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1 Misc

1.1 2-SAT

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct TWO_SAT {
5     int n, N;
6     vector<vector<int>> G, rev_G;
7     deque<bool> used;
8     vector<int> order, comp;
9     deque<bool> assignment;
10    void init(int _n) {
11        n = _n;
12        N = _n * 2;
13        G.resize(N + 5);
14        rev_G.resize(N + 5);
15    }
16    void dfs1(int v) {
17        used[v] = true;
18        for (int u : G[v]) {
19            if (!used[u])
20                dfs1(u);
21        }
22        order.push_back(v);
23    }
24    void dfs2(int v, int c1) {
25        comp[v] = c1;
26        for (int u : rev_G[v]) {
27            if (comp[u] == -1)
28                dfs2(u, c1);
29        }
30    }
31    bool solve() {
32        order.clear();
33        used.assign(N, false);
34        for (int i = 0; i < N; ++i) {
35            if (!used[i])
36                dfs1(i);
37        }
38        comp.assign(N, -1);
39        for (int i = 0, j = 0; i < N; ++i) {
40            int v = order[N - i - 1];
41            if (comp[v] == -1)
42                dfs2(v, j++);
43        }
44        assignment.assign(n, false);
45        for (int i = 0; i < N; i += 2) {
46            if (comp[i] == comp[i + 1])
47                return false;
48            assignment[i / 2] = (comp[i] > comp[i + 1]);
49        }
50        return true;
51    }
52    void add_disjunction(int a, bool na, int b, bool nb) { //
53        // A or B
54        // na means whether a is negative or not
55        // nb means whether b is negative or not
56        a = 2 * a ^ na;
57        b = 2 * b ^ nb;
58        int neg_a = a ^ 1;
59        int neg_b = b ^ 1;

```

```

59        G[neg_a].push_back(b);
60        G[neg_b].push_back(a);
61        rev_G[b].push_back(neg_a);
62        rev_G[a].push_back(neg_b);
63        return;
64    }
65    void get_result(vector<int>& res) {
66        res.clear();
67        for (int i = 0; i < n; i++)
68            res.push_back(assignment[i]);
69    }
70 };
71 /* CSES Giant Pizza
72 3 5
73 + 1 + 2
74 - 1 + 3
75
76 - + + + -
77 */
78 int main() {
79     int n, m;
80     cin >> n >> m;
81     TWO_SAT E;
82     E.init(m);
83
84     char c1, c2;
85     int inp1, inp2;
86     for (int i = 0; i < n; i++) {
87         cin >> c1 >> inp1;
88         cin >> c2 >> inp2;
89         E.add_disjunction(inp1 - 1, c1 == '-', inp2 - 1, c2
90             == '-');
91     }
92
93     bool able = E.solve();
94     if (able) {
95         vector<int> ans;
96         E.get_result(ans);
97         for (int i : ans)
98             cout << (i == true ? '+' : '-') << ' ';
99     } else {
100         cout << "IMPOSSIBLE\n";
101     }
102
103     return 0;
104 }

```

1.2 Custom Set PQ Sort

```

1 // priority_queue · 務必檢查相等的 case · 給所有元素一個排序的
2 依據
3 struct cmp{
4     bool operator () (Data a, Data b){
5         return a.x<b.x;
6     }
7 };
8
9 // set · 務必檢查相等的 case · 給所有元素一個排序的依據
10 struct Data{
11     int x;

```

```

12
13     bool operator < (const Data &b) const {
14         return x<b.x;
15     }
16 };

```

1.3 Default Code New

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 #define int long long
4
5 const int MAX_N = 5e5 + 10;
6 const int INF = 2e18;
7
8 void solve(){
9
10 }
11
12 signed main(){
13     ios::sync_with_stdio(0), cin.tie(0);
14
15     int t = 1;
16     while (t--){
17         solve();
18     }
19
20     return 0;
21 }

```

1.4 Default Code Old

```

1 #include <bits/stdc++.h>
2 #define int long long
3 #define ALL(x) x.begin(), x.end()
4 #define SZ(x) ((int)x.size())
5 #define fastio ios::sync_with_stdio(0), cin.tie(0);
6 using namespace std;
7
8 #ifdef LOCAL
9 #define cout cout << "\033[0;32m"
10 #define cerr cerr << "\033[0;31m"
11 #define endl endl << "\033[0m"
12 #else
13 #pragma GCC optimize("O3,unroll-loops")
14 #pragma GCC target("avx,avx2,sse,sse2,sse3,sse4,popcnt")
15 #define endl "\n"
16 #endif
17
18 const int MAX_N = 5e5+10;
19 const int INF = 2e18;
20
21 void solve1(){
22
23     return;
24 }
25
26 signed main(){
27     fastio;
28

```

```

29 |
30 |     int t = 1;
31 |     while (t--){
32 |         solve1();
33 |     }
34 |
35 |     return 0;
36 | }

```

1.5 Enumerate Subset

```

1 | // 時間複雜度  $O(3^n)$ 
2 | // 枚舉每個 mask 的子集
3 | for (int mask=0; mask<(1<<n); mask++){
4 |     for (int s=mask; s>=0; s=(s-1)&m){
5 |         // s 是 mask 的子集
6 |         if (s==0) break;
7 |     }
8 | }

```

1.6 Fast Input

```

1 | // fast IO
2 | // 6f8879
3 | inline char readchar(){
4 |     static char buffer[BUFSIZ], *now = buffer + BUFSIZ, *
5 |         end = buffer + BUFSIZ;
6 |     if (now == end)
7 |     {
8 |         if (end < buffer + BUFSIZ)
9 |             return EOF;
10 |        end = (buffer + fread(buffer, 1, BUFSIZ, stdin));
11 |        now = buffer;
12 |    }
13 |    return *now++;
14 | }
15 | inline int nextint(){
16 |     int x = 0, c = readchar(), neg = false;
17 |     while (('0' > c || c > '9') && c!='-' && c!=EOF) c =
18 |         readchar();
19 |     if (c == '-') neg = true, c = readchar();
20 |     while ('0' <= c && c <= '9') x = (x<<3) + (x<<1) + (c-'0')
21 |         , c = readchar();
22 |     if (neg) x = -x;
23 |     return x; // returns 0 if EOF
24 | }

```

1.7 Radix Sort

```

1 | // 值域限制:  $0 \sim 1073741823(2^{30}-1)$ 
2 | inline void radix_sort(vector<int> &a, int n){
3 |     static int cnt[32768] = {0};
4 |     vector<int> tmpa(n);
5 |     for (int i = 0; i < n; ++i)
6 |         ++cnt[a[i] & 32767];
7 |     for (int i = 1; i < 32768; ++i)

```

```

8 |         cnt[i] += cnt[i-1];
9 |     static int temp;
10 |    for (int i = n-1; i >= 0; --i){
11 |        temp = a[i] & 32767;
12 |        --cnt[temp];
13 |        tmpa[cnt[temp]] = a[i];
14 |    }
15 |
16 |    static int cnt2[32768] = {0};
17 |    for (int i = 0; i < n; ++i)
18 |        ++cnt2[(tmpa[i]>>15)];
19 |    for (int i = 1; i < 32768; ++i)
20 |        cnt2[i] += cnt2[i-1];
21 |
22 |    for (int i = n-1; i >= 0; --i){
23 |        temp = (tmpa[i]>>15);
24 |        --cnt2[temp];
25 |        a[cnt2[temp]] = tmpa[i];
26 |    }
27 |    return;
28 | }

```

1.8 Random Int

```

1 | mt19937 seed(chrono::steady_clock::now().time_since_epoch()).
2 |     count());
3 | int rng(int l, int r){
4 |     return uniform_int_distribution<int>(l, r)(seed);
5 | }

```

1.9 Xor Basis

```

1 | vector<int> basis;
2 | void add_vector(int x){
3 |     for (auto v : basis){
4 |         x=min(x, x^v);
5 |     }
6 |     if (x) basis.push_back(x);
7 | }
8 |
9 | // 給一數字集合 S，求能不能 XOR 出 x
10 | bool check(int x){
11 |     for (auto v : basis){
12 |         x=min(x, x^v);
13 |     }
14 |     return x;
15 | }
16 |
17 | // 給一數字集合 S，求能 XOR 出多少數字
18 | // 答案等於  $2^{\{basis\} \text{ 的大小}}$ 
19 |
20 | // 給一數字集合 S，求 XOR 出最大的數字
21 | int get_max(){
22 |     int ans=0;
23 |     for (auto v : basis){
24 |         ans=max(ans, ans^v);
25 |     }
26 |     return ans;
27 | }

```

1.10 run

```

1 | import os
2 | p = os.listdir(".")
3 | f = input("input: ")
4 |
5 | if os.system(f"g++ {f}.cpp -std=c++17 -Wall -Wextra -Wshadow
6 |     -O2 -DLOCAL -g -fsanitize=undefined,address -o {f}") !=
7 |     0:
8 |     print("CE")
9 |     exit(1)
10 |
11 | for x in p:
12 |     if x[:len(f)]==f and x[-3:]!=".in":
13 |         print(x)
14 |         if os.system(f"./{f} < {x}")!=0:
15 |             print("RE")
16 |             exit(1)
17 |         print()

```

1.11 setup

```

1 | se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
2 |
3 | :inoremap " ""<Esc>i
4 | :inoremap {<CR> {<CR><Esc>ko
5 | :inoremap { {<ESC>i
6 |
7 | function! F(...)
8 |     execute '!./%:r < ./' . a:1
9 | endfunction
10 | command! -nargs=* R call F(<f-args>)
11 |
12 | map <F7> :w<bar>!g++ "%" -o %:r -std=c++17 -Wall -Wextra -
13 |     Wshadow -O2 -DLOCAL -g -fsanitize=undefined,address<CR>
14 | map <F8> :!./%:r<CR>
15 | map <F9> :!./%:r < ./%:r.in<CR>
16 |
17 | ca hash w !cpp -dD -P -fpreprocessed \\\ tr -d "[:space:]" \\\
18 |     md5sum \\\ cut -c-6
19 |
20 | " i+<esc>25A---+<esc>
21 | " o|<esc>25A |<esc>
22 | " ggVGyG35pGdd

```

2 Convolution

2.1 FFT any mod

```

1 | /*
2 | 修改 const int MOD = 998244353 更改要取餘的數字
3 | PolyMul(a, b) 回傳多項式乘法的結果 (c_k = \sum_{i+j=k} a_i b_j
4 |     mod MOD)
5 |
6 | 大約可以支援  $5e5 \cdot a_i, b_i$  皆在 MOD 以下的非負整數
7 | */
8 | const int MOD = 998244353;

```

```

8 typedef complex<double> cd;
9
10 // b9c90a
11 void FFT(vector<cd> &a) {
12     int n = a.size(), L = 31-__builtin_clz(n);
13     vector<complex<long double>> R(2, 1);
14     vector<cd> rt(2, 1);
15     for (int k=2; k<n; k*=2){
16         R.resize(n);
17         rt.resize(n);
18         auto x = polar(1.0L, acos(-1.0L) / k);
19         for (int i=k; i<2*k; i++){
20             rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
21         }
22     }
23
24     vector<int> rev(n);
25     for (int i=0; i<n; i++){
26         rev[i] = (rev[i/2] | (i&1)<<L)/2;
27     }
28     for (int i=0; i<n; i++){
29         if (i<rev[i]) swap(a[i], a[rev[i]]);
30     }
31     for (int k=1; k<n; k*=2){
32         for (int i=0; i<n; i+=2*k){
33             for (int j=0; j<k; j++){
34                 auto x = (double *)&rt[j+k];
35                 auto y = (double *)&a[i+j+k];
36                 cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
37                 a[i+j+k] = a[i+j]-z;
38                 a[i+j] += z;
39             }
40         }
41     }
42     return;
43 }
44
45 // d3c65e
46 vector<int> PolyMul(vector<int> a, vector<int> b){
47     if (a.empty() || b.empty()) return {};
48
49     vector<int> res(a.size()+b.size()-1);
50     int B = 32-__builtin_clz(res.size()), n = (1<<B), cut =
51         int(sqrt(MOD));
52     vector<cd> L(n), R(n), outs(n), outl(n);
53
54     for (int i=0; i<a.size(); i++){
55         L[i] = cd((int) a[i]/cut, (int)a[i]%cut);
56     }
57     for (int i=0; i<b.size(); i++){
58         R[i] = cd((int) b[i]/cut, (int)b[i]%cut);
59     }
60     FFT(L);
61     FFT(R);
62     for (int i=0; i<n; i++){
63         int j = -i&(n-1);
64         outl[j] = (L[i]+conj(L[j])) * R[i]/(2.0*n);
65         outs[j] = (L[i]-conj(L[j])) * R[i]/(2.0*n)/1i;
66     }
67     FFT(outl);
68     FFT(outs);
69     for (int i=0; i<res.size(); i++){
70         int av = (int)(real(outl[i])+0.5), cv = (int)(imag(
71             outs[i])+0.5);

```

2.2 FFT new

```

1 typedef complex<double> cd;
2
3 void FFT(vector<cd> &a) {
4     int n = a.size(), L = 31-__builtin_clz(n);
5     vector<complex<long double>> R(2, 1);
6     vector<cd> rt(2, 1);
7     for (int k=2; k<n; k*=2){
8         R.resize(n);
9         rt.resize(n);
10        auto x = polar(1.0L, acos(-1.0L) / k);
11        for (int i=k; i<2*k; i++){
12            rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
13        }
14    }
15
16    vector<int> rev(n);
17    for (int i=0; i<n; i++){
18        rev[i] = (rev[i/2] | (i&1)<<L)/2;
19    }
20    for (int i=0; i<n; i++){
21        if (i<rev[i]) swap(a[i], a[rev[i]]);
22    }
23    for (int k=1; k<n; k*=2){
24        for (int i=0; i<n; i+=2*k){
25            for (int j=0; j<k; j++){
26                auto x = (double *)&rt[j+k];
27                auto y = (double *)&a[i+j+k];
28                cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
29                a[i+j+k] = a[i+j]-z;
30                a[i+j] += z;
31            }
32        }
33    }
34    return;
35 }
36
37 vector<double> PolyMul(const vector<double> a, const vector<
38     double> b){
39     if (a.empty() || b.empty()) return {};
40     vector<double> res(a.size()+b.size()-1);
41     int L = 32 - __builtin_clz(res.size()), n = 1 << L;
42     vector<cd> in(n), out(n);
43
44     copy(a.begin(), a.end(), begin(in));
45     for (int i=0; i<b.size(); i++){
46         in[i].imag(b[i]);
47     }
48     FFT(in);
49     for (cd& x : in) x *= x;
50     for (int i=0; i<n; i++){
51         out[i] = in[-i & (n - 1)] - conj(in[i]);

```

```

52     FFT(out);
53
54     for (int i=0; i<res.size(); i++){
55         res[i] = imag(out[i]) / (4 * n);
56     }
57
58     return res;
59 }

```

2.3 FFT old

```

1 typedef complex<double> cd;
2 const double PI = acos(-1);
3
4 void FFT(vector<cd> &a, bool inv){
5
6     int n = a.size();
7
8     for (int i=1, j=0; i<n; i++){
9         int bit = (n>>1);
10        for (; j<bit; bit>=1){
11            j ^= bit;
12        }
13        j ^= bit;
14        if (i<j){
15            swap(a[i], a[j]);
16        }
17    }
18
19    for (int len=2; len<=n; len<=1){
20        cd wlen = polar(1.0, (inv ? 2 : -2)*PI/len);
21
22        for (int i=0; i<n; i+=len){
23            cd w(1);
24            for (int j=0; j<len/2; j++){
25                cd u = a[i+j];
26                cd v = a[i+j+len/2]*w;
27                a[i+j] = u+v;
28                a[i+j+len/2] = u-v;
29                w *= wlen;
30            }
31        }
32    }
33
34    if (inv){
35        for (auto &x : a){
36            x /= n;
37        }
38    }
39
40    return;
41 }
42
43 vector<cd> polyMul(vector<cd> a, vector<cd> b){
44     int sa = a.size(), sb = b.size(), n = 1;
45
46     while (n<sa+sb-1) n *= 2;
47     a.resize(n);
48     b.resize(n);
49     vector<cd> c(n);
50
51     FFT(a, 0);
52     FFT(b, 0);

```

```

53 for (int i=0 ; i<n ; i++) c[i] = a[i]*b[i];
54 FFT(c, 1);
55
56 c.resize(sa+sb-1);
57
58 return c;
59 }

```

2.4 NTT mod 998244353

```

1 const int MOD = (119 << 23) + 1, ROOT = 62; // = 998244353
2 // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
3 // 21
4 // and 483 << 21 (same root). The last two are > 10^9.
5 // 9cd58a
6 void NTT(vector<int> &a) {
7     int n = a.size();
8     int L = 31-__builtin_clz(n);
9     vector<int> rt(2, 1);
10    for (int k=2, s=2 ; k<n ; k*=2, s++){
11        rt.resize(n);
12        int z[] = {1, qp(ROOT, MOD>>s)};
13        for (int i=k ; i<2*k ; i++){
14            rt[i] = rt[i/2]*z[i&1]%MOD;
15        }
16    }
17
18    vector<int> rev(n);
19    for (int i=0 ; i<n ; i++){
20        rev[i] = (rev[i/2]|(i&1)<<L)/2;
21    }
22    for (int i=0 ; i<n ; i++){
23        if (i<rev[i]){
24            swap(a[i], a[rev[i]]);
25        }
26    }
27
28    for (int k=1 ; k<n ; k*=2){
29        for (int i=0 ; i<n ; i+=2*k){
30            for (int j=0 ; j<k ; j++){
31                int z = rt[j+k]*a[i+j+k]%MOD, &ai = a[i+j];
32                a[i+j+k] = ai-z+(z>ai ? MOD : 0);
33                ai += (ai+z)>MOD ? z-MOD : z;
34            }
35        }
36    }
37 }
38
39 // 0b0e99
40 vector<int> polyMul(vector<int> &a, vector<int> &b){
41     if (a.empty() || b.empty()) return {};
42     int s = a.size()+b.size()-1, B = 32-__builtin_clz(s), n =
43         1<<B;
44     int inv = qp(n, MOD-2);
45
46     vector<int> L(a), R(b), out(n);
47     L.resize(n), R.resize(n);
48     NTT(L), NTT(R);
49     for (int i=0 ; i<n ; i++){
50         out[-i&(n-1)] = L[i]*R[i]%MOD*inv%MOD;
51     }
52     NTT(out);

```

```

52
53 out.resize(s);
54 return out;
55 }

```

3 Data-Structure

3.1 BIT

```

1 vector<int> BIT(MAX_SIZE);
2 void update(int pos, int val){
3     for (int i=pos ; i<MAX_SIZE ; i+=i&-i){
4         BIT[i]+=val;
5     }
6 }
7
8 int query(int pos){
9     int ret=0;
10    for (int i=pos ; i>0 ; i-=i&-i){
11        ret+=BIT[i];
12    }
13    return ret;
14 }
15
16 // const int MAX_N = (1<<20)
17 int k_th(int k){ // 回傳 BIT 中第 k 小的元素 (based-1)
18     int res = 0;
19     for (int i=MAX_N>>1 ; i>=1 ; i>=>1)
20         if (bit[res+i]<k)
21             k -= bit[res+i];
22     return res+1;
23 }

```

3.2 Disjoint Set Persistent

```

1 struct Persistent_Disjoint_Set{
2     Persistent_Segment_Tree arr, sz;
3
4     void init(int n){
5         arr.init(n);
6         vector<int> v1;
7         for (int i=0 ; i<n ; i++){
8             v1.push_back(i);
9         }
10        arr.build(v1, 0);
11
12        sz.init(n);
13        vector<int> v2;
14        for (int i=0 ; i<n ; i++){
15            v2.push_back(1);
16        }
17        sz.build(v2, 0);
18    }
19
20    int find(int a){
21        int res = arr.query_version(a, a+1, arr.version.size()
22            (-1).val;
23        if (res==a) return a;

```

```

23         return find(res);
24     }
25
26     bool unite(int a, int b){
27         a = find(a);
28         b = find(b);
29
30         if (a!=b){
31
32             int sz1 = sz.query_version(a, a+1, arr.version.
33                 size()-1).val;
34             int sz2 = sz.query_version(b, b+1, arr.version.
35                 size()-1).val;
36
37             if (sz1<sz2){
38                 arr.update_version(a, b, arr.version.size()
39                     -1);
40                 sz.update_version(b, sz1+sz2, arr.version.
41                     size()-1);
42             }else{
43                 arr.update_version(b, a, arr.version.size()
44                     -1);
45                 sz.update_version(a, sz1+sz2, arr.version.
46                     size()-1);
47             }
48             return true;
49         }
50         return false;
51     }
52 }

```

3.3 PBDS GP Hash Table

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 typedef tree<int, null_type, less<int>, rb_tree_tag,
4     tree_order_statistics_node_update> order_set;
5 struct custom_hash {
6     static uint64_t splitmix64(uint64_t x) {
7         // http://xorshift.di.unimi.it/splitmix64.c
8         x += 0x9e3779b97f4a7c15;
9         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
10        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
11        return x ^ (x >> 31);
12    }
13
14    size_t operator()(uint64_t x) const {
15        static const uint64_t FIXED_RANDOM = chrono::
16            steady_clock::now().time_since_epoch().count();
17        return splitmix64(x + FIXED_RANDOM);
18    }
19 };
20 gp_hash_table<int, int, custom_hash> ss;

```

3.4 PBDS Order Set

```

1 /*
2 .find_by_order(k) 回傳第 k 小的值 (based-0)

```

```

3 | .order_of_key(k) 回傳有多少元素比 k 小
4 | 不能在 #define int Long Long 後 #include 檔案
5 | */
6 |
7 | #include <ext/pb_ds/assoc_container.hpp>
8 | #include <ext/pb_ds/tree_policy.hpp>
9 | using namespace __gnu_pbds;
10 | typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> order_set;

```

3.5 Segment Tree Add Set

```

1 | // [ll, rr), based-0
2 | // 使用前記得 init(陣列大小), build(陣列名稱)
3 | // add(ll, rr): 區間修改
4 | // set(ll, rr): 區間賦值
5 | // query(ll, rr): 區間求和 / 求最大值
6 | struct SegmentTree{
7 |     struct node{
8 |         int add_tag = 0;
9 |         int set_tag = 0;
10 |        int sum = 0;
11 |        int ma = 0;
12 |    };
13 |
14 |    vector<node> arr;
15 |
16 |    SegmentTree(int n){
17 |        arr.resize(n<<2);
18 |    }
19 |
20 |    node pull(node A, node B){
21 |        node C;
22 |        C.sum = A.sum+B.sum;
23 |        C.ma = max(A.ma, B.ma);
24 |        return C;
25 |    }
26 |
27 |    // cce0c8
28 |    void push(int idx, int ll, int rr){
29 |        if (arr[idx].set_tag!=0){
30 |            arr[idx].sum = (rr-ll)*arr[idx].set_tag;
31 |            arr[idx].ma = arr[idx].set_tag;
32 |            if (rr-ll>1){
33 |                arr[idx*2+1].add_tag = 0;
34 |                arr[idx*2+1].set_tag = arr[idx].set_tag;
35 |                arr[idx*2+2].add_tag = 0;
36 |                arr[idx*2+2].set_tag = arr[idx].set_tag;
37 |            }
38 |            arr[idx].set_tag = 0;
39 |        }
40 |        if (arr[idx].add_tag!=0){
41 |            arr[idx].sum += (rr-ll)*arr[idx].add_tag;
42 |            arr[idx].ma += arr[idx].add_tag;
43 |            if (rr-ll>1){
44 |                arr[idx*2+1].add_tag += arr[idx].add_tag;
45 |                arr[idx*2+2].add_tag += arr[idx].add_tag;
46 |            }
47 |            arr[idx].add_tag = 0;
48 |        }
49 |    }
50 | }

```

```

51 | void build(vector<int> &v, int idx = 0, int ll = 0, int
    rr = n){
52 |     if (rr-ll==1){
53 |         arr[idx].sum = v[ll];
54 |         arr[idx].ma = v[ll];
55 |     }else{
56 |         int mid = (ll+rr)/2;
57 |         build(v, idx*2+1, ll, mid);
58 |         build(v, idx*2+2, mid, rr);
59 |         arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
60 |     }
61 | }
62 |
63 | void add(int ql, int qr, int val, int idx = 0, int ll =
    0, int rr = n){
64 |     push(idx, ll, rr);
65 |     if (rr<=ql || qr<=ll) return;
66 |     if (ql<=ll && rr<=qr){
67 |         arr[idx].add_tag += val;
68 |         push(idx, ll, rr);
69 |         return;
70 |     }
71 |     int mid = (ll+rr)/2;
72 |     add(ql, qr, val, idx*2+1, ll, mid);
73 |     add(ql, qr, val, idx*2+2, mid, rr);
74 |     arr[idx]=pull(arr[idx*2+1], arr[idx*2+2]);
75 | }
76 |
77 | void set(int ql, int qr, int val, int idx=0, int ll=0,
    int rr=n){
78 |     push(idx, ll, rr);
79 |     if (rr<=ql || qr<=ll) return;
80 |     if (ql<=ll && rr<=qr){
81 |         arr[idx].add_tag = 0;
82 |         arr[idx].set_tag = val;
83 |         push(idx, ll, rr);
84 |         return;
85 |     }
86 |     int mid = (ll+rr)/2;
87 |     set(ql, qr, val, idx*2+1, ll, mid);
88 |     set(ql, qr, val, idx*2+2, mid, rr);
89 |     arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
90 | }
91 |
92 | node query(int ql, int qr, int idx = 0, int ll = 0, int
    rr = n){
93 |     push(idx, ll, rr);
94 |     if (rr<=ql || qr<=ll) return node();
95 |     if (ql<=ll && rr<=qr) return arr[idx];
96 |
97 |     int mid = (ll+rr)/2;
98 |     return pull(query(ql, qr, idx*2+1, ll, mid), query(ql
    , qr, idx*2+2, mid, rr));
99 | }
100 | } ST;

```

3.6 Segment Tree Li Chao

```

1 | /*
2 | 全部都是 0-based
3 |
4 | 宣告

```

```

5 | LC_Segment_Tree st(n);
6 |
7 | 函式：
8 | update(val)：將一個 pair <a, b> 代表插入一條 y=ax+b 的直線
9 | query(x)：查詢所有直線在位置 x 的最小值
10 | */
11 | const int MAX_V = 1e6+10; // 值域最大值
12 |
13 | struct LC_Segment_Tree{
14 |     struct Node{ // y = ax+b
15 |         int a = 0;
16 |         int b = INF;
17 |
18 |         int y(int x){
19 |             return a*x+b;
20 |         }
21 |     };
22 |     vector<Node> arr;
23 |
24 |     LC_Segment_Tree(int n = 0){
25 |         arr.resize(4*n);
26 |     }
27 |
28 |     void update(Node val, int idx = 0, int ll = 0, int rr =
        MAX_V){
29 |         if (rr-ll==1){
30 |             if (val.y(ll)<arr[idx].y(ll)){
31 |                 arr[idx] = val;
32 |             }
33 |             return;
34 |         }
35 |
36 |         int mid = (ll+rr)/2;
37 |         if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
            的線斜率要比較小
38 |         if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
39 |             update(val, idx*2+1, ll, mid);
40 |         }else{ // 交點在右邊
41 |             swap(arr[idx], val); // 在左子樹中，新線比舊線還
                要好
42 |             update(val, idx*2+2, mid, rr);
43 |         }
44 |         return;
45 |     }
46 |
47 |     int query(int x, int idx = 0, int ll = 0, int rr = MAX_V)
        {
48 |         if (rr-ll==1){
49 |             return arr[idx].y(ll);
50 |         }
51 |
52 |         int mid = (ll+rr)/2;
53 |         if (x<mid){
54 |             return min(arr[idx].y(x), query(x, idx*2+1, ll,
                mid));
55 |         }else{
56 |             return min(arr[idx].y(x), query(x, idx*2+2, mid,
                rr));
57 |         }
58 |     }
59 | };

```

3.7 Segment Tree Persistent

```

1  /*
2  全部都是 0-based
3
4  宣告
5  Persistent_Segment_Tree st(n+q);
6  st.build(v, 0);
7
8  函式：
9  update_version(pos, val, ver) : 對版本 ver 的 pos 位置改成 val
10 query_version(ql, qr, ver) : 對版本 ver 查詢 [ql, qr) 的區間和
11 clone_version(ver) : 複製版本 ver 到最新的版本
12 */
13 struct Persistent_Segment_Tree{
14     int node_cnt = 0;
15     struct Node{
16         int lc = -1;
17         int rc = -1;
18         int val = 0;
19     };
20     vector<Node> arr;
21     vector<int> version;
22
23     Persistent_Segment_Tree(int sz){
24         arr.resize(32*sz);
25         version.push_back(node_cnt++);
26         return;
27     }
28
29     void pull(Node &c, Node a, Node b){
30         c.val = a.val+b.val;
31         return;
32     }
33
34     void build(vector<int> &v, int idx, int ll = 0, int rr =
35         n){
36         auto &now = arr[idx];
37
38         if (rr-ll==1){
39             now.val = v[ll];
40             return;
41         }
42
43         int mid = (ll+rr)/2;
44         now.lc = node_cnt++;
45         now.rc = node_cnt++;
46         build(v, now.lc, ll, mid);
47         build(v, now.rc, mid, rr);
48         pull(now, arr[now.lc], arr[now.rc]);
49         return;
50     }
51
52     void update(int pos, int val, int idx, int ll = 0, int rr
53         = n){
54         auto &now = arr[idx];
55
56         if (rr-ll==1){
57             now.val = val;
58             return;
59         }
60
61         int mid = (ll+rr)/2;
62         if (pos<mid){

```

```

61         arr[node_cnt] = arr[now.lc];
62         now.lc = node_cnt;
63         node_cnt++;
64         update(pos, val, now.lc, ll, mid);
65     }else{
66         arr[node_cnt] = arr[now.rc];
67         now.rc = node_cnt;
68         node_cnt++;
69         update(pos, val, now.rc, mid, rr);
70     }
71     pull(now, arr[now.lc], arr[now.rc]);
72     return;
73 }
74
75 void update_version(int pos, int val, int ver){
76     update(pos, val, version[ver]);
77 }
78
79 Node query(int ql, int qr, int idx, int ll = 0, int rr =
80     n){
81     auto &now = arr[idx];
82
83     if (ql<=ll && rr<=qr) return now;
84     if (rr<=ql || qr<=ll) return Node();
85
86     int mid = (ll+rr)/2;
87
88     Node ret;
89     pull(ret, query(ql, qr, now.lc, ll, mid), query(ql,
90         qr, now.rc, mid, rr));
91     return ret;
92 }
93
94 Node query_version(int ql, int qr, int ver){
95     return query(ql, qr, version[ver]);
96 }
97
98 void clone_version(int ver){
99     version.push_back(node_cnt);
100     arr[node_cnt] = arr[version[ver]];
101     node_cnt++;
102 }
103
104 };

```

3.8 Sparse Table

```

1 struct SparseTable{
2     vector<vector<int>> st;
3     void build(vector<int> v){
4         int h = __lg(v.size());
5         st.resize(h+1);
6         st[0] = v;
7
8         for (int i=1 ; i<=h ; i++){
9             int gap = (1<<(i-1));
10             for (int j=0 ; j+gap<st[i-1].size() ; j++){
11                 st[i].push_back(min(st[i-1][j], st[i-1][j+gap
12                     ]));
13             }
14         }
15     }
16
17     // 回傳 [ll, rr) 的最小值

```

```

17     int query(int ll, int rr){
18         int h = __lg(rr-ll);
19         return min(st[h][ll], st[h][rr-(1<<h)]);
20     }
21 };

```

3.9 Treap

```

1 struct Treap{
2     Treap *l = nullptr, *r = nullptr;
3     int pri = rand(), val = 0, sz = 1;
4
5     Treap(int _val){
6         val = _val;
7     }
8 };
9
10 int size(Treap *t){return t ? t->sz : 0;}
11 void pull(Treap *t){
12     t->sz = size(t->l)+size(t->r)+1;
13 }
14
15 Treap* merge(Treap *a, Treap *b){
16     if (!a || !b) return a ? a : b;
17
18     if (a->pri>b->pri){
19         a->r = merge(a->r, b);
20         pull(a);
21         return a;
22     }else{
23         b->l = merge(a, b->l);
24         pull(b);
25         return b;
26     }
27 }
28
29 pair<Treap*, Treap*> split(Treap *&t, int k){ // 1-based <前
30     k 個元素, 其他元素>
31     if (!t) return {};
32     if (size(t->l)>=k){
33         auto pa = split(t->l, k);
34         t->l = pa.second;
35         pull(t);
36         return {pa.first, t};
37     }else{
38         auto pa = split(t->r, k-size(t->l)-1);
39         t->r = pa.first;
40         pull(t);
41         return {t, pa.second};
42     }
43 }
44
45 // functions
46 Treap* build(vector<int> v){
47     Treap* ret;
48     for (int i=0 ; i<SZ(v) ; i++){
49         ret = merge(ret, new Treap(v[i]));
50     }
51     return ret;
52 }
53

```



```

54 array<Treap*, 3> cut(Treap *t, int l, int r){ // 1-based <前
    1~l-1 個元素, l~r 個元素, r+1 個元素>
55     array<Treap*, 3> ret;
56     tie(ret[1], ret[2]) = split(t, r);
57     tie(ret[0], ret[1]) = split(ret[1], l-1);
58     return ret;
59 }
60
61 void print(Treap *t, bool flag = true){
62     if (t->l!=0) print(t->l, false);
63     cout << t->val;
64     if (t->r!=0) print(t->r, false);
65     if (flag) cout << endl;
66 }

```

3.10 Trie

```

1 struct Trie{
2     struct Data{
3         int nxt[2]={0, 0};
4     };
5
6     int sz=0;
7     vector<Data> arr;
8
9     void init(int n){
10         arr.resize(n);
11     }
12
13     void insert(int n){
14         int now=0;
15         for (int i=N; i>=0; i--){
16             int v=(n>>i)&1;
17             if (!arr[now].nxt[v]){
18                 arr[now].nxt[v]=++sz;
19             }
20             now=arr[now].nxt[v];
21         }
22     }
23
24     int query(int n){
25         int now=0, ret=0;
26         for (int i=N; i>=0; i--){
27             int v=(n>>i)&1;
28             if (arr[now].nxt[1-v]){
29                 ret+=(1<<i);
30                 now=arr[now].nxt[1-v];
31             }else if (arr[now].nxt[v]){
32                 now=arr[now].nxt[v];
33             }else{
34                 return ret;
35             }
36         }
37         return ret;
38     }
39 } tr;

```

4 Dynamic-Programming

4.1 Digit DP

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 long long l, r;
5 long long dp[20][10][2][2]; // dp[pos][pre][limit] = 後 pos
    位 · pos 前一位是 pre · (是/否) 有上界 · (是/否) 有前綴零
    的答案數量
6
7 long long memorize_search(string &s, int pos, int pre, bool
    limit, bool lead){
8
9     // 已經被找過了 · 直接回傳值
10    if (dp[pos][pre][limit][lead]!=-1) return dp[pos][pre][
    limit][lead];
11
12    // 已經搜尋完畢 · 紀錄答案並回傳
13    if (pos==(int)s.size()){
14        return dp[pos][pre][limit][lead] = 1;
15    }
16
17    // 枚舉目前的位數數字是多少
18    long long ans = 0;
19    for (int now=0; now<=(limit ? s[pos]-'0' : 9); now++){
20        if (now==pre){
21
22            // 1~9 絕對不能連續出現
23            if (pre!=0) continue;
24
25            // 如果已經不在前綴零的範圍內 · 0 不能連續出現
26            if (lead==false) continue;
27        }
28
29        ans += memorize_search(s, pos+1, now, limit&(now==(s[
    pos]-'0')), lead&(now==0));
30    }
31
32    // 已經搜尋完畢 · 紀錄答案並回傳
33    return dp[pos][pre][limit][lead] = ans;
34 }
35
36 // 回傳 [0, n] 有多少數字符合條件
37 long long find_answer(long long n){
38     memset(dp, -1, sizeof(dp));
39     string tmp = to_string(n);
40
41     return memorize_search(tmp, 0, 0, true, true);
42 }
43
44 int main(){
45
46     // input
47     cin >> l >> r;
48
49     // output - 計算 [l, r] 有多少數字任意兩個位數都不相同
50     cout << find_answer(r)-find_answer(l-1) << "\n";
51
52     return 0;
53 }

```

4.2 SOS DP

```

1 // 總時間複雜度為 O(n 2^n)
2 // 計算 dp[i] = i 所有 bit mask 子集的和
3 for (int i=0; i<n; i++){
4     for (int mask=0; mask<(1<<n); mask++){
5         if ((mask>>i)&1){
6             dp[mask] += dp[mask^(1<<i)];
7         }
8     }
9 }

```

4.3 Integer Partition

$dp[i][x]$ = 要將整數 x 拆成 i 堆的「組合數」

$dp[i+1][x+1] += dp[i][x]$ (創造新的一堆)
 $dp[i][x+i] += dp[i][x]$ (把每一堆都增加 1)

5 Geometry

5.1 Geometry Struct

```

1 // 判斷數值正負：{1:正數,0:零,-1:負數}
2 int sign(long long x) {return (x >= 0) ? ((bool)x) : -1; }
3 int sign(double x) {
4     return (abs(x) < 1e-9) ? 0 : (x > 0 ? 1 : -1);
5 }
6
7 template<typename T>
8 struct point {
9     T x, y;
10    point() {}
11    point(const T &x, const T &y) : x(x), y(y) {}
12
13    point operator+(point b) {return {x+b.x, y+b.y}; }
14    point operator-(point b) {return {x-b.x, y-b.y}; }
15    point operator*(T b) {return {x*b, y*b}; }
16    point operator/(T b) {return {x/b, y/b}; }
17    bool operator==(point b) {return x==b.x && y==b.y; }
18    // 逆時針極角排序
19    bool operator<(point &b) {return (x*b.y > b.x*y); }
20    friend ostream& operator<<(ostream& os, point p) {
21        os << "(" << p.x << ", " << p.y << ")";
22        return os;
23    }
24
25    // 判斷 ab 到 ac 的方向：{1:逆時鐘,0:重疊,-1:順時鐘}
26    friend int ori(point a, point b, point c) {
27        return sign((b-a)^(c-a));
28    }
29
30    friend int btw(point a, point b, point c) {
31        return ori(a, b, c) == 0 && sign((a-c)*(b-c)) <= 0;
32    }
33
34    // 判斷線段 ab, cd 是否相交
35    friend bool banana(point a, point b, point c, point d) {
36        int s1 = ori(a, b, c);
37        int s2 = ori(a, b, d);
38        int s3 = ori(c, d, a);

```



```

36     int s4 = ori(c, d, b);
37     if (btw(a, b, c) || btw(a, b, d) || btw(c, d, a) ||
        btw(c, d, b)) return 1;
38     return (s1 * s2 < 0) && (s3 * s4 < 0);
39 }
40
41 T operator*(point b) {return x * b.x + y * b.y; }
42 T operator^(point b) {return x * b.y - y * b.x; }
43 T abs2() {return (*this) * (*this); }
44
45 // 旋轉 Arg(b) 的角度 (小心溢位)
46 point rotate(point b) {return {x*b.x - y*b.y, x*b.y + y*b
    .x}; }
47 };
48
49 template<typename T>
50 struct line {
51     point<T> p1, p2;
52     // ax + by + c = 0
53     T a, b, c; // |a|, |b| ≤ 2C, |c| ≤ 8C²
54     line() {}
55     line(const point<T> &x, const point<T> &y) : p1(x), p2(y)
        {
56         build();
57     }
58     void build() {
59         a = p1.y - p2.y;
60         b = p2.x - p1.x;
61         c = (-a*p1.x) - b*p1.y;
62     }
63     // 判斷點和有向直線的關係：{1:左邊,0:在線上,-1:右邊}
64     int ori(point<T> &p) {
65         return sign((p2-p1) ^ (p-p1));
66     }
67     // 判斷直線斜率是否相同
68     bool parallel(line &l) {
69         return ((p1-p2) ^ (l.p1-l.p2)) == 0;
70     }
71     // 兩直線交點
72     point<long double> line_intersection(line &l) {
73         using P = point<long double>;
74         point<T> a = p2-p1, b = l.p2-l.p1, s = l.p1-p1;
75         return P(p1.x, p1.y) + P(a.x, a.y) * (((long double)(s^b)
            ) / (a^b));
76     }
77 };
78
79 template<typename T>
80 struct polygon {
81     vector<point<T>> v;
82     polygon() {}
83     polygon(const vector<point<T>> &u) : v(u) {}
84     // simple 為 true 的時候會回傳任意三點不共線的凸包
85     void make_convex_hull(int simple) {
86         auto cmp = [&](point<T> &p, point<T> &q) {
87             return (p.x == q.x) ? (p.y < q.y) : (p.x < q.x);
88         };
89         simple = (bool)simple;
90         sort(v.begin(), v.end(), cmp);
91         v.resize(unique(v.begin(), v.end()) - v.begin());
92         vector<point<T>> hull;
93         for (int t = 0; t < 2; ++t) {
94             int sz = hull.size();
95             for (auto &i:v) {

```

```

96         while (hull.size() >= sz+2 && ori(hull[hull.
            size()-2], hull.back(), i) < simple) {
97             hull.pop_back();
98         }
99         hull.push_back(i);
100     }
101     hull.pop_back();
102     reverse(v.begin(), v.end());
103 }
104 swap(hull, v);
105 }
106 // 可以在有 n 個點的簡單多邊形內·用 O(n) 的時間回傳：
107 // {1 : 在多邊形內, 0 : 在多邊形上, -1 : 在多邊形外}
108 int in_polygon(point<T> a) {
109     const T MAX_POS = (1e9 + 5); // [記得修改] 座標的最大
        值
110     point<T> pre = v.back(), b(MAX_POS, a.y + 1);
111     int cnt = 0;
112
113     for (auto &i:v) {
114         if (btw(pre, i, a)) return 0;
115         if (banana(a, b, pre, i)) cnt++;
116         pre = i;
117     }
118
119     return cnt%2 ? 1 : -1;
120 }
121 // 凸包專用的環狀二分搜·回傳 0-based index
122 int cycle_search(auto f, int tar) {
123     /// TO DO
124 }
125 // 可以在有 n 個點的凸包內·用 O(Log n) 的時間回傳：
126 // {1 : 在凸包內, 0 : 在凸包邊上, -1 : 在凸包外}
127 int in_convex(point<T> p) {
128     /// TO DO
129 }
130 // 可以在有 n 個點的凸包內·用 O(Log n) 的時間回傳：
131 // {1 : 穿過凸包, 0 : 剛好切過凸包, -1 : 沒碰到凸包}
132 int line_cut_convex(line<T> p) {
133     /// TO DO
134 }
135 int segment_cut_convex(line<T> p) {
136     /// TO DO
137 }
138 // 回傳點過凸包的兩條切線的切點 index
139 pair<int,int> point_tangent(point<T> p) {
140     /// TO DO
141     // 注意特判：戳到凸包頂點的 case
142 }
143
144 friend int halfplane_intersection(vector<line<T>> &s,
    polygon<T> &P) {
145     #define neg(p) ((p.y == 0 ? p.x : p.y) < 0)
146     auto angle_cmp = [&](line<T> &A, line<T> &B) {
147         point<T> a = A.p2-A.p1, b = B.p2-B.p1;
148         return neg(a) < neg(b) || (neg(a) == neg(b) && (a
            ^b) > 0);
149     };
150     #undef neg
151     sort(s.begin(), s.end(), angle_cmp); // 線段左側為該
        線段半平面
152     int L, R, n = s.size();
153     vector<point<T>> px(n);
154     vector<line<T>> q(n);
155     q[L = R = 0] = s[0];

```

```

156     for(int i = 1; i < n; ++i) {
157         while(L < R && s[i].ori(px[R-1]) <= 0) --R;
158         while(L < R && s[i].ori(px[L]) <= 0) ++L;
159         q[++R] = s[i];
160         if(q[R].parallel(q[R-1])) {
161             --R;
162             if(q[R].ori(s[i].p1) > 0) q[R] = s[i];
163         }
164         if(L < R) px[R-1] = q[R-1].line_intersection(q[R
            ]);
165     }
166     while(L < R && q[L].ori(px[R-1]) <= 0) --R;
167     P.v.clear();
168     if(R - L <= 1) return 0;
169     px[R] = q[R].line_intersection(q[L]);
170     for(int i = L; i <= R; ++i) P.v.push_back(px[i]);
171     return R - L + 1;
172 }
173 /// TO DO : .svg maker

```

5.2 Geometry 卦長

```

1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
3 struct point{
4     T x,y;
5     point(){}
6     point(const T&x,const T&y):x(x),y(y){}
7     point operator+(const point &b)const{
8         return point(x+b.x,y+b.y); }
9     point operator-(const point &b)const{
10        return point(x-b.x,y-b.y); }
11     point operator*(const T &b)const{
12        return point(x*b,y*b); }
13     point operator/(const T &b)const{
14        return point(x/b,y/b); }
15     bool operator==(const point &b)const{
16        return x==b.x&&y==b.y; }
17     T dot(const point &b)const{
18        return x*b.x+y*b.y; }
19     T cross(const point &b)const{
20        return x*b.y-y*b.x; }
21     point normal()const{//求法向量
22        return point(-y,x); }
23     T abs2()const{//向量長度的平方
24        return dot(*this); }
25     T rad(const point &b)const{//兩向量的弧度
26     return fabs(atan2(fabs(cross(b)),dot(b))); }
27     T getA()const{//對x軸的弧度
28         T A=atan2(y,x);//超過180度會變負的
29         if(A<=-PI/2)A+=PI*2;
30         return A;
31     }
32 };
33 template<typename T>
34 struct line{
35     line(){}
36     point<T> p1,p2;
37     T a,b,c;//ax+by+c=0
38     line(const point<T>&x,const point<T>&y):p1(x),p2(y){}
39     void pton()//轉成一般式

```

```

40  a=p1.y-p2.y;
41  b=p2.x-p1.x;
42  c=-a*p1.x-b*p1.y;
43  }
44  T ori(const point<T> &p) const { // 點和有向直線的關係 · >0 左
    邊、=0 在線上 <0 右邊
    return (p2-p1).cross(p-p1);
45  }
46  T btw(const point<T> &p) const { // 點投影落在線段上 <=0
    return (p1-p).dot(p2-p);
47  }
48  bool point_on_segment(const point<T> &p) const { // 點是否在線段
    上
    return ori(p)==0 && btw(p)<=0;
49  }
50  T dis2(const point<T> &p, bool is_segment=0) const { // 點跟直線
    /線段的距離平方
    point<T> v=p2-p1, v1=p-p1;
    if(is_segment){
51      point<T> v2=p-p2;
52      if(v.dot(v1)<=0) return v1.abs2();
53      if(v.dot(v2)>=0) return v2.abs2();
54      }
55      T tmp=v.cross(v1);
56      return tmp*tmp/v.abs2();
57  }
58  T seg_dis2(const line<T> &l) const { // 兩線段距離平方
    return min({dis2(l.p1,1), dis2(l.p2,1), l.dis2(p1,1), l.dis2(p2,1)});
59  }
60  point<T> projection(const point<T> &p) const { // 點對直線的投
    影
    point<T> n=(p2-p1).normal();
    return p-n*(p-p1).dot(n)/n.abs2();
61  }
62  point<T> mirror(const point<T> &p) const {
    // 點對直線的鏡射 · 要先呼叫 pton 轉成一般式
    point<T> R;
    T d=a*b+b*c;
    R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
    R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
    return R;
63  }
64  bool equal(const line &l) const { // 直線相等
    return ori(l.p1)==0 && ori(l.p2)==0;
65  }
66  bool parallel(const line &l) const {
    return (p1-p2).cross(l.p1-l.p2)==0;
67  }
68  bool cross_seg(const line &l) const {
    return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;
    // 直線是否交線段
69  }
70  int line_intersect(const line &l) const { // 直線相交情況 · -1 無
    限多點、1 交於一點、0 不相交
    return parallel(l)?(ori(l.p1)==0?-1:0):1;
71  }
72  int seg_intersect(const line &l) const {
    T c1=ori(l.p1), c2=ori(l.p2);
    T c3=l.ori(p1), c4=l.ori(p2);
    if(c1==0 && c2==0) { // 共線
    bool b1=btw(l.p1)>=0, b2=btw(l.p2)>=0;
    T a3=l.btw(p1), a4=l.btw(p2);
73      if(b1 && b2 && a3==0 && a4>=0) return 2;
74      if(b1 && b2 && a3>=0 && a4==0) return 3;
75      if(b1 && b2 && a3>=0 && a4>=0) return 0;
76      return -1; // 無限交點
    } else if(c1*c2<=0 && c3*c4<=0) return 1;
    return 0; // 不相交
77  }
78  point<T> line_intersection(const line &l) const { // 直線交點
    point<T> a=p2-p1, b=l.p2-l.p1, s=l.p1-p1;
    // if(a.cross(b)==0) return INF;
    return p1+a*(s.cross(b)/a.cross(b));
79  }
80  point<T> seg_intersection(const line &l) const { // 線段交點
    int res=seg_intersect(l);
    if(res<=0) assert(0);
    if(res==2) return p1;
    if(res==3) return p2;
    return line_intersection(l);
81  }
82  };
83  template<typename T>
84  struct polygon {
    polygon() {}
    vector<point<T>> > p; // 逆時針順序
    T area() const { // 面積
    T ans=0;
    for(int i=p.size()-1, j=0; j<(int)p.size(); i=j++)
    ans+=p[i].cross(p[j]);
    return ans/2;
85  }
86  }
87  point<T> center_of_mass() const { // 重心
    T cx=0, cy=0, w=0;
    for(int i=p.size()-1, j=0; j<(int)p.size(); i=j++){
    T a=p[i].cross(p[j]);
    cx+=(p[i].x+p[j].x)*a;
    cy+=(p[i].y+p[j].y)*a;
    w+=a;
88  }
89  return point<T>(cx/3/w, cy/3/w);
90  }
91  char ahas(const point<T> &t) const { // 點是否在簡單多邊形內 ·
    是的話回傳 1、在邊上回傳 -1、否則回傳 0
    bool c=0;
    for(int i=0, j=p.size()-1; i<p.size(); j=i++){
    if(line<T>(p[i], p[j]).point_on_segment(t)) return -1;
    else if((p[i].y>t.y) != (p[j].y>t.y) &&
    t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j].y-p[i].y)+p[i].x)
    c=!c;
    return c;
92  }
93  char point_in_convex(const point<T> &x) const {
    int l=1, r=(int)p.size()-2;
    while(l<=r) { // 點是否在凸多邊形內 · 是的話回傳 1、在邊上回傳
    -1、否則回傳 0
    int mid=(l+r)/2;
    T a1=(p[mid]-p[0]).cross(x-p[0]);
    T a2=(p[mid+1]-p[0]).cross(x-p[0]);
    if(a1>=0 && a2<=0) {
    T res=(p[mid+1]-p[mid]).cross(x-p[mid]);
    return res>0?1:(res>=0?-1:0);
    } else if(a1<0) r=mid-1;
    else l=mid+1;
94  }
95  }
96  return 0;
97  }
98  vector<T> getA() const { // 凸包邊對 x 軸的夾角
    vector<T> res; // 一定是遞增的
    for(size_t i=0; i<p.size(); ++i)
    res.push_back((p[(i+1)%p.size()]-p[i]).getA());
    return res;
99  }
100  bool line_intersect(const vector<T> &A, const line<T> &l)
    const { // 0 (LogN)
    int f1=upper_bound(A.begin(), A.end(), (l.p1-l.p2).getA())-
    A.begin();
    int f2=upper_bound(A.begin(), A.end(), (l.p2-l.p1).getA())-
    A.begin();
    return l.cross_seg(line<T>(p[f1], p[f2]));
101  }
102  polygon cut(const line<T> &l) const { // 凸包對直線切割 · 得到直
    線 L 左側的凸包
    polygon ans;
    for(int n=p.size(), i=n-1, j=0; j<n; i=j++){
    if(l.ori(p[i])>=0) {
    ans.p.push_back(p[i]);
    if(l.ori(p[j])<0)
    ans.p.push_back(l.line_intersection(line<T>(p[i], p[j]),
    l));
    } else if(l.ori(p[j])>0)
    ans.p.push_back(l.line_intersection(line<T>(p[i], p[j]),
    l));
    }
    return ans;
103  }
104  static bool monotone_chain_cmp(const point<T> &a, const
    point<T> &b) { // 凸包排序函數
    return (a.x<b.x) || (a.x==b.x && a.y<b.y);
105  }
106  void monotone_chain(vector<point<T>> &s) { // 凸包
    sort(s.begin(), s.end(), monotone_chain_cmp);
    p.resize(s.size()+1);
    int m=0;
    for(size_t i=0; i<s.size(); ++i) {
    while(m>=2 && (p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0) --m;
    p[m++]=s[i];
    }
    for(int i=s.size()-2, t=m+1; i>=0; --i) {
    while(m>=t && (p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0) --m;
    p[m++]=s[i];
    }
    if(s.size()>1) --m;
    p.resize(m);
107  }
108  T diam() { // 直徑
    int n=p.size(), t=1;
    T ans=0; p.push_back(p[0]);
    for(int i=0; i<n; ++i) {
    point<T> now=p[i+1]-p[i];
    while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i])) t=(t
    +1)%n;
    ans=max(ans, (p[i]-p[t]).abs2());
    }
    return p.pop_back(), ans;
109  }
110  T min_cover_rectangle() { // 最小覆蓋矩形
    int n=p.size(), t=1, r=1, l;
    if(n<3) return 0; // 也可以做最小周長矩形
111  }

```

```

213 T ans=1e99;p.push_back(p[0]);
214 for(int i=0;i<n;i++){
215     point<T> now=p[i+1]-p[i];
216     while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t
        +1)%n;
217     while(now.dot(p[r+1]-p[i])>now.dot(p[r]-p[i]))r=(r+1)%n
        ;
218     if(!i)l=r;
219     while(now.dot(p[l+1]-p[i])<=now.dot(p[l]-p[i]))l=(l+1)%
        n;
220     T d=now.abs2();
221     T tmp=now.cross(p[t]-p[i])*(now.dot(p[r]-p[i])-now.dot(
        p[l]-p[i]))/d;
222     ans=min(ans,tmp);
223 }
224 return p.pop_back(),ans;
225 }
226 T dis2(polygon &p1){//凸包最近距離平方
227     vector<point<T>> > &P=p,&Q=p1.p;
228     int n=P.size(),m=Q.size(),l=0,r=0;
229     for(int i=0;i<n;++i)if(P[i].y<P[l].y)l=i;
230     for(int i=0;i<m;++i)if(Q[i].y<Q[r].y)r=i;
231     P.push_back(P[0]),Q.push_back(Q[0]);
232     T ans=1e99;
233     for(int i=0;i<n;++i){
234         while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])<0)r=(r+1)%m;
235         ans=min(ans,line<T>(P[l],P[l+1]).seg_dis2(line<T>(Q[r],
        Q[r+1])));
236         l=(l+1)%n;
237     }
238     return P.pop_back(),Q.pop_back(),ans;
239 }
240 static char sign(const point<T>&t){
241     return (t.y==0?t.x:t.y)<0;
242 }
243 static bool angle_cmp(const line<T>& A,const line<T>& B){
244     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
245     return sign(a)<sign(b)||((sign(a)==sign(b)&&a.cross(b)>0));
246 }
247 int halfplane_intersection(vector<line<T>> &s){//半平面交
248     sort(s.begin(),s.end(),angle_cmp);//線段左側為該線段半平
        面
249     int L,R,n=s.size();
250     vector<point<T>> > px(n);
251     vector<line<T>> > q(n);
252     q[L=R=0]=s[0];
253     for(int i=1;i<n;++i){
254         while(L<R&&s[i].ori(px[R-1])<=0)--R;
255         while(L<R&&s[i].ori(px[L])<=0)++L;
256         q[++R]=s[i];
257         if(q[R].parallel(q[R-1])){
258             --R;
259             if(q[R].ori(s[i].p1)>0)q[R]=s[i];
260         }
261         if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
262     }
263     while(L<R&&q[L].ori(px[R-1])<=0)--R;
264     p.clear();
265     if(R-L<=1)return 0;
266     px[R]=q[R].line_intersection(q[L]);
267     for(int i=L;i<R;++i)p.push_back(px[i]);
268     return R-L+1;
269 }
270 };
271 template<typename T>
272 struct triangle{
273     point<T> a,b,c;
274     triangle(){
275         triangle(const point<T> &a,const point<T> &b,const point<T>
        &c):a(a),b(b),c(c){
276             T area()const{
277                 T t=(b-a).cross(c-a)/2;
278                 return t>0?t:-t;
279             }
280             point<T> barycenter()const{//重心
281                 return (a+b+c)/3;
282             }
283             point<T> circumcenter()const{//外心
284                 static line<T> u,v;
285                 u.p1=(a+b)/2;
286                 u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
287                 v.p1=(a+c)/2;
288                 v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
289                 return u.line_intersection(v);
290             }
291             point<T> incenter()const{//內心
292                 T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).
        abs2());
293                 return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B
        +C);
294             }
295             point<T> perpcenter()const{//垂心
296                 return barycenter()*3-circumcenter()*2;
297             }
298 };
299 template<typename T>
300 struct point3D{
301     T x,y,z;
302     point3D(){
303         point3D(const T&x,const T&y,const T&z):x(x),y(y),z(z){}
304         point3D operator+(const point3D &b)const{
305             return point3D(x+b.x,y+b.y,z+b.z);
306         }
307         point3D operator-(const point3D &b)const{
308             return point3D(x-b.x,y-b.y,z-b.z);
309         }
310         point3D operator*(const T &b)const{
311             return point3D(x*b,y*b,z*b);
312         }
313         point3D operator/(const T &b)const{
314             return point3D(x/b,y/b,z/b);
315         }
316         bool operator==(const point3D &b)const{
317             return x==b.x&&y==b.y&&z==b.z;
318         }
319         T dot(const point3D &b)const{
320             return x*b.x+y*b.y+z*b.z;
321         }
322         point3D cross(const point3D &b)const{
323             return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
324         }
325         T abs2()const{//向量長度的平方
326             return dot(*this);
327         }
328         T area2(const point3D &b)const{//和b、原點圍成面積的平方
329             return cross(b).abs2()/4;
330         }
331 };
332 template<typename T>
333 struct line3D{
334     point3D<T> p1,p2;
335     line3D(){
336         line3D(const point3D<T> &p1,const point3D<T> &p2):p1(p1),p2
        (p2){}
337     T dis2(const point3D<T> &p,bool is_segment=0)const{//點跟直
        線/線段的距離平方
338     point3D<T> v=p2-p1,v1=p-p1;
339     if(is_segment){
340         point3D<T> v2=p-p2;
341         if(v.dot(v1)<=0)return v1.abs2();
342         if(v.dot(v2)>=0)return v2.abs2();
343     }
344     point3D<T> tmp=v.cross(v1);
345     return tmp.abs2()/v.abs2();
346 }
347 pair<point3D<T>,point3D<T>> > closest_pair(const line3D<T> &
        l)const{
348     point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
349     point3D<T> N=v1.cross(v2),ab(p1-l.p1);
350     //if(N.abs2()==0)return NULL;平行或重合
351     T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//最近點對距離
352     point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2),G=l.p1-p1
        ;
353     T t1=(G.cross(d2)).dot(D)/D.abs2();
354     T t2=(G.cross(d1)).dot(D)/D.abs2();
355     return make_pair(p1+d1*t1,l.p1+d2*t2);
356 }
357 bool same_side(const point3D<T> &a,const point3D<T> &b)
        const{
358     return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
359 }
360 };
361 template<typename T>
362 struct plane{
363     point3D<T> p0,n;//平面上的點和法向量
364     plane(){
365         plane(const point3D<T> &p0,const point3D<T> &n):p0(p0),n(n)
        {}
366     T dis2(const point3D<T> &p)const{//點到平面距離的平方
367         T tmp=(p-p0).dot(n);
368         return tmp*tmp/n.abs2();
369     }
370     point3D<T> projection(const point3D<T> &p)const{
371         return p-n*(p-p0).dot(n)/n.abs2();
372     }
373     point3D<T> line_intersection(const line3D<T> &l)const{
374         T tmp=n.dot(l.p2-l.p1);//等於0表示平行或重合該平面
375         return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
376     }
377     line3D<T> plane_intersection(const plane &p1)const{
378         point3D<T> e=n.cross(p1.n),v=n.cross(e);
379         T tmp=p1.n.dot(v);//等於0表示平行或重合該平面
380         point3D<T> q=p0+(v*(p1.p0-p0))/tmp;
381         return line3D<T>(q,q+e);
382     }
383 };
384 template<typename T>
385 struct triangle3D{
386     point3D<T> a,b,c;
387     triangle3D(){
388         triangle3D(const point3D<T> &a,const point3D<T> &b,const
        point3D<T> &c):a(a),b(b),c(c){}
389     bool point_in(const point3D<T> &p)const{//點在該平面上的投
        影在三角形中
390         return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).
        same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
391     }
392 };
393 template<typename T>
394 struct tetrahedron{//四面體
395     point3D<T> a,b,c,d;
396     tetrahedron(){

```

```

388 tetrahedron(const point3D<T> &a,const point3D<T> &b,const
    point3D<T> &c,const point3D<T> &d):a(a),b(b),c(c),d(d)
    {}
389 T volume6()const{//體積的六倍
    return (d-a).dot((b-a).cross(c-a));
390 }
391 point3D<T> centroid()const{
392     return (a+b+c+d)/4;
393 }
394 bool point_in(const point3D<T> &p)const{
395     return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,
396         d,a).point_in(p);
397 }
398 };
399 template<typename T>
400 struct convexhull3D{
401     static const int MAXN=1005;
402     struct face{
403         int a,b,c;
404         face(int a,int b,int c):a(a),b(b),c(c){}
405     };
406     vector<point3D<T>> pt;
407     vector<face> ans;
408     int fid[MAXN][MAXN];
409     void build(){
410         int n=pt.size();
411         ans.clear();
412         memset(fid,0,sizeof(fid));
413         ans.emplace_back(0,1,2);//注意不能共線
414         ans.emplace_back(2,1,0);
415         int ftop = 0;
416         for(int i=3, ftop=1; i<n; ++i,++ftop){
417             vector<face> next;
418             for(auto &f:ans){
419                 T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[f.a]).cross(pt[f.
420                     c]-pt[f.a]));
421                 if(d<=0) next.push_back(f);
422                 int ff=0;
423                 if(d>0) ff=ftop;
424                 else if(d<0) ff=-ftop;
425                 fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c][f.a]=ff;
426             }
427             for(auto &f:ans){
428                 if(fid[f.a][f.b]>0 && fid[f.a][f.b]!=fid[f.b][f.a])
429                     next.emplace_back(f.a,f.b,i);
430                 if(fid[f.b][f.c]>0 && fid[f.b][f.c]!=fid[f.c][f.b])
431                     next.emplace_back(f.b,f.c,i);
432                 if(fid[f.c][f.a]>0 && fid[f.c][f.a]!=fid[f.a][f.c])
433                     next.emplace_back(f.c,f.a,i);
434             }
435             ans=next;
436         }
437     point3D<T> centroid()const{
438         point3D<T> res(0,0,0);
439         T vol=0;
440         for(auto &f:ans){
441             T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]));
442             res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
443             vol+=tmp;
444         }
445         return res/(vol*4);
446     }
447 };

```

5.3 Pick's Theorem

給定頂點坐標均是整點的簡單多邊形 · 面積 = 內部格點數 + 邊上格點數/2 - 1

6 Graph

6.1 Bridge BCC

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 200005;
5 vector<int> G[N];
6 int low[N], depth[N];
7 bool vis[N];
8 vector<vector<int>> bcc;
9 stack<int> stk;
10
11 void dfs(int v, int p) {
12     stk.push(v);
13     vis[v] = true;
14     low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
15     for (int u : G[v]) {
16         if (u == p) continue;
17         if (!vis[u]) {
18             // (v, u) 是樹邊
19             dfs(u, v);
20             low[v] = min(low[v], low[u]);
21         } else {
22             // (v, u) 是回邊
23             low[v] = min(low[v], depth[u]);
24         }
25     }
26     // v 在不依靠父邊的情況下永遠沒辦法走到它的祖先
27     if (low[v] == depth[v]) {
28         bcc.emplace_back();
29         while (stk.top() != v) {
30             bcc.back().push_back(stk.top());
31             stk.pop();
32         }
33         bcc.back().push_back(stk.top());
34         stk.pop();
35     }
36 }

```

6.2 Cut BCC

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 200005;
5 vector<int> G[N];
6 int low[N], depth[N];
7 bool vis[N];
8 vector<vector<int>> bcc;
9 stack<int> stk;
10
11 void dfs(int v, int p) {

```

```

12     stk.push(v);
13     vis[v] = true;
14     low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
15     for (int u : G[v]) {
16         if (u == p) continue;
17         if (!vis[u]) {
18             // (v, u) 是樹邊
19             dfs(u, v);
20             low[v] = min(low[v], low[u]);
21             // u 無法在不經過父邊的情況走到 v 的祖先
22             if (low[u] >= depth[v]) {
23                 bcc.emplace_back();
24                 while (stk.top() != u) {
25                     bcc.back().push_back(stk.top());
26                     stk.pop();
27                 }
28                 bcc.back().push_back(stk.top());
29                 stk.pop();
30                 bcc.back().push_back(v);
31             }
32         } else {
33             // (v, u) 是回邊
34             low[v] = min(low[v], depth[u]);
35         }
36     }
37 }

```

6.3 Dijkstra

```

1 // 可以在  $O(E \log E)$  的時間複雜度解決在無負權有向圖單點源最短路
2 const int INF = 2e18; // 要確保 INF 開的足夠大
3
4 vector<vector<pair<int, int>>> G(n); // G[i] = <節點, 權重>
5 vector<int> dis(n, INF);
6 priority_queue<pair<int, int>, vector<pair<int, int>>,
    greater<pair<int, int>>> pq;
7 dis[s] = 0;
8 pq.push({0, s});
9
10 while (pq.size()){
11     int now_dis = pq.top().first;
12     int now_node = pq.top().second;
13     pq.pop();
14
15     if (now_dis > dis[now_node]) continue;
16
17     for (auto x : G[now_node]){
18         if (now_dis+x.second < dis[x.first]){
19             dis[x.first] = now_dis+x.second;
20             pq.push({dis[x.first], x.first});
21         }
22     }
23 }

```

6.4 Dinic

```

1 // 一般圖:  $O(EV^2)$ 
2 // 二分圖:  $O(EV)$ 
3 struct Flow{
4     struct Edge{
5         int v, rc, rid;
6     };
7     vector<vector<Edge>> G;
8     void add(int u, int v, int c){
9         G[u].push_back({v, c, G[v].size()});
10        G[v].push_back({u, 0, G[u].size()-1});
11    }
12    vector<int> dis, it;
13
14    Flow(int n){
15        G.resize(n);
16        dis.resize(n);
17        it.resize(n);
18    }
19
20    int dfs(int u, int t, int f){
21        if (u==t || f==0) return f;
22        for (int &i=it[u] ; i<G[u].size() ; i++){
23            auto &[v, rc, rid] = G[u][i];
24            if (dis[v]!=dis[u]+1) continue;
25            int df = dfs(v, t, min(f, rc));
26            if (df<=0) continue;
27            rc -= df;
28            G[v][rid].rc += df;
29            return df;
30        }
31        return 0;
32    }
33
34    int flow(int s, int t){
35        int ans = 0;
36        while (true){
37            fill(dis.begin(), dis.end(), INF);
38            queue<int> q;
39            q.push(s);
40            dis[s] = 0;
41
42            while (q.size()){
43                int u = q.front(); q.pop();
44                for (auto [v, rc, rid] : G[u]){
45                    if (rc<=0 || dis[v]<INF) continue;
46                    dis[v] = dis[u]+1;
47                    q.push(v);
48                }
49            }
50            if (dis[t]==INF) break;
51
52            fill(it.begin(), it.end(), 0);
53            while (true){
54                int df = dfs(s, t, INF);
55                if (df<=0) break;
56                ans += df;
57            }
58        }
59        return ans;
60    }
61    // the code below constructs minimum cut
62    void dfs_mincut(int now, vector<bool> &vis){
63        vis[now] = true;
64        for (auto &[v, rc, rid] : G[now]){
65            if (vis[v]==false && rc>0){

```

```

66            dfs_mincut(v, vis);
67        }
68    }
69 }
70
71 vector<pair<int, int>> construct(int n, int s, vector<pair<
72     int,int>> &E){
73     // E is G without capacity
74     vector<bool> vis(n);
75     dfs_mincut(s, vis);
76     vector<pair<int, int>> ret;
77     for (auto &[u, v] : E){
78         if (vis[u]==true && vis[v]==false){
79             ret.emplace_back(u, v);
80         }
81     }
82     return ret;
83 };

```

6.5 Dinic with double

```

1 const double double_INF = 1e18;
2 const int INF = (int)(1e9 + 10);
3
4 struct Flow{
5     const double eps = 1e-9;
6     struct Edge{
7         int v; double rc; int rid;
8     };
9     vector<vector<Edge>> G;
10    void add(int u, int v, double c){
11        G[u].push_back({v, c, G[v].size()});
12        G[v].push_back({u, 0, G[u].size()-1});
13    }
14    vector<int> dis, it;
15
16    Flow(int n){
17        G.resize(n);
18        dis.resize(n);
19        it.resize(n);
20    }
21
22    double dfs(int u, int t, double f){
23        if (u == t || abs(f) < eps) return f;
24        for (int &i=it[u] ; i<G[u].size() ; i++){
25            auto &[v, rc, rid] = G[u][i];
26            if (dis[v]!=dis[u]+1) continue;
27            double df = dfs(v, t, min(f, rc));
28            if (abs(df) <= eps) continue;
29            rc -= df;
30            G[v][rid].rc += df;
31            return df;
32        }
33        return 0;
34    }
35
36    double flow(int s, int t){
37        double ans = 0;
38        while (true){
39            fill(dis.begin(), dis.end(), INF);
40            queue<int> q;
41            q.push(s);

```

```

42            dis[s] = 0;
43
44            while (q.size()){
45                int u = q.front(); q.pop();
46                for (auto [v, rc, rid] : G[u]){
47                    if (abs(rc) <= eps || dis[v] < INF)
48                        continue;
49                    dis[v] = dis[u] + 1;
50                    q.push(v);
51                }
52            }
53            if (dis[t]==INF) break;
54
55            fill(it.begin(), it.end(), 0);
56            while (true){
57                double df = dfs(s, t, double_INF);
58                if (abs(df) <= eps) break;
59                ans += df;
60            }
61        }
62        return ans;
63    }
64    // the code below constructs minimum cut
65    void dfs_mincut(int now, vector<bool> &vis){
66        vis[now] = true;
67        for (auto &[v, rc, rid] : G[now]){
68            if (vis[v] == false && rc > eps){
69                dfs_mincut(v, vis);
70            }
71        }
72    }
73
74    vector<pair<int, int>> construct(int n, int s, vector<
75        pair<int,int>> &E){
76        // E is G without capacity
77        vector<bool> vis(n);
78        dfs_mincut(s, vis);
79        vector<pair<int, int>> ret;
80        for (auto &[u, v] : E){
81            if (vis[u] == true && vis[v] == false){
82                ret.emplace_back(u, v);
83            }
84        }
85        return ret;
86    }
87 };

```

6.6 Find Bridge

```

1 vector<int> dep(MAX_N), low(MAX_N);
2 vector<pair<int, int>> bridge;
3 bitset<MAX_N> vis;
4
5 void dfs(int now, int pre){
6     vis[now] = 1;
7     low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
8
9     for (auto x : G[now]){
10        if (x==pre){
11            continue;
12        }else if (vis[x]==0){
13            // 沒有走過的節點

```



```

14     dfs(x, now);
15     low[now] = min(low[now], low[x]);
16 } else if (vis[x]==1){
17     low[now] = min(low[now], dep[x]);
18 }
19 }
20
21 if (now!=1 && low[now]==dep[now]){
22     bridge.push_back({now, pre});
23 }
24 return;
25 }

```

6.7 HLD

```

1 #include <bits/stdc++.h>
2 #define int long long
3 using namespace std;
4
5 const int N = 100005;
6 vector<int> G[N];
7 struct HLD {
8     vector<int> pa, sz, depth, mxson, topf, id;
9     int n, idcnt = 0;
10    HLD(int _n) : n(_n), pa(_n + 1), sz(_n + 1), depth(_n + 1), mxson(_n + 1), topf(_n + 1), id(_n + 1) {}
11    void dfs1(int v = 1, int p = -1) {
12        pa[v] = p; sz[v] = 1; mxson[v] = 0;
13        depth[v] = (p == -1 ? 0 : depth[p] + 1);
14        for (int u : G[v]) {
15            if (u == p) continue;
16            dfs1(u, v);
17            sz[v] += sz[u];
18            if (sz[u] > sz[mxson[v]]) mxson[v] = u;
19        }
20    }
21    void dfs2(int v = 1, int top = 1) {
22        id[v] = ++idcnt;
23        topf[v] = top;
24        if (mxson[v]) dfs2(mxson[v], top);
25        for (int u : G[v]) {
26            if (u == mxson[v] || u == pa[v]) continue;
27            dfs2(u, u);
28        }
29    }
30    // query 為區間資料結構
31    int path_query(int a, int b) {
32        int res = 0;
33        while (topf[a] != topf[b]) { /// 若不在同一條鍊上
34            if (depth[topf[a]] < depth[topf[b]]) swap(a, b);
35            res = max(res, 011); // query : L = id[topf[a]], r = id[a]
36            a = pa[topf[a]];
37        }
38        /// 此時已在同一條鍊上
39        if (depth[a] < depth[b]) swap(a, b);
40        res = max(res, 011); // query : L = id[b], r = id[a]
41        return res;
42    }
43 };

```

6.8 Kosaraju to DAG

```

1 /*
2 給定一個有向圖，迴傳傳縮點後的圖、SCC 的資訊
3 所有點都以 based-0 編號
4
5 函式：
6 SCC_compress G(n): 宣告一個有 n 個點的圖
7 .add_edge(u, v): 加上一條邊 u -> v
8 .compress: O(n log n) 計算 G3、SCC、SCC_id 的資訊，並把縮點後
   的結果存在 result 裡
9
10 SCC[i] = 某個 SCC 中的所有點
11 SCC_id[i] = 第 i 個點在第幾個 SCC
12 */
13 // c8b146
14 struct SCC_compress{
15     int n = 0, m = 0;
16     vector<vector<int>> G, inv_G, result;
17     vector<pair<int, int>> edges;
18     vector<bool> vis;
19     vector<int> order;
20
21     vector<vector<int>> SCC;
22     vector<int> SCC_id;
23
24     SCC_compress(int _n){
25         n = _n;
26         G.resize(n);
27         inv_G.resize(n);
28         result.resize(n);
29         vis.resize(n);
30         SCC_id.resize(n);
31     }
32
33     void add_edge(int u, int v){
34         G[u].push_back(v);
35         inv_G[v].push_back(u);
36         edges.push_back({u, v});
37         m++;
38     }
39
40     void dfs1(vector<vector<int>> &G, int now){
41         vis[now] = 1;
42         for (auto x : G[now]){
43             if (vis[x]==0){
44                 dfs1(G, x);
45             }
46         }
47         order.push_back(now);
48         return;
49     }
50
51     void dfs2(vector<vector<int>> &G, int now){
52         SCC_id[now] = SCC.size()-1;
53         SCC.back().push_back(now);
54         vis[now] = 1;
55
56         for (auto x : G[now]){
57             if (vis[x]==0){
58                 dfs2(G, x);
59             }
60         }
61         return;

```

```

62     }
63
64     void compress(){
65         fill(vis.begin(), vis.end(), 0);
66         for (int i=0; i<n; i++){
67             if (vis[i]==0){
68                 dfs1(G, i);
69             }
70         }
71
72         fill(vis.begin(), vis.end(), 0);
73         reverse(order.begin(), order.end());
74         for (int i=0; i<n; i++){
75             if (vis[order[i]]==0){
76                 SCC.push_back(vector<int>());
77                 dfs2(inv_G, order[i]);
78             }
79         }
80
81         for (int i=0; i<m; i++){
82             if (SCC_id[edges[i].first]!=SCC_id[edges[i].second]){
83                 result[SCC_id[edges[i].first]].push_back(SCC_id[edges[i].second]);
84             }
85         }
86         for (int i=0; i<SCC.size(); i++){
87             sort(result[i].begin(), result[i].end());
88             result[i].resize(unique(result[i].begin(), result[i].end())-result[i].begin());
89         }
90     }
91 };

```

6.9 MCMF

```

1 struct Flow {
2     struct Edge {
3         int u, rc, k, rv;
4     };
5
6     vector<vector<Edge>> G;
7     vector<int> par, par_eid;
8     Flow(int n) : G(n+1), par(n+1), par_eid(n+1) {}
9
10    // v->u, capacity: c, cost: k
11    void add(int v, int u, int c, int k){
12        G[v].push_back({u, c, k, SZ(G[u])});
13        G[u].push_back({v, 0, -k, SZ(G[v])-1});
14    }
15
16    // 3701d6
17    int spfa(int s, int t){
18        fill(ALL(par), -1);
19        vector<int> dis(SZ(par), INF);
20        vector<bool> in_q(SZ(par), false);
21        queue<int> Q;
22        dis[s] = 0;
23        in_q[s] = true;
24        Q.push(s);
25
26        while (!Q.empty()){
27            int v = Q.front();

```

```

28     Q.pop();
29     in_q[v] = false;
30
31     for (int i=0 ; i<SZ(G[v]) ; i++){
32         auto [u, rc, k, rv] = G[v][i];
33         if (rc>0 && dis[v]+k<dis[u]){
34             dis[u] = dis[v]+k;
35             par[u] = v;
36             par_eid[u] = i;
37             if (!in_q[u]) Q.push(u);
38             in_q[u] = true;
39         }
40     }
41 }
42
43 return dis[t];
44 }
45
46 // return <max flow, min cost>, 150093
47 pair<int, int> flow(int s, int t){
48     int fl = 0, cost = 0, d;
49     while ((d = spfa(s, t))<INF){
50         int cur = INF;
51         for (int v=t ; v!=s ; v=par[v])
52             cur = min(cur, G[par[v]][par_eid[v]].rc);
53         fl += cur;
54         cost += d*cur;
55         for (int v=t ; v!=s ; v=par[v]){
56             G[par[v]][par_eid[v]].rc -= cur;
57             G[v][G[par[v]][par_eid[v]].rv].rc += cur;
58         }
59     }
60     return {fl, cost};
61 }
62
63 vector<pair<int, int>> construct(){
64     vector<pair<int, int>> ret;
65     for (int i=0 ; i<n ; i++){
66         for (auto x : G[i]){
67             if (x.rc==0){
68                 ret.push_back({i+1, x.u-n+1});
69                 break;
70             }
71         }
72     }
73     return ret;
74 }
75 };

```

6.10 Tarjan Find AP

```

1 vector<int> dep(MAX_N), low(MAX_N), AP;
2 bitset<MAX_N> vis;
3
4 void dfs(int now, int pre){
5     int cnt = 0;
6     bool ap = 0;
7     vis[now] = 1;
8     low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
9
10    for (auto x : G[now]){
11        if (x==pre){
12            continue;

```

```

13        }else if (vis[x]==0){
14            cnt++;
15            dfs(x, now);
16            low[now] = min(low[now], low[x]);
17            if (low[x]>=dep[now]) ap=1;
18        }else{
19            low[now] = min(low[now], dep[x]);
20        }
21    }
22
23    if ((now==pre && cnt>=2) || (now!=pre && ap)){
24        AP.push_back(now);
25    }
26 }

```

6.11 Tree Isomorphism

```

1 #include <bits/stdc++.h>
2 #pragma GCC optimize("O3,unroll-loops")
3 #define fastio ios::sync_with_stdio(0), cin.tie(0), cout.tie(0)
4 #define dbg(x) cerr << #x << " = " << x << endl
5 #define int long long
6 using namespace std;
7
8 // declare
9 const int MAX_SIZE = 2e5+5;
10 const int INF = 9e18;
11 const int MOD = 1e9+7;
12 const double EPS = 1e-6;
13 typedef vector<vector<int>> Graph;
14 typedef map<vector<int>, int> Hash;
15
16 int n, a, b;
17 int id1, id2;
18 pair<int, int> c1, c2;
19 vector<int> sz1(MAX_SIZE), sz2(MAX_SIZE);
20 vector<int> we1(MAX_SIZE), we2(MAX_SIZE);
21 Graph g1(MAX_SIZE), g2(MAX_SIZE);
22 Hash m1, m2;
23 int testcase=0;
24
25 void centroid(Graph &g, vector<int> &s, vector<int> &w, pair<
    int, int> &rec, int now, int pre){
26     s[now]=1;
27     w[now]=0;
28     for (auto x : g[now]){
29         if (x!=pre){
30             centroid(g, s, w, rec, x, now);
31             s[now]+=s[x];
32             w[now]=max(w[now], s[x]);
33         }
34     }
35
36     w[now]=max(w[now], n-s[now]);
37     if (w[now]<=n/2){
38         if (rec.first==0) rec.first=now;
39         else rec.second=now;
40     }
41 }
42
43 int dfs(Graph &g, Hash &m, int &id, int now, int pre){
44     vector<int> v;

```

```

45     for (auto x : g[now]){
46         if (x!=pre){
47             int add=dfs(g, m, id, x, now);
48             v.push_back(add);
49         }
50     }
51     sort(v.begin(), v.end());
52
53     if (m.find(v)!=m.end()){
54         return m[v];
55     }else{
56         m[v]=++id;
57         return id;
58     }
59 }
60
61 void solve1(){
62     // init
63     id1=0;
64     id2=0;
65     c1={0, 0};
66     c2={0, 0};
67     fill(sz1.begin(), sz1.begin()+n+1, 0);
68     fill(sz2.begin(), sz2.begin()+n+1, 0);
69     fill(we1.begin(), we1.begin()+n+1, 0);
70     fill(we2.begin(), we2.begin()+n+1, 0);
71     for (int i=1 ; i<=n ; i++){
72         g1[i].clear();
73         g2[i].clear();
74     }
75     m1.clear();
76     m2.clear();
77
78     // input
79     cin >> n;
80     for (int i=0 ; i<n-1 ; i++){
81         cin >> a >> b;
82         g1[a].push_back(b);
83         g1[b].push_back(a);
84     }
85     for (int i=0 ; i<n-1 ; i++){
86         cin >> a >> b;
87         g2[a].push_back(b);
88         g2[b].push_back(a);
89     }
90
91     // get tree centroid
92     centroid(g1, sz1, we1, c1, 1, 0);
93     centroid(g2, sz2, we2, c2, 1, 0);
94
95     // process
96     int res1=0, res2=0, res3=0;
97     if (c2.second!=0){
98         res1=dfs(g1, m1, id1, c1.first, 0);
99         m2=m1;
100         id2=id1;
101         res2=dfs(g2, m1, id1, c2.first, 0);
102         res3=dfs(g2, m2, id2, c2.second, 0);
103     }else if (c1.second!=0){
104         res1=dfs(g2, m1, id1, c2.first, 0);
105         m2=m1;
106         id2=id1;
107         res2=dfs(g1, m1, id1, c1.first, 0);
108         res3=dfs(g1, m2, id2, c1.second, 0);
109     }
110 }

```



```

111 }else{
112     res1=dfs(g1, m1, id1, c1.first, 0);
113     res2=dfs(g2, m1, id1, c2.first, 0);
114 }
115 // output
116 cout << (res1==res2 || res1==res3 ? "YES" : "NO") << endl;
117 ;
118 return;
119 }
120 }
121 signed main(void){
122     fastio;
123
124     int t=1;
125     cin >> t;
126     while (t--){
127         solve1();
128     }
129     return 0;
130 }
131 }

```

6.12 tarjan

```

1 struct tarjan_SCC {
2     int now_T, now_SCCs;
3     vector<int> dfn, low, SCC;
4     stack<int> S;
5     vector<vector<int>> E;
6     vector<bool> vis, in_stack;
7
8     tarjan_SCC(int n) {
9         init(n);
10    }
11    void init(int n) {
12        now_T = now_SCCs = 0;
13        dfn = low = SCC = vector<int>(n);
14        E = vector<vector<int>>(n);
15        S = stack<int>();
16        vis = in_stack = vector<bool>(n);
17    }
18    void add(int u, int v) {
19        E[u].push_back(v);
20    }
21    void build() {
22        for (int i = 0; i < dfn.size(); ++i) {
23            if (!dfn[i]) dfs(i);
24        }
25    }
26    void dfs(int v) {
27        now_T++;
28        vis[v] = in_stack[v] = true;
29        dfn[v] = low[v] = now_T;
30        S.push(v);
31        for (auto &i:E[v]) {
32            if (!vis[i]) {
33                vis[i] = true;
34                dfs(i);
35                low[v] = min(low[v], low[i]);
36            }
37            else if (in_stack[i]) {
38                low[v] = min(low[v], dfn[i]);

```

```

39    }
40    }
41    if (low[v] == dfn[v]) {
42        int tmp;
43        do {
44            tmp = S.top();
45            S.pop();
46            SCC[tmp] = now_SCCs;
47            in_stack[tmp] = false;
48        } while (tmp != v);
49        now_SCCs += 1;
50    }
51    }
52 };

```

6.13 圓方樹

```

1 #include <bits/stdc++.h>
2 #define lp(i,a,b) for(int i=(a);i<(b);i++)
3 #define pii pair<int,int>
4 #define pb push_back
5 #define ins insert
6 #define ff first
7 #define ss second
8 #define opa(x) cerr << #x << " = " << x << ", ";
9 #define op(x) cerr << #x << " = " << x << endl;
10 #define ops(x) cerr << x;
11 #define etr cerr << endl;
12 #define spc cerr << ' ';
13 #define BAE(x) (x).begin(), (x).end()
14 #define STL(x) cerr << #x << " : "; for(auto &qwe:x) cerr <<
15     qwe << ' '; cerr << endl;
16 #define deb1 cerr << "deb1" << endl;
17 #define deb2 cerr << "deb2" << endl;
18 #define deb3 cerr << "deb3" << endl;
19 #define deb4 cerr << "deb4" << endl;
20 #define deb5 cerr << "deb5" << endl;
21 #define bye exit(0);
22 using namespace std;
23
24 const int mxn = (int)(2e5) + 10;
25 const int mxlg = 17;
26 int last_special_node = (int)(1e5) + 1;
27 vector<int> E[mxn], F[mxn];
28 struct edg{
29     int fr, to;
30     edg(int _fr, int _to){
31         fr = _fr;
32         to = _to;
33     }
34 };
35 ostream& operator<<(ostream& os, edg x){os << x.fr << "--" <<
36     x.to;}
37 vector<edg> EV;
38 void tarjan(int v, int par, stack<int>& S){
39     static vector<int> dfn(mxn), low(mxn);
40     static vector<bool> to_add(mxn);
41     static int nowT = 0;
42
43     int child = 0;
44     nowT += 1;

```

```

45     dfn[v] = low[v] = nowT;
46     for(auto &ne:E[v]){
47         int i = EV[ne].to;
48         if(i == par) continue;
49         if(!dfn[i]){
50             S.push(ne);
51             tarjan(i, v, S);
52             child += 1;
53             low[v] = min(low[v], low[i]);
54
55             if(par >= 0 && low[i] >= dfn[v]){
56                 vector<int> bcc;
57                 int tmp;
58                 do{
59                     tmp = S.top(); S.pop();
60                     if(!to_add[EV[tmp].fr]){
61                         to_add[EV[tmp].fr] = true;
62                         bcc.pb(EV[tmp].fr);
63                     }
64                     if(!to_add[EV[tmp].to]){
65                         to_add[EV[tmp].to] = true;
66                         bcc.pb(EV[tmp].to);
67                     }
68                 }while(tmp != ne);
69                 for(auto &j:bcc){
70                     to_add[j] = false;
71                     F[last_special_node].pb(j);
72                     F[j].pb(last_special_node);
73                 }
74                 last_special_node += 1;
75             }
76         }
77     }
78     else{
79         low[v] = min(low[v], dfn[i]);
80         if(dfn[i] < dfn[v]){ // edge i--v will be visited
81             // twice at here, but we only need one.
82             S.push(ne);
83         }
84     }
85 }
86 int dep[mxn], jmp[mxn][mxlg];
87 void dfs_lca(int v, int par, int depth){
88     dep[v] = depth;
89     for(auto &i:F[v]){
90         if(i == par) continue;
91         jmp[i][0] = v;
92         dfs_lca(i, v, depth + 1);
93     }
94 }
95
96 inline void build_lca(){
97     jmp[1][0] = 1;
98     dfs_lca(1, -1, 1);
99     lp(j,1,mxlg){
100         lp(i,1,mxn){
101             jmp[i][j] = jmp[jmp[i][j-1]][j-1];
102         }
103     }
104 }
105
106 inline int lca(int x, int y){
107     if(dep[x] < dep[y]){ swap(x, y); }
108
109     int diff = dep[x] - dep[y];

```

```

110 lp(j,0,mxlg){
111     if((diff >> j) & 1){
112         x = jmp[x][j];
113     }
114 }
115 if(x == y) return x;
116
117 for(int j = mxlg - 1; j >= 0; j--){
118     if(jmp[x][j] != jmp[y][j]){
119         x = jmp[x][j];
120         y = jmp[y][j];
121     }
122 }
123 return jmp[x][0];
124 }
125
126 inline bool can_reach(int fr, int to){
127     if(dep[to] > dep[fr]) return false;
128
129     int diff = dep[fr] - dep[to];
130     lp(j,0,mxlg){
131         if((diff >> j) & 1){
132             fr = jmp[fr][j];
133         }
134     }
135     return fr == to;
136 }
137
138 int main(){
139     ios::sync_with_stdio(false); cin.tie(0);
140     // freopen("test_input.txt", "r", stdin);
141     int n, m, q; cin >> n >> m >> q;
142     lp(i,0,m){
143         int u, v; cin >> u >> v;
144         E[u].pb(EV.size());
145         EV.pb(edg(u, v));
146         E[v].pb(EV.size());
147         EV.pb(edg(v, u));
148     }
149     E[0].pb(EV.size());
150     EV.pb(edg(0, 1));
151     stack<int> S;
152     tarjan(0, -1, S);
153     build_lca();
154
155     lp(queries,0,q){
156         int fr, to, relay; cin >> fr >> to >> relay;
157         if(fr == relay || to == relay){
158             cout << "NO\n";
159             continue;
160         }
161         if((can_reach(fr, relay) || can_reach(to, relay)) &&
162             dep[relay] >= dep[lca(fr, to)]){
163             cout << "NO\n";
164             continue;
165         }
166         cout << "YES\n";
167     }
168 }

```

6.14 最大權閉合圖

```

2 Problem:
3 Given w = [w_0, w_1, ..., w_{n-1}] (which can be
4 either positive or negative or 0), you can choose
5 to take w_i (0 < i < n) or not, but if edge u -> v
6 exists, you must take w_v if you want to take w_u
7 (in other words, you can't take w_u without taking
8 w_v), this function returns the maximum value(> 0)
9 you can get. If you need a construction, you can
10 output the minimum cut of the S(source) side.
11 Complexity:
12 MaxFlow(n, m) (Non-Biparte:O(n^2m) / Bipartite:O(mvn))
13 */
14 int maximum_closure(vector<int> w, vector<pair<int,int>> EV)
15 {
16     int n = w.size(), S = n + 1, T = n + 2;
17     Flow G(T + 5); // Graph/Dinic.cpp
18     int sum = 0;
19     for (int i = 0; i < n; ++i) {
20         if (w[i] > 0) {
21             G.add(S, i, w[i]);
22             sum += w[i];
23         }
24         else if (w[i] < 0) {
25             G.add(i, T, abs(w[i]));
26         }
27     }
28     for (auto &[u, v] : EV) { // You should make sure that
29         INF > S/w_i
30         G.add(u, v, INF);
31     }
32     int cut = G.flow(S, T);
33     return sum - cut;
34 }

```

6.15 Theorem

- 最小點覆蓋 = 最大匹配 = n - 最大點獨立集
 - 最小點覆蓋：選最少點讓所有的邊都有碰到一個點
 - 最大點獨立集：選最多不共邊的點
- 只有邊帶權的二分圖的定理（可能不重要）
 - w-vertex-cover（帶權點覆蓋）：每條邊的兩個連接點被選中的次數總和至少要是 w_e 。
 - w-weight matching（帶權匹配）
 - minimum vertex count of w-vertex-cover = maximum weight count of w-weight matching（一個點可以被選很多次，但邊不行）
- 點、邊都帶權的二分圖的定理（可能不重要）
 - b-matching：假設 v 的點權是 b_v ，那所有 v 的匹配邊 e 的權重都要滿足 $\sum w_e \leq b_v$ 。
 - The maximum w-weight of a b-matching equals the minimum b-weight of vertices in a w-vertex-cover.

7 Math

7.1 CRT m Coprime

```

1 vector<int> a, m;
2
3 int extgcd(int a, int b, int &x, int &y){
4     if (b==0){
5         x=1, y=0;
6         return a;
7     }
8
9     int ret=extgcd(b, a%b, y, x);
10    y-=a/b*x;
11    return ret;
12 }
13
14 // n = 有幾個式子 · 求解 x \equiv a_i \pmod{m_i}
15 int CRT(int n, vector<int> &a, vector<int> &m){
16     int p=1, ans=0;
17
18     vector<int> M(n), inv_M(n);
19
20     for (int i=0; i<n; i++){
21         int p*=m[i];
22         for (int i=0; i<n; i++){
23             M[i]=p/m[i];
24             int tmp;
25             extgcd(M[i], m[i], inv_M[i], tmp);
26             ans+=a[i]*inv_M[i]*M[i];
27             ans%=p;
28         }
29     }
30     return (ans%p+p)%p;
31 }

```

7.2 CRT m Not Coprime

```

1 int extgcd(int a, int b, int &x, int &y){
2     if (b==0){
3         x=1, y=0;
4         return a;
5     }
6
7     int ret=extgcd(b, a%b, y, x);
8     y-=a/b*x;
9     return ret;
10 }
11
12 // 對於方程組的式子兩兩求解
13 // {是否有解, {a, m}}
14 pair<bool, pair<int, int>> CRT(int a1, int m1, int a2, int m2)
15 {
16     int g=__gcd(m1, m2);
17     if ((a2-a1)%g!=0) return {0, {-1, -1}};
18
19     int x, y;
20     extgcd(m1, m2, x, y);
21
22     x=(a2-a1)*x/g; // 兩者不能相反
23     a1=x*m1+a1;

```

```

23 | m1=m1*m2/g;
24 | a1=(a1*m1+m1)%m1;
25 | return {1, {a1, m1}};
26 | }

```

7.3 Fraction

```

1 | #include <bits/stdc++.h>
2 | using namespace std;
3 |
4 | /// Fraction template starts ///
5 | #define fraction_template_bonus_check
6 | const long long ll_overflow_warning_value = (long long)(3e9);
7 |
8 | long long gcd(long long a, long long b){
9 |     if(a == 0) return 0;
10 |    if(b == 0) return a;
11 |    if(a < b) return gcd(b,a);
12 |    return gcd(b, a%b);
13 | }
14 | struct frac{
15 |     long long a, b;
16 |     frac(long long _a = 0, long long _b = 1){
17 |         a = _a; b = _b;
18 |         if(b == 0){
19 |             cerr << "Error: division by zero\n";
20 |             cerr << "Called : Constructor(" << _a << ", " <<
                _b << ")\n";
21 |             return;
22 |         }
23 |         if(a == 0){b = 1; return;}
24 |         if(b < 0){a = -a; b = -b;}
25 |         long long gcd_ab = gcd(std::abs(a), b);
26 |         if(gcd_ab != 1){a /= gcd_ab; b /= gcd_ab;}
27 |
28 |         #ifdef fraction_template_bonus_check
29 |         if(std::abs(a) > ll_overflow_warning_value || b >
                ll_overflow_warning_value){
30 |             cerr << "Overflow warning : " << a << "/" << b <<
                "\n";
31 |         }
32 |         #endif /// fraction_template_bonus_check
33 |     }
34 |     frac operator+(frac const &B){
35 |         return frac(a*(B.b)+(B.a)*b, b*(B.b));
36 |     }
37 |     frac operator-(frac const &B){
38 |         return frac(a*(B.b)-(B.a)*b, b*(B.b));
39 |     }
40 |     frac operator*(frac const &B){
41 |         return frac(a*(B.a), b*(B.b));
42 |     }
43 |     frac operator/(frac const &B){
44 |         return frac(a*(B.b), b*(B.a));
45 |     }
46 |     frac operator+=(frac const &B){
47 |         *this = frac(a*(B.b)+(B.a)*b, b*(B.b));
48 |     }
49 |     frac operator-=(frac const &B){
50 |         *this = frac(a*(B.b)-(B.a)*b, b*(B.b));
51 |     }
52 |     frac operator*=(frac const &B){
53 |         *this = frac(a*(B.a), b*(B.b));
54 |     }
55 |     frac operator/=(frac const &B){
56 |         *this = frac(a*(B.b), b*(B.a));
57 |     }
58 |     long long abs(){
59 |         a = std::abs(a);
60 |         return *this;

```

```

54 | }
55 |
56 | bool operator<(frac const &B){
57 |     return a*B.b < B.a*b;
58 | }
59 | bool operator<=(frac const &B){
60 |     return a*B.b <= B.a*b;
61 | }
62 | bool operator>(frac const &B){
63 |     return a*B.b > B.a*b;
64 | }
65 | bool operator>=(frac const &B){
66 |     return a*B.b >= B.a*b;
67 | }
68 | bool operator==(frac const &B){
69 |     return a * B.b == B.a * b;
70 | }
71 | bool operator!=(frac const &B){
72 |     return a * B.b != B.a * b;
73 | }
74 | };
75 | ostream& operator<<(ostream &os, const frac& A){
76 |     os << A.a << "/" << A.b;
77 |     return os;
78 | }
79 | /// Fraction template ends ///
80 | void test(frac A, frac B){
81 |     cout << "A = " << A << endl;
82 |     cout << "B = " << B << endl;
83 |     cout << endl;
84 |     cout << "A + B = " << A + B << endl;
85 |     cout << "A - B = " << A - B << endl;
86 |     cout << "A * B = " << A * B << endl;
87 |     cout << "A / B = " << A / B << endl;
88 |     cout << endl;
89 |     cout << "(A < B) = " << (A < B) << endl;
90 |     cout << "(A <= B) = " << (A <= B) << endl;
91 |     cout << "(A > B) = " << (A > B) << endl;
92 |     cout << "(A >= B) = " << (A >= B) << endl;
93 |     cout << "(A == B) = " << (A == B) << endl;
94 |     cout << "(A != B) = " << (A != B) << endl;
95 |     cout << "-----\n";
96 |     return;
97 | }
98 | int main(){
99 |     frac tmp1(-7, 2);
100 |    frac tmp2(5, 3);
101 |    test(tmp1, tmp2);
102 |
103 |    frac tmp3(-7);
104 |    frac tmp4(0);
105 |    test(tmp3, tmp4);
106 |    return 0;

```

7.4 Josephus Problem

```

1 | /// 有 n 個人，第偶數個報數的人被刪掉，問第 k 個被踢掉的是誰
2 | int solve(int n, int k){
3 |     if (n==1) return 1;
4 |     if (k<=(n+1)/2){
5 |         if (2*k>n) return 2*k%n;
6 |         else return 2*k;
7 |     }else{
8 |         int res=solve(n/2, k-(n+1)/2);
9 |         if (n&1) return 2*res+1;
10 |        else return 2*res-1;

```

```

11 | }
12 | }

```

7.5 Lagrange Any x

```

1 | /// init: (x1, y1), (x2, y2) in a vector
2 | struct Lagrange{
3 |     int n;
4 |     vector<pair<int, int>> v;
5 |
6 |     Lagrange(vector<pair<int, int>> &_v){
7 |         n = _v.size();
8 |         v = _v;
9 |     }
10 |
11 |     /// O(n^2 log MAX_A)
12 |     int solve(int x){
13 |         int ret = 0;
14 |         for (int i=0; i<n; i++){
15 |             int now = v[i].second;
16 |             for (int j=0; j<n; j++){
17 |                 if (i==j) continue;
18 |                 now *= ((x-v[j].first+MOD)%MOD);
19 |                 now %= MOD;
20 |                 now *= (qp((v[i].first-v[j].first+MOD)%MOD,
                MOD-2)+MOD)%MOD;
21 |                 now %= MOD;
22 |             }
23 |             ret = (ret+now)%MOD;
24 |         }
25 |         return ret;
26 |     }
27 | };
28 | };

```

7.6 Lagrange Continuous x

```

1 | #include <bits/stdc++.h>
2 | using namespace std;
3 |
4 | const int MAX_N = 5e5 + 10;
5 | const int mod = 1e9 + 7;
6 |
7 | long long inv_fac[MAX_N];
8 |
9 | inline int fp(long long x, int y) {
10 |     int ret = 1;
11 |     for (; y; y >= 1) {
12 |         ret = (y & 1) ? (ret * x % mod) : ret;
13 |         x = x * x % mod;
14 |     }
15 |     return ret;
16 | }
17 |
18 | /// TO USE THIS TEMPLATE, YOU MUST MAKE SURE THAT THE MOD
19 | NUMBER IS A PRIME.
20 | struct Lagrange {
21 |     /* Initialize a polynomial with f(x_0), f(x_0 + 1), ..., f(x_0 + n).

```

```

22  This determines a polynomial  $f(x)$  whose degree is at most  $n$ .
23  Then you can call sample(x) and you get the value of  $f(x)$ .
24  Complexity of init() and sample() are both  $O(n)$ .
25  */
26  int m, shift; //  $m = n + 1$ 
27  vector<int> v, mul;
28  // You can use this function if you don't have inv_fac array
   already.
29  void construct_inv_fac() {
30      long long fac = 1;
31      for (int i = 2; i < MAX_N; ++i) {
32          fac = fac * i % mod;
33      }
34      inv_fac[MAX_N - 1] = fp(fac, mod - 2);
35      for (int i = MAX_N - 1; i >= 1; --i) {
36          inv_fac[i - 1] = inv_fac[i] * i % mod;
37      }
38  }
39  // You call init() many times without having a second
   instance of this struct.
40  void init(int X_0, vector<int> &u) {
41      v = u;
42      shift = ((1 - X_0) % mod + mod) % mod;
43      if (v.size() == 1) v.push_back(v[0]);
44      m = v.size();
45      mul.resize(m);
46  }
47  // You can use sample(x) instead of sample(x % mod).
48  int sample(int x) {
49      x = ((long long)x + shift) % mod;
50      x = (x < 0) ? (x + mod) : x;
51      long long now = 1;
52      for (int i = m; i >= 1; --i) {
53          mul[i - 1] = now;
54          now = now * (x - i) % mod;
55      }
56      int ret = 0;
57      bool neg = (m - 1) & 1;
58      now = 1;
59      for (int i = 1; i <= m; ++i) {
60          int up = now * mul[i - 1] % mod;
61          int down = inv_fac[m - i] * inv_fac[i - 1] % mod;
62          int tmp = ((long long)v[i - 1] * up % mod) * down
63                  % mod;
64          ret += (neg && tmp) ? (mod - tmp) : (tmp);
65          ret = (ret >= mod) ? (ret - mod) : ret;
66          now = now * (x - i) % mod;
67          neg ^= 1;
68      }
69      return ret;
70  };
71
72  int main() {
73      int n; cin >> n;
74      vector<int> v(n);
75      for (int i = 0; i < n; ++i) {
76          cin >> v[i];
77      }
78      Lagrange L;
79      L.construct_inv_fac();
80      L.init(0, v);
81      int x; cin >> x;
82      cout << L.sample(x);

```

7.7 Lucas's Theorem

```

1  // 對於很大的  $C^n_m$  對質數  $p$  取模。只要  $p$  不大就可以用。
2  int Lucas(int n, int m, int p){
3      if (m==0) return 1;
4      return (C(n%p, m%p, p)*Lucas(n/p, m/p, p)%p);
5  }

```

7.8 Matrix

```

1  struct Matrix{
2      int n, m;
3      vector<vector<int>> arr;
4
5      Matrix(int _n, int _m){
6          n = _n;
7          m = _m;
8          arr.resize(n, vector<int>(m));
9      }
10
11      Matrix operator * (Matrix b){
12          Matrix b_t(b.m, b.n);
13          for (int i=0; i<b.n; i++){
14              for (int j=0; j<b.m; j++){
15                  b_t.arr[j][i] = b.arr[i][j];
16              }
17          }
18
19          Matrix ret(n, b.m);
20          for (int i=0; i<n; i++){
21              for (int j=0; j<b.m; j++){
22                  for (int k=0; k<m; k++){
23                      ret.arr[i][j] += arr[i][k]*b_t.arr[j][k];
24                      ret.arr[i][j] %= MOD;
25                  }
26              }
27          }
28          return ret;
29      }
30
31      Matrix pow(int p){
32          Matrix ret(n, n), mul = *this;
33          for (int i=0; i<n; i++){
34              ret.arr[i][i] = 1;
35          }
36
37          for (; p; p>>=1){
38              if (p&1) ret = ret*mul;
39              mul = mul*mul;
40          }
41
42          return ret;
43      }
44
45      int det(){
46          vector<vector<int>> arr = this->arr;
47          bool flag = false;

```

```

49      for (int i=0; i<n; i++){
50          int target = -1;
51          for (int j=i; j<n; j++){
52              if (arr[j][i]){
53                  target = j;
54                  break;
55              }
56          }
57          if (target==-1) return 0;
58          if (i!=target){
59              swap(arr[i], arr[target]);
60              flag = !flag;
61          }
62
63          for (int j=i+1; j<n; j++){
64              if (!arr[j][i]) continue;
65              int freq = arr[j][i]*qp(arr[i][i], MOD-2)%MOD;
66              for (int k=i; k<n; k++){
67                  arr[j][k] -= freq*arr[i][k];
68                  arr[j][k] = (arr[j][k]%MOD+MOD)%MOD;
69              }
70          }
71
72          int ret = !flag ? 1 : MOD-1;
73          for (int i=0; i<n; i++){
74              ret *= arr[i][i];
75              ret %= MOD;
76          }
77          return ret;
78      }
79  };
80

```

7.9 Matrix 01

```

1  const int MAX_N = (1LL<<12);
2  struct Matrix{
3      int n, m;
4      vector<bitset<MAX_N>> arr;
5
6      Matrix(int _n, int _m){
7          n = _n;
8          m = _m;
9          arr.resize(n);
10     }
11
12     Matrix operator * (Matrix b){
13         Matrix b_t(b.m, b.n);
14         for (int i=0; i<b.n; i++){
15             for (int j=0; j<b.m; j++){
16                 b_t.arr[j][i] = b.arr[i][j];
17             }
18         }
19
20         Matrix ret(n, b.m);
21         for (int i=0; i<n; i++){
22             for (int j=0; j<b.m; j++){
23                 ret.arr[i][j] = ((arr[i]&b_t.arr[j]).count()
24                                 &1);
25             }
26         }
27         return ret;

```

```
27 |     }
28 | };
```

7.10 Miller Rabin

```
1 // O(Log n)
2 typedef UInt unsigned long long
3 UInt modmul(UInt a, UInt b, UInt m) {
4     int ret = a*b - m*(UInt)((long double)a*b/m);
5     return ret + m*(ret < 0) - m*(ret >= (int)m);
6 }
7
8 int qp(int b, int p, int m){
9     int ret = 1;
10    for ( ; p ; p>=1){
11        if (p&1){
12            ret = modmul(ret, b, m);
13        }
14        b = modmul(b, b, m);
15    }
16    return ret;
17 }
18
19 // ed23aa
20 vector<int> llsprp = {2, 325, 9375, 28178, 450775, 9780504,
21                    1795265022};
22 bool isprime(int n, vector<int> sprp = llsprp){
23     if (n==2) return 1;
24     if (n<2 || n%2==0) return 0;
25
26     int t = 0;
27     int u = n-1;
28     for ( ; u%2==0 ; t++) u>>=1;
29
30     for (int i=0 ; i<sprp.size() ; i++){
31         int a = sprp[i]%n;
32         if (a==0 || a==1 || a==n-1) continue;
33         int x = qp(a, u, n);
34         if (x==1 || x==n-1) continue;
35         for (int j=0 ; j<t ; j++){
36             x = modmul(x, x, n);
37             if (x==1) return 0;
38             if (x==n-1) break;
39         }
40         if (x==n-1) continue;
41         return 0;
42     }
43
44     return 1;
45 }
```

7.11 Pollard Rho

```
1 mt19937 seed(chrono::steady_clock::now().time_since_epoch().
2 count());
3 int rnd(int l, int r){
4     return uniform_int_distribution<int>(l, r)(seed);
5 }
```

```
6 // O(n^{1/4}) 回傳 1 或自己的因數、記得先判斷 n 是不是質數
7 // (用 Miller-Rabin)
8 // c1670c
9 int Pollard_Rho(int n){
10    int s = 0, t = 0;
11    int c = rnd(1, n-1);
12
13    int step = 0, goal = 1;
14    int val = 1;
15
16    for (goal=1 ; ; goal<=1, s=t, val=1){
17        for (step=1 ; step<=goal ; step++){
18            t = ((__int128)t*t+c)%n;
19            val = (__int128)val*abs(t-s)%n;
20
21            if ((step % 127) == 0){
22                int d = __gcd(val, n);
23                if (d>1) return d;
24            }
25        }
26
27        int d = __gcd(val, n);
28        if (d>1) return d;
29    }
30 }
```

7.12 Quick Pow

```
1 int qp(int b, int p, int m = MOD){
2     int ret = 1;
3     for ( ; p ; p>=1){
4         if (p&1) ret = ret*b%m;
5         b = b*b%m;
6     }
7     return ret;
8 }
```

7.13 數論分塊

```
1 /*
2 時間複雜度為 O(sqrt(n))
3 區間為 [L, r]
4 */
5 for(int i=1 ; i<=n ; i++){
6     int l = i, r = n/(n/i);
7     i = r;
8     ans.push_back(r);
9 }
```

7.14 最大質因數

```
1 void max_fac(int n, int &ret){
2     if (n<=ret || n<2) return;
3     if (isprime(n)){
4         ret = max(ret, n);
5     }
```

```
5     return;
6 }
7
8 int p = Pollard_Rho(n);
9 max_fac(p, ret), max_fac(n/p, ret);
10 }
```

7.15 歐拉公式

```
1 // phi(n) = 小於 n 並與 n 互質的正整數數量。
2 // O(sqrt(n)) · 回傳 phi(n)
3 int phi(int n){
4     int ret = n;
5
6     for (int i=2 ; i*i<=n ; i++){
7         if (n%i==0){
8             while (n%i==0) n /= i;
9             ret = ret*(i-1)/i;
10        }
11    }
12    if (n>1) ret = ret*(n-1)/n;
13
14    return ret;
15 }
16
17 // O(n log n) · 回傳 1~n 的 phi 值
18 vector<int> phi_1_to_n(int n){
19     vector<int> phi(n+1);
20     phi[0]=0;
21     phi[1]=1;
22
23     for (int i=2 ; i<=n ; i++){
24         phi[i]=i-1;
25     }
26
27     for (int i=2 ; i<=n ; i++){
28         for (int j=2*i ; j<=n ; j+=i){ // 枚舉所有倍數
29             phi[j]-=phi[i];
30         }
31     }
32
33     return phi;
34 }
```

7.16 線性篩

```
1 const int MAX_N = 5e5;
2
3 // Lpf[i] = i 的最小質因數
4 vector<int> prime, lpf(MAX_N);
5
6 void prime_init(){
7     for (int i=2 ; i<MAX_N ; i++){
8         if (lpf[i]==0){
9             lpf[i]=i;
10            prime.push_back(i);
11        }
12
13        for (int j : prime){
```

```

14         if (i*j>=MAX_N) break;
15         lpf[i*j]=j;
16         if (lpf[i]==j) break;
17     }
18 }
19 }

```

7.17 Burnside's Lemma

$$\sum_{k=1}^n \frac{c(k)}{n}$$

- n : 有多少種置換方式 (例如 : 旋轉方式)
- $c(k)$: 所有可能中, 經過 k 次旋轉後, 仍不會和別人相同的方式的數量

7.18 Catalan Number

任意括號序列 : $C_n = \frac{1}{n+1} \binom{2n}{n}$

7.19 Matrix Tree Theorem

目標 : 給定一張無向圖, 問他的生成樹數量。
方法 : 先把所有自環刪掉, 定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg(v_i) & \text{if } i = j \\ -(邊v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第一個 row 跟 column, 它的 determinant 就是答案。
目標 : 給定一張有向圖, 問他的以 r 為根, 可以走到所有點生成樹數量。

方法 : 先把所有自環刪掉, 定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg_{in}(v_i) & \text{if } i = j \\ -(邊v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第 r 個 row 跟 column, 它的 determinant 就是答案。

7.20 Stirling's formula

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

7.21 Theorem

- $1 \sim x$ 質數的數量 $\approx \frac{x}{\ln x}$
- $1 \sim x$ 的因數的數量 $\approx x^{\frac{1}{3}}$
- x 的質因數的數量 $\approx \log \log x$
- p is a prime number $\Leftrightarrow (p-1)! \equiv -1 \pmod{p}$
- 每個正整數都可以表示成四個整數的平方和
- 任何大於 2 的整數都可以表示成兩個質數的和

7.22 二元一次方程式

$$\begin{cases} ax + by = c \\ dx + ey = f \end{cases} = \begin{cases} x = \frac{ed-bf}{ad-bc} \\ y = \frac{af-ec}{ad-bc} \end{cases}$$

若 $x = \frac{0}{0}$ 且 $y = \frac{0}{0}$, 則代表無限多組解。若 $x = \frac{*}{0}$ 且 $y = \frac{*}{0}$, 則代表無解。

7.23 歐拉定理

若 a, m 互質, 則 :

$$a^n \bmod m = a^{n \bmod \varphi(m)} \bmod m$$

若 a, m 可能是任何數, 則 :

$$a^{\varphi(m) + [n \bmod \varphi(m)]} \bmod m$$

7.24 錯排公式

錯排公式 : (n 個人中, 每個人皆不再原來位置的組合數)

$$dp_i = \begin{cases} 1 & i = 0 \\ 0 & i = 1 \\ (i-1)(dp_{i-1} + dp_{i-2}) & \text{otherwise} \end{cases}$$

8 String

8.1 Hash

```

1 int A = rng(1e5, 8e8);
2 const int B = 1e9+7;
3
4 struct RollingHash{
5     vector<int> Pow, Pre;
6     RollingHash(string s = ""){
7         Pow.resize(s.size());
8         Pre.resize(s.size());
9
10        for (int i=0 ; i<s.size() ; i++){
11            if (i==0){
12                Pow[i] = 1;
13                Pre[i] = s[i];
14            }else{
15                Pow[i] = Pow[i-1]*A%B;
16                Pre[i] = (Pre[i-1]*A+s[i])%B;
17            }
18        }
19
20        return;
21    }
22
23    int get(int l, int r){ // 取得 [l, r] 的數值
24        if (l==0) return Pre[r];
25        int res = (Pre[r]-Pre[l-1]*Pow[r-l+1])%B;

```

```

26        if (res<0) res += B;
27        return res;
28    }
29 };

```

8.2 KMP

```

1 // 給一個字串 S, 定義函數 \pi(i) = k 代表 S[1 ... k] = S[i-k
2 //  +1 ... i]
3 // 4c61a3
4 vector<int> KMP(string &s){
5     n = SZ(s);
6     vector<int> ret(n);
7     int now = 0;
8     for (int i=1 ; i<n ; i++){
9         int j = ret[i-1];
10        while (j>0 && s[i]!=s[j]){
11            j = ret[j-1];
12        }
13        if (s[i]==s[j]) j++;
14        ret[i] = j;
15    }
16    return ret;

```

8.3 Manacher

```

1 string Manacher(string str) {
2     string tmp = "$#";
3     for(char i : str) {
4         tmp += i;
5         tmp += '#';
6     }
7
8     vector<int> p(tmp.size(), 0);
9     int mx = 0, id = 0, len = 0, center = 0;
10    for(int i=1 ; i<(int)tmp.size() ; i++) {
11        p[i] = mx > i ? min(p[id*2-i], mx-i) : 1;
12
13        while(tmp[i+p[i]] == tmp[i-p[i]]) p[i]++;
14        if(mx<i+p[i]) mx = i+p[i], id = i;
15        if(len<p[i]) len = p[i], center = i;
16    }
17    return str.substr((center-len)/2, len-1);
18 }

```

8.4 Min Rotation

```

1 // 9d296f
2 int minRotation(string s) {
3     int a=0, N=s.size(); s += s;
4     for (int b=0; b<N; b++){
5         for (int k=0; k<N; k++){
6             if (a+k == b || s[a+k] < s[b+k]) {b += max(0LL, k
7                 -1); break;}
8             if (s[a+k] > s[b+k]) {a = b; break;}
9         }
10    }
11    return a;

```

8.5 Suffix Array

```

1 // 注意·當 |s|=1 時·lcp 不會有值·務必測試 |s|=1 的 case
2 struct SuffixArray {
3     string s;
4     vector<int> sa, lcp;
5     SuffixArray(string _s, int lim = 256) {
6         s = _s;
7         int n = s.size()+1, k = 0, a, b;
8         vector<int> x(s.begin(), s.end()), y(n), ws(max(n,
9             lim)), rank(n);
10        x.push_back(0);
11        sa = lcp = y;
12        iota(sa.begin(), sa.end(), 0);
13        for (int j=0, p=0; p<n; j=max(1LL, j*2), lim=p) {
14            p = j;
15            iota(y.begin(), y.end(), n-j);
16            for (int i=0; i<n; i++) if (sa[i] >= j) y[p++]
17                = sa[i] - j;
18            fill(ws.begin(), ws.end(), 0);
19            for (int i=0; i<n; i++) ws[x[i]]++;
20            for (int i=1; i<lim; i++) ws[i] += ws[i - 1];
21            for (int i = n; i--;) sa[--ws[x[i]]] = y[i];
22            swap(x, y), p = 1, x[sa[0]] = 0;
23            for (int i=1; i<n; i++){
24                a = sa[i - 1];
25                b = sa[i];
26                x[b] = (y[a] == y[b] && y[a + j] == y[b + j])
27                    ? p - 1 : p++;
28            }
29            for (int i=1; i<n; i++) rank[sa[i]] = i;
30            for (int i=0, j; i<n-1; lcp[rk[i++]]=k)
31                for (k && k--, j=sa[rank[i]-1]; i+k<s.size() &&
32                    j+k<s.size() && s[i+k]==s[j+k]; k++);
33            sa.erase(sa.begin());
34            lcp.erase(lcp.begin(), lcp.begin()+2);
35        }
36        vector<int> pos; // pos[i] = i 這個值在 pos 的哪個地方
37        SparseTable st;
38        void init_lcp(){
39            pos.resize(sa.size());
40            for (int i=0; i<sa.size(); i++){
41                pos[sa[i]] = i;

```

```

42        if (lcp.size()){
43            st.build(lcp);
44        }
45    }
46
47    // 用之前記得 init
48    // 回傳 [l1, r1] 跟 [l2, r2] 的 lcp · 0-based
49    int get_lcp(int l1, int r1, int l2, int r2){
50        int pos_1 = pos[l1], len_1 = r1-l1+1;
51        int pos_2 = pos[l2], len_2 = r2-l2+1;
52        if (pos_1>pos_2){
53            swap(pos_1, pos_2);
54            swap(len_1, len_2);
55        }
56
57        if (l1==l2){
58            return min(len_1, len_2);
59        }else{
60            return min({st.query(pos_1, pos_2), len_1, len_2
61                });
62        }
63    }
64
65    // 檢查 [l1, r1] 跟 [l2, r2] 的大小關係 · 0-based
66    // 如果前者小於後者·就回傳 <0·相等就回傳 =0·否則回傳
67    >0
68    int substring_cmp(int l1, int r1, int l2, int r2){
69        int len_1 = r1-l1+1;
70        int len_2 = r2-l2+1;
71        int res = get_lcp(l1, r1, l2, r2);
72
73        if (res<len_1 && res<len_2){
74            return s[l1+res]-s[l2+res];
75        }else if (len_1==res && len_2==res){
76            // 如果不需要以 index 作為次要排序參數·這裡要回
77            傳 0
78            return l1-l2;
79        }else{
80            return len_1==res ? -1 : 1;
81        }
82    }
83
84    // 對於位置在 <=p 的後綴·找離他左邊/右邊最接近位置 >p 的
85    後綴的 lcp · 0-based
86    // pre[i] = s[i] 離他左邊最接近位置 >p 的後綴的 lcp · 0-
87    based
88    // suf[i] = s[i] 離他右邊最接近位置 >p 的後綴的 lcp · 0-
89    based
90    pair<vector<int>, vector<int>> get_left_and_right_lcp(int
91        p){
92        vector<int> pre(p+1);
93        vector<int> suf(p+1);
94
95        { // build pre
96            int now = 0;
97            for (int i=0; i<s.size(); i++){
98                if (sa[i]<=p){
99                    pre[sa[i]] = now;
100                    if (i<lcp.size()) now = min(now, lcp[i]);
101                }else{
102                    if (i<lcp.size()) now = lcp[i];
103                }
104            }
105        }
106
107        { // build suf
108            int now = 0;
109            for (int i=s.size()-1; i>=0; i--){
110                if (sa[i]<=p){
111                    suf[sa[i]] = now;
112                    if (i-1>=0) now = min(now, lcp[i-1]);
113                }else{
114                    if (i-1>=0) now = lcp[i-1];
115                }
116            }
117        }
118        return {pre, suf};
119    }

```

```

99    { // build suf
100        int now = 0;
101        for (int i=s.size()-1; i>=0; i--){
102            if (sa[i]<=p){
103                suf[sa[i]] = now;
104                if (i-1>=0) now = min(now, lcp[i-1]);
105            }else{
106                if (i-1>=0) now = lcp[i-1];
107            }
108        }
109    }
110
111    return {pre, suf};
112 }
113 };

```

8.6 Z Algorithm

```

1 // 定義一個長度為 n 的文本為 T·則陣列 Z 的 Z[i] 代表 T[0:n]
2 和 T[i:n] 最長共同前綴
3 // bcfbd6
4 vector<int> z_function(string s){
5     vector<int> ret(s.size());
6     int ll = 0, rr = 0;
7
8     for (int i=1; i<s.size(); i++){
9         int j = 0;
10
11         if (i<rr) j = min(ret[i-ll], rr-i);
12         while (s[j]==s[i+j]) j++;
13         ret[i] = j;
14
15         if (i+j>rr){
16             ll = i;
17             rr = i+j;
18         }
19     }
20
21     ret[0] = s.size();
22     return ret;

```