120     return {pre, suf};
121 }
122 };

```

## 8.7 Z Algorithm [bcfbd6]

```

1 // 定義一個長度為 n 的文本為 T · 則陣列 Z 的 Z[i] 代表 T[0:n]
2 // 和 T[i:n] 最長共同前綴
3 // bcfbd6
4 vector<int> z_function(string s){
5     vector<int> ret(s.size());
6     int ll = 0, rr = 0;
7
8     for (int i=1 ; i<s.size() ; i++){
9         int j = 0;
10
11         if (i<rr) j = min(ret[i-ll], rr-i);
12         while (s[j]==s[i+j]) j++;
13         ret[i] = j;
14
15         if (i+j>rr){
16             ll = i;
17             rr = i+j;
18         }
19     }
20 }

```

```

19 ret[0] = s.size();
20 return ret;
21 }
22 }

```

## 8.8 k-th Substring1 [61f66b]

```

1 // 回傳 s 所有子字串 (完全不同) 中 · 第 k 大的
2 string k_th_substring(string &s, int k){
3     int n = s.size();
4     SuffixArray sa(s);
5     sa.init_lcp();
6
7     int prePrefix = 0, nowRank = 0;
8     for (int i=0 ; i<n ; i++){
9         int len = n-sa[i];
10        int add = len-prePrefix;
11
12        if (nowRank+add>=k){
13            return s.substr(sa[i], prePrefix+k-nowRank);
14        }
15
16        prePrefix = sa.lcp[i];
17        nowRank += add;
18    }
19 }

```