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1 Misc

1.1 2-SAT

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct TWO_SAT {
5     int n, N;
6     vector<vector<int>> G, rev_G;
7     deque<bool> used;
8     vector<int> order, comp;
9     deque<bool> assignment;
10    void init(int _n) {
11        n = _n;
12        N = _n * 2;
13        G.resize(N + 5);
14        rev_G.resize(N + 5);
15    }
16    void dfs1(int v) {
17        used[v] = true;
18        for (int u : G[v]) {
19            if (!used[u])
20                dfs1(u);
21        }
22        order.push_back(v);
23    }
24    void dfs2(int v, int c1) {
25        comp[v] = c1;
26        for (int u : rev_G[v]) {
27            if (comp[u] == -1)
28                dfs2(u, c1);
29        }
30    }
31    bool solve() {
32        order.clear();
33        used.assign(N, false);
34        for (int i = 0; i < N; ++i) {
35            if (!used[i])
36                dfs1(i);
37        }
38        comp.assign(N, -1);
39        for (int i = 0, j = 0; i < N; ++i) {
40            int v = order[N - i - 1];
41            if (comp[v] == -1)
42                dfs2(v, j++);
43        }
44        assignment.assign(n, false);
45        for (int i = 0; i < N; i += 2) {
46            if (comp[i] == comp[i + 1])
47                return false;
48            assignment[i / 2] = (comp[i] > comp[i + 1]);
49        }
50        return true;
51    }
52    void add_disjunction(int a, bool na, int b, bool nb) { //
53        // A or B 都是 0-based
54        // na means whether a is negative or not
55        // nb means whether b is negative or not
56        a = 2 * a ^ na;
57        b = 2 * b ^ nb;
58        int neg_a = a ^ 1;
59        int neg_b = b ^ 1;

```

```

59        G[neg_a].push_back(b);
60        G[neg_b].push_back(a);
61        rev_G[b].push_back(neg_a);
62        rev_G[a].push_back(neg_b);
63        return;
64    }
65    void get_result(vector<int>& res) {
66        res.clear();
67        for (int i = 0; i < n; i++)
68            res.push_back(assignment[i]);
69    }
70    };
71    /* CSES Giant Pizza
72    3 5
73    + 1 + 2
74    - 1 + 3
75
76    - + + + -
77    */
78    int main() {
79        int n, m;
80        cin >> n >> m;
81        TWO_SAT E;
82        E.init(m);
83
84        char c1, c2;
85        int inp1, inp2;
86        for (int i = 0; i < n; i++) {
87            cin >> c1 >> inp1;
88            cin >> c2 >> inp2;
89            E.add_disjunction(inp1 - 1, c1 == '-', inp2 - 1, c2
90                             == '-');
91        }
92        bool able = E.solve();
93        if (able) {
94            vector<int> ans;
95            E.get_result(ans);
96            for (int i : ans)
97                cout << (i == true ? '+' : '-') << ' ';
98            cout << '\n';
99        } else {
100            cout << "IMPOSSIBLE\n";
101        }
102    }
103    return 0;
104 }

```

1.2 Custom Set PQ Sort

```

1 // priority_queue · 務必檢查相等的 case · 給所有元素一個排序的
2 // 依據
3 struct cmp{
4     bool operator () (Data a, Data b){
5         return a.x<b.x;
6     }
7 };
8 priority_queue<Data, vector<Data>, cmp> pq;
9
10 // set · 務必檢查相等的 case · 給所有元素一個排序的依據
11 struct Data{
12     int x;

```

```

12
13     bool operator < (const Data &b) const {
14         return x<b.x;
15     }
16 };

```

1.3 Default Code New

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 #define int long long
4
5 const int MAX_N = 5e5 + 10;
6 const int INF = 2e18;
7
8 void solve(){
9
10 }
11
12 signed main(){
13     ios::sync_with_stdio(0), cin.tie(0);
14
15     int t = 1;
16     while (t--){
17         solve();
18     }
19
20     return 0;
21 }

```

1.4 Default Code Old

```

1 #include <bits/stdc++.h>
2 #define int long long
3 #define ALL(x) x.begin(), x.end()
4 #define SZ(x) ((int)x.size())
5 #define fastio ios::sync_with_stdio(0), cin.tie(0);
6 using namespace std;
7
8 #ifdef LOCAL
9 #define cout cout << "\033[0;32m"
10 #define cerr cerr << "\033[0;31m"
11 #define endl endl << "\033[0m"
12 #else
13 #pragma GCC optimize("O3,unroll-loops")
14 #pragma GCC target("avx,avx2,sse,sse2,sse3,sse4,popcnt")
15 #define endl "\n"
16 #endif
17
18 const int MAX_N = 5e5+10;
19 const int INF = 2e18;
20
21 void solve1(){
22
23     return;
24 }
25
26 signed main(){
27     fastio;
28

```

```

29 |
30 |     int t = 1;
31 |     while (t--){
32 |         solve1();
33 |     }
34 |
35 |     return 0;
36 | }

```

1.5 Enumerate Subset

```

1 | // 時間複雜度  $O(3^n)$ 
2 | // 枚舉每個 mask 的子集
3 | for (int mask=0; mask<(1<<n); mask++){
4 |     for (int s=mask; s>=0; s=(s-1)&m){
5 |         // s 是 mask 的子集
6 |         if (s==0) break;
7 |     }
8 | }

```

1.6 Fast Input

```

1 | // fast IO
2 | // 6f8879
3 | inline char readchar(){
4 |     static char buffer[BUFSIZ], *now = buffer + BUFSIZ, *
5 |         end = buffer + BUFSIZ;
6 |     if (now == end)
7 |     {
8 |         if (end < buffer + BUFSIZ)
9 |             return EOF;
10 |        end = (buffer + fread(buffer, 1, BUFSIZ, stdin));
11 |        now = buffer;
12 |    }
13 |    return *now++;
14 | }
15 | inline int nextint(){
16 |     int x = 0, c = readchar(), neg = false;
17 |     while (('0' > c || c > '9') && c != '-' && c != EOF) c =
18 |         readchar();
19 |     if (c == '-') neg = true, c = readchar();
20 |     while ('0' <= c && c <= '9') x = (x<<3) + (x<<1) + (c-'0')
21 |         , c = readchar();
22 |     if (neg) x = -x;
23 |     return x; // returns 0 if EOF

```

1.7 Radix Sort

```

1 | // 值域限制:  $0 \sim 1073741823(2^{30}-1)$ 
2 | inline void radix_sort(vector<int> &a, int n){
3 |     static int cnt[32768] = {0};
4 |     vector<int> tmpa(n);
5 |     for(int i = 0; i < n; ++i)
6 |         ++cnt[a[i] & 32767];
7 |     for(int i = 1; i < 32768; ++i)

```

```

8 |         cnt[i] += cnt[i-1];
9 |     static int temp;
10 |    for(int i = n-1; i >= 0; --i){
11 |        temp = a[i] & 32767;
12 |        --cnt[temp];
13 |        tmpa[cnt[temp]] = a[i];
14 |    }
15 |
16 |    static int cnt2[32768] = {0};
17 |    for(int i = 0; i < n; ++i)
18 |        ++cnt2[(tmpa[i]>>15)];
19 |    for(int i = 1; i < 32768; ++i)
20 |        cnt2[i] += cnt2[i-1];
21 |
22 |    for(int i = n-1; i >= 0; --i){
23 |        temp = (tmpa[i]>>15);
24 |        --cnt2[temp];
25 |        a[cnt2[temp]] = tmpa[i];
26 |    }
27 |    return;
28 | }

```

1.8 Xor Basis

```

1 | vector<int> basis;
2 | void add_vector(int x){
3 |     for (auto v : basis){
4 |         x=min(x, x^v);
5 |     }
6 |     if (x) basis.push_back(x);
7 | }
8 |
9 | // 給一數字集合 S · 求能不能 XOR 出 x
10 | bool check(int x){
11 |     for (auto v : basis){
12 |         x=min(x, x^v);
13 |     }
14 |     return x;
15 | }
16 |
17 | // 給一數字集合 S · 求能 XOR 出多少數字
18 | // 答案等於  $2^{\text{basis 的大小}}$ 
19 |
20 | // 給一數字集合 S · 求 XOR 出最大的數字
21 | int get_max(){
22 |     int ans=0;
23 |     for (auto v : basis){
24 |         ans=max(ans, ans^v);
25 |     }
26 |     return ans;
27 | }

```

1.9 random int

```

1 | mt19937 seed(chrono::steady_clock::now().time_since_epoch().
2 |     count());
3 | int rng(int l, int r){
4 |     return uniform_int_distribution<int>(l, r)(seed);

```

1.10 hash command

```

1 | cat file.cpp | cpp -dD -P -fpreprocessed | tr -d "[:space:]"
2 | | md5sum | cut -c-6

```

1.11 run

```

1 | import os
2 | p = os.listdir(".")
3 | f = input("input: ")
4 |
5 | if os.system(f"g++ {f}.cpp -std=c++17 -Wall -Wextra -Wshadow
6 |     -O2 -DLOCAL -g -fsanitize=undefined,address -o {f}") !=
7 |     0:
8 |     print("CE")
9 |     exit(1)
10 |
11 | for x in p:
12 |     if x[:len(f)]==f and x[-3:]=="_in":
13 |         print(x)
14 |         if os.system(f"./{f} < {x}")!=0:
15 |             print("RE")
16 |             exit(1)
17 |         print()

```

1.12 run2

```

1 | import os
2 |
3 | f = "pA"
4 |
5 | while 1:
6 |     i = input("input: ")
7 |     p = os.listdir(".")
8 |     if i != "":
9 |         f = i
10 |        print(f"file = {f}")
11 |        if os.system(f"g++ {f}.cpp -std=c++17 -Wall -Wextra -
12 |            Wshadow -O2 -D LOCAL -g -fsanitize=undefined,address
13 |            -o {f}")!=0:
14 |            print("CE")
15 |            continue
16 |        os.system("clear")
17 |
18 | for x in sorted(p):
19 |     if f in x and ".in" in x:
20 |         print(x)
21 |         if os.system(f"./{f} < {x}")!=0:
22 |             print("RE")
23 |             print()

```

1.13 setup

```

1 se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
2
3 :inoremap " ""<Esc>i
4 :inoremap {<CR> {<CR><Esc>ko
5 :inoremap [{ {<ESC>i
6
7 function! F(...)
8     execute '!./%:r < ./' . a:1
9 endfunction
10 command! -nargs=* R call F(<f-args>)
11
12 map <F7> :w<bar>!g++ "%" -o %:r -std=c++17 -Wall -Wextra -
13     Wshadow -O2 -DLOCAL -g -fsanitize=undefined,address<CR>
14 map <F8> :!./%:r<CR>
15 map <F9> :!./%:r < ./%:r.in<CR>
16
17 ca hash w !cpp -dD -P -fpreprocessed \ | tr -d "[:space:]" \
18     md5sum \ | cut -c-6
19
20 " i+<esc>25A---+<esc>
21 " o|<esc>25A |<esc>
22 " "ggVGyG35pGdd

```

2 Convolution

2.1 FFT any mod

```

1 /*
2 修改 const int MOD = 998244353 更改要取餘的數字
3 PolyMul(a, b) 回傳多項式乘法的結果 (c_k = \sum_{i+j} a_i+b_j
4     mod MOD)
5
6 大約可以支援 5e5 · a_i, b_i 皆在 MOD 以下的非負整數
7 */
8 const int MOD = 998244353;
9 typedef complex<double> cd;
10
11 // b9c90a
12 void FFT(vector<cd> &a) {
13     int n = a.size(), L = 31-__builtin_clz(n);
14     vector<complex<long double>> R(2, 1);
15     vector<cd> rt(2, 1);
16     for (int k=2; k<n; k*=2){
17         R.resize(n);
18         rt.resize(n);
19         auto x = polar(1.0L, acos(-1.0L) / k);
20         for (int i=k; i<2*k; i++){
21             rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
22         }
23     }
24     vector<int> rev(n);
25     for (int i=0; i<n; i++){
26         rev[i] = (rev[i/2] | (i&1)<<L)/2;
27     }
28     for (int i=0; i<n; i++){
29         if (i<rev[i]) swap(a[i], a[rev[i]]);

```

```

30     }
31     for (int k=1; k<n; k*=2){
32         for (int i=0; i<n; i+=2*k){
33             for (int j=0; j<k; j++){
34                 auto x = (double *)&rt[j+k];
35                 auto y = (double *)&a[i+j+k];
36                 cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*
37                     y[0]);
38                 a[i+j+k] = a[i+j]-z;
39                 a[i+j] += z;
40             }
41         }
42     }
43     return;
44 }
45 // d3c65e
46 vector<int> PolyMul(vector<int> a, vector<int> b){
47     if (a.empty() || b.empty()) return {};
48
49     vector<int> res(a.size()+b.size()-1);
50     int B = 32-__builtin_clz(res.size()), n = (1<<B), cut =
51         int(sqrt(MOD));
52     vector<cd> L(n), R(n), outs(n), outl(n);
53
54     for (int i=0; i<a.size(); i++){
55         L[i] = cd((int) a[i]/cut, (int)a[i]%cut);
56     }
57     for (int i=0; i<b.size(); i++){
58         R[i] = cd((int) b[i]/cut, (int)b[i]%cut);
59     }
60     FFT(L);
61     FFT(R);
62     for (int i=0; i<n; i++){
63         int j = -i&(n-1);
64         outl[j] = (L[i]+conj(L[j])) * R[i]/(2.0*n);
65         outs[j] = (L[i]-conj(L[j])) * R[i]/(2.0*n)/1i;
66     }
67     FFT(outl);
68     FFT(outs);
69     for (int i=0; i<res.size(); i++){
70         int av = (int)(real(outl[i])+0.5), cv = (int)(imag(
71             outs[i])+0.5);
72         int bv = (int)(imag(outl[i])+0.5) + (int)(real(outs[i]
73             )+0.5);
74         res[i] = ((av%MOD*cut+bv) % MOD*cut+cv) % MOD;
75     }
76     return res;
77 }

```

2.2 FFT new

```

1 typedef complex<double> cd;
2
3 void FFT(vector<cd> &a) {
4     int n = a.size(), L = 31-__builtin_clz(n);
5     vector<complex<long double>> R(2, 1);
6     vector<cd> rt(2, 1);
7     for (int k=2; k<n; k*=2){
8         R.resize(n);
9         rt.resize(n);
10        auto x = polar(1.0L, acos(-1.0L) / k);

```

```

11        for (int i=k; i<2*k; i++){
12            rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
13        }
14    }
15    vector<int> rev(n);
16    for (int i=0; i<n; i++){
17        rev[i] = (rev[i/2] | (i&1)<<L)/2;
18    }
19    for (int i=0; i<n; i++){
20        if (i<rev[i]) swap(a[i], a[rev[i]]);
21    }
22    for (int k=1; k<n; k*=2){
23        for (int i=0; i<n; i+=2*k){
24            for (int j=0; j<k; j++){
25                auto x = (double *)&rt[j+k];
26                auto y = (double *)&a[i+j+k];
27                cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*
28                    y[0]);
29                a[i+j+k] = a[i+j]-z;
30                a[i+j] += z;
31            }
32        }
33    }
34    return;
35 }
36
37 vector<double> PolyMul(const vector<double> a, const vector<
38     double> b){
39     if (a.empty() || b.empty()) return {};
40     vector<double> res(a.size()+b.size()-1);
41     int L = 32 - __builtin_clz(res.size()), n = 1 << L;
42     vector<cd> in(n), out(n);
43
44     copy(a.begin(), a.end(), begin(in));
45     for (int i=0; i<b.size(); i++){
46         in[i].imag(b[i]);
47     }
48     FFT(in);
49     for (cd& x : in) x *= x;
50     for (int i=0; i<n; i++){
51         out[i] = in[-i & (n - 1)] - conj(in[i]);
52     }
53     FFT(out);
54     for (int i=0; i<res.size(); i++){
55         res[i] = imag(out[i]) / (4 * n);
56     }
57     return res;
58 }
59 }

```

2.3 FFT old

```

1 typedef complex<double> cd;
2 const double PI = acos(-1);
3
4 void FFT(vector<cd> &a, bool inv){
5
6     int n = a.size();
7
8     for (int i=1, j=0; i<n; i++){
9         int bit = (n>>1);

```

```

10     for ( ; j&bit ; bit>>=1){
11         j ^= bit;
12     }
13     j ^= bit;
14     if (i<j){
15         swap(a[i], a[j]);
16     }
17 }
18
19 for (int len=2 ; len<=n ; len<=1){
20     cd wlen = polar(1.0, (inv ? 2 : -2)*PI/len);
21
22     for (int i=0 ; i<n ; i+=len){
23         cd w(1);
24         for (int j=0 ; j<len/2 ; j++){
25             cd u = a[i+j];
26             cd v = a[i+j+len/2]*w;
27             a[i+j] = u+v;
28             a[i+j+len/2] = u-v;
29             w *= wlen;
30         }
31     }
32 }
33
34 if (inv){
35     for (auto &x : a){
36         x /= n;
37     }
38 }
39
40 return;
41 }
42
43 vector<cd> polyMul(vector<cd> a, vector<cd> b){
44     int sa = a.size(), sb = b.size(), n = 1;
45
46     while (n<sa+sb-1) n *= 2;
47     a.resize(n);
48     b.resize(n);
49     vector<cd> c(n);
50
51     FFT(a, 0);
52     FFT(b, 0);
53     for (int i=0 ; i<n ; i++) c[i] = a[i]*b[i];
54     FFT(c, 1);
55
56     c.resize(sa+sb-1);
57
58     return c;
59 }

```

2.4 NTT mod 998244353

```

1 const int MOD = (119 << 23) + 1, ROOT = 62; // = 998244353
2 // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
3 // and 483 << 21 (same root). The last two are > 10^9.
4
5 // 9cd58a
6 void NTT(vector<int> &a) {
7     int n = a.size();
8     int L = 31-__builtin_clz(n);
9     vector<int> rt(2, 1);

```

```

10     for (int k=2, s=2 ; k<n ; k*=2, s++){
11         rt.resize(n);
12         int z[] = {1, qp(ROOT, MOD>>s)};
13         for (int i=k ; i<2*k ; i++){
14             rt[i] = rt[i/2]*z[i&1]%MOD;
15         }
16     }
17
18     vector<int> rev(n);
19     for (int i=0 ; i<n ; i++){
20         rev[i] = (rev[i/2] | (i&1)<<L)/2;
21     }
22     for (int i=0 ; i<n ; i++){
23         if (i<rev[i]){
24             swap(a[i], a[rev[i]]);
25         }
26     }
27
28     for (int k=1 ; k<n ; k*=2){
29         for (int i=0 ; i<n ; i+=2*k){
30             for (int j=0 ; j<k ; j++){
31                 int z = rt[j+k]*a[i+j+k]%MOD, &ai = a[i+j];
32                 a[i+j+k] = ai-z+(z>ai ? MOD : 0);
33                 ai += (ai+z>MOD ? z-MOD : z);
34             }
35         }
36     }
37 }
38
39 // 0b0e99
40 vector<int> polyMul(vector<int> &a, vector<int> &b){
41     if (a.empty() || b.empty()) return {};
42     int s = a.size()+b.size()-1, B = 32-__builtin_clz(s), n = 1<<B;
43     int inv = qp(n, MOD-2);
44
45     vector<int> L(a), R(b), out(n);
46     L.resize(n), R.resize(n);
47     NTT(L), NTT(R);
48     for (int i=0 ; i<n ; i++){
49         out[-i&(n-1)] = L[i]*R[i]%MOD*inv%MOD;
50     }
51     NTT(out);
52
53     out.resize(s);
54     return out;
55 }

```

3 Data-Structure

3.1 BIT

```

1 vector<int> BIT(MAX_SIZE);
2 void update(int pos, int val){
3     for (int i=pos ; i<MAX_SIZE ; i+=i&-i){
4         BIT[i]+=val;
5     }
6 }
7
8 int query(int pos){
9     int ret=0;

```

```

10     for (int i=pos ; i>0 ; i-=i&-i){
11         ret+=BIT[i];
12     }
13     return ret;
14 }
15
16 // const int MAX_N = (1<<20)
17 int k_th(int k){ // 回傳 BIT 中第 k 小的元素 (based-1)
18     int res = 0;
19     for (int i=MAX_N>>1 ; i>=1 ; i>=1)
20         if (bit[res+i]<k)
21             k -= bit[res+i];
22     return res+1;
23 }

```

3.2 Disjoint Set Persistent

```

1 struct Persistent_Disjoint_Set{
2     Persistent_Segment_Tree arr, sz;
3
4     void init(int n){
5         arr.init(n);
6         vector<int> v1;
7         for (int i=0 ; i<n ; i++){
8             v1.push_back(i);
9         }
10        arr.build(v1, 0);
11
12        sz.init(n);
13        vector<int> v2;
14        for (int i=0 ; i<n ; i++){
15            v2.push_back(1);
16        }
17        sz.build(v2, 0);
18    }
19
20    int find(int a){
21        int res = arr.query_version(a, a+1, arr.version.size()-1).val;
22        if (res==a) return a;
23        return find(res);
24    }
25
26    bool unite(int a, int b){
27        a = find(a);
28        b = find(b);
29
30        if (a!=b){
31
32            int sz1 = sz.query_version(a, a+1, arr.version.size()-1).val;
33            int sz2 = sz.query_version(b, b+1, arr.version.size()-1).val;
34
35            if (sz1<sz2){
36                arr.update_version(a, b, arr.version.size()-1);
37                sz.update_version(b, sz1+sz2, arr.version.size()-1);
38            }else{
39                arr.update_version(b, a, arr.version.size()-1);

```

```

40         sz.update_version(a, sz1+sz2, arr.version.
41             size()-1);
42     }
43     return true;
44 }
45 return false;
46 };

```

3.3 PBDS GP Hash Table

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 typedef tree<int, null_type, less<int>, rb_tree_tag,
4     tree_order_statistics_node_update> order_set;
5 struct custom_hash {
6     static uint64_t splitmix64(uint64_t x) {
7         // http://xorshift.di.unimi.it/splitmix64.c
8         x += 0x9e3779b97f4a7c15;
9         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
10        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
11        return x ^ (x >> 31);
12    }
13    size_t operator()(uint64_t x) const {
14        static const uint64_t FIXED_RANDOM = chrono::
15            steady_clock::now().time_since_epoch().count();
16        return splitmix64(x + FIXED_RANDOM);
17    }
18 };
19 gp_hash_table<int, int, custom_hash> ss;

```

3.4 PBDS Order Set

```

1 /*
2  .find_by_order(k) 回傳第 k 小的值 (based-0)
3  .order_of_key(k) 回傳有多少元素比 k 小
4  不能在 #define int Long Long 後 #include 檔案
5  */
6
7 #include <ext/pb_ds/assoc_container.hpp>
8 #include <ext/pb_ds/tree_policy.hpp>
9 using namespace __gnu_pbds;
10 typedef tree<int, null_type, less<int>, rb_tree_tag,
11     tree_order_statistics_node_update> order_set;

```

3.5 Segment Tree Add Set

```

1 // [ll, rr), based-0
2 // 使用前記得 init(陣列大小), build(陣列名稱)
3 // add(ll, rr): 區間修改
4 // set(ll, rr): 區間賦值
5 // query(ll, rr): 區間求和 / 求最大值
6 struct SegmentTree{

```

```

7     struct node{
8         int add_tag = 0;
9         int set_tag = 0;
10        int sum = 0;
11        int ma = 0;
12    };
13
14    vector<node> arr;
15
16    SegmentTree(int n){
17        arr.resize(n<<2);
18    }
19
20    node pull(node A, node B){
21        node C;
22        C.sum = A.sum+B.sum;
23        C.ma = max(A.ma, B.ma);
24        return C;
25    }
26
27    // cce0c8
28    void push(int idx, int ll, int rr){
29        if (arr[idx].set_tag!=0){
30            arr[idx].sum = (rr-ll)*arr[idx].set_tag;
31            arr[idx].ma = arr[idx].set_tag;
32            if (rr-ll>1){
33                arr[idx*2+1].add_tag = 0;
34                arr[idx*2+1].set_tag = arr[idx].set_tag;
35                arr[idx*2+2].add_tag = 0;
36                arr[idx*2+2].set_tag = arr[idx].set_tag;
37            }
38            arr[idx].set_tag = 0;
39        }
40        if (arr[idx].add_tag!=0){
41            arr[idx].sum += (rr-ll)*arr[idx].add_tag;
42            arr[idx].ma += arr[idx].add_tag;
43            if (rr-ll>1){
44                arr[idx*2+1].add_tag += arr[idx].add_tag;
45                arr[idx*2+2].add_tag += arr[idx].add_tag;
46            }
47            arr[idx].add_tag = 0;
48        }
49    }
50
51    void build(vector<int> &v, int idx = 0, int ll = 0, int
52        rr = n){
53        if (rr-ll==1){
54            arr[idx].sum = v[ll];
55            arr[idx].ma = v[ll];
56        }else{
57            int mid = (ll+rr)/2;
58            build(v, idx*2+1, ll, mid);
59            build(v, idx*2+2, mid, rr);
60            arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
61        }
62    }
63
64    void add(int ql, int qr, int val, int idx = 0, int ll =
65        0, int rr = n){
66        push(idx, ll, rr);
67        if (rr<=ql || qr<=ll) return;
68        if (ql<=ll && rr<=qr){
69            arr[idx].add_tag += val;
70            push(idx, ll, rr);
71            return;

```

```

71        int mid = (ll+rr)/2;
72        add(ql, qr, val, idx*2+1, ll, mid);
73        add(ql, qr, val, idx*2+2, mid, rr);
74        arr[idx]=pull(arr[idx*2+1], arr[idx*2+2]);
75    }
76
77    void set(int ql, int qr, int val, int idx=0, int ll=0,
78        int rr=n){
79        push(idx, ll, rr);
80        if (rr<=ql || qr<=ll) return;
81        if (ql<=ll && rr<=qr){
82            arr[idx].add_tag = 0;
83            arr[idx].set_tag = val;
84            push(idx, ll, rr);
85            return;
86        }
87        int mid = (ll+rr)/2;
88        set(ql, qr, val, idx*2+1, ll, mid);
89        set(ql, qr, val, idx*2+2, mid, rr);
90        arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
91    }
92
93    node query(int ql, int qr, int idx = 0, int ll = 0, int
94        rr = n){
95        push(idx, ll, rr);
96        if (rr<=ql || qr<=ll) return node();
97        if (ql<=ll && rr<=qr) return arr[idx];
98
99        int mid = (ll+rr)/2;
100       return pull(query(ql, qr, idx*2+1, ll, mid), query(ql
101           , qr, idx*2+2, mid, rr));
102    }
103 } ST;

```

3.6 Segment Tree Li Chao Line

```

1 /*
2  全部都是 0-based
3
4  宣告
5  LC_Segment_Tree st(n);
6
7  函式:
8  update({a, b}): 插入一條 y=ax+b 的全域直線
9  query(x): 查詢所有直線在位置 x 的最小值
10 */
11 const int MAX_V = 1e6+10; // 值域最大值
12
13 struct LC_Segment_Tree{
14     struct Node{ // y = ax+b
15         int a = 0;
16         int b = INF;
17
18         int y(int x){
19             return a*x+b;
20         }
21     };
22     vector<Node> arr;
23
24     LC_Segment_Tree(int n = 0){
25         arr.resize(4*n);
26     }

```

```

27 void update(Node val, int idx = 0, int ll = 0, int rr =
28     MAX_V){
29     if (rr-ll==0) return;
30     if (rr-ll==1){
31         if (val.y(ll)<arr[idx].y(ll)){
32             arr[idx] = val;
33         }
34         return;
35     }
36     int mid = (ll+rr)/2;
37     if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
38     // 的線斜率要比較小
39     if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
40         update(val, idx*2+1, ll, mid);
41     }else{ // 交點在右邊
42         swap(arr[idx], val); // 在左子樹中，新線比舊線還
43         // 要好
44         update(val, idx*2+2, mid, rr);
45     }
46     return;
47 }
48 int query(int x, int idx = 0, int ll = 0, int rr = MAX_V)
49 {
50     if (rr-ll==0) return INF;
51     if (rr-ll==1){
52         return arr[idx].y(ll);
53     }
54     int mid = (ll+rr)/2;
55     if (x<mid){
56         return min(arr[idx].y(x), query(x, idx*2+1, ll,
57             mid));
58     }else{
59         return min(arr[idx].y(x), query(x, idx*2+2, mid,
60             rr));
61     }
62 }

```

3.7 Segment Tree Li Chao Segment

```

1  /*
2  全部都是 0-based
3
4  宣告
5  LC_Segment_Tree st(n);
6
7  函式：
8  update_segment({a, b}, ql, qr)：在 [ql, qr) 插入一條 y=ax+b
9  // 的線段
10 query(x)：查詢所有直線在位置 x 的最小值
11 */
12 const int MAX_V = 1e6+10; // 值域最大值
13 struct LC_Segment_Tree{
14     struct Node{ // y = ax+b
15         int a = 0;
16         int b = INF;

```

```

17     int y(int x){
18         return a*x+b;
19     }
20 };
21 vector<Node> arr;
22
23 LC_Segment_Tree(int n = 0){
24     arr.resize(4*n);
25 }
26
27 void update(Node val, int idx = 0, int ll = 0, int rr =
28     MAX_V){
29     if (rr-ll==0) return;
30     if (rr-ll==1){
31         if (val.y(ll)<arr[idx].y(ll)){
32             arr[idx] = val;
33         }
34         return;
35     }
36     int mid = (ll+rr)/2;
37     if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
38     // 的線斜率要比較小
39     if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
40         update(val, idx*2+1, ll, mid);
41     }else{ // 交點在右邊
42         swap(arr[idx], val); // 在左子樹中，新線比舊線還
43         // 要好
44         update(val, idx*2+2, mid, rr);
45     }
46     return;
47 }
48
49 // 在 [ql, qr) 加上一條 val 的線段
50 void update_segment(Node val, int ql, int qr, int idx =
51     0, int ll = 0, int rr = MAX_V){
52     if (rr-ll==0) return;
53     if (rr-ql || qr<=ll) return;
54     if (ql<=ll && rr<=qr){
55         update(val, idx, ll, rr);
56         return;
57     }
58     int mid = (ll+rr)/2;
59     update_segment(val, ql, qr, idx*2+1, ll, mid);
60     update_segment(val, ql, qr, idx*2+2, mid, rr);
61     return;
62 }
63
64 int query(int x, int idx = 0, int ll = 0, int rr = MAX_V)
65 {
66     if (rr-ll==0) return INF;
67     if (rr-ll==1){
68         return arr[idx].y(ll);
69     }
70     int mid = (ll+rr)/2;
71     if (x<mid){
72         return min(arr[idx].y(x), query(x, idx*2+1, ll,
73             mid));
74     }else{
75         return min(arr[idx].y(x), query(x, idx*2+2, mid,
76             rr));
77     }
78 }

```

```

75 }
76 };

```

3.8 Segment Tree Persistent

```

1  /*
2  全部都是 0-based
3
4  宣告
5  Persistent_Segment_Tree st(n+q);
6  st.build(v, 0);
7
8  函式：
9  update_version(pos, val, ver)：對版本 ver 的 pos 位置改成 val
10 query_version(ql, qr, ver)：對版本 ver 查詢 [ql, qr) 的區間和
11 clone_version(ver)：複製版本 ver 到最新的版本
12 */
13 struct Persistent_Segment_Tree{
14     int node_cnt = 0;
15     struct Node{
16         int lc = -1;
17         int rc = -1;
18         int val = 0;
19     };
20     vector<Node> arr;
21     vector<int> version;
22
23     Persistent_Segment_Tree(int sz){
24         arr.resize(32*sz);
25         version.push_back(node_cnt++);
26         return;
27     }
28
29     void pull(Node &c, Node a, Node b){
30         c.val = a.val+b.val;
31         return;
32     }
33
34     void build(vector<int> &v, int idx, int ll = 0, int rr =
35         n){
36         auto &now = arr[idx];
37
38         if (rr-ll==1){
39             now.val = v[ll];
40             return;
41         }
42
43         int mid = (ll+rr)/2;
44         now.lc = node_cnt++;
45         now.rc = node_cnt++;
46         build(v, now.lc, ll, mid);
47         build(v, now.rc, mid, rr);
48         pull(now, arr[now.lc], arr[now.rc]);
49         return;
50     }
51
52     void update(int pos, int val, int idx, int ll = 0, int rr
53         = n){
54         auto &now = arr[idx];
55
56         if (rr-ll==1){
57             now.val = val;

```



```

56     return;
57 }
58
59 int mid = (ll+rr)/2;
60 if (pos<mid){
61     arr[node_cnt] = arr[now.lc];
62     now.lc = node_cnt;
63     node_cnt++;
64     update(pos, val, now.lc, ll, mid);
65 }else{
66     arr[node_cnt] = arr[now.rc];
67     now.rc = node_cnt;
68     node_cnt++;
69     update(pos, val, now.rc, mid, rr);
70 }
71 pull(now, arr[now.lc], arr[now.rc]);
72 return;
73 }
74
75 void update_version(int pos, int val, int ver){
76     update(pos, val, version[ver]);
77 }
78
79 Node query(int ql, int qr, int idx, int ll = 0, int rr =
80 n){
81     auto &now = arr[idx];
82
83     if (ql<=ll && rr<=qr) return now;
84     if (rr<=ql || qr<=ll) return Node();
85
86     int mid = (ll+rr)/2;
87
88     Node ret;
89     pull(ret, query(ql, qr, now.lc, ll, mid), query(ql,
90 qr, now.rc, mid, rr));
91     return ret;
92 }
93
94 Node query_version(int ql, int qr, int ver){
95     return query(ql, qr, version[ver]);
96 }
97
98 void clone_version(int ver){
99     version.push_back(node_cnt);
100     arr[node_cnt] = arr[version[ver]];
101     node_cnt++;
102 }
103
104 };

```

3.9 Sparse Table

```

1 struct SparseTable{
2     vector<vector<int>> st;
3     void build(vector<int> v){
4         int h = __lg(v.size());
5         st.resize(h+1);
6         st[0] = v;
7
8         for (int i=1 ; i<=h ; i++){
9             int gap = (1<<(i-1));
10            for (int j=0 ; j+gap<st[i-1].size() ; j++){
11                st[i].push_back(min(st[i-1][j], st[i-1][j+gap
12                ]));
13            }
14        }
15    }
16 }

```

```

12     }
13 }
14 }
15
16 // 回傳 [ll, rr) 的最小值
17 int query(int ll, int rr){
18     int h = __lg(rr-ll);
19     return min(st[h][ll], st[h][rr-(1<<h)]);
20 }
21 };

```

3.10 Treap

```

1 struct Treap{
2     Treap *l = nullptr, *r = nullptr;
3     int pri = rand(), val = 0, sz = 1;
4
5     Treap(int _val){
6         val = _val;
7     }
8 };
9
10 int size(Treap *t){return t ? t->sz : 0;}
11 void pull(Treap *t){
12     t->sz = size(t->l)+size(t->r)+1;
13 }
14
15 Treap* merge(Treap *a, Treap *b){
16     if (!a || !b) return a ? a : b;
17
18     if (a->pri>b->pri){
19         a->r = merge(a->r, b);
20         pull(a);
21         return a;
22     }else{
23         b->l = merge(a, b->l);
24         pull(b);
25         return b;
26     }
27 }
28
29 pair<Treap*, Treap*> split(Treap *t, int k){ // 1-based <前
30     k 個元素, 其他元素>
31     if (!t) return {};
32     if (size(t->l)>=k){
33         auto pa = split(t->l, k);
34         t->l = pa.second;
35         pull(t);
36         return {pa.first, t};
37     }else{
38         auto pa = split(t->r, k-size(t->l)-1);
39         t->r = pa.first;
40         pull(t);
41         return {t, pa.second};
42     }
43 }
44
45 // functions
46 Treap* build(vector<int> v){
47     Treap* ret;
48     for (int i=0 ; i<SZ(v) ; i++){

```

```

49         ret = merge(ret, new Treap(v[i]));
50     }
51     return ret;
52 }
53
54 array<Treap*, 3> cut(Treap *t, int l, int r){ // 1-based <前
55     1~l-1 個元素, l~r 個元素, r+1 個元素>
56     array<Treap*, 3> ret;
57     tie(ret[1], ret[2]) = split(t, r);
58     tie(ret[0], ret[1]) = split(ret[1], l-1);
59     return ret;
60 }
61
62 void print(Treap *t, bool flag = true){
63     if (t->l!=0) print(t->l, false);
64     cout << t->val;
65     if (t->r!=0) print(t->r, false);
66     if (flag) cout << endl;
67 }

```

3.11 Trie

```

1 struct Trie{
2     struct Data{
3         int nxt[2]={0, 0};
4     };
5
6     int sz=0;
7     vector<Data> arr;
8
9     void init(int n){
10         arr.resize(n);
11     }
12
13     void insert(int n){
14         int now=0;
15         for (int i=N ; i>=0 ; i--){
16             int v=(n>>i)&1;
17             if (!arr[now].nxt[v]){
18                 arr[now].nxt[v]=++sz;
19             }
20             now=arr[now].nxt[v];
21         }
22     }
23
24     int query(int n){
25         int now=0, ret=0;
26         for (int i=N ; i>=0 ; i--){
27             int v=(n>>i)&1;
28             if (arr[now].nxt[1-v]){
29                 ret+=(1<<i);
30                 now=arr[now].nxt[1-v];
31             }else if (arr[now].nxt[v]){
32                 now=arr[now].nxt[v];
33             }else{
34                 return ret;
35             }
36         }
37         return ret;
38     }
39 }
40 } tr;

```


4 Dynamic-Programming

4.1 Digit DP

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 long long l, r;
5 long long dp[20][10][2][2]; // dp[pos][pre][limit] = 後 pos
    位 · pos 前一位是 pre · (是/否) 有上界 · (是/否) 有前綴零
    的答案數量
6
7 long long memorize_search(string &s, int pos, int pre, bool
    limit, bool lead){
8
9     // 已經被找過了 · 直接回傳值
10    if (dp[pos][pre][limit][lead] != -1) return dp[pos][pre][
        limit][lead];
11
12    // 已經搜尋完畢 · 紀錄答案並回傳
13    if (pos == (int)s.size()){
14        return dp[pos][pre][limit][lead] = 1;
15    }
16
17    // 枚舉目前的位數數字是多少
18    long long ans = 0;
19    for (int now = 0; now <= (limit ? s[pos] - '0' : 9); now++){
20        if (now == pre){
21
22            // 1~9 絕對不能連續出現
23            if (pre != 0) continue;
24
25            // 如果已經不在前綴零的範圍內 · 0 不能連續出現
26            if (lead == false) continue;
27        }
28
29        ans += memorize_search(s, pos+1, now, limit & (now == (s[
            pos] - '0')), lead & (now == 0));
30    }
31
32    // 已經搜尋完畢 · 紀錄答案並回傳
33    return dp[pos][pre][limit][lead] = ans;
34 }
35
36 // 回傳 [0, n] 有多少數字符合條件
37 long long find_answer(long long n){
38     memset(dp, -1, sizeof(dp));
39     string tmp = to_string(n);
40
41     return memorize_search(tmp, 0, 0, true, true);
42 }
43
44 int main(){
45
46     // input
47     cin >> l >> r;
48
49     // output - 計算 [l, r] 有多少數字任意兩個位數都不相同
50     cout << find_answer(r) - find_answer(l-1) << "\n";
51
52     return 0;
53 }

```

4.2 SOS DP

```

1 // 總時間複雜度為  $O(n \cdot 2^n)$ 
2 // 計算  $dp[i] = i$  所有 bit mask 子集的和
3 for (int i = 0; i < n; i++){
4     for (int mask = 0; mask < (1 << n); mask++){
5         if ((mask >> i) & 1){
6             dp[mask] += dp[mask ^ (1 << i)];
7         }
8     }
9 }

```

4.3 Integer Partition

$dp[i][x]$ = 要將整數 x 拆成 i 堆的「組合數」

$dp[i+1][x+1] + = dp[i][x]$ (創造新的一堆)
 $dp[i][x+i] + = dp[i][x]$ (把每一堆都增加 1)

5 Geometry

5.1 Geometry Struct

```

1 // 判斷數值正負：{1:正數, 0:零, -1:負數}
2 int sign(long long x) {return (x >= 0) ? ((bool)x) : -1; }
3 int sign(double x) {
4     return (abs(x) < 1e-9) ? 0 : (x > 0 ? 1 : -1);
5 }
6
7 template<typename T>
8 struct point {
9     T x, y;
10    point() {}
11    point(const T &x, const T &y) : x(x), y(y) {}
12
13    point operator+(point b) {return {x+b.x, y+b.y}; }
14    point operator-(point b) {return {x-b.x, y-b.y}; }
15    point operator*(T b) {return {x*b, y*b}; }
16    point operator/(T b) {return {x/b, y/b}; }
17    bool operator==(point b) {return x==b.x && y==b.y; }
18    // 逆時針極角排序
19    bool operator<(point &b) {return (x*b.y > b.x*y); }
20    friend ostream& operator<<(ostream& os, point p) {
21        os << "(" << p.x << ", " << p.y << ")";
22        return os;
23    }
24
25    // 判斷 ab 到 ac 的方向：{1:逆時鐘, 0:重疊, -1:順時鐘}
26    friend int ori(point a, point b, point c) {
27        return sign((b-a)^(c-a));
28    }
29
30    friend bool btw(point a, point b, point c) {
31        return ori(a, b, c) == 0 && sign((a-c)*(b-c)) <= 0;
32    }
33
34    // 判斷線段 ab, cd 是否相交
35    friend bool banana(point a, point b, point c, point d) {
36        int s1 = ori(a, b, c);
37        int s2 = ori(a, b, d);
38        int s3 = ori(c, d, a);

```

```

36    int s4 = ori(c, d, b);
37    if (btw(a, b, c) || btw(a, b, d) || btw(c, d, a) ||
        btw(c, d, b)) return 1;
38    return (s1 * s2 < 0) && (s3 * s4 < 0);
39 }
40
41 T operator*(point b) {return x * b.x + y * b.y; }
42 T operator^(point b) {return x * b.y - y * b.x; }
43 T abs2() {return (*this) * (*this); }
44
45 // 旋轉 Arg(b) 的角度 (小心溢位)
46 point rotate(point b) {return {x*b.x - y*b.y, x*b.y + y*b
    .x}; }
47
48
49 template<typename T>
50 struct line {
51    point<T> p1, p2;
52    // ax + by + c = 0
53    T a, b, c; // |a|, |b| ≤ 2C, |c| ≤ 8C^2
54    line() {}
55    line(const point<T> &x, const point<T> &y) : p1(x), p2(y){
56        build();
57    }
58    void build() {
59        a = p1.y - p2.y;
60        b = p2.x - p1.x;
61        c = (-a*p1.x) - b*p1.y;
62    }
63
64    // 判斷點和有向直線的關係：{1:左邊, 0:在線上, -1:右邊}
65    int ori(point<T> &p) {
66        return sign((p2-p1) ^ (p-p1));
67    }
68
69    // 判斷直線斜率是否相同
70    bool parallel(line &l) {
71        return ((p1-p2) ^ (l.p1-l.p2)) == 0;
72    }
73
74    // 兩直線交點
75    point<long double> line_intersection(line &l) {
76        using P = point<long double>;
77        point<T> a = p2-p1, b = l.p2-l.p1, s = l.p1-p1;
78        return P(p1.x, p1.y) + P(a.x, a.y) * (((long double)(s^b))
79            / (a^b));
80    }
81
82 template<typename T>
83 struct polygon {
84    vector<point<T>> v;
85    polygon() {}
86    polygon(const vector<point<T>> &u) : v(u) {}
87    // simple 為 true 的時候會回傳任意三點不共線的凸包
88    void make_convex_hull(int simple) {
89        auto cmp = [&](point<T> &p, point<T> &q) {
90            return (p.x == q.x) ? (p.y < q.y) : (p.x < q.x);
91        };
92        simple = (bool)simple;
93        sort(v.begin(), v.end(), cmp);
94        v.resize(unique(v.begin(), v.end()) - v.begin());
95        vector<point<T>> hull;
96        for (int t = 0; t < 2; ++t){
97            int sz = hull.size();
98            for (auto &i:v) {
99                while (hull.size() >= sz+2 && ori(hull[hull.
100                    size()-2], hull.back(), i) < simple) {

```

```

97         hull.pop_back();
98     }
99     hull.push_back(i);
100 }
101 hull.pop_back();
102 reverse(v.begin(), v.end());
103 }
104 swap(hull, v);
105 }
106 // 可以在有 n 個點的簡單多邊形內，用 O(n) 判斷一個點：
107 // {1 : 在多邊形內，0 : 在多邊形上，-1 : 在多邊形外}
108 int in_polygon(point<T> a){
109     const T MAX_POS = 1e9 + 5; // [記得修改] 座標的最大值
110     point<T> pre = v.back(), b(MAX_POS, a.y + 1);
111     int cnt = 0;
112
113     for (auto &i:v) {
114         if (btw(pre, i, a)) return 0;
115         if (banana(a, b, pre, i)) cnt++;
116         pre = i;
117     }
118
119     return cnt%2 ? 1 : -1;
120 }
121 /// 警告：以下所有凸包專用的函式都只接受逆時針排序且任三點不
122 /// 共線的凸包 ///
123 // 可以在有 n 個點的凸包內，用 O(Log n) 判斷一個點：
124 // {1 : 在凸包內，0 : 在凸包邊上，-1 : 在凸包外}
125 int in_convex(point<T> p) {
126     int n = v.size();
127     int a = ori(v[0], v[1], p), b = ori(v[0], v[n-1], p);
128     if (a < 0 || b > 0) return -1;
129     if (btw(v[0], v[1], p)) return 0;
130     if (btw(v[0], v[n-1], p)) return 0;
131     int l = 1, r = n - 1, mid;
132     while (l + 1 < r) {
133         mid = (l + r) >> 1;
134         if (ori(v[0], v[mid], p) >= 0) l = mid;
135         else r = mid;
136     }
137     int k = ori(v[l], v[r], p);
138     if (k <= 0) return k;
139     return 1;
140 }
141 // 凸包專用的環狀二分搜，回傳 0-based index
142 int cycle_search(auto &f) {
143     int n = v.size(), l = 0, r = n;
144     bool rv = f(1, 0);
145     while (r - l > 1) {
146         int m = (l + r) / 2;
147         if (f(0, m) ? rv : f(m, (m + 1) % n)) r = m;
148         else l = m;
149     }
150     return f(1, r % n) ? 1 : r % n;
151 }
152 // 可以在有 n 個點的凸包內，用 O(Log n) 判斷一條直線：
153 // {1 : 穿過凸包，0 : 剛好切過凸包，-1 : 沒碰到凸包}
154 int line_cut_convex(line<T> L) {
155     point<T> p(L.a, L.b); // 記得 L 要 build
156     auto gt = [&](int neg) {
157         auto f = [&](int x, int y) {
158             return sign((v[x] - v[y]) * p) == neg;
159         };
160         return -(v[cycle_search(f)] * p);
161     };
162     T x = gt(1), y = gt(-1);
163     if (L.c < x || y < L.c) return -1;
164     return not (L.c == x || L.c == y);
165 }
166 // 可以在有 n 個點的凸包內，用 O(Log n) 判斷一個線段：
167 // {1 : 存在一個凸包上的邊可以把這個線段切成兩半，
168 // 0 : 有碰到凸包但沒有任何凸包上的邊可以把它切成兩半，
169 // -1 : 沒碰到凸包}
170 /// 除非線段兩端點都不在凸包邊上，否則此函數回傳 0 的時候不一
171 /// 定表示線段沒有通過凸包內部 ///
172 int segment_across_convex(line<T> L) {
173     point<T> p(L.a, L.b); // 記得 L 要 build
174     auto gt = [&](int neg) {
175         auto f = [&](int x, int y) {
176             return sign((v[x] - v[y]) * p) == neg;
177         };
178         return cycle_search(f);
179     };
180     int i = gt(1), j = gt(-1), n = v.size();
181     T x = -(v[i] * p), y = -(v[j] * p);
182     if (L.c < x || y < L.c) return -1;
183     if (L.c == x || L.c == y) return 0;
184
185     if (i > j) swap(i, j);
186     auto g = [&](int x, int lim) {
187         int now = 0, nxt;
188         for (int i = 1 << __lg(lim); i > 0; i /= 2) {
189             if (now + i > lim) continue;
190             nxt = (x + i) % n;
191             if (L.ori(v[x], v[nxt]) * L.ori(v[nxt], v[(x + 1) % n]) >= 0) {
192                 x = nxt;
193                 now += i;
194             }
195         }
196         // ↓ BE CAREFUL
197         return -(ori(v[x], v[(x + 1) % n], L.p1) * ori(v[
198             x], v[(x + 1) % n], L.p2));
199     };
200     return max(g(i, j - i), g(j, n - (j - i)));
201 }
202 // 可以在有 n 個點的凸包內，用 O(Log n) 判斷一個線段：
203 // {1 : 線段上存在某一點位於凸包內部（邊上不算），
204 // 0 : 線段上存在某一點碰到凸包的邊但線段上任一點均不在凸包
205 // 內部，
206 // -1 : 線段完全在凸包外面}
207 int segment_pass_convex_interior(line<T> L) {
208     if (in_convex(L.p1) == 1 || in_convex(L.p2) == 1)
209         return 1;
210     point<T> p(L.a, L.b); // 記得 L 要 build
211     auto gt = [&](int neg) {
212         auto f = [&](int x, int y) {
213             return sign((v[x] - v[y]) * p) == neg;
214         };
215         return cycle_search(f);
216     };
217     int i = gt(1), j = gt(-1), n = v.size();
218     T x = -(v[i] * p), y = -(v[j] * p);
219     if (L.c < x || y < L.c) return -1;
220     if (L.c == x || L.c == y) return 0;
221
222     if (i > j) swap(i, j);
223     auto g = [&](int x, int lim) {
224         int now = 0, nxt;
225         for (int i = 1 << __lg(lim); i > 0; i /= 2) {
226             if (now + i > lim) continue;
227             nxt = (x + i) % n;
228             if (L.ori(v[x], v[nxt]) * L.ori(v[nxt], v[(x + 1) % n]) >= 0) {
229                 x = nxt;
230                 now += i;
231             }
232         }
233         // ↓ BE CAREFUL
234         return -(ori(v[x], v[(x + 1) % n], L.p1) * ori(v[
235             x], v[(x + 1) % n], L.p2));
236     };
237     return max(g(i, j - i), g(j, n - (j - i)));
238 }
239 // 回傳點過凸包的兩條切線的切點的 0-based index (不保證兩條
240 // 切線的順逆時針關係)
241 pair<int, int> convex_tangent_point(point<T> p) {
242     int n = v.size(), z = -1, edg = -1;
243     auto gt = [&](int neg) {
244         auto check = [&](int x) {
245             if (v[x] == p) z = x;
246             if (btw(v[x], v[(x + 1) % n], p)) edg = x;
247             if (btw(v[(x + n - 1) % n], v[x], p)) edg = (
248                 x + n - 1) % n;
249         };
250         auto f = [&](int x, int y) {
251             check(x); check(y);
252             return ori(p, v[x], v[y]) == neg;
253         };
254         return cycle_search(f);
255     };
256     int x = gt(1), y = gt(-1);
257     if (z != -1) {
258         return {(z + n - 1) % n, (z + 1) % n};
259     }
260     else if (edg != -1) {
261         return {edg, (edg + 1) % n};
262     }
263     else {
264         return {x, y};
265     }
266 }
267 friend int halfplane_intersection(vector<line<T>> &s,
268     polygon<T> &P) {
269     #define neg(p) ((p.y == 0 ? p.x : p.y) < 0)
270     auto angle_cmp = [&](line<T> &A, line<T> &B) {
271         point<T> a = A.p2-A.p1, b = B.p2-B.p1;
272         return neg(a) < neg(b) || (neg(a) == neg(b) && (a
273             ^b) > 0);
274     };
275     #undef neg
276     sort(s.begin(), s.end(), angle_cmp); // 線段左側為該
277     // 線段半平面
278     int L, R, n = s.size();
279     vector<point<T>> px(n);
280     vector<line<T>> q(n);
281     q[L = R = 0] = s[0];
282     for (int i = 1; i < n; ++i) {
283         while (L < R && s[i].ori(px[R-1]) <= 0) --R;
284         while (L < R && s[i].ori(px[L]) <= 0) ++L;
285         q[++R] = s[i];
286         if (q[R].parallel(q[R-1])) {
287             --R;
288             if (q[R].ori(s[i].p1) > 0) q[R] = s[i];
289         }
290     }
291 }

```

```

278         if(L < R) px[R-1] = q[R-1].line_intersection(q[R
279             ]);
280     }
281     while(L < R && q[L].ori(px[R-1]) <= 0) --R;
282     P.v.clear();
283     if(R - L <= 1) return 0;
284     px[R] = q[R].line_intersection(q[L]);
285     for(int i = L; i <= R; ++i) P.v.push_back(px[i]);
286     return R - L + 1;
287 }
};

```

5.2 Geometry 卦長

```

1  const double PI=atan2(0.0,-1.0);
2  template<typename T>
3  struct point{
4      T x,y;
5      point(){}
6      point(const T&x,const T&y):x(x),y(y){}
7      point operator+(const point &b)const{
8          return point(x+b.x,y+b.y); }
9      point operator-(const point &b)const{
10         return point(x-b.x,y-b.y); }
11     point operator*(const T &b)const{
12         return point(x*b,y*b); }
13     point operator/(const T &b)const{
14         return point(x/b,y/b); }
15     bool operator==(const point &b)const{
16         return x==b.x&&y==b.y; }
17     T dot(const point &b)const{
18         return x*b.x+y*b.y; }
19     T cross(const point &b)const{
20         return x*b.y-y*b.x; }
21     point normal()const{//求法向量
22         return point(-y,x); }
23     T abs2()const{//向量長度的平方
24         return dot(*this); }
25     T rad(const point &b)const{//兩向量的弧度
26     return fabs(atan2(fabs(cross(b)),dot(b))); }
27     T getA()const{//對x軸的弧度
28         T A=atan2(y,x);//超過180度會變負的
29         if(A<=-PI/2)A+=PI*2;
30         return A;
31     }
32 };
33 template<typename T>
34 struct line{
35     line(){}
36     point<T> p1,p2;
37     T a,b,c;//ax+by+c=0
38     line(const point<T>&x,const point<T>&y):p1(x),p2(y){}
39     void pton()const{//轉成一般式
40         a=p1.y-p2.y;
41         b=p2.x-p1.x;
42         c=-a*p1.x-b*p1.y;
43     }
44     T ori(const point<T> &p)const{//點和有向直線的關係，>0左
45         邊、=0在線上<0右邊
46         return (p2-p1).cross(p-p1);

```

```

47     T btw(const point<T> &p)const{//點投影落在線段上<=0
48         return (p1-p).dot(p2-p);
49     }
50     bool point_on_segment(const point<T>&p)const{//點是否在線段
51         上
52         return ori(p)==0&&btw(p)<=0;
53     }
54     T dis2(const point<T> &p,bool is_segment=0)const{//點跟直線
55         //線段的距離平方
56         point<T> v=p2-p1,v1=p-p1;
57         if(is_segment){
58             point<T> v2=p-p2;
59             if(v.dot(v1)<=0)return v1.abs2();
60             if(v.dot(v2)>=0)return v2.abs2();
61         }
62         T tmp=v.cross(v1);
63         return tmp*tmp/v.abs2();
64     }
65     T seg_dis2(const line<T> &l)const{//兩線段距離平方
66         return min({dis2(l.p1,1),dis2(l.p2,1),l.dis2(p1,1),l.dis2
67             (p2,1)});
68     }
69     point<T> projection(const point<T> &p)const{//點對直線的投
70         影
71         point<T> n=(p2-p1).normal();
72         return p-n*(p-p1).dot(n)/n.abs2();
73     }
74     point<T> mirror(const point<T> &p)const{
75         //點對直線的鏡射，要先呼叫pton轉成一般式
76         point<T> R;
77         T d=a*a+b*b;
78         R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
79         R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
80         return R;
81     }
82     bool equal(const line &l)const{//直線相等
83         return ori(l.p1)==0&&ori(l.p2)==0;
84     }
85     bool parallel(const line &l)const{
86         return (p1-p2).cross(l.p1-l.p2)==0;
87     }
88     bool cross_seg(const line &l)const{
89         return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;
90         //直線是否交線段
91     }
92     int line_intersect(const line &l)const{//直線相交情況，-1無
93         限多點、1交於一點、0不相交
94         return parallel(l)?(ori(l.p1)==0?-1:0):1;
95     }
96     int seg_intersect(const line &l)const{
97         T c1=ori(l.p1), c2=ori(l.p2);
98         T c3=l.ori(p1), c4=l.ori(p2);
99         if(c1==0&&c2==0){//共線
100             bool b1=btw(l.p1)>=0,b2=btw(l.p2)>=0;
101             T a3=l.btw(p1),a4=l.btw(p2);
102             if(b1&&b2&&a3==0&&a4>=0) return 2;
103             if(b1&&b2&&a3>=0&&a4==0) return 3;
104             if(b1&&b2&&a3>=0&&a4>=0) return 0;
105             return -1;//無限交點
106         }else if(c1*c2<=0&&c3*c4<=0)return 1;
107         return 0;//不相交
108     }
109     point<T> line_intersection(const line &l)const{//直線交點*/

```

```

104     point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
105     //if(a.cross(b)==0)return INF;
106     return p1+a*(s.cross(b)/a.cross(b));
107 }
108 point<T> seg_intersection(const line &l)const{//線段交點
109     int res=seg_intersect(l);
110     if(res<=0) assert(0);
111     if(res==2) return p1;
112     if(res==3) return p2;
113     return line_intersection(l);
114 }
115 };
116 template<typename T>
117 struct polygon{
118     polygon(){}
119     vector<point<T> > p;//逆時針順序
120     T area()const{//面積
121         T ans=0;
122         for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
123             ans+=p[i].cross(p[j]);
124         }
125         return ans/2;
126     }
127     point<T> center_of_mass()const{//重心
128         T cx=0,cy=0,w=0;
129         for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
130             T a=p[i].cross(p[j]);
131             cx+=(p[i].x+p[j].x)*a;
132             cy+=(p[i].y+p[j].y)*a;
133             w+=a;
134         }
135         return point<T>(cx/3/w,cy/3/w);
136     }
137     char ahas(const point<T> &t)const{//點是否在簡單多邊形內，
138         是的話回傳1、在邊上回傳-1、否則回傳0
139         bool c=0;
140         for(int i=0,j=p.size()-1;i<p.size();j=i++){
141             if(line<T>(p[i],p[j]).point_on_segment(t))return -1;
142             else if((p[i].y>t.y)!=p[j].y>t.y)&&
143                 t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j].y-p[i].y)+p[i].x
144             )
145                 c=!c;
146             return c;
147         }
148     }
149     char point_in_convex(const point<T>&x)const{
150         int l=1,r=(int)p.size()-2;
151         while(l<r){//點是否在凸多邊形內，是的話回傳1、在邊上回傳
152             -1、否則回傳0
153             int mid=(l+r)/2;
154             T a1=(p[mid]-p[0]).cross(x-p[0]);
155             T a2=(p[mid+1]-p[0]).cross(x-p[0]);
156             if(a1>0&&a2<=0){
157                 T res=(p[mid+1]-p[mid]).cross(x-p[mid]);
158                 return res>0?1:(res>0?-1:0);
159             }else if(a1<0)r=mid-1;
160             else l=mid+1;
161         }
162         return 0;
163     }
164     vector<T> getA()const{//凸包邊對x軸的夾角
165         vector<T>res;//一定是遞增的
166         for(size_t i=0;i<p.size();++i)
167             res.push_back((p[(i+1)%p.size()]-p[i]).getA());
168         return res;
169     }

```

```

165 bool line_intersect(const vector<T>&A, const line<T> &l) 219
166     const{//O(LogN)
167     int f1=upper_bound(A.begin(),A.end(),(l.p1-l.p2).getA())- 220
168         A.begin();
169     int f2=upper_bound(A.begin(),A.end(),(l.p2-l.p1).getA())- 221
170         A.begin();
171     return l.cross_seg(line<T>(p[f1],p[f2])); 222
172 }
173 polygon cut(const line<T> &l) const{//凸包對直線切割，得到直 223
174     線L左側的凸包
175     polygon ans; 224
176     for(int n=p.size(),i=n-1,j=0;j<n;i=j++){ 225
177         if(l.ori(p[i])>=0){ 226
178             ans.p.push_back(p[i]); 227
179             if(l.ori(p[j])<0) 228
180                 ans.p.push_back(l.line_intersection(line<T>(p[i],p[j]),l)); 229
181         }else if(l.ori(p[j])>0) 230
182             ans.p.push_back(l.line_intersection(line<T>(p[i],p[j]),l)); 231
183         } 232
184     } 233
185     return ans; 234
186 } 235
187 static bool monotone_chain_cmp(const point<T>& a, const 236
188     point<T>& b){//凸包排序函數
189     return (a.x<b.x)|| (a.x==b.x&&a.y<b.y); 237
190 } 238
191 void monotone_chain(vector<point<T> > &s){//凸包 239
192     sort(s.begin(),s.end(),monotone_chain_cmp);
193     p.resize(s.size()+1);
194     int m=0;
195     for(size_t i=0;i<s.size();i++){
196         while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
197         p[m++]=s[i];
198     }
199     for(int i=s.size()-2,t=m+1;i>=0;--i){
200         while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
201         p[m++]=s[i];
202     }
203     if(s.size()>1)--m;
204     p.resize(m);
205 }
206 T diam(){//直徑
207     int n=p.size(),t=1;
208     T ans=0;p.push_back(p[0]);
209     for(int i=0;i<n;i++){
210         point<T> now=p[i+1]-p[i];
211         while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t+1)%n;
212         ans=max(ans,(p[i]-p[t]).abs2());
213     }
214     return p.pop_back(),ans;
215 }
216 T min_cover_rectangle(){//最小覆蓋矩形
217     int n=p.size(),t=1,r=1,l;
218     if(n<3)return 0;//也可以做最小周長矩形
219     T ans=1e99;p.push_back(p[0]);
220     for(int i=0;i<n;i++){
221         point<T> now=p[i+1]-p[i];
222         while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t+1)%n;
223         while(now.dot(p[r+1]-p[i])>now.dot(p[r]-p[i]))r=(r+1)%n;
224         if(!i)l=r;
225     }
226     while(now.dot(p[l+1]-p[i])<=now.dot(p[l]-p[i]))l=(l+1)%n;
227     T d=now.abs2();
228     T tmp=now.cross(p[t]-p[i])*(now.dot(p[r]-p[i])-now.dot(p[l]-p[i]))/d;
229     ans=min(ans,tmp);
230 }
231 return p.pop_back(),ans;
232 }
233 T dis2(polygon &p1){//凸包最近距離平方
234     vector<point<T> > &P=p,&Q=p1.p;
235     int n=P.size(),m=Q.size(),l=0,r=0;
236     for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;
237     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;
238     P.push_back(P[0]),Q.push_back(Q[0]);
239     T ans=1e99;
240     for(int i=0;i<n;++i){
241         while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])<0)r=(r+1)%m;
242         ans=min(ans,line<T>(P[l],P[l+1]).seg_dis2(line<T>(Q[r],Q[r+1])));
243         l=(l+1)%n;
244     }
245     return P.pop_back(),Q.pop_back(),ans;
246 }
247 static char sign(const point<T>&t){
248     return (t.y==0?t.x:t.y)<0;
249 }
250 static bool angle_cmp(const line<T>& A, const line<T>& B){
251     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
252     return sign(a)<sign(b)|| (sign(a)==sign(b)&&a.cross(b)>0);
253 }
254 int halfplane_intersection(vector<line<T> > &s){//半平面交
255     sort(s.begin(),s.end(),angle_cmp);//線段左側為該線段半平面
256     int L,R,n=s.size();
257     vector<point<T> > px(n);
258     vector<line<T> > q(n);
259     q[L=R=0]=s[0];
260     for(int i=1;i<n;++i){
261         while(L<R&&s[i].ori(px[R-1])<=0)--R;
262         while(L<R&&s[i].ori(px[L])<=0)++L;
263         q[++R]=s[i];
264         if(q[R].parallel(q[R-1])){
265             --R;
266             if(q[R].ori(s[i].p1)>0)q[R]=s[i];
267         }
268         if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
269     }
270     while(L<R&&q[L].ori(px[R-1])<=0)--R;
271     p.clear();
272     if(R-L<=1)return 0;
273     px[R]=q[R].line_intersection(q[L]);
274     for(int i=L;i<R;++i)p.push_back(px[i]);
275     return R-L+1;
276 }
277 }
278 template<typename T>
279 struct triangle{
280     point<T> a,b,c;
281     triangle(){
282         triangle(const point<T> &a, const point<T> &b, const point<T> &c):a(a),b(b),c(c){
283             T area()const{
284                 T t=(b-a).cross(c-a)/2;
285                 return t>0?t:-t;
286             }
287         }
288     };
289     point<T> barycenter()const{//重心
290         return (a+b+c)/3;
291     }
292     point<T> circumcenter()const{//外心
293         static line<T> u,v;
294         u.p1=(a+b)/2;
295         u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
296         v.p1=(a+c)/2;
297         v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
298         return u.line_intersection(v);
299     }
300     point<T> incenter()const{//內心
301         T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).abs2());
302         return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B+C);
303     }
304     point<T> perpercenter()const{//垂心
305         return barycenter()*3-circumcenter()*2;
306     }
307 };
308 template<typename T>
309 struct point3D{
310     T x,y,z;
311     point3D(){
312         point3D(const T&x, const T&y, const T&z):x(x),y(y),z(z){}
313     }
314     point3D operator+(const point3D &b)const{
315         return point3D(x+b.x,y+b.y,z+b.z);
316     }
317     point3D operator-(const point3D &b)const{
318         return point3D(x-b.x,y-b.y,z-b.z);
319     }
320     point3D operator*(const T &b)const{
321         return point3D(x*b,y*b,z*b);
322     }
323     point3D operator/(const T &b)const{
324         return point3D(x/b,y/b,z/b);
325     }
326     bool operator==(const point3D &b)const{
327         return x==b.x&&y==b.y&&z==b.z;
328     }
329     T dot(const point3D &b)const{
330         return x*b.x+y*b.y+z*b.z;
331     }
332     point3D cross(const point3D &b)const{
333         return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
334     }
335     T abs2()const{//向量長度的平方
336         return dot(*this);
337     }
338     T area2(const point3D &b)const{//和b、原點圍成面積的平方
339         return cross(b).abs2()/4;
340     }
341 };
342 template<typename T>
343 struct line3D{
344     point3D<T> p1,p2;
345     line3D(){
346         line3D(const point3D<T> &p1, const point3D<T> &p2):p1(p1),p2(p2){}
347     }
348     T dis2(const point3D<T> &p, bool is_segment=0)const{//點跟直線/線段的距離平方
349         point3D<T> v=p2-p1,v1=v-p1;
350         if(is_segment){
351             point3D<T> v2=p-p1;
352             if(v.dot(v1)<=0)return v1.abs2();
353             if(v.dot(v2)>=0)return v2.abs2();
354         }
355         point3D<T> tmp=v.cross(v1);
356         return tmp.abs2()/v.abs2();
357     }
358 };

```



```

338 pair<point3D<T>,point3D<T> > closest_pair(const line3D<T> & 393
      1)const{
339     point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
340     point3D<T> N=v1.cross(v2),ab(p1-l.p1);
341     //if(N.abs2()==0)return NULL; 平行或重合
342     T tmp=N.dot(ab),ans=tmp*tmp/N.abs2(); //最近點對距離
343     point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2),G=l.p1-p1
      ;
344     T t1=(G.cross(d2)).dot(D)/D.abs2();
345     T t2=(G.cross(d1)).dot(D)/D.abs2();
346     return make_pair(p1+d1*t1,l.p1+d2*t2);
347 }
348 bool same_side(const point3D<T> &a,const point3D<T> &b)
      const{
349     return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
350 }
351 };
352 template<typename T>
353 struct plane{
354     point3D<T> p0,n; //平面上的點和法向量
355     plane(){}
356     plane(const point3D<T> &p0,const point3D<T> &n):p0(p0),n(n)
      {}
357     T dis2(const point3D<T> &p)const{ //點到平面距離的平方
358         T tmp=(p-p0).dot(n);
359         return tmp*tmp/n.abs2();
360     }
361     point3D<T> projection(const point3D<T> &p)const{
362         return p-n*(p-p0).dot(n)/n.abs2();
363     }
364     point3D<T> line_intersection(const line3D<T> &l)const{
365         T tmp=n.dot(l.p2-l.p1); //等於0表示平行或重合該平面
366         return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
367     }
368     line3D<T> plane_intersection(const plane &pl)const{
369         point3D<T> e=n.cross(pl.n),v=n.cross(e);
370         T tmp=pl.n.dot(v); //等於0表示平行或重合該平面
371         point3D<T> q=p0+(v*(pl.n.dot(pl.p0-p0))/tmp);
372         return line3D<T>(q,q+e);
373     }
374 };
375 template<typename T>
376 struct triangle3D{
377     point3D<T> a,b,c;
378     triangle3D(){}
379     triangle3D(const point3D<T> &a,const point3D<T> &b,const
      point3D<T> &c):a(a),b(b),c(c){}
380     bool point_in(const point3D<T> &p)const{ //點在該平面上的投
      影在三角形中
381         return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).
      same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
382     }
383 };
384 template<typename T>
385 struct tetrahedron{ //四面體
386     point3D<T> a,b,c,d;
387     tetrahedron(){}
388     tetrahedron(const point3D<T> &a,const point3D<T> &b,const
      point3D<T> &c,const point3D<T> &d):a(a),b(b),c(c),d(d)
      {}
389     T volume6()const{ //體積的六倍
390         return (d-a).dot((b-a).cross(c-a));
391     }
392     point3D<T> centroid()const{

```

```

      return (a+b+c+d)/4;
393 }
394 bool point_in(const point3D<T> &p)const{
395     return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,
      d,a).point_in(p);
396 }
397 };
398 template<typename T>
399 struct convexhull3D{
400     static const int MAXN=1005;
401     struct face{
402         int a,b,c;
403         face(int a,int b,int c):a(a),b(b),c(c){}
404     };
405     vector<point3D<T>> pt;
406     vector<face> ans;
407     int fid[MAXN][MAXN];
408     void build(){
409         int n=pt.size();
410         ans.clear();
411         memset(fid,0,sizeof(fid));
412         ans.emplace_back(0,1,2); //注意不能共線
413         ans.emplace_back(2,1,0);
414         int ftop = 0;
415         for(int i=3; ftop=1; i<n; ++i,++ftop){
416             vector<face> next;
417             for(auto &f:ans){
418                 T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[f.a]).cross(pt[f.
      c]-pt[f.a]));
419                 if(d<=0) next.push_back(f);
420                 int ff=0;
421                 if(d>0) ff=ftop;
422                 else if(d<0) ff=-ftop;
423                 fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c][f.a]=ff;
424             }
425             for(auto &f:ans){
426                 if(fid[f.a][f.b]>0 && fid[f.a][f.b]!=fid[f.b][f.a])
427                     next.emplace_back(f.a,f.b,i);
428                 if(fid[f.b][f.c]>0 && fid[f.b][f.c]!=fid[f.c][f.b])
429                     next.emplace_back(f.b,f.c,i);
430                 if(fid[f.c][f.a]>0 && fid[f.c][f.a]!=fid[f.a][f.c])
431                     next.emplace_back(f.c,f.a,i);
432             }
433             ans=next;
434         }
435     }
436     point3D<T> centroid()const{
437         point3D<T> res(0,0,0);
438         T vol=0;
439         for(auto &f:ans){
440             T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]));
441             res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
442             vol+=tmp;
443         }
444         return res/(vol*4);
445     }
446 };

```

5.3 Pick's Theorem

給定頂點坐標均是整點的簡單多邊形 · 面積 = 內部格點數 + 邊上格點數/2 - 1

6 Graph

6.1 Augment Path

```

1 struct AugmentPath{
2     int n, m;
3     vector<vector<int>> G;
4     vector<int> mx, my;
5     vector<int> visx, visy;
6     int stamp;
7
8     AugmentPath(int _n, int _m) : n(_n), m(_m), G(n), mx(n,
      -1), my(m, -1), visx(n), visy(m){
9         stamp = 0;
10    }
11
12    void add(int x, int y){
13        G[x].push_back(y);
14    }
15
16    // bb03e2
17    bool dfs1(int now){
18        visx[now] = stamp;
19
20        for (auto x : G[now]){
21            if (my[x]==-1){
22                mx[now] = x;
23                my[x] = now;
24                return true;
25            }
26        }
27        for (auto x : G[now]){
28            if (visx[my[x]]!=stamp && dfs1(my[x])){
29                mx[now] = x;
30                my[x] = now;
31                return true;
32            }
33        }
34        return false;
35    }
36
37    vector<pair<int, int>> find_max_matching(){
38        vector<pair<int, int>> ret;
39
40        while (true){
41            stamp++;
42            int tmp = 0;
43            for (int i=0 ; i<n ; i++){
44                if (mx[i]==-1 && dfs1(i)) tmp++;
45            }
46            if (tmp==0) break;
47        }
48
49        for (int i=0 ; i<n ; i++){
50            if (mx[i]!=-1){
51                ret.push_back({i, mx[i]});
52            }
53        }
54        return ret;
55    }
56
57    // 645577
58    void dfs2(int now){

```

```

59     visx[now] = true;
60
61     for (auto x : G[now]){
62         if (my[x]!=-1 && visy[x]==false){
63             visy[x] = true;
64             dfs2(my[x]);
65         }
66     }
67 }
68
69 // 要先執行 find_max_matching 一次
70 vector<pair<int, int>> find_min_vertex_cover(){
71     fill(visx.begin(), visx.end(), false);
72     fill(visy.begin(), visy.end(), false);
73
74     vector<pair<int, int>> ret;
75     for (int i=0 ; i<n ; i++){
76         if (mx[i]==-1) dfs2(i);
77     }
78
79     for (int i=0 ; i<n ; i++){
80         if (visx[i]==false) ret.push_back({1, i});
81     }
82     for (int i=0 ; i<m ; i++){
83         if (visy[i]==true) ret.push_back({2, i});
84     }
85
86     return ret;
87 }
88 };

```

6.2 Bridge BCC

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 200005;
5 vector<int> G[N];
6 int low[N], depth[N];
7 bool vis[N];
8 vector<vector<int>> bcc;
9 stack<int> stk;
10
11 void dfs(int v, int p) {
12     stk.push(v);
13     vis[v] = true;
14     low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
15     for (int u : G[v]) {
16         if (u == p) continue;
17         if (!vis[u]) {
18             /// (v, u) 是樹邊
19             dfs(u, v);
20             low[v] = min(low[v], low[u]);
21         } else {
22             /// (v, u) 是回邊
23             low[v] = min(low[v], depth[u]);
24         }
25     }
26     /// v 在不依靠父邊的情況下永遠沒辦法走到它的祖先
27     if (low[v] == depth[v]) {
28         bcc.emplace_back();
29         while (stk.top() != v) {

```

```

30         bcc.back().push_back(stk.top());
31         stk.pop();
32     }
33     bcc.back().push_back(stk.top());
34     stk.pop();
35 }
36 }

```

6.3 Cut BCC

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 200005;
5 vector<int> G[N];
6 int low[N], depth[N];
7 bool vis[N];
8 vector<vector<int>> bcc;
9 stack<int> stk;
10
11 void dfs(int v, int p) {
12     stk.push(v);
13     vis[v] = true;
14     low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
15     for (int u : G[v]) {
16         if (u == p) continue;
17         if (!vis[u]) {
18             /// (v, u) 是樹邊
19             dfs(u, v);
20             low[v] = min(low[v], low[u]);
21             /// u 無法在不經過父邊的情況走到 v 的祖先
22             if (low[u] >= depth[v]) {
23                 bcc.emplace_back();
24                 while (stk.top() != u) {
25                     bcc.back().push_back(stk.top());
26                     stk.pop();
27                 }
28                 bcc.back().push_back(stk.top());
29                 stk.pop();
30                 bcc.back().push_back(v);
31             }
32         } else {
33             /// (v, u) 是回邊
34             low[v] = min(low[v], depth[u]);
35         }
36     }
37 }

```

6.4 Dijkstra

```

1 // 可以在  $O(E \log E)$  的時間複雜度解決在無負權有向圖單點源最短
   路
2 const int INF = 2e18; // 要確保 INF 開的足夠大
3
4 vector<vector<pair<int, int>>> G(n); // G[i] = <節點, 權重>
5 vector<int> dis(n, INF);
6 priority_queue<pair<int, int>, vector<pair<int, int>>,
   greater<pair<int, int>>> pq;
7 dis[s] = 0;

```

```

8 pq.push({0, s});
9
10 while (pq.size()){
11     int now_dis = pq.top().first;
12     int now_node = pq.top().second;
13     pq.pop();
14
15     if (now_dis > dis[now_node]) continue;
16
17     for (auto x : G[now_node]){
18         if (now_dis+x.second < dis[x.first]){
19             dis[x.first] = now_dis+x.second;
20             pq.push({dis[x.first], x.first});
21         }
22     }
23 }

```

6.5 Dinic

```

1 // 一般圖:  $O(EV^2)$ 
2 // 二分圖:  $O(E\sqrt{V})$ 
3 struct Flow{
4     struct Edge{
5         int v, rc, rid;
6     };
7     vector<vector<Edge>> G;
8     void add(int u, int v, int c){
9         G[u].push_back({v, c, G[v].size()});
10        G[v].push_back({u, 0, G[u].size()-1});
11    }
12    vector<int> dis, it;
13
14    Flow(int n){
15        G.resize(n);
16        dis.resize(n);
17        it.resize(n);
18    }
19
20    int dfs(int u, int t, int f){
21        if (u==t || f==0) return f;
22        for (int &i=it[u] ; i<G[u].size() ; i++){
23            auto &[v, rc, rid] = G[u][i];
24            if (dis[v]!=dis[u]+1) continue;
25            int df = dfs(v, t, min(f, rc));
26            if (df==0) continue;
27            rc -= df;
28            G[v][rid].rc += df;
29            return df;
30        }
31        return 0;
32    }
33
34    int flow(int s, int t){
35        int ans = 0;
36        while (true){
37            fill(dis.begin(), dis.end(), INF);
38            queue<int> q;
39            q.push(s);
40            dis[s] = 0;
41
42            while (q.size()){
43                int u = q.front(); q.pop();
44                for (auto [v, rc, rid] : G[u]){

```

```

45         if (rc <= 0 || dis[v] < INF) continue;
46         dis[v] = dis[u] + 1;
47         q.push(v);
48     }
49 }
50 if (dis[t] == INF) break;
51
52 fill(it.begin(), it.end(), 0);
53 while (true){
54     int df = dfs(s, t, INF);
55     if (df <= 0) break;
56     ans += df;
57 }
58 return ans;
59 }
60 // the code below constructs minimum cut
61 void dfs_mincut(int now, vector<bool> &vis){
62     vis[now] = true;
63     for (auto &[v, rc, rid] : G[now]){
64         if (vis[v] == false && rc > 0){
65             dfs_mincut(v, vis);
66         }
67     }
68 }
69 vector<pair<int, int>> construct(int n, int s, vector<pair<
70     int, int>> &E){
71     // E is G without capacity
72     vector<bool> vis(n);
73     dfs_mincut(s, vis);
74     vector<pair<int, int>> ret;
75     for (auto &[u, v] : E){
76         if (vis[u] == true && vis[v] == false){
77             ret.emplace_back(u, v);
78         }
79     }
80     return ret;
81 }
82 };
83

```

6.6 Dinic with double

```

1 const double double_INF = 1e18;
2 const int INF = (int)(1e9 + 10);
3
4 struct Flow{
5     const double eps = 1e-9;
6     struct Edge{
7         int v; double rc; int rid;
8     };
9     vector<vector<Edge>> G;
10    void add(int u, int v, double c){
11        G[u].push_back({v, c, G[v].size()});
12        G[v].push_back({u, 0, G[u].size()-1});
13    }
14    vector<int> dis, it;
15
16    Flow(int n){
17        G.resize(n);
18        dis.resize(n);
19        it.resize(n);
20    }

```

```

21    double dfs(int u, int t, double f){
22        if (u == t || abs(f) < eps) return f;
23        for (int &i = it[u] ; i < G[u].size() ; i++){
24            auto &[v, rc, rid] = G[u][i];
25            if (dis[v] != dis[u] + 1) continue;
26            double df = dfs(v, t, min(f, rc));
27            if (abs(df) <= eps) continue;
28            rc -= df;
29            G[v][rid].rc += df;
30            return df;
31        }
32        return 0;
33    }
34
35    double flow(int s, int t){
36        double ans = 0;
37        while (true){
38            fill(dis.begin(), dis.end(), INF);
39            queue<int> q;
40            q.push(s);
41            dis[s] = 0;
42
43            while (q.size()){
44                int u = q.front(); q.pop();
45                for (auto [v, rc, rid] : G[u]){
46                    if (abs(rc) <= eps || dis[v] < INF)
47                        continue;
48                    dis[v] = dis[u] + 1;
49                    q.push(v);
50                }
51            }
52            if (dis[t] == INF) break;
53
54            fill(it.begin(), it.end(), 0);
55            while (true){
56                double df = dfs(s, t, double_INF);
57                if (abs(df) <= eps) break;
58                ans += df;
59            }
60        }
61        return ans;
62    }
63
64    // the code below constructs minimum cut
65    void dfs_mincut(int now, vector<bool> &vis){
66        vis[now] = true;
67        for (auto &[v, rc, rid] : G[now]){
68            if (vis[v] == false && rc > eps){
69                dfs_mincut(v, vis);
70            }
71        }
72    }
73
74    vector<pair<int, int>> construct(int n, int s, vector<
75        pair<int, int>> &E){
76        // E is G without capacity
77        vector<bool> vis(n);
78        dfs_mincut(s, vis);
79        vector<pair<int, int>> ret;
80        for (auto &[u, v] : E){
81            if (vis[u] == true && vis[v] == false){
82                ret.emplace_back(u, v);
83            }
84        }
85        return ret;
86    }

```

```

85     }
86 };

```

6.7 Dominator Tree

```

1 /*
2  全部都是 0-based
3  一開始要初始化 G(N, root) · 代表有 N 個節點 · 根是 root
4  用完之後要 build
5  G[i] = i 的 idom · 也就是從 root 走到 i 時 · 一定要走到的點且離
6      i 最近
7 */
8 struct DominatorTree{
9     int N;
10    vector<vector<int>> G;
11    vector<vector<int>> buckets, rg;
12    // dfn[x] = the DFS order of x
13    // rev[x] = the vertex with DFS order x
14    // par[x] = the parent of x
15    vector<int> dfn, rev, par;
16    vector<int> sdom, dom, idom;
17    vector<int> fa, val;
18    int stamp;
19    int root;
20
21    int operator [] (int x){
22        return idom[x];
23    }
24
25    DominatorTree(int _N, int _root) :
26        N(_N),
27        G(N), buckets(N), rg(N),
28        dfn(N, -1), rev(N, -1), par(N, -1),
29        sdom(N, -1), dom(N, -1), idom(N, -1),
30        fa(N, -1), val(N, -1)
31    {
32        stamp = 0;
33        root = _root;
34    }
35
36    void add_edge(int u, int v){
37        G[u].push_back(v);
38    }
39
40    void dfs(int x){
41        rev[dfn[x]] = stamp = x;
42        fa[stamp] = sdom[stamp] = val[stamp] = stamp;
43        stamp++;
44
45        for (int u : G[x]){
46            if (dfn[u] == -1){
47                dfs(u);
48                par[dfn[u]] = dfn[x];
49                rg[dfn[u]].push_back(dfn[x]);
50            }
51        }
52    }
53
54    int eval(int x, bool first){
55        if (fa[x] == x) return !first ? -1 : x;
56        int p = eval(fa[x], false);

```



```

57     if (p==-1) return x;
58     if (sdom[val[x]]>sdom[val[fa[x]]]) val[x] = val[fa[x]]];
59     fa[x] = p;
60
61     return !first ? p : val[x];
62 }
63
64 void link(int x, int y){
65     fa[x] = y;
66 }
67
68 void build(){
69     dfs(root);
70
71     for (int x=stamp-1 ; x>=0 ; x--){
72         for (int y : rg[x]){
73             sdom[x] = min(sdom[x], sdom[eval(y, true)]);
74         }
75         if (x>0) buckets[sdom[x]].push_back(x);
76         for (int u : buckets[x]){
77             int p = eval(u, true);
78             if (sdom[p]==x) dom[u] = x;
79             else dom[u] = p;
80         }
81         if (x>0) link(x, par[x]);
82     }
83
84     idom[root] = root;
85     for (int x=1 ; x<stamp ; x++){
86         if (sdom[x]!=dom[x]) dom[x] = dom[dom[x]];
87     }
88     for (int i=1 ; i<stamp ; i++) idom[rev[i]] = rev[dom[i]];
89 }
90 };

```

6.8 Find Bridge

```

1 vector<int> dep(MAX_N), low(MAX_N);
2 vector<pair<int, int>> bridge;
3 bitset<MAX_N> vis;
4
5 void dfs(int now, int pre){
6     vis[now] = 1;
7     low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
8
9     for (auto x : G[now]){
10         if (x==pre){
11             continue;
12         }else if (vis[x]==0){
13             // 沒有走過的節點
14             dfs(x, now);
15             low[now] = min(low[now], low[x]);
16         }else if (vis[x]==1){
17             low[now] = min(low[now], dep[x]);
18         }
19     }
20
21     if (now!=1 && low[now]==dep[now]){
22         bridge.push_back({now, pre});
23     }
24     return;

```

6.9 HLD

```

1 #include <bits/stdc++.h>
2 #define int long long
3 using namespace std;
4
5 const int N = 100005;
6 vector<int> G[N];
7 struct HLD {
8     vector<int> pa, sz, depth, mxson, topf, id;
9     int n, idcnt = 0;
10     HLD(int _n) : n(_n), pa(_n + 1), sz(_n + 1), depth(_n + 1), mxson(_n + 1), topf(_n + 1), id(_n + 1) {}
11     void dfs1(int v = 1, int p = -1) {
12         pa[v] = p; sz[v] = 1; mxson[v] = 0;
13         depth[v] = (p == -1 ? 0 : depth[p] + 1);
14         for (int u : G[v]) {
15             if (u == p) continue;
16             dfs1(u, v);
17             sz[v] += sz[u];
18             if (sz[u] > sz[mxson[v]]) mxson[v] = u;
19         }
20     }
21     void dfs2(int v = 1, int top = 1) {
22         id[v] = ++idcnt;
23         topf[v] = top;
24         if (mxson[v]) dfs2(mxson[v], top);
25         for (int u : G[v]) {
26             if (u == mxson[v] || u == pa[v]) continue;
27             dfs2(u, u);
28         }
29     }
30     // query 為區間資料結構
31     int path_query(int a, int b) {
32         int res = 0;
33         while (topf[a] != topf[b]) { /// 若不在同一條鍊上
34             if (depth[topf[a]] < depth[topf[b]]) swap(a, b);
35             res = max(res, 0ll); // query : L = id[topf[a]], R = id[a]
36             a = pa[topf[a]];
37         }
38         /// 此時已在同一條鍊上
39         if (depth[a] < depth[b]) swap(a, b);
40         res = max(res, 0ll); // query : L = id[b], R = id[a]
41         return res;
42     }
43 };

```

6.10 Kosaraju

```

1 /*
2 給定一個有向圖，迴傳傳縮點後的圖、SCC 的資訊
3 所有點都以 based-0 編號
4
5 函式：
6 SCC_compress G(n): 宣告一個有 n 個點的圖

```

```

7 .add_edge(u, v): 加上一條邊 u -> v
8 .compress: O(n log n) 計算 G3、SCC、SCC_id 的資訊，並把縮點後的結果存在 result 裡
9
10 SCC[i] = 某個 SCC 中的所有點
11 SCC_id[i] = 第 i 個點在第幾個 SCC
12 */
13 struct SCC_compress{
14     int N, M, sz;
15     vector<vector<int>> G, inv_G, result;
16     vector<pair<int, int>> edges;
17     vector<bool> vis;
18     vector<int> order;
19
20     vector<vector<int>> SCC;
21     vector<int> SCC_id;
22
23     SCC_compress(int _N) :
24         N(_N), M(0), sz(0),
25         G(N), inv_G(N),
26         vis(N), SCC_id(N)
27     {}
28
29     vector<int> operator [] (int x){
30         return result[x];
31     }
32
33     void add_edge(int u, int v){
34         G[u].push_back(v);
35         inv_G[v].push_back(u);
36         edges.push_back({u, v});
37         M++;
38     }
39
40     void dfs1(vector<vector<int>> &G, int now){
41         vis[now] = 1;
42         for (auto x : G[now]) if (!vis[x]) dfs1(G, x);
43         order.push_back(now);
44     }
45
46     void dfs2(vector<vector<int>> &G, int now){
47         SCC_id[now] = SCC.size()-1;
48         SCC.back().push_back(now);
49         vis[now] = 1;
50         for (auto x : G[now]) if (!vis[x]) dfs2(G, x);
51     }
52
53     void compress(){
54         fill(vis.begin(), vis.end(), 0);
55         for (int i=0 ; i<N ; i++) if (!vis[i]) dfs1(G, i);
56
57         fill(vis.begin(), vis.end(), 0);
58         reverse(order.begin(), order.end());
59         for (int i=0 ; i<N ; i++){
60             if (!vis[order[i]]){
61                 SCC.push_back(vector<int>());
62                 dfs2(inv_G, order[i]);
63             }
64         }
65
66         result.resize(SCC.size());
67         sz = SCC.size();
68         for (auto [u, v] : edges){
69             if (SCC_id[u]!=SCC_id[v]) result[SCC_id[u]].push_back(SCC_id[v]);

```

```

70     }
71     for (int i=0 ; i<SCC.size() ; i++){
72         sort(result[i].begin(), result[i].end());
73         result[i].resize(unique(result[i].begin(), result
74             [i].end())-result[i].begin());
75     }
76 };

```

6.11 Kuhn Munkres

```

1 struct KuhnMunkres{
2     int n; // max(n, m)
3     vector<vector<int>>> G;
4     vector<int> match, lx, ly, visx, visy;
5     vector<int> slack;
6     int stamp = 0;
7
8     KuhnMunkres(int n) : n(n), G(n, vector<int>(n)), lx(n),
9         ly(n), slack(n), match(n), visx(n), visy(n) {}
10
11     void add(int x, int y, int w){
12         G[x][y] = max(G[x][y], w);
13     }
14
15     bool dfs(int i, bool aug){ // aug = true 表示要更新 match
16         if (visx[i]==stamp) return false;
17         visx[i] = stamp;
18
19         for (int j=0 ; j<n ; j++){
20             if (visy[j]==stamp) continue;
21             int d = lx[i]+ly[j]-G[i][j];
22
23             if (d==0){
24                 visy[j] = stamp;
25                 if (match[j]==-1 || dfs(match[j], aug)){
26                     if (aug){
27                         match[j] = i;
28                     }
29                     return true;
30                 }
31             }else{
32                 slack[j] = min(slack[j], d);
33             }
34         }
35         return false;
36     }
37
38     bool augment(){
39         for (int j=0 ; j<n ; j++){
40             if (visy[j]!=stamp && slack[j]==0){
41                 visy[j] = stamp;
42                 if (match[j]==-1 || dfs(match[j], false)){
43                     return true;
44                 }
45             }
46         }
47         return false;
48     }
49
50     void relabel(){
51         int delta = INF;
52         for (int j=0 ; j<n ; j++){

```

```

52         if (visy[j]!=stamp) delta = min(delta, slack[j]);
53     }
54     for (int i=0 ; i<n ; i++){
55         if (visx[i]==stamp) lx[i] -= delta;
56     }
57     for (int j=0 ; j<n ; j++){
58         if (visy[j]==stamp) ly[j] += delta;
59         else slack[j] -= delta;
60     }
61 }
62
63 int solve(){
64
65     for (int i=0 ; i<n ; i++){
66         lx[i] = 0;
67         for (int j=0 ; j<n ; j++){
68             lx[i] = max(lx[i], G[i][j]);
69         }
70     }
71
72     fill(ly.begin(), ly.end(), 0);
73     fill(match.begin(), match.end(), -1);
74
75     for(int i = 0; i < n; i++) {
76         fill(slack.begin(), slack.end(), INF);
77         stamp++;
78         if(dfs(i, true)) continue;
79
80         while(augment()==false) relabel();
81         stamp++;
82         dfs(i, true);
83     }
84
85     int ans = 0;
86     for (int j=0 ; j<n ; j++){
87         if (match[j]!=-1){
88             ans += G[match[j]][j];
89         }
90     }
91     return ans;
92 }
93 };

```

6.12 MCMF

```

1 struct Flow {
2     struct Edge {
3         int u, rc, k, rv;
4     };
5
6     vector<vector<Edge>>> G;
7     vector<int> par, par_eid;
8     Flow(int n) : G(n+1), par(n+1), par_eid(n+1) {}
9
10    // v->u, capacity: c, cost: k
11    void add(int v, int u, int c, int k){
12        G[v].push_back({u, c, k, SZ(G[u])});
13        G[u].push_back({v, 0, -k, SZ(G[v])-1});
14    }
15
16    // 3701d6
17    int spfa(int s, int t){
18        fill(ALL(par), -1);

```

```

19        vector<int> dis(SZ(par), INF);
20        vector<bool> in_q(SZ(par), false);
21        queue<int> Q;
22        dis[s] = 0;
23        in_q[s] = true;
24        Q.push(s);
25
26        while (!Q.empty()){
27            int v = Q.front();
28            Q.pop();
29            in_q[v] = false;
30
31            for (int i=0 ; i<SZ(G[v]) ; i++){
32                auto [u, rc, k, rv] = G[v][i];
33                if (rc>0 && dis[v]+k<dis[u]){
34                    dis[u] = dis[v]+k;
35                    par[u] = v;
36                    par_eid[u] = i;
37                    if (!in_q[u]) Q.push(u);
38                    in_q[u] = true;
39                }
40            }
41        }
42
43        return dis[t];
44    }
45
46    // return <max flow, min cost>, 150093
47    pair<int, int> flow(int s, int t){
48        int fl = 0, cost = 0, d;
49        while ((d = spfa(s, t))<INF){
50            int cur = INF;
51            for (int v=t ; v!=s ; v=par[v]){
52                cur = min(cur, G[par[v]][par_eid[v]].rc);
53            }
54            fl += cur;
55            cost += d*cur;
56            for (int v=t ; v!=s ; v=par[v]){
57                G[par[v]][par_eid[v]].rc -= cur;
58                G[v][G[par[v]][par_eid[v]].rv].rc += cur;
59            }
60        }
61        return {fl, cost};
62    }
63
64    vector<pair<int, int>> construct(){
65        vector<pair<int, int>> ret;
66        for (int i=0 ; i<n ; i++){
67            for (auto x : G[i]){
68                if (x.rc==0){
69                    ret.push_back({i+1, x.u-n+1});
70                    break;
71                }
72            }
73        }
74        return ret;
75    }

```

6.13 Tarjan

```

1 struct tarjan_SCC {
2     int now_T, now_SCCs;
3     vector<int> dfn, low, SCC;

```

```

4   stack<int> S;
5   vector<vector<int>> E;
6   vector<bool> vis, in_stack;
7
8   tarjan_SCC(int n) {
9       init(n);
10  }
11  void init(int n) {
12      now_T = now_SCCs = 0;
13      dfn = low = SCC = vector<int>(n);
14      E = vector<vector<int>>(n);
15      S = stack<int>();
16      vis = in_stack = vector<bool>(n);
17  }
18  void add(int u, int v) {
19      E[u].push_back(v);
20  }
21  void build() {
22      for (int i = 0; i < dfn.size(); ++i) {
23          if (!dfn[i]) dfs(i);
24      }
25  }
26  void dfs(int v) {
27      now_T++;
28      vis[v] = in_stack[v] = true;
29      dfn[v] = low[v] = now_T;
30      S.push(v);
31      for (auto &i:E[v]) {
32          if (!vis[i]) {
33              vis[i] = true;
34              dfs(i);
35              low[v] = min(low[v], low[i]);
36          }
37          else if (in_stack[i]) {
38              low[v] = min(low[v], dfn[i]);
39          }
40      }
41      if (low[v] == dfn[v]) {
42          int tmp;
43          do {
44              tmp = S.top();
45              S.pop();
46              SCC[tmp] = now_SCCs;
47              in_stack[tmp] = false;
48          } while (tmp != v);
49          now_SCCs += 1;
50      }
51  }
52  };

```

6.14 Tarjan Find AP

```

1  vector<int> dep(MAX_N), low(MAX_N), AP;
2  bitset<MAX_N> vis;
3
4  void dfs(int now, int pre){
5      int cnt = 0;
6      bool ap = 0;
7      vis[now] = 1;
8      low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
9
10     for (auto x : G[now]){
11         if (x==pre){

```

```

12         continue;
13     }else if (vis[x]==0){
14         cnt++;
15         dfs(x, now);
16         low[now] = min(low[now], low[x]);
17         if (low[x]>=dep[now]) ap=1;
18     }else{
19         low[now] = min(low[now], dep[x]);
20     }
21 }
22
23 if ((now==pre && cnt>=2) || (now!=pre && ap)){
24     AP.push_back(now);
25 }
26 }

```

6.15 Tree Isomorphism

```

1  #include <bits/stdc++.h>
2  #pragma GCC optimize("O3,unroll-loops")
3  #define fastio ios::sync_with_stdio(0), cin.tie(0), cout.tie(0)
4  #define dbg(x) cerr << #x << " = " << x << endl
5  #define int long long
6  using namespace std;
7
8  // declare
9  const int MAX_SIZE = 2e5+5;
10 const int INF = 9e18;
11 const int MOD = 1e9+7;
12 const double EPS = 1e-6;
13 typedef vector<vector<int>> Graph;
14 typedef map<vector<int>, int> Hash;
15
16 int n, a, b;
17 int id1, id2;
18 pair<int, int> c1, c2;
19 vector<int> sz1(MAX_SIZE), sz2(MAX_SIZE);
20 vector<int> we1(MAX_SIZE), we2(MAX_SIZE);
21 Graph g1(MAX_SIZE), g2(MAX_SIZE);
22 Hash m1, m2;
23 int testcase=0;
24
25 void centroid(Graph &g, vector<int> &s, vector<int> &w, pair<
    int, int> &rec, int now, int pre){
26     s[now]=1;
27     w[now]=0;
28     for (auto x : g[now]){
29         if (x!=pre){
30             centroid(g, s, w, rec, x, now);
31             s[now]+=s[x];
32             w[now]=max(w[now], s[x]);
33         }
34     }
35
36     w[now]=max(w[now], n-s[now]);
37     if (w[now]<=n/2){
38         if (rec.first==0) rec.first=now;
39         else rec.second=now;
40     }
41 }
42
43 int dfs(Graph &g, Hash &m, int &id, int now, int pre){

```

```

44     vector<int> v;
45     for (auto x : g[now]){
46         if (x!=pre){
47             int add=dfs(g, m, id, x, now);
48             v.push_back(add);
49         }
50     }
51     sort(v.begin(), v.end());
52
53     if (m.find(v)!=m.end()){
54         return m[v];
55     }else{
56         m[v]=++id;
57         return id;
58     }
59 }
60
61 void solve1(){
62
63     // init
64     id1=0;
65     id2=0;
66     c1={0, 0};
67     c2={0, 0};
68     fill(sz1.begin(), sz1.begin()+n+1, 0);
69     fill(sz2.begin(), sz2.begin()+n+1, 0);
70     fill(we1.begin(), we1.begin()+n+1, 0);
71     fill(we2.begin(), we2.begin()+n+1, 0);
72     for (int i=1; i<=n; i++){
73         g1[i].clear();
74         g2[i].clear();
75     }
76     m1.clear();
77     m2.clear();
78
79     // input
80     cin >> n;
81     for (int i=0; i<n-1; i++){
82         cin >> a >> b;
83         g1[a].push_back(b);
84         g1[b].push_back(a);
85     }
86     for (int i=0; i<n-1; i++){
87         cin >> a >> b;
88         g2[a].push_back(b);
89         g2[b].push_back(a);
90     }
91
92     // get tree centroid
93     centroid(g1, sz1, we1, c1, 1, 0);
94     centroid(g2, sz2, we2, c2, 1, 0);
95
96     // process
97     int res1=0, res2=0, res3=0;
98     if (c2.second!=0){
99         res1=dfs(g1, m1, id1, c1.first, 0);
100         m2=m1;
101         id2=id1;
102         res2=dfs(g2, m1, id1, c2.first, 0);
103         res3=dfs(g2, m2, id2, c2.second, 0);
104     }else if (c1.second!=0){
105         res1=dfs(g2, m1, id1, c2.first, 0);
106         m2=m1;
107         id2=id1;
108         res2=dfs(g1, m1, id1, c1.first, 0);
109     }

```

```

110     res3=dfs(g1, m2, id2, c1.second, 0);
111 }else{
112     res1=dfs(g1, m1, id1, c1.first, 0);
113     res2=dfs(g2, m1, id1, c2.first, 0);
114 }
115
116 // output
117 cout << (res1==res2 || res1==res3 ? "YES" : "NO") << endl;
118 ;
119 return;
120 }
121
122 signed main(void){
123     fastio;
124
125     int t=1;
126     cin >> t;
127     while (t--){
128         solve1();
129     }
130     return 0;
131 }

```

6.16 圓方樹

```

1 #include <bits/stdc++.h>
2 #define lp(i,a,b) for(int i=a;i<(b);i++)
3 #define pii pair<int,int>
4 #define pb push_back
5 #define ins insert
6 #define ff first
7 #define ss second
8 #define opa(x) cerr << #x << " = " << x << ", ";
9 #define op(x) cerr << #x << " = " << x << endl;
10 #define ops(x) cerr << x;
11 #define etr cerr << endl;
12 #define spc cerr << ' ';
13 #define BAE(x) (x).begin(), (x).end()
14 #define STL(x) cerr << #x << " : "; for(auto &qwe:x) cerr <<
15     qwe << ' '; cerr << endl;
16 #define deb1 cerr << "deb1" << endl;
17 #define deb2 cerr << "deb2" << endl;
18 #define deb3 cerr << "deb3" << endl;
19 #define deb4 cerr << "deb4" << endl;
20 #define deb5 cerr << "deb5" << endl;
21 #define bye exit(0);
22 using namespace std;
23
24 const int mxn = (int)(2e5) + 10;
25 const int mxlg = 17;
26 int last_special_node = (int)(1e5) + 1;
27 vector<int> E[mxn], F[mxn];
28
29 struct edg{
30     int fr, to;
31     edg(int _fr, int _to){
32         fr = _fr;
33         to = _to;
34     };
35 ostream& operator<<(ostream& os, edg x){os << x.fr << "--" <<
36     x.to;};

```

```

36 vector<edg> EV;
37
38 void tarjan(int v, int par, stack<int>& S){
39     static vector<int> dfn(mxn), low(mxn);
40     static vector<bool> to_add(mxn);
41     static int nowT = 0;
42
43     int childs = 0;
44     nowT += 1;
45     dfn[v] = low[v] = nowT;
46     for(auto &ne:E[v]){
47         int i = EV[ne].to;
48         if(i == par) continue;
49         if(!dfn[i]){
50             S.push(ne);
51             tarjan(i, v, S);
52             childs += 1;
53             low[v] = min(low[v], low[i]);
54
55             if(par >= 0 && low[i] >= dfn[v]){
56                 vector<int> bcc;
57                 int tmp;
58                 do{
59                     tmp = S.top(); S.pop();
60                     if(!to_add[EV[tmp].fr]){
61                         to_add[EV[tmp].fr] = true;
62                         bcc.pb(EV[tmp].fr);
63                     }
64                     if(!to_add[EV[tmp].to]){
65                         to_add[EV[tmp].to] = true;
66                         bcc.pb(EV[tmp].to);
67                     }
68                 }while(tmp != ne);
69                 for(auto &j:bcc){
70                     to_add[j] = false;
71                     F[last_special_node].pb(j);
72                     F[j].pb(last_special_node);
73                 }
74                 last_special_node += 1;
75             }
76         }
77         else{
78             low[v] = min(low[v], dfn[i]);
79             if(dfn[i] < dfn[v]){ // edge i--v will be visited
80                 twice at here, but we only need one.
81                 S.push(ne);
82             }
83         }
84     }
85
86     int dep[mxn], jmp[mxn][mxlg];
87     void dfs_lca(int v, int par, int depth){
88         dep[v] = depth;
89         for(auto &i:F[v]){
90             if(i == par) continue;
91             jmp[i][0] = v;
92             dfs_lca(i, v, depth + 1);
93         }
94     }
95
96     inline void build_lca(){
97         jmp[1][0] = 1;
98         dfs_lca(1, -1, 1);
99         lp(j,1,mxlg){
100             lp(i,1,mxn){

```

```

101             jmp[i][j] = jmp[jmp[i][j-1]][j-1];
102         }
103     }
104
105     inline int lca(int x, int y){
106         if(dep[x] < dep[y]){ swap(x, y); }
107
108         int diff = dep[x] - dep[y];
109         lp(j,0,mxlg){
110             if((diff >> j) & 1){
111                 x = jmp[x][j];
112             }
113         }
114         if(x == y) return x;
115
116         for(int j = mxlg - 1; j >= 0; j--){
117             if(jmp[x][j] != jmp[y][j]){
118                 x = jmp[x][j];
119                 y = jmp[y][j];
120             }
121         }
122         return jmp[x][0];
123     }
124
125     inline bool can_reach(int fr, int to){
126         if(dep[to] > dep[fr]) return false;
127
128         int diff = dep[fr] - dep[to];
129         lp(j,0,mxlg){
130             if((diff >> j) & 1){
131                 fr = jmp[fr][j];
132             }
133         }
134         return fr == to;
135     }
136
137     int main(){
138         ios::sync_with_stdio(false); cin.tie(0);
139         // freopen("test_input.txt", "r", stdin);
140         int n, m, q; cin >> n >> m >> q;
141         lp(i,0,m){
142             int u, v; cin >> u >> v;
143             E[u].pb(EV.size());
144             EV.pb(edg(u, v));
145             E[v].pb(EV.size());
146             EV.pb(edg(v, u));
147         }
148         E[0].pb(EV.size());
149         EV.pb(edg(0, 1));
150         stack<int> S;
151         tarjan(0, -1, S);
152         build_lca();
153
154         lp(queries,0,q){
155             int fr, to, relay; cin >> fr >> to >> relay;
156             if(fr == relay || to == relay){
157                 cout << "NO\n";
158                 continue;
159             }
160             if((can_reach(fr, relay) || can_reach(to, relay)) &&
161                 dep[relay] >= dep[lca(fr, to)]){
162                 cout << "NO\n";
163                 continue;
164             }
165             cout << "YES\n";

```

```

166 }
167 }

```

6.17 最大權閉合圖

```

1  /*
2  Problem:
3  Given w = [w_0, w_1, ..., w_{n-1}] (which can be
4  either positive or negative or 0), you can choose
5  to take w_i (0 < i < n) or not, but if edge u -> v
6  exists, you must take w_v if you want to take w_u
7  (in other words, you can't take w_u without taking
8  w_v), this function returns the maximum value(> 0)
9  you can get. If you need a construction, you can
10 output the minimum cut of the S(source) side.
11 Complexity:
12 MaxFlow(n, m) (Non-Biparte:O(n^2m) / Bipartite:O(m√n))
13 */
14 int maximum_closure(vector<int> w, vector<pair<int,int>> EV)
15 {
16     int n = w.size(), S = n + 1, T = n + 2;
17     Flow G(T + 5); // Graph/Dinic.cpp
18     int sum = 0;
19     for (int i = 0; i < n; ++i) {
20         if (w[i] > 0) {
21             G.add(S, i, w[i]);
22             sum += w[i];
23         }
24         else if (w[i] < 0) {
25             G.add(i, T, abs(w[i]));
26         }
27     }
28     for (auto &[u, v] : EV) { // You should make sure that
29         INF > Σ w_i /
30         G.add(u, v, INF);
31     }
32     int cut = G.flow(S, T);
33     return sum - cut;
34 }

```

6.18 Theorem

- 任意圖
 - 不能有孤點 · 最大匹配 + 最小邊覆蓋 = n - 點覆蓋的補集是獨立集 · 最小點覆蓋 + 最大獨立集 = n
- 二分圖
 - 最小點覆蓋 = 最大匹配 = n - 最大獨立集
- 只有邊帶權的二分圖
 - w-vertex-cover (帶權點覆蓋): 每條邊的兩個連接點被選中的次數總和至少要是 w_e ·
 - w-weight matching (帶權匹配)
 - minimum vertex count of w-vertex-cover = maximum weight count of w-weight matching (一個點可以被選很多次 · 但邊不行)
- 點、邊都帶權的二分圖的定理

- b-matching: 假設 v 的點權是 b_v · 那所有 v 的匹配邊 e 的權重都要滿足 $\sum w_e \leq b_v$ ·
- The maximum w-weight of a b-matching equals the minimum b-weight of vertices in a w-vertex-cover.

7 Math

7.1 CRT m Coprime

```

1 vector<int> a, m;
2
3 int extgcd(int a, int b, int &x, int &y){
4     if (b==0){
5         x=1, y=0;
6         return a;
7     }
8
9     int ret=extgcd(b, a%b, y, x);
10    y-=a/b*x;
11    return ret;
12 }
13
14 // n = 有幾個式子 · 求解 x \equiv a_i \pmod{m_i}
15 int CRT(int n, vector<int> &a, vector<int> &m){
16     int p=1, ans=0;
17
18     vector<int> M(n), inv_M(n);
19
20     for (int i=0; i<n; i++) p*=m[i];
21     for (int i=0; i<n; i++){
22         M[i]=p/m[i];
23         int tmp;
24         extgcd(M[i], m[i], inv_M[i], tmp);
25         ans+=a[i]*inv_M[i]*M[i];
26         ans%=p;
27     }
28
29     return (ans%p+p)%p;
30 }

```

7.2 CRT m Not Coprime

```

1 int extgcd(int a, int b, int &x, int &y){
2     if (b==0){
3         x=1, y=0;
4         return a;
5     }
6
7     int ret=extgcd(b, a%b, y, x);
8     y-=a/b*x;
9     return ret;
10 }
11
12 // 對於方程組的式子兩兩求解
13 // {是否有解, {a, m}}
14 pair<bool, pair<int, int>> CRT(int a1, int m1, int a2, int m2)
15 {
16 }

```

```

15 int g=__gcd(m1, m2);
16 if ((a2-a1)%g!=0) return {0, {-1, -1}};
17
18 int x, y;
19 extgcd(m1, m2, x, y);
20
21 x=(a2-a1)*x/g; // 兩者不能相反
22 a1=x*m1+a1;
23 m1=m1*m2/g;
24 a1=(a1%m1+m1)%m1;
25 return {1, {a1, m1}};
26 }

```

7.3 Fraction

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 // Fraction template starts //
5 #define fraction_template_bonus_check
6 const long long ll_overflow_warning_value = (long long)(3e9);
7
8 long long gcd(long long a, long long b){
9     if(a == 0) return 0;
10    if(b == 0) return a;
11    if(a < b) return gcd(b,a);
12    return gcd(b, a%b);
13 }
14 struct frac{
15     long long a, b;
16     frac(long long _a = 0, long long _b = 1){
17         a = _a; b = _b;
18         if(b == 0){
19             cerr << "Error: division by zero\n";
20             cerr << "Called : Constructor(" << _a << ", " <<
21                 _b << ")\n";
22             return;
23         }
24         if(a == 0){b = 1; return;}
25         if(b < 0){a = -a; b = -b;}
26         long long gcd_ab = gcd(std::abs(a), b);
27         if(gcd_ab != 1){a /= gcd_ab; b /= gcd_ab;}
28
29         #ifdef fraction_template_bonus_check
30         if(std::abs(a) > ll_overflow_warning_value || b >
31             ll_overflow_warning_value){
32             cerr << "Overflow warning : " << a << "/" << b <<
33                 "\n";
34             #endif // fraction_template_bonus_check
35         }
36         frac operator+(frac const &B){
37             return frac(a*(B.b)+(B.a)*b, b*(B.b));
38         }
39         frac operator-(frac const &B){
40             return frac(a*(B.b)-(B.a)*b, b*(B.b));
41         }
42         frac operator*(frac const &B){
43             return frac(a*(B.a), b*(B.b));
44         }
45         frac operator/(frac const &B){
46             return frac(a*(B.b), b*(B.a));
47         }
48
49         frac operator+=(frac const &B){
50             *this = frac(a*(B.b)+(B.a)*b, b*(B.b));
51         }
52         frac operator-=(frac const &B){
53             *this = frac(a*(B.b)-(B.a)*b, b*(B.b));
54         }
55     }
56 };

```

```

46 frac operator*=(frac const &B){
47     *this = frac(a*(B.a), b*(B.b));}
48 frac operator/=(frac const &B){
49     *this = frac(a*(B.b), b*(B.a));}
50
51 frac abs(){
52     a = std::abs(a);
53     return *this;
54 }
55
56 bool operator<(frac const &B){
57     return a*B.b < B.a*b;}
58 bool operator<=(frac const &B){
59     return a*B.b <= B.a*b;}
60 bool operator>(frac const &B){
61     return a*B.b > B.a*b;}
62 bool operator>=(frac const &B){
63     return a*B.b >= B.a*b;}
64 bool operator==(frac const &B){
65     return a * B.b == B.a * b;}
66 bool operator!=(frac const &B){
67     return a * B.b != B.a * b;}
68 };
69 ostream& operator<<(ostream &os, const frac& A){
70     os << A.a << "/" << A.b;
71     return os;
72 }
73 /// Fraction template ends ///
74
75 void test(frac A, frac B){
76     cout << "A = " << A << endl;
77     cout << "B = " << B << endl;
78     cout << endl;
79     cout << "A + B = " << A + B << endl;
80     cout << "A - B = " << A - B << endl;
81     cout << "A * B = " << A * B << endl;
82     cout << "A / B = " << A / B << endl;
83     cout << endl;
84     cout << "(A < B) = " << (A < B) << endl;
85     cout << "(A <= B) = " << (A <= B) << endl;
86     cout << "(A > B) = " << (A > B) << endl;
87     cout << "(A >= B) = " << (A >= B) << endl;
88     cout << "(A == B) = " << (A == B) << endl;
89     cout << "(A != B) = " << (A != B) << endl;
90     cout << "-----\n";
91     return;
92 }
93
94 int main(){
95     frac tmp1(-7, 2);
96     frac tmp2(5, 3);
97     test(tmp1, tmp2);
98
99     frac tmp3(-7);
100    frac tmp4(0);
101    test(tmp3, tmp4);
102    return 0;
103 }

```

7.4 Josephus Problem

```

1 /// 有 n 個人，第偶數個報數的人被刪掉，問第 k 個被踢掉的是誰
2 int solve(int n, int k){

```

```

3     if (n==1) return 1;
4     if (k<=(n+1)/2){
5         if (2*k>n) return 2*k%n;
6         else return 2*k;
7     }else{
8         int res=solve(n/2, k-(n+1)/2);
9         if (n&1) return 2*res+1;
10        else return 2*res-1;
11    }
12 }

```

7.5 Lagrange any x

```

1 /// init: (x1, y1), (x2, y2) in a vector
2 struct Lagrange{
3     int n;
4     vector<pair<int, int>> v;
5
6     Lagrange(vector<pair<int, int>> &_v){
7         n = _v.size();
8         v = _v;
9     }
10
11    /// O(n^2 log MAX_A)
12    int solve(int x){
13        int ret = 0;
14        for (int i=0 ; i<n ; i++){
15            int now = v[i].second;
16            for (int j=0 ; j<n ; j++){
17                if (i==j) continue;
18                now *= ((x-v[j].first+MOD)%MOD);
19                now %= MOD;
20                now *= (qp((v[i].first-v[j].first+MOD)%MOD,
21                    MOD-2)+MOD)%MOD;
22                now %= MOD;
23            }
24            ret = (ret+now)%MOD;
25        }
26        return ret;
27    }
28 };

```

7.6 Lagrange continuous x

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAX_N = 5e5 + 10;
5 const int mod = 1e9 + 7;
6
7 long long inv_fac[MAX_N];
8
9 inline int fp(long long x, int y) {
10     int ret = 1;
11     for (; y; y >>= 1) {
12         ret = (y & 1) ? (ret * x % mod) : ret;
13         x = x * x % mod;
14     }
15     return ret;

```

```

16 }
17
18 /// TO USE THIS TEMPLATE, YOU MUST MAKE SURE THAT THE MOD
19 NUMBER IS A PRIME.
20 struct Lagrange {
21     ///
22     Initialize a polynomial with f(x_0), f(x_0 + 1), ..., f(x_0 + n).
23     This determines a polynomial f(x) whose degree is at most
24     n.
25     Then you can call sample(x) and you get the value of f(x)
26     .
27     Complexity of init() and sample() are both O(n).
28     */
29     int m, shift; /// m = n + 1
30     vector<int> v, mul;
31     /// You can use this function if you don't have inv_fac array
32     already.
33     void construct_inv_fac() {
34         long long fac = 1;
35         for (int i = 2; i < MAX_N; ++i) {
36             fac = fac * i % mod;
37         }
38         inv_fac[MAX_N - 1] = fp(fac, mod - 2);
39         for (int i = MAX_N - 1; i >= 1; --i) {
40             inv_fac[i - 1] = inv_fac[i] * i % mod;
41         }
42     }
43     /// You call init() many times without having a second
44     instance of this struct.
45     void init(int X_0, vector<int> &u) {
46         v = u;
47         shift = ((1 - X_0) % mod + mod) % mod;
48         if (v.size() == 1) v.push_back(v[0]);
49         m = v.size();
50         mul.resize(m);
51     }
52     /// You can use sample(x) instead of sample(x % mod).
53     int sample(int x) {
54         x = ((long long)x + shift) % mod;
55         x = (x < 0) ? (x + mod) : x;
56         long long now = 1;
57         for (int i = m; i >= 1; --i) {
58             mul[i - 1] = now;
59             now = now * (x - i) % mod;
60         }
61         int ret = 0;
62         bool neg = (m - 1) & 1;
63         now = 1;
64         for (int i = 1; i <= m; ++i) {
65             int up = now * mul[i - 1] % mod;
66             int down = inv_fac[m - i] * inv_fac[i - 1] % mod;
67             int tmp = ((long long)v[i - 1] * up % mod) * down
68                 % mod;
69             ret += (neg && tmp) ? (mod - tmp) : (tmp);
70             ret = (ret >= mod) ? (ret - mod) : ret;
71             now = now * (x - i) % mod;
72             neg ^= 1;
73         }
74         return ret;
75     }
76 };
77
78 int main() {
79     int n; cin >> n;
80     vector<int> v(n);

```



```

75   for (int i = 0; i < n; ++i) {
76       cin >> v[i];
77   }
78   Lagrange L;
79   L.construct_inv_fac();
80   L.init(0, v);
81   int x; cin >> x;
82   cout << L.sample(x);
83 }

```

7.7 Lucas's Theorem

```

1 // 對於很大的  $C^n_m$  對質數  $p$  取模，只要  $p$  不大就可以用。
2 int Lucas(int n, int m, int p){
3     if (m==0) return 1;
4     return (C(n%p, m%p, p)*Lucas(n/p, m/p, p)%p);
5 }

```

7.8 Matrix

```

1 struct Matrix{
2     int n, m;
3     vector<vector<int>> arr;
4
5     Matrix(int _n, int _m){
6         n = _n;
7         m = _m;
8         arr.resize(n, vector<int>(m));
9     }
10
11     Matrix operator * (Matrix b){
12         Matrix b_t(b.m, b.n);
13         for (int i=0 ; i<b.n ; i++){
14             for (int j=0 ; j<b.m ; j++){
15                 b_t.arr[j][i] = b.arr[i][j];
16             }
17         }
18
19         Matrix ret(n, b.m);
20         for (int i=0 ; i<n ; i++){
21             for (int j=0 ; j<b.m ; j++){
22                 for (int k=0 ; k<m ; k++){
23                     ret.arr[i][j] += arr[i][k]*b_t.arr[j][k];
24                     ret.arr[i][j] %= MOD;
25                 }
26             }
27         }
28         return ret;
29     }
30
31     Matrix pow(int p){
32         Matrix ret(n, n), mul = *this;
33         for (int i=0 ; i<n ; i++){
34             ret.arr[i][i] = 1;
35         }
36
37         for ( ; p ; p>>=1){
38             if (p&1) ret = ret*mul;
39             mul = mul*mul;
40         }

```

```

41     return ret;
42 }
43
44 int det(){
45     vector<vector<int>> arr = this->arr;
46     bool flag = false;
47     for (int i=0 ; i<n ; i++){
48         int target = -1;
49         for (int j=i ; j<n ; j++){
50             if (arr[j][i]){
51                 target = j;
52                 break;
53             }
54         }
55         if (target==-1) return 0;
56         if (i!=target){
57             swap(arr[i], arr[target]);
58             flag = !flag;
59         }
60         for (int j=i+1 ; j<n ; j++){
61             if (!arr[j][i]) continue;
62             int freq = arr[j][i]*qp(arr[i][i], MOD-2)%MOD;
63             for (int k=i+1 ; k<n ; k++){
64                 arr[j][k] -= freq*arr[i][k];
65                 arr[j][k] = (arr[j][k]%MOD+MOD)%MOD;
66             }
67         }
68         int ret = !flag ? 1 : MOD-1;
69         for (int i=0 ; i<n ; i++){
70             ret *= arr[i][i];
71             ret %= MOD;
72         }
73         return ret;
74     }
75 }
76
77 }
78
79 };

```

7.9 Matrix 01

```

1 const int MAX_N = (1LL<<12);
2 struct Matrix{
3     int n, m;
4     vector<bitset<MAX_N>> arr;
5
6     Matrix(int _n, int _m){
7         n = _n;
8         m = _m;
9         arr.resize(n);
10    }
11
12    Matrix operator * (Matrix b){
13        Matrix b_t(b.m, b.n);
14        for (int i=0 ; i<b.n ; i++){
15            for (int j=0 ; j<b.m ; j++){
16                b_t.arr[j][i] = b.arr[i][j];
17            }
18        }
19    }

```

```

20    Matrix ret(n, b.m);
21    for (int i=0 ; i<n ; i++){
22        for (int j=0 ; j<b.m ; j++){
23            ret.arr[i][j] = ((arr[i]&b_t.arr[j]).count()
24                &1);
25        }
26    }
27    return ret;
28 }

```

7.10 Miller Rabin

```

1 //  $O(\log n)$ 
2 typedef Uint unsigned long long
3 Uint modmul(Uint a, Uint b, Uint m) {
4     int ret = a*b - m*(Uint)((long double)a*b/m);
5     return ret + m*(ret < 0) - m*(ret>=(int)m);
6 }
7
8 int qp(int b, int p, int m){
9     int ret = 1;
10    for ( ; p ; p>>=1){
11        if (p&1){
12            ret = modmul(ret, b, m);
13        }
14        b = modmul(b, b, m);
15    }
16    return ret;
17 }
18
19 // ed23aa
20 vector<int> llsprp = {2, 325, 9375, 28178, 450775, 9780504,
21     1795265022};
22 bool isprime(int n, vector<int> sprp = llsprp){
23     if (n==2) return 1;
24     if (n<2 || n%2==0) return 0;
25
26     int t = 0;
27     int u = n-1;
28     for ( ; u%2==0 ; t++) u>>=1;
29
30     for (int i=0 ; i<sprp.size() ; i++){
31         int a = sprp[i]%n;
32         if (a==0 || a==1 || a==n-1) continue;
33         int x = qp(a, u, n);
34         if (x==1 || x==n-1) continue;
35         for (int j=0 ; j<t ; j++){
36             x = modmul(x, x, n);
37             if (x==1) return 0;
38             if (x==n-1) break;
39         }
40         if (x==n-1) continue;
41         return 0;
42     }
43     return 1;
44 }

```


7.11 Pollard Rho

```

1 | mt19937 seed(chrono::steady_clock::now().time_since_epoch().
    count());
2 | int rnd(int l, int r){
3 |     return uniform_int_distribution<int>(l, r)(seed);
4 | }
5 |
6 | // O(n^{1/4}) 回傳 1 或自己的因數、記得先判斷 n 是不是質數
    (用 Miller-Rabin)
7 | // c1670c
8 | int Pollard_Rho(int n){
9 |     int s = 0, t = 0;
10 |    int c = rnd(1, n-1);
11 |
12 |    int step = 0, goal = 1;
13 |    int val = 1;
14 |
15 |    for (goal=1 ; ; goal<=1, s=t, val=1){
16 |        for (step=1 ; step<=goal ; step++){
17 |
18 |            t = ((__int128)t*t+c)%n;
19 |            val = ((__int128)val*abs(t-s)%n;
20 |
21 |            if ((step % 127) == 0){
22 |                int d = __gcd(val, n);
23 |                if (d>1) return d;
24 |            }
25 |        }
26 |
27 |        int d = __gcd(val, n);
28 |        if (d>1) return d;
29 |    }
30 | }

```

7.12 Quick Pow

```

1 | int qp(int b, int p, int m = MOD){
2 |     int ret = 1;
3 |     for ( ; p ; p>>=1){
4 |         if (p&1) ret = ret*b%m;
5 |         b = b*b%m;
6 |     }
7 |     return ret;
8 | }

```

7.13 數論分塊

```

1 | /*
2 | 時間複雜度為 O(sqrt(n))
3 | 區間為 [L, r]
4 | */
5 | for(int i=1 ; i<=n ; i++){
6 |     int l = i, r = n/(n/i);
7 |     i = r;
8 |     ans.push_back(r);
9 | }

```

7.14 最大質因數

```

1 | void max_fac(int n, int &ret){
2 |     if (n<=ret || n<2) return;
3 |     if (isprime(n)){
4 |         ret = max(ret, n);
5 |         return;
6 |     }
7 |
8 |     int p = Pollard_Rho(n);
9 |     max_fac(p, ret), max_fac(n/p, ret);
10 | }

```

7.15 歐拉公式

```

1 | // phi(n) = 小於 n 並與 n 互質的正整數數量。
2 | // O(sqrt(n)) · 回傳 phi(n)
3 | int phi(int n){
4 |     int ret = n;
5 |
6 |     for (int i=2 ; i*i<=n ; i++){
7 |         if (n%i==0){
8 |             while (n%i==0) n /= i;
9 |             ret = ret*(i-1)/i;
10 |        }
11 |    }
12 |    if (n>1) ret = ret*(n-1)/n;
13 |
14 |    return ret;
15 | }
16 |
17 | // O(n Log n) · 回傳 1~n 的 phi 值
18 | vector<int> phi_1_to_n(int n){
19 |     vector<int> phi(n+1);
20 |     phi[0]=0;
21 |     phi[1]=1;
22 |
23 |     for (int i=2 ; i<=n ; i++){
24 |         phi[i]=i-1;
25 |     }
26 |
27 |     for (int i=2 ; i<=n ; i++){
28 |         for (int j=2*i ; j<=n ; j+=i){ // 枚舉所有倍數
29 |             phi[j]-=phi[i];
30 |         }
31 |     }
32 |
33 |     return phi;
34 | }

```

7.16 線性篩

```

1 | const int MAX_N = 5e5;
2 |
3 | // lpf[i] = i 的最小質因數
4 | vector<int> prime, lpf(MAX_N);
5 |
6 | void prime_init(){

```

```

7 |     for (int i=2 ; i<MAX_N ; i++){
8 |         if (lpf[i]==0){
9 |             lpf[i] = i;
10 |            prime.push_back(i);
11 |        }
12 |
13 |        for (int j : prime){
14 |            if (i*j>=MAX_N) break;
15 |            lpf[i*j] = j;
16 |            if (i%j==0) break;
17 |        }
18 |    }
19 | }

```

7.17 Burnside's Lemma

$$\sum_{k=1}^n \frac{c(k)}{n}$$

- n : 有多少種置換方式 (例如: 旋轉方式)
- $c(k)$: 所有可能中, 經過 k 次旋轉後, 仍不會和別人相同的方式的數量

7.18 Catalan Number

任意括號序列: $C_n = \frac{1}{n+1} \binom{2n}{n}$

7.19 Matrix Tree Theorem

目標: 給定一張無向圖, 問他的生成樹數量。
方法: 先把所有自環刪掉, 定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg(v_i) & \text{if } i = j \\ -(\text{邊 } v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第一個 row 跟 column, 它的 determinant 就是答案。
目標: 給定一張有向圖, 問他的以 r 為根, 可以走到所有點生成樹數量。

方法: 先把所有自環刪掉, 定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg_{in}(v_i) & \text{if } i = j \\ -(\text{邊 } v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第 r 個 row 跟 column, 它的 determinant 就是答案。

7.20 Stirling's formula

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

7.21 Theorem

1. $1 \sim x$ 質數的數量 $\approx \frac{x}{\ln x}$
2. $1 \sim x$ 的因數的數量 $\approx x^{\frac{1}{3}}$
3. x 的質因數的數量 $\approx \log \log x$
4. p is a prime number $\Leftrightarrow (p-1)! \equiv -1 \pmod{p}$
5. 每個正整數都可以表示成四個整數的平方和
6. 任何大於 2 的整數都可以表示成兩個質數的和

7.22 二元一次方程式

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases} = \begin{cases} x = \frac{ed-bf}{ad-bc} \\ y = \frac{af-ec}{ad-bc} \end{cases}$$

若 $x = \frac{0}{0}$ 且 $y = \frac{0}{0}$ ，則代表無限多組解。若 $x = \frac{*}{0}$ 且 $y = \frac{*}{0}$ ，則代表無解。

7.23 歐拉定理

若 a, m 互質，則：

$$a^n \equiv a^{n \bmod \varphi(m)} \pmod{m}$$

若 a, m 不互質，則：

$$a^n \equiv a^{\varphi(m) + [n \bmod \varphi(m)]} \pmod{m}$$

7.24 錯排公式

錯排公式：(n 個人中，每個人皆不再原來位置的組合數)

$$dp_i = \begin{cases} 1 & i = 0 \\ 0 & i = 1 \\ (i-1)(dp_{i-1} + dp_{i-2}) & \text{otherwise} \end{cases}$$

8 String

8.1 Hash

```
1 int A = rng(1e5, 8e8);
2 const int B = 1e9+7;
3
4 struct RollingHash{
5     vector<int> Pow, Pre;
6     RollingHash(string s = ""){
7         Pow.resize(s.size());
8         Pre.resize(s.size());
9     }
```

```
10     for (int i=0 ; i<s.size() ; i++){
11         if (i==0){
12             Pow[i] = 1;
13             Pre[i] = s[i];
14         }else{
15             Pow[i] = Pow[i-1]*A%B;
16             Pre[i] = (Pre[i-1]*A+s[i])%B;
17         }
18     }
19
20     return;
21 }
22
23 int get(int l, int r){ // 取得 [l, r] 的數值
24     if (l==0) return Pre[r];
25     int res = (Pre[r]-Pre[l-1]*Pow[r-l+1])%B;
26     if (res<0) res += B;
27     return res;
28 }
29 ;
```

8.2 KMP

```
1 // 給一個字串 S，定義函數 \pi(i) = k 代表 S[1 ... k] = S[i-k
2 // +1 ... i] (最長真前後綴)
3 // e5b7ce
4 vector<int> KMP(string &s){
5     int n = s.size();
6     vector<int> ret(n);
7     for (int i=1 ; i<n ; i++){
8         int j = ret[i-1];
9         while (j>0 && s[i]!=s[j]) j = ret[j-1];
10        j += (s[i]==s[j]);
11        ret[i] = j;
12    }
13    return ret;
14 }
```

8.3 Manacher

```
1 string Manacher(string str) {
2     string tmp = "$#";
3     for(char i : str) {
4         tmp += i;
5         tmp += '#';
6     }
7
8     vector<int> p(tmp.size(), 0);
9     int mx = 0, id = 0, len = 0, center = 0;
10    for(int i=1 ; i<(int)tmp.size() ; i++) {
11        p[i] = mx > i ? min(p[id*2-i], mx-i) : 1;
12
13        while(tmp[i+p[i]] == tmp[i-p[i]]) p[i]++;
14        if(mx<i+p[i]) mx = i+p[i], id = i;
15        if(len<p[i]) len = p[i], center = i;
16    }
17    return str.substr((center-len)/2, len-1);
18 }
```

8.4 Min Rotation

```
1 // 9d296f
2 int minRotation(string s) {
3     int a=0, N=SZ(s); s += s;
4     for (int b=0 ; b<N ; b++){
5         for (int k=0 ; k<N ; k++){
6             if (a+k == b || s[a+k] < s[b+k]) {b += max(0LL, k
7                 -1); break;}
8             if (s[a+k] > s[b+k]) {a = b; break;}
9         }
10        return a;
11    }
```

8.5 Suffix Array

```
1 // 注意，當 |s|=1 時，lcp 不會有值，務必測試 |s|=1 的 case
2 struct SuffixArray {
3     string s;
4     vector<int> sa, lcp;
5     SuffixArray(string _s, int lim = 256) {
6         s = _s;
7         int n = s.size()+1, k = 0, a, b;
8         vector<int> x(s.begin(), s.end()), y(n), ws(max(n,
9             lim)), rank(n);
10        x.push_back(0);
11        sa = lcp = y;
12        iota(sa.begin(), sa.end(), 0);
13        for (int j=0, p=0 ; p<n ; j=max(1LL, j*2), lim=p) {
14            p = j;
15            iota(y.begin(), y.end(), n-j);
16            for (int i=0 ; i<n ; i++) if (sa[i] >= j) y[p++]
17                = sa[i] - j;
18            fill(ws.begin(), ws.end(), 0);
19            for (int i=0 ; i<n ; i++) ws[x[i]]++;
20            for (int i=1 ; i<lim ; i++) ws[i] += ws[i-1];
21            for (int i = n; i--;) sa[--ws[x[i]]] = i;
22            swap(x, y), p = 1, x[sa[0]] = 0;
23            for (int i=1 ; i<n ; i++){
24                a = sa[i-1];
25                b = sa[i];
26                x[b] = (y[a] == y[b] && y[a+j] == y[b+j])
27                    ? p-1 : p++;
28            }
29
30            for (int i=1 ; i<n ; i++) rank[sa[i]] = i;
31            for (int i=0, j ; i<n-1 ; lcp[rank[i++]]=k)
32                for (k && k--, j=sa[rank[i]-1] ; i+k<s.size() &&
33                    j+k<s.size() && s[i+k]==s[j+k] ; k++);
34            sa.erase(sa.begin());
35            lcp.erase(lcp.begin(), lcp.begin()+2);
36        }
37
38        vector<int> pos; // pos[i] = i 這個值在 pos 的哪個地方
39        SparseTable st;
40        void init_lcp(){
41            pos.resize(sa.size());
42            for (int i=0 ; i<sa.size() ; i++){
43                pos[sa[i]] = i;
44            }
45        }
```

```

42     if (lcp.size()){
43         st.build(lcp);
44     }
45 }
46
47 // 用之前記得 init
48 // 回傳 [l1, r1] 跟 [l2, r2] 的 lcp · 0-based
49 int get_lcp(int l1, int r1, int l2, int r2){
50     int pos_1 = pos[l1], len_1 = r1-l1+1;
51     int pos_2 = pos[l2], len_2 = r2-l2+1;
52     if (pos_1>pos_2){
53         swap(pos_1, pos_2);
54         swap(len_1, len_2);
55     }
56
57     if (l1==l2){
58         return min(len_1, len_2);
59     }else{
60         return min({st.query(pos_1, pos_2), len_1, len_2
61             });
62     }
63 }
64
65 // 檢查 [l1, r1] 跟 [l2, r2] 的大小關係 · 0-based
66 // 如果前者小於後者 · 就回傳 <0 · 相等就回傳 =0 · 否則回傳
67 // >0
68 int substring_cmp(int l1, int r1, int l2, int r2){
69     int len_1 = r1-l1+1;
70     int len_2 = r2-l2+1;
71     int res = get_lcp(l1, r1, l2, r2);
72
73     if (res<len_1 && res<len_2){
74         return s[l1+res]-s[l2+res];
75     }else if (len_1==res && len_2==res){
76         // 如果不需要以 index 作為次要排序參數 · 這裡要回
77         // 傳 0
78         return l1-l2;
79     }else{
80         return len_1==res ? -1 : 1;
81     }
82 }
83
84 // 對於位置在 <=p 的後綴 · 找離他左邊/右邊最接近位置 >p 的
85 // 後綴的 lcp · 0-based
86 // pre[i] = s[i] 離他左邊最接近位置 >p 的後綴的 lcp · 0-
87 // based
88 // suf[i] = s[i] 離他右邊最接近位置 >p 的後綴的 lcp · 0-
89 // based
90 pair<vector<int>, vector<int>> get_left_and_right_lcp(int
91     p){
92     vector<int> pre(p+1);
93     vector<int> suf(p+1);
94
95     { // build pre
96         int now = 0;
97         for (int i=0 ; i<s.size() ; i++){
98             if (sa[i]<=p){
99                 pre[sa[i]] = now;
100                 if (i<lcp.size()) now = min(now, lcp[i]);
101             }else{
102                 if (i<lcp.size()) now = lcp[i];
103             }
104         }
105     }
106
107     { // build suf
108         int now = 0;
109         for (int i=s.size()-1 ; i>=0 ; i--){
110             if (sa[i]<=p){
111                 suf[sa[i]] = now;
112                 if (i-1>=0) now = min(now, lcp[i-1]);
113             }else{
114                 if (i-1>=0) now = lcp[i-1];
115             }
116         }
117     }
118
119     return {pre, suf};
120 }

```

8.6 Z Algorithm

```

1 // 定義一個長度為 n 的文本為 T · 則陣列 Z 的 Z[i] 代表 T[0:n]
2 // 和 T[i:n] 最長共同前綴
3 // bcfbd6
4 vector<int> z_function(string s){
5     vector<int> ret(s.size());
6     int ll = 0, rr = 0;
7
8     for (int i=1 ; i<s.size() ; i++){
9         int j = 0;
10
11         if (i<rr) j = min(ret[i-ll], rr-i);
12         while (s[j]==s[i+j]) j++;
13         ret[i] = j;
14
15         if (i+j>rr){
16             ll = i;
17             rr = i+j;
18         }
19     }
20
21     ret[0] = s.size();
22     return ret;
23 }

```