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1 Misc

1.1 2-SAT

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 struct TWO_SAT {
5     int n, N;
6     vector<vector<int>> G, rev_G;
7     deque<bool> used;
8     vector<int> order, comp;
9     deque<bool> assignment;
10    void init(int _n) {
11        n = _n;
12        N = _n * 2;
13        G.resize(N + 5);
14        rev_G.resize(N + 5);
15    }
16    void dfs1(int v) {
17        used[v] = true;
18        for (int u : G[v]) {
19            if (!used[u])
20                dfs1(u);
21        }
22        order.push_back(v);
23    }
24    void dfs2(int v, int c1) {
25        comp[v] = c1;
26        for (int u : rev_G[v]) {
27            if (comp[u] == -1)
28                dfs2(u, c1);
29        }
30    }
31    bool solve() {
32        order.clear();
33        used.assign(N, false);
34        for (int i = 0; i < N; ++i) {
35            if (!used[i])
36                dfs1(i);
37        }
38        comp.assign(N, -1);
39        for (int i = 0, j = 0; i < N; ++i) {
40            int v = order[N - i - 1];
41            if (comp[v] == -1)
42                dfs2(v, j++);
43        }
44        assignment.assign(n, false);
45        for (int i = 0; i < N; i += 2) {
46            if (comp[i] == comp[i + 1])
47                return false;
48            assignment[i / 2] = (comp[i] > comp[i + 1]);
49        }
50        return true;
51    }
52    void add_disjunction(int a, bool na, int b, bool nb) { //
53        // A or B
54        // na means whether a is negative or not
55        // nb means whether b is negative or not
56        a = 2 * a ^ na;
57        b = 2 * b ^ nb;
58        int neg_a = a ^ 1;
59        int neg_b = b ^ 1;

```

```

59        G[neg_a].push_back(b);
60        G[neg_b].push_back(a);
61        rev_G[b].push_back(neg_a);
62        rev_G[a].push_back(neg_b);
63        return;
64    }
65    void get_result(vector<int>& res) {
66        res.clear();
67        for (int i = 0; i < n; i++)
68            res.push_back(assignment[i]);
69    }
70 };
71 /* CSES Giant Pizza
72 3 5
73 + 1 + 2
74 - 1 + 3
75
76 - + + + -
77 */
78 int main() {
79     int n, m;
80     cin >> n >> m;
81     TWO_SAT E;
82     E.init(m);
83
84     char c1, c2;
85     int inp1, inp2;
86     for (int i = 0; i < n; i++) {
87         cin >> c1 >> inp1;
88         cin >> c2 >> inp2;
89         E.add_disjunction(inp1 - 1, c1 == '-', inp2 - 1, c2
90             == '-');
91     }
92
93     bool able = E.solve();
94     if (able) {
95         vector<int> ans;
96         E.get_result(ans);
97         for (int i : ans)
98             cout << (i == true ? '+' : '-') << ' ';
99     } else {
100         cout << "IMPOSSIBLE\n";
101     }
102
103     return 0;
104 }

```

1.2 Custom Set PQ Sort

```

1 // priority_queue · 務必檢查相等的 case · 給所有元素一個排序的
2 // 依據
3 struct cmp{
4     bool operator () (Data a, Data b){
5         return a.x<b.x;
6     }
7 };
8
9 // set · 務必檢查相等的 case · 給所有元素一個排序的依據
10 struct Data{
11     int x;

```

```

12
13     bool operator < (const Data &b) const {
14         return x<b.x;
15     }
16 };

```

1.3 Default Code New

```

1 #include <bits/stdc++.h>
2 using namespace std;
3 #define int long long
4
5 const int MAX_N = 5e5 + 10;
6 const int INF = 2e18;
7
8 void solve(){
9
10 }
11
12 signed main(){
13     ios::sync_with_stdio(0), cin.tie(0);
14
15     int t = 1;
16     while (t--){
17         solve();
18     }
19
20     return 0;
21 }

```

1.4 Default Code Old

```

1 #include <bits/stdc++.h>
2 #define int long long
3 #define ALL(x) x.begin(), x.end()
4 #define SZ(x) ((int)x.size())
5 #define fastio ios::sync_with_stdio(0), cin.tie(0);
6 using namespace std;
7
8 #ifdef LOCAL
9 #define cout cout << "\033[0;32m"
10 #define cerr cerr << "\033[0;31m"
11 #define endl endl << "\033[0m"
12 #else
13 #pragma GCC optimize("O3,unroll-loops")
14 #pragma GCC target("avx,avx2,sse,sse2,sse3,sse4,popcnt")
15 #define endl "\n"
16 #endif
17
18 const int MAX_N = 5e5+10;
19 const int INF = 2e18;
20
21 void solve1(){
22
23     return;
24 }
25
26 signed main(){
27     fastio;

```

```

29
30     int t = 1;
31     while (t--){
32         solve1();
33     }
34
35     return 0;
36 }

```

1.5 Enumerate Subset

```

1 // 時間複雜度  $O(3^n)$ 
2 // 枚舉每個 mask 的子集
3 for (int mask=0; mask<(1<<n); mask++){
4     for (int s=mask; s>=0; s=(s-1)&m){
5         // s 是 mask 的子集
6         if (s==0) break;
7     }
8 }

```

1.6 Fast Input

```

1 // fast IO
2 // 6f8879
3 inline char readchar(){
4     static char buffer[BUFSIZ], *now = buffer + BUFSIZ, *
5         end = buffer + BUFSIZ;
6     if (now == end)
7     {
8         if (end < buffer + BUFSIZ)
9             return EOF;
10        end = (buffer + fread(buffer, 1, BUFSIZ, stdin));
11        now = buffer;
12    }
13    return *now++;
14 }
15 inline int nextint(){
16     int x = 0, c = readchar(), neg = false;
17     while (('0' > c || c > '9') && c != '-' && c != EOF) c =
18         readchar();
19     if (c == '-') neg = true, c = readchar();
20     while ('0' <= c && c <= '9') x = (x<<3) + (x<<1) + (c-'0')
21         , c = readchar();
22     if (neg) x = -x;
23     return x; // returns 0 if EOF
24 }

```

1.7 Radix Sort

```

1 // 值域限制:  $0 \sim 1073741823(2^{30}-1)$ 
2 inline void radix_sort(vector<int> &a, int n){
3     static int cnt[32768] = {0};
4     vector<int> tmpa(n);
5     for(int i = 0; i < n; ++i)
6         ++cnt[a[i] & 32767];
7     for(int i = 1; i < 32768; ++i)

```

```

8         cnt[i] += cnt[i-1];
9     static int temp;
10    for(int i = n-1; i >= 0; --i){
11        temp = a[i] & 32767;
12        --cnt[temp];
13        tmpa[cnt[temp]] = a[i];
14    }
15
16    static int cnt2[32768] = {0};
17    for(int i = 0; i < n; ++i)
18        ++cnt2[(tmpa[i]>>15)];
19    for(int i = 1; i < 32768; ++i)
20        cnt2[i] += cnt2[i-1];
21
22    for(int i = n-1; i >= 0; --i){
23        temp = (tmpa[i]>>15);
24        --cnt2[temp];
25        a[cnt2[temp]] = tmpa[i];
26    }
27    return;
28 }

```

1.8 Xor Basis

```

1 vector<int> basis;
2 void add_vector(int x){
3     for (auto v : basis){
4         x=min(x, x^v);
5     }
6     if (x) basis.push_back(x);
7 }
8
9 // 給一數字集合 S，求能不能 XOR 出 x
10 bool check(int x){
11     for (auto v : basis){
12         x=min(x, x^v);
13     }
14     return x;
15 }
16
17 // 給一數字集合 S，求能 XOR 出多少數字
18 // 答案等於  $2^{\{basis\} \text{ 的大小}}$ 
19
20 // 給一數字集合 S，求 XOR 出最大的數字
21 int get_max(){
22     int ans=0;
23     for (auto v : basis){
24         ans=max(ans, ans^v);
25     }
26     return ans;
27 }

```

1.9 random int

```

1 mt19937 seed(chrono::steady_clock::now().time_since_epoch().
2     count());
3 int rng(int l, int r){
4     return uniform_int_distribution<int>(l, r)(seed);
5 }

```

1.10 run

```

1 import os
2 p = os.listdir(".")
3 f = input("input: ")
4
5 if os.system(f"g++ {f}.cpp -std=c++17 -Wall -Wextra -Wshadow
6     -O2 -DLOCAL -g -fsanitize=undefined,address -o {f}") !=
7     0:
8     print("CE")
9     exit(1)
10
11 for x in p:
12     if x[:len(f)]==f and x[-3:]!=".in":
13         print(x)
14         if os.system(f"./{f} < {x}")!=0:
15             print("RE")
16             exit(1)
17         print()

```

1.11 setup

```

1 se nu rnu bs=2 sw=4 ts=4 hls ls=2 si acd bo=all mouse=a
2
3 :inoremap " ""<Esc>i
4 :inoremap {<CR> {<CR><Esc>ko
5 :inoremap [{ {<ESC>i
6
7 function! F(...)
8     execute '!./%:r < ./' . a:1
9 endfunction
10 command! -nargs=* R call F(<f-args>)
11
12 map <F7> :w<bar>!g++ "%" -o %:r -std=c++17 -Wall -Wextra -
13     Wshadow -O2 -DLOCAL -g -fsanitize=undefined,address<CR>
14 map <F8> :!./%:r<CR>
15 map <F9> :!./%:r < ./%:r.in<CR>
16
17 ca hash w !cpp -dD -P -fpreprocessed \\\ tr -d "[:space:]" \\\
18     md5sum \\\ cut -c-6
19
20 " i+<esc>25A---<esc>
21 " o|<esc>25A |<esc>
22 " ggVGyG35pGdd

```

2 Convolution

2.1 FFT any mod

```

1 /*
2 修改 const int MOD = 998244353 更改要取餘的數字
3 PolyMul(a, b) 回傳多項式乘法的結果 (c_k = \sum_{i+j=k} a_i b_j
4     mod MOD)
5
6 大約可以支援  $5e5 \cdot a_i, b_i$  皆在 MOD 以下的非負整數
7 */
8 const int MOD = 998244353;

```

```

8 typedef complex<double> cd;
9
10 // b9c90a
11 void FFT(vector<cd> &a) {
12     int n = a.size(), L = 31-__builtin_clz(n);
13     vector<complex<long double>> R(2, 1);
14     vector<cd> rt(2, 1);
15     for (int k=2; k<n; k*=2){
16         R.resize(n);
17         rt.resize(n);
18         auto x = polar(1.0L, acos(-1.0L) / k);
19         for (int i=k; i<2*k; i++){
20             rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
21         }
22     }
23
24     vector<int> rev(n);
25     for (int i=0; i<n; i++){
26         rev[i] = (rev[i/2] | (i&1)<<L)/2;
27     }
28     for (int i=0; i<n; i++){
29         if (i<rev[i]) swap(a[i], a[rev[i]]);
30     }
31     for (int k=1; k<n; k*=2){
32         for (int i=0; i<n; i+=2*k){
33             for (int j=0; j<k; j++){
34                 auto x = (double *)&rt[j+k];
35                 auto y = (double *)&a[i+j+k];
36                 cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
37                 a[i+j+k] = a[i+j]-z;
38                 a[i+j] += z;
39             }
40         }
41     }
42     return;
43 }
44
45 // d3c65e
46 vector<int> PolyMul(vector<int> a, vector<int> b){
47     if (a.empty() || b.empty()) return {};
48
49     vector<int> res(a.size()+b.size()-1);
50     int B = 32-__builtin_clz(res.size()), n = (1<<B), cut =
51         int(sqrt(MOD));
52     vector<cd> L(n), R(n), outs(n), outl(n);
53
54     for (int i=0; i<a.size(); i++){
55         L[i] = cd((int) a[i]/cut, (int)a[i]%cut);
56     }
57     for (int i=0; i<b.size(); i++){
58         R[i] = cd((int) b[i]/cut, (int)b[i]%cut);
59     }
60     FFT(L);
61     FFT(R);
62     for (int i=0; i<n; i++){
63         int j = -i&(n-1);
64         outl[j] = (L[i]+conj(L[j])) * R[i]/(2.0*n);
65         outs[j] = (L[i]-conj(L[j])) * R[i]/(2.0*n)/1i;
66     }
67     FFT(outl);
68     FFT(outs);
69     for (int i=0; i<res.size(); i++){
70         int av = (int)(real(outl[i])+0.5), cv = (int)(imag(
71             outs[i])+0.5);

```

2.2 FFT new

```

1 typedef complex<double> cd;
2
3 void FFT(vector<cd> &a) {
4     int n = a.size(), L = 31-__builtin_clz(n);
5     vector<complex<long double>> R(2, 1);
6     vector<cd> rt(2, 1);
7     for (int k=2; k<n; k*=2){
8         R.resize(n);
9         rt.resize(n);
10        auto x = polar(1.0L, acos(-1.0L) / k);
11        for (int i=k; i<2*k; i++){
12            rt[i] = R[i] = (i&1 ? R[i/2]*x : R[i/2]);
13        }
14    }
15
16    vector<int> rev(n);
17    for (int i=0; i<n; i++){
18        rev[i] = (rev[i/2] | (i&1)<<L)/2;
19    }
20    for (int i=0; i<n; i++){
21        if (i<rev[i]) swap(a[i], a[rev[i]]);
22    }
23    for (int k=1; k<n; k*=2){
24        for (int i=0; i<n; i+=2*k){
25            for (int j=0; j<k; j++){
26                auto x = (double *)&rt[j+k];
27                auto y = (double *)&a[i+j+k];
28                cd z(x[0]*y[0] - x[1]*y[1], x[0]*y[1] + x[1]*y[0]);
29                a[i+j+k] = a[i+j]-z;
30                a[i+j] += z;
31            }
32        }
33    }
34    return;
35 }
36
37 vector<double> PolyMul(const vector<double> a, const vector<
38     double> b){
39     if (a.empty() || b.empty()) return {};
40     vector<double> res(a.size()+b.size()-1);
41     int L = 32 - __builtin_clz(res.size()), n = 1 << L;
42     vector<cd> in(n), out(n);
43
44     copy(a.begin(), a.end(), begin(in));
45     for (int i=0; i<b.size(); i++){
46         in[i].imag(b[i]);
47     }
48     FFT(in);
49     for (cd& x : in) x *= x;
50     for (int i=0; i<n; i++){
51         out[i] = in[-i & (n - 1)] - conj(in[i]);

```

```

52     FFT(out);
53
54     for (int i=0; i<res.size(); i++){
55         res[i] = imag(out[i]) / (4 * n);
56     }
57
58     return res;
59 }

```

2.3 FFT old

```

1 typedef complex<double> cd;
2 const double PI = acos(-1);
3
4 void FFT(vector<cd> &a, bool inv){
5
6     int n = a.size();
7
8     for (int i=1, j=0; i<n; i++){
9         int bit = (n>>1);
10        for (; j<bit; bit>=1){
11            j ^= bit;
12        }
13        j ^= bit;
14        if (i<j){
15            swap(a[i], a[j]);
16        }
17    }
18
19    for (int len=2; len<=n; len<=1){
20        cd wlen = polar(1.0, (inv ? 2 : -2)*PI/len);
21
22        for (int i=0; i<n; i+=len){
23            cd w(1);
24            for (int j=0; j<len/2; j++){
25                cd u = a[i+j];
26                cd v = a[i+j+len/2]*w;
27                a[i+j] = u+v;
28                a[i+j+len/2] = u-v;
29                w *= wlen;
30            }
31        }
32    }
33
34    if (inv){
35        for (auto &x : a){
36            x /= n;
37        }
38    }
39
40    return;
41 }
42
43 vector<cd> polyMul(vector<cd> a, vector<cd> b){
44     int sa = a.size(), sb = b.size(), n = 1;
45
46     while (n<sa+sb-1) n *= 2;
47     a.resize(n);
48     b.resize(n);
49     vector<cd> c(n);
50
51     FFT(a, 0);
52     FFT(b, 0);

```

```

53 for (int i=0 ; i<n ; i++) c[i] = a[i]*b[i];
54 FFT(c, 1);
55
56 c.resize(sa+sb-1);
57
58 return c;
59 }

```

2.4 NTT mod 998244353

```

1 const int MOD = (119 << 23) + 1, ROOT = 62; // = 998244353
2 // For p < 2^30 there is also e.g. 5 << 25, 7 << 26, 479 <<
3 // 21
4 // and 483 << 21 (same root). The last two are > 10^9.
5 // 9cd58a
6 void NTT(vector<int> &a) {
7     int n = a.size();
8     int L = 31-__builtin_clz(n);
9     vector<int> rt(2, 1);
10    for (int k=2, s=2 ; k<n ; k*=2, s++){
11        rt.resize(n);
12        int z[] = {1, qp(ROOT, MOD>>s)};
13        for (int i=k ; i<2*k ; i++){
14            rt[i] = rt[i/2]*z[i&1]%MOD;
15        }
16    }
17
18    vector<int> rev(n);
19    for (int i=0 ; i<n ; i++){
20        rev[i] = (rev[i/2]|(i&1)<<L)/2;
21    }
22    for (int i=0 ; i<n ; i++){
23        if (i<rev[i]){
24            swap(a[i], a[rev[i]]);
25        }
26    }
27
28    for (int k=1 ; k<n ; k*=2){
29        for (int i=0 ; i<n ; i+=2*k){
30            for (int j=0 ; j<k ; j++){
31                int z = rt[j+k]*a[i+j+k]%MOD, &ai = a[i+j];
32                a[i+j+k] = ai-z+(z>ai ? MOD : 0);
33                ai += (ai+z)>MOD ? z-MOD : z;
34            }
35        }
36    }
37 }
38
39 // 0b0e99
40 vector<int> polyMul(vector<int> &a, vector<int> &b){
41     if (a.empty() || b.empty()) return {};
42     int s = a.size()+b.size()-1, B = 32-__builtin_clz(s), n =
43         1<<B;
44     int inv = qp(n, MOD-2);
45
46     vector<int> L(a), R(b), out(n);
47     L.resize(n), R.resize(n);
48     NTT(L), NTT(R);
49     for (int i=0 ; i<n ; i++){
50         out[-i&(n-1)] = L[i]*R[i]%MOD*inv%MOD;
51     }
52     NTT(out);

```

```

52
53 out.resize(s);
54 return out;
55 }

```

3 Data-Structure

3.1 BIT

```

1 vector<int> BIT(MAX_SIZE);
2 void update(int pos, int val){
3     for (int i=pos ; i<MAX_SIZE ; i+=i&-i){
4         BIT[i]+=val;
5     }
6 }
7
8 int query(int pos){
9     int ret=0;
10    for (int i=pos ; i>0 ; i-=i&-i){
11        ret+=BIT[i];
12    }
13    return ret;
14 }
15
16 // const int MAX_N = (1<<20)
17 int k_th(int k){ // 回傳 BIT 中第 k 小的元素 (based-1)
18     int res = 0;
19     for (int i=MAX_N>>1 ; i>=1 ; i>=>1)
20         if (bit[res+i]<k)
21             k -= bit[res+i];
22     return res+1;
23 }

```

3.2 Disjoint Set Persistent

```

1 struct Persistent_Disjoint_Set{
2     Persistent_Segment_Tree arr, sz;
3
4     void init(int n){
5         arr.init(n);
6         vector<int> v1;
7         for (int i=0 ; i<n ; i++){
8             v1.push_back(i);
9         }
10        arr.build(v1, 0);
11
12        sz.init(n);
13        vector<int> v2;
14        for (int i=0 ; i<n ; i++){
15            v2.push_back(1);
16        }
17        sz.build(v2, 0);
18    }
19
20    int find(int a){
21        int res = arr.query_version(a, a+1, arr.version.size()
22            (-1).val;
23        if (res==a) return a;

```

```

23         return find(res);
24     }
25
26     bool unite(int a, int b){
27         a = find(a);
28         b = find(b);
29
30         if (a!=b){
31
32             int sz1 = sz.query_version(a, a+1, arr.version.
33                 size()-1).val;
34             int sz2 = sz.query_version(b, b+1, arr.version.
35                 size()-1).val;
36
37             if (sz1<sz2){
38                 arr.update_version(a, b, arr.version.size()
39                     -1);
40                 sz.update_version(b, sz1+sz2, arr.version.
41                     size()-1);
42             }else{
43                 arr.update_version(b, a, arr.version.size()
44                     -1);
45                 sz.update_version(a, sz1+sz2, arr.version.
46                     size()-1);
47             }
48             return true;
49         }
50         return false;
51     }
52 }

```

3.3 PBDS GP Hash Table

```

1 #include <ext/pb_ds/assoc_container.hpp>
2 using namespace __gnu_pbds;
3 typedef tree<int, null_type, less<int>, rb_tree_tag,
4     tree_order_statistics_node_update> order_set;
5 struct custom_hash {
6     static uint64_t splitmix64(uint64_t x) {
7         // http://xorshift.di.unimi.it/splitmix64.c
8         x += 0x9e3779b97f4a7c15;
9         x = (x ^ (x >> 30)) * 0xbf58476d1ce4e5b9;
10        x = (x ^ (x >> 27)) * 0x94d049bb133111eb;
11        return x ^ (x >> 31);
12    }
13
14    size_t operator()(uint64_t x) const {
15        static const uint64_t FIXED_RANDOM = chrono::
16            steady_clock::now().time_since_epoch().count();
17        return splitmix64(x + FIXED_RANDOM);
18    }
19 };
20 gp_hash_table<int, int, custom_hash> ss;

```

3.4 PBDS Order Set

```

1 /*
2 .find_by_order(k) 回傳第 k 小的值 (based-0)

```

```

3 | .order_of_key(k) 回傳有多少元素比 k 小
4 | 不能在 #define int Long Long 後 #include 檔案
5 | */
6 |
7 | #include <ext/pb_ds/assoc_container.hpp>
8 | #include <ext/pb_ds/tree_policy.hpp>
9 | using namespace __gnu_pbds;
10 | typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> order_set;

```

3.5 Segment Tree Add Set

```

1 | // [ll, rr), based-0
2 | // 使用前記得 init(陣列大小), build(陣列名稱)
3 | // add(ll, rr): 區間修改
4 | // set(ll, rr): 區間賦值
5 | // query(ll, rr): 區間求和 / 求最大值
6 | struct SegmentTree{
7 |     struct node{
8 |         int add_tag = 0;
9 |         int set_tag = 0;
10 |        int sum = 0;
11 |        int ma = 0;
12 |    };
13 |
14 |    vector<node> arr;
15 |
16 |    SegmentTree(int n){
17 |        arr.resize(n<<2);
18 |    }
19 |
20 |    node pull(node A, node B){
21 |        node C;
22 |        C.sum = A.sum+B.sum;
23 |        C.ma = max(A.ma, B.ma);
24 |        return C;
25 |    }
26 |
27 |    // cce0c8
28 |    void push(int idx, int ll, int rr){
29 |        if (arr[idx].set_tag!=0){
30 |            arr[idx].sum = (rr-ll)*arr[idx].set_tag;
31 |            arr[idx].ma = arr[idx].set_tag;
32 |            if (rr-ll>1){
33 |                arr[idx*2+1].add_tag = 0;
34 |                arr[idx*2+1].set_tag = arr[idx].set_tag;
35 |                arr[idx*2+2].add_tag = 0;
36 |                arr[idx*2+2].set_tag = arr[idx].set_tag;
37 |            }
38 |            arr[idx].set_tag = 0;
39 |        }
40 |        if (arr[idx].add_tag!=0){
41 |            arr[idx].sum += (rr-ll)*arr[idx].add_tag;
42 |            arr[idx].ma += arr[idx].add_tag;
43 |            if (rr-ll>1){
44 |                arr[idx*2+1].add_tag += arr[idx].add_tag;
45 |                arr[idx*2+2].add_tag += arr[idx].add_tag;
46 |            }
47 |            arr[idx].add_tag = 0;
48 |        }
49 |    }
50 | }

```

```

51 | void build(vector<int> &v, int idx = 0, int ll = 0, int
    rr = n){
52 |     if (rr-ll==1){
53 |         arr[idx].sum = v[ll];
54 |         arr[idx].ma = v[ll];
55 |     }else{
56 |         int mid = (ll+rr)/2;
57 |         build(v, idx*2+1, ll, mid);
58 |         build(v, idx*2+2, mid, rr);
59 |         arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
60 |     }
61 | }
62 |
63 | void add(int ql, int qr, int val, int idx = 0, int ll =
    0, int rr = n){
64 |     push(idx, ll, rr);
65 |     if (rr<=ql || qr<=ll) return;
66 |     if (ql<=ll && rr<=qr){
67 |         arr[idx].add_tag += val;
68 |         push(idx, ll, rr);
69 |         return;
70 |     }
71 |     int mid = (ll+rr)/2;
72 |     add(ql, qr, val, idx*2+1, ll, mid);
73 |     add(ql, qr, val, idx*2+2, mid, rr);
74 |     arr[idx]=pull(arr[idx*2+1], arr[idx*2+2]);
75 | }
76 |
77 | void set(int ql, int qr, int val, int idx=0, int ll=0,
    int rr=n){
78 |     push(idx, ll, rr);
79 |     if (rr<=ql || qr<=ll) return;
80 |     if (ql<=ll && rr<=qr){
81 |         arr[idx].add_tag = 0;
82 |         arr[idx].set_tag = val;
83 |         push(idx, ll, rr);
84 |         return;
85 |     }
86 |     int mid = (ll+rr)/2;
87 |     set(ql, qr, val, idx*2+1, ll, mid);
88 |     set(ql, qr, val, idx*2+2, mid, rr);
89 |     arr[idx] = pull(arr[idx*2+1], arr[idx*2+2]);
90 | }
91 |
92 | node query(int ql, int qr, int idx = 0, int ll = 0, int
    rr = n){
93 |     push(idx, ll, rr);
94 |     if (rr<=ql || qr<=ll) return node();
95 |     if (ql<=ll && rr<=qr) return arr[idx];
96 |
97 |     int mid = (ll+rr)/2;
98 |     return pull(query(ql, qr, idx*2+1, ll, mid), query(ql
    , qr, idx*2+2, mid, rr));
99 | }
100 | } ST;

```

3.6 Segment Tree Li Chao

```

1 | /*
2 | 全部都是 0-based
3 |
4 | 宣告

```

```

5 | LC_Segment_Tree st(n);
6 |
7 | 函式：
8 | update(val)：將一個 pair <a, b> 代表插入一條 y=ax+b 的直線
9 | query(x)：查詢所有直線在位置 x 的最小值
10 | */
11 | const int MAX_V = 1e6+10; // 值域最大值
12 |
13 | struct LC_Segment_Tree{
14 |     struct Node{ // y = ax+b
15 |         int a = 0;
16 |         int b = INF;
17 |
18 |         int y(int x){
19 |             return a*x+b;
20 |         }
21 |     };
22 |     vector<Node> arr;
23 |
24 |     LC_Segment_Tree(int n = 0){
25 |         arr.resize(4*n);
26 |     }
27 |
28 |     void update(Node val, int idx = 0, int ll = 0, int rr =
        MAX_V){
29 |         if (rr-ll==1){
30 |             if (val.y(ll)<arr[idx].y(ll)){
31 |                 arr[idx] = val;
32 |             }
33 |             return;
34 |         }
35 |
36 |         int mid = (ll+rr)/2;
37 |         if (arr[idx].a > val.a) swap(arr[idx], val); // 原本
            的線斜率要比較小
38 |         if (arr[idx].y(mid) < val.y(mid)){ // 交點在左邊
39 |             update(val, idx*2+1, ll, mid);
40 |         }else{ // 交點在右邊
41 |             swap(arr[idx], val); // 在左子樹中，新線比舊線還
                要好
42 |             update(val, idx*2+2, mid, rr);
43 |         }
44 |         return;
45 |     }
46 |
47 |     int query(int x, int idx = 0, int ll = 0, int rr = MAX_V)
        {
48 |         if (rr-ll==1){
49 |             return arr[idx].y(ll);
50 |         }
51 |
52 |         int mid = (ll+rr)/2;
53 |         if (x<mid){
54 |             return min(arr[idx].y(x), query(x, idx*2+1, ll,
                mid));
55 |         }else{
56 |             return min(arr[idx].y(x), query(x, idx*2+2, mid,
                rr));
57 |         }
58 |     }
59 | };

```

3.7 Segment Tree Persistent

```

1  /*
2  全部都是 0-based
3
4  宣告
5  Persistent_Segment_Tree st(n+q);
6  st.build(v, 0);
7
8  函式：
9  update_version(pos, val, ver) : 對版本 ver 的 pos 位置改成 val
10 query_version(ql, qr, ver) : 對版本 ver 查詢 [ql, qr) 的區間和
11 clone_version(ver) : 複製版本 ver 到最新的版本
12 */
13 struct Persistent_Segment_Tree{
14     int node_cnt = 0;
15     struct Node{
16         int lc = -1;
17         int rc = -1;
18         int val = 0;
19     };
20     vector<Node> arr;
21     vector<int> version;
22
23     Persistent_Segment_Tree(int sz){
24         arr.resize(32*sz);
25         version.push_back(node_cnt++);
26         return;
27     }
28
29     void pull(Node &c, Node a, Node b){
30         c.val = a.val+b.val;
31         return;
32     }
33
34     void build(vector<int> &v, int idx, int ll = 0, int rr =
35         n){
36         auto &now = arr[idx];
37
38         if (rr-ll==1){
39             now.val = v[ll];
40             return;
41         }
42
43         int mid = (ll+rr)/2;
44         now.lc = node_cnt++;
45         now.rc = node_cnt++;
46         build(v, now.lc, ll, mid);
47         build(v, now.rc, mid, rr);
48         pull(now, arr[now.lc], arr[now.rc]);
49         return;
50     }
51
52     void update(int pos, int val, int idx, int ll = 0, int rr
53         = n){
54         auto &now = arr[idx];
55
56         if (rr-ll==1){
57             now.val = val;
58             return;
59         }
60
61         int mid = (ll+rr)/2;
62         if (pos<mid){

```

```

61         arr[node_cnt] = arr[now.lc];
62         now.lc = node_cnt;
63         node_cnt++;
64         update(pos, val, now.lc, ll, mid);
65     }else{
66         arr[node_cnt] = arr[now.rc];
67         now.rc = node_cnt;
68         node_cnt++;
69         update(pos, val, now.rc, mid, rr);
70     }
71     pull(now, arr[now.lc], arr[now.rc]);
72     return;
73 }
74
75 void update_version(int pos, int val, int ver){
76     update(pos, val, version[ver]);
77 }
78
79 Node query(int ql, int qr, int idx, int ll = 0, int rr =
80     n){
81     auto &now = arr[idx];
82
83     if (ql<=ll && rr<=qr) return now;
84     if (rr<=ql || qr<=ll) return Node();
85
86     int mid = (ll+rr)/2;
87
88     Node ret;
89     pull(ret, query(ql, qr, now.lc, ll, mid), query(ql,
90         qr, now.rc, mid, rr));
91     return ret;
92 }
93
94 Node query_version(int ql, int qr, int ver){
95     return query(ql, qr, version[ver]);
96 }
97
98 void clone_version(int ver){
99     version.push_back(node_cnt);
100     arr[node_cnt] = arr[version[ver]];
101     node_cnt++;
102 }
103
104 };

```

3.8 Sparse Table

```

1 struct SparseTable{
2     vector<vector<int>> st;
3     void build(vector<int> v){
4         int h = __lg(v.size());
5         st.resize(h+1);
6         st[0] = v;
7
8         for (int i=1 ; i<=h ; i++){
9             int gap = (1<<(i-1));
10             for (int j=0 ; j+gap<st[i-1].size() ; j++){
11                 st[i].push_back(min(st[i-1][j], st[i-1][j+gap
12                     ]));
13             }
14         }
15     }
16
17     // 回傳 [ll, rr) 的最小值

```

```

17     int query(int ll, int rr){
18         int h = __lg(rr-ll);
19         return min(st[h][ll], st[h][rr-(1<<h)]);
20     }
21 };

```

3.9 Treap

```

1 struct Treap{
2     Treap *l = nullptr, *r = nullptr;
3     int pri = rand(), val = 0, sz = 1;
4
5     Treap(int _val){
6         val = _val;
7     }
8 };
9
10 int size(Treap *t){return t ? t->sz : 0;}
11 void pull(Treap *t){
12     t->sz = size(t->l)+size(t->r)+1;
13 }
14
15 Treap* merge(Treap *a, Treap *b){
16     if (!a || !b) return a ? a : b;
17
18     if (a->pri>b->pri){
19         a->r = merge(a->r, b);
20         pull(a);
21         return a;
22     }else{
23         b->l = merge(a, b->l);
24         pull(b);
25         return b;
26     }
27 }
28
29 pair<Treap*, Treap*> split(Treap *t, int k){ // 1-based <前
30     k 個元素, 其他元素>
31     if (!t) return {};
32     if (size(t->l)>=k){
33         auto pa = split(t->l, k);
34         t->l = pa.second;
35         pull(t);
36         return {pa.first, t};
37     }else{
38         auto pa = split(t->r, k-size(t->l)-1);
39         t->r = pa.first;
40         pull(t);
41         return {t, pa.second};
42     }
43 }
44
45 // functions
46 Treap* build(vector<int> v){
47     Treap* ret;
48     for (int i=0 ; i<SZ(v) ; i++){
49         ret = merge(ret, new Treap(v[i]));
50     }
51     return ret;
52 }
53

```



```

54 array<Treap*, 3> cut(Treap *t, int l, int r){ // 1-based <前
    1~l-1 個元素, l~r 個元素, r+1 個元素>
55     array<Treap*, 3> ret;
56     tie(ret[1], ret[2]) = split(t, r);
57     tie(ret[0], ret[1]) = split(ret[1], l-1);
58     return ret;
59 }
60
61 void print(Treap *t, bool flag = true){
62     if (t->l!=0) print(t->l, false);
63     cout << t->val;
64     if (t->r!=0) print(t->r, false);
65     if (flag) cout << endl;
66 }

```

3.10 Trie

```

1 struct Trie{
2     struct Data{
3         int nxt[2]={0, 0};
4     };
5
6     int sz=0;
7     vector<Data> arr;
8
9     void init(int n){
10         arr.resize(n);
11     }
12
13     void insert(int n){
14         int now=0;
15         for (int i=N; i>=0; i--){
16             int v=(n>>i)&1;
17             if (!arr[now].nxt[v]){
18                 arr[now].nxt[v]=++sz;
19             }
20             now=arr[now].nxt[v];
21         }
22     }
23
24     int query(int n){
25         int now=0, ret=0;
26         for (int i=N; i>=0; i--){
27             int v=(n>>i)&1;
28             if (arr[now].nxt[1-v]){
29                 ret+=(1<<i);
30                 now=arr[now].nxt[1-v];
31             }else if (arr[now].nxt[v]){
32                 now=arr[now].nxt[v];
33             }else{
34                 return ret;
35             }
36         }
37         return ret;
38     }
39 } tr;
40

```

4 Dynamic-Programming

4.1 Digit DP

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 long long l, r;
5 long long dp[20][10][2][2]; // dp[pos][pre][limit] = 後 pos
    位 · pos 前一位是 pre · (是/否) 有上界 · (是/否) 有前綴零
    的答案數量
6
7 long long memorize_search(string &s, int pos, int pre, bool
    limit, bool lead){
8
9     // 已經被找過了 · 直接回傳值
10    if (dp[pos][pre][limit][lead]!=-1) return dp[pos][pre][
    limit][lead];
11
12    // 已經搜尋完畢 · 紀錄答案並回傳
13    if (pos==(int)s.size()){
14        return dp[pos][pre][limit][lead] = 1;
15    }
16
17    // 枚舉目前的位數數字是多少
18    long long ans = 0;
19    for (int now=0; now<=(limit ? s[pos]-'0' : 9); now++){
20        if (now==pre){
21
22            // 1~9 絕對不能連續出現
23            if (pre!=0) continue;
24
25            // 如果已經不在前綴零的範圍內 · 0 不能連續出現
26            if (lead==false) continue;
27
28            ans += memorize_search(s, pos+1, now, limit&(now==(s[
    pos]-'0')), lead&(now==0));
29        }
30    }
31
32    // 已經搜尋完畢 · 紀錄答案並回傳
33    return dp[pos][pre][limit][lead] = ans;
34 }
35
36 // 回傳 [0, n] 有多少數字符合條件
37 long long find_answer(long long n){
38     memset(dp, -1, sizeof(dp));
39     string tmp = to_string(n);
40
41     return memorize_search(tmp, 0, 0, true, true);
42 }
43
44 int main(){
45
46     // input
47     cin >> l >> r;
48
49     // output - 計算 [l, r] 有多少數字任意兩個位數都不相同
50     cout << find_answer(r)-find_answer(l-1) << "\n";
51
52     return 0;
53 }

```

4.2 SOS DP

```

1 // 總時間複雜度為 O(n 2^n)
2 // 計算 dp[i] = i 所有 bit mask 子集的和
3 for (int i=0; i<n; i++){
4     for (int mask=0; mask<(1<<n); mask++){
5         if ((mask>>i)&1){
6             dp[mask] += dp[mask^(1<<i)];
7         }
8     }
9 }

```

4.3 Integer Partition

$dp[i][x]$ = 要將整數 x 拆成 i 堆的「組合數」

$dp[i+1][x+1] += dp[i][x]$ (創造新的一堆)
 $dp[i][x+i] += dp[i][x]$ (把每一堆都增加 1)

5 Geometry

5.1 Geometry Struct

```

1 // 判斷數值正負：{1:正數,0:零,-1:負數}
2 int sign(long long x) {return (x >= 0) ? ((bool)x) : -1; }
3 int sign(double x) {
4     return (abs(x) < 1e-9) ? 0 : (x > 0 ? 1 : -1);
5 }
6
7 template<typename T>
8 struct point {
9     T x, y;
10    point() {}
11    point(const T &x, const T &y) : x(x), y(y) {}
12
13    point operator+(point b) {return {x+b.x, y+b.y}; }
14    point operator-(point b) {return {x-b.x, y-b.y}; }
15    point operator*(T b) {return {x*b, y*b}; }
16    point operator/(T b) {return {x/b, y/b}; }
17    bool operator==(point b) {return x==b.x && y==b.y; }
18    // 逆時針極角排序
19    bool operator<(point &b) {return (x*b.y > b.x*y); }
20    friend ostream& operator<<(ostream& os, point p) {
21        os << "(" << p.x << ", " << p.y << ")";
22        return os;
23    }
24
25    // 判斷 ab 到 ac 的方向：{1:逆時鐘,0:重疊,-1:順時鐘}
26    friend int ori(point a, point b, point c) {
27        return sign((b-a)^(c-a));
28    }
29
30    friend int btw(point a, point b, point c) {
31        return ori(a, b, c) == 0 && sign((a-c)*(b-c)) <= 0;
32    }
33
34    // 判斷線段 ab, cd 是否相交
35    friend bool banana(point a, point b, point c, point d) {
36        int s1 = ori(a, b, c);
37        int s2 = ori(a, b, d);
38        int s3 = ori(c, d, a);
39

```



```

36     int s4 = ori(c, d, b);
37     if (btw(a, b, c) || btw(a, b, d) || btw(c, d, a) ||
        btw(c, d, b)) return 1;
38     return (s1 * s2 < 0) && (s3 * s4 < 0);
39 }
40
41 T operator*(point b) {return x * b.x + y * b.y; }
42 T operator^(point b) {return x * b.y - y * b.x; }
43 T abs2() {return (*this) * (*this); }
44
45 // 旋轉 Arg(b) 的角度 (小心溢位)
46 point rotate(point b) {return {x*b.x - y*b.y, x*b.y + y*b
    .x}; }
47 };
48
49 template<typename T>
50 struct line {
51     point<T> p1, p2;
52     // ax + by + c = 0
53     T a, b, c; // |a|, |b| ≤ 2C, |c| ≤ 8C²
54     line() {}
55     line(const point<T> &x, const point<T> &y) : p1(x), p2(y){
56         build();
57     }
58     void build() {
59         a = p1.y - p2.y;
60         b = p2.x - p1.x;
61         c = (-a*p1.x) - b*p1.y;
62     }
63     // 判斷點和有向直線的關係：{1:左邊,0:在線上,-1:右邊}
64     int ori(point<T> &p) {
65         return sign((p2-p1) ^ (p-p1));
66     }
67     // 判斷直線斜率是否相同
68     bool parallel(line &l) {
69         return ((p1-p2) ^ (l.p1-l.p2)) == 0;
70     }
71     // 兩直線交點
72     point<long double> line_intersection(line &l) {
73         using P = point<long double>;
74         point<T> a = p2-p1, b = l.p2-l.p1, s = l.p1-p1;
75         return P(p1.x, p1.y) + P(a.x, a.y) * (((long double)(s^b))
            / (a^b));
76     }
77 };
78
79 template<typename T>
80 struct polygon {
81     vector<point<T>> v;
82     polygon() {}
83     polygon(const vector<point<T>> &u) : v(u) {}
84     // simple 為 true 的時候會回傳任意三點不共線的凸包
85     void make_convex_hull(int simple) {
86         auto cmp = [&](point<T> &p, point<T> &q) {
87             return (p.x == q.x) ? (p.y < q.y) : (p.x < q.x);
88         };
89         simple = (bool)simple;
90         sort(v.begin(), v.end(), cmp);
91         v.resize(unique(v.begin(), v.end()) - v.begin());
92         vector<point<T>> hull;
93         for (int t = 0; t < 2; ++t){
94             int sz = hull.size();
95             for (auto &i:v) {
96                 while (hull.size() >= sz+2 && ori(hull[hull.
                    size()-2], hull.back(), i) < simple) {

```

```

97                 hull.pop_back();
98             }
99             hull.push_back(i);
100         }
101         hull.pop_back();
102         reverse(v.begin(), v.end());
103     }
104     swap(hull, v);
105 }
106 // 可以在有 n 個點的簡單多邊形內·用 O(n) 判斷一個點：
107 // {1：在多邊形內，0：在多邊形上，-1：在多邊形外}
108 int in_polygon(point<T> a){
109     const T MAX_POS = 1e9 + 5; // [記得修改] 座標的最大值
110     point<T> pre = v.back(), b(MAX_POS, a.y + 1);
111     int cnt = 0;
112
113     for (auto &i:v) {
114         if (btw(pre, i, a)) return 0;
115         if (banana(a, b, pre, i)) cnt++;
116         pre = i;
117     }
118
119     return cnt%2 ? 1 : -1;
120 }
121 // 警告：所有凸包專用的函式都只接受逆時針排序且任三點不共線
    的凸包 ///
122 // 可以在有 n 個點的凸包內·用 O(Log n) 判斷一個點：
123 // {1：在凸包內，0：在凸包邊上，-1：在凸包外}
124 int in_convex(point<T> p) {
125     int n = v.size();
126     int a = ori(v[0], v[1], p), b = ori(v[0], v[n-1], p);
127     if (a < 0 || b > 0) return -1;
128     if (btw(v[0], v[1], p)) return 0;
129     if (btw(v[0], v[n-1], p)) return 0;
130     int l = 1, r = n - 1, mid;
131     while (l + 1 < r) {
132         mid = (l + r) >> 1;
133         if (ori(v[0], v[mid], p) >= 0) l = mid;
134         else r = mid;
135     }
136     int k = ori(v[l], v[r], p);
137     if (k <= 0) return k;
138     return 1;
139 }
140 // 凸包專用的環狀二分搜·回傳 0-based index
141 int cycle_search(auto &f) {
142     int i = 0, n = v.size();
143     for (int j = 1 << __lg(n); j > 0; j >= 1) {
144         int nxt = (i + j) % n;
145         for (int k = 0; k < 2; ++k) {
146             if (f(i, nxt)) {
147                 i = nxt;
148                 break;
149             }
150             nxt = (i + n - j) % n;
151         }
152     }
153     return i;
154 }
155 // 可以在有 n 個點的凸包內·用 O(Log n) 判斷一條直線：
156 // {1：穿過凸包，0：剛好切過凸包，-1：沒碰到凸包}
157 int line_cut_convex(line<T> p) {
158     /// TO DO
159 }

```

```

160 int segment_cut_convex(line<T> p) {
161     /// TO DO
162 }
163 // 回傳點過凸包的兩條切線的切點的 0-based index
164 pair<int,int> convex_tangent_point(point<T> p) {
165     auto gt = [&](int neg) {
166         auto f = [&](int x, int y) {
167             return ori(p, v[x], v[y]) == neg;
168         };
169         return cycle_search(f);
170     };
171     int x = gt(1), y = gt(-1), n = v.size();
172     int z = (v[x] == p) ? x : y;
173     if (v[z] == p) {
174         return {(z + n - 1) % n, (z + 1) % n};
175     }
176     else {
177         return {x, y};
178     }
179 }
180 friend int halfplane_intersection(vector<line<T>> &s,
    polygon<T> &P) {
181     #define neg(p) ((p.y == 0 ? p.x : p.y) < 0)
182     auto angle_cmp = [&](line<T> &A, line<T> &B) {
183         point<T> a = A.p2-A.p1, b = B.p2-B.p1;
184         return neg(a) < neg(b) || (neg(a) == neg(b) && (a
            ^b) > 0);
185     };
186     #undef neg
187     sort(s.begin(), s.end(), angle_cmp); // 線段左側為該
        線段半平面
188     int L, R, n = s.size();
189     vector<point<T>> px(n);
190     vector<line<T>> q(n);
191     q[L = R = 0] = s[0];
192     for(int i = 1; i < n; ++i) {
193         while(L < R && s[i].ori(px[R-1]) <= 0) --R;
194         while(L < R && s[i].ori(px[L]) <= 0) ++L;
195         q[++R] = s[i];
196         if(q[R].parallel(q[R-1])) {
197             --R;
198             if(q[R].ori(s[i].p1) > 0) q[R] = s[i];
199         }
200         if(L < R) px[R-1] = q[R-1].line_intersection(q[R
            ]));
201     }
202     while(L < R && q[L].ori(px[R-1]) <= 0) --R;
203     P.v.clear();
204     if(R - L <= 1) return 0;
205     px[R] = q[R].line_intersection(q[L]);
206     for(int i = L; i <= R; ++i) P.v.push_back(px[i]);
207     return R - L + 1;
208 }
209 };
210 /// TO DO : .svg maker

```

5.2 Geometry 卦長

```

1 const double PI=atan2(0.0,-1.0);
2 template<typename T>
3 struct point{
4     T x,y;

```

```

5 point(){}
6 point(const T&x, const T&y):x(x),y(y){}
7 point operator+(const point &b) const{
8     return point(x+b.x,y+b.y); }
9 point operator-(const point &b) const{
10    return point(x-b.x,y-b.y); }
11 point operator*(const T &b) const{
12    return point(x*b,y*b); }
13 point operator/(const T &b) const{
14    return point(x/b,y/b); }
15 bool operator==(const point &b) const{
16    return x==b.x&&y==b.y; }
17 T dot(const point &b) const{
18    return x*b.x+y*b.y; }
19 T cross(const point &b) const{
20    return x*b.y-y*b.x; }
21 point normal() const{//求法向量
22    return point(-y,x); }
23 T abs2() const{//向量長度的平方
24    return dot(*this); }
25 T rad(const point &b) const{//兩向量的弧度
26    return fabs(atan2(fabs(cross(b)),dot(b))); }
27 T getA() const{//對x軸的弧度
28    T A=atan2(y,x); //超過180度會變負的
29    if(A<=-PI/2) A+=PI*2;
30    return A;
31 }
32 };
33 template<typename T>
34 struct line{
35     line(){}
36     point<T> p1,p2;
37     T a,b,c;//ax+by+c=0
38     line(const point<T>&x, const point<T>&y):p1(x),p2(y){}
39     void pton() {//轉成一般式
40         a=p1.y-p2.y;
41         b=p2.x-p1.x;
42         c=-a*p1.x-b*p1.y;
43     }
44     T ori(const point<T> &p) const{//點和有向直線的關係・>0左
45         //邊・=0在線上<0右邊
46         return (p2-p1).cross(p-p1);
47     }
48     T btw(const point<T> &p) const{//點投影落在線段上<=0
49         return (p1-p).dot(p2-p);
50     }
51     bool point_on_segment(const point<T>&p) const{//點是否在線段
52         //上
53         return ori(p)==0&&btw(p)<=0;
54     }
55     T dis2(const point<T> &p, bool is_segment=0) const{//點跟直線
56         //線段的距離平方
57         point<T> v=p2-p1,v1=p-p1;
58         if(is_segment){
59             point<T> v2=p-p2;
60             if(v.dot(v1)<=0) return v1.abs2();
61             if(v.dot(v2)>=0) return v2.abs2();
62         }
63         T tmp=v.cross(v1);
64         return tmp*tmp/v.abs2();
65     }
66     T seg_dis2(const line<T> &l) const{//兩線段距離平方
67         return min({dis2(l.p1,1),dis2(l.p2,1),l.dis2(p1,1),l.dis2
68             (p2,1)});
69     }
70     point<T> projection(const point<T> &p) const{//點對直線的投
71         //影
72         point<T> n=(p2-p1).normal();
73         return p-n*(p-p1).dot(n)/n.abs2();
74     }
75     point<T> mirror(const point<T> &p) const{
76         //點對直線的鏡射・要先呼叫pton轉成一般式
77         point<T> R;
78         T d=a*b+b*b;
79         R.x=(b*b*p.x-a*a*p.x-2*a*b*p.y-2*a*c)/d;
80         R.y=(a*a*p.y-b*b*p.y-2*a*b*p.x-2*b*c)/d;
81         return R;
82     }
83     bool equal(const line &l) const{//直線相等
84         return ori(l.p1)==0&&ori(l.p2)==0;
85     }
86     bool parallel(const line &l) const{
87         return (p1-p2).cross(l.p1-l.p2)==0;
88     }
89     bool cross_seg(const line &l) const{
90         return (p2-p1).cross(l.p1-p1)*(p2-p1).cross(l.p2-p1)<=0;
91         //直線是否交線段
92     }
93     int line_intersect(const line &l) const{//直線相交情況・-1無
94         //限多點・1交於一點・0不相交
95         return parallel(l)?(ori(l.p1)==0?-1:0):1;
96     }
97     int seg_intersect(const line &l) const{
98         T c1=ori(l.p1), c2=ori(l.p2);
99         T c3=l.ori(p1), c4=l.ori(p2);
100         if(c1==0&&c2==0){ //共線
101             bool b1=btw(l.p1)>=0,b2=btw(l.p2)>=0;
102             T a3=l.btw(p1),a4=l.btw(p2);
103             if(b1&&b2&&a3==0&&a4>=0) return 2;
104             if(b1&&b2&&a3>=0&&a4==0) return 3;
105             if(b1&&b2&&a3>=0&&a4>=0) return 0;
106             return -1; //無限交點
107         } else if(c1*c2<=0&&c3*c4<=0) return 1;
108         return 0; //不相交
109     }
110     point<T> line_intersection(const line &l) const{/*直線交點*/
111         point<T> a=p2-p1,b=l.p2-l.p1,s=l.p1-p1;
112         //if(a.cross(b)==0) return INF;
113         return p1+a*(s.cross(b)/a.cross(b));
114     }
115     point<T> seg_intersection(const line &l) const{//線段交點
116         int res=seg_intersect(l);
117         if(res<=0) assert(0);
118         if(res==2) return p1;
119         if(res==3) return p2;
120         return line_intersection(l);
121     }
122 };
123 template<typename T>
124 struct polygon{
125     polygon(){}
126     vector<point<T>> p; //逆時針順序
127     T area() const{//面積
128         T ans=0;
129         for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
130             ans+=p[i].cross(p[j]);
131         }
132         return ans/2;
133     }
134     point<T> center_of_mass() const{//重心
135         T cx=0,cy=0,w=0;
136         for(int i=p.size()-1,j=0;j<(int)p.size();i=j++){
137             T a=p[i].cross(p[j]);
138             cx+=(p[i].x+p[j].x)*a;
139             cy+=(p[i].y+p[j].y)*a;
140             w+=a;
141         }
142         return point<T>(cx/3/w,cy/3/w);
143     }
144     char ahas(const point<T> &t) const{//點是否在簡單多邊形內・
145         //是的話回傳1・在邊上回傳-1・否則回傳0
146         bool c=0;
147         for(int i=0,j=p.size()-1;i<p.size();j=i++){
148             if(line<T>(p[i],p[j]).point_on_segment(t)) return -1;
149             else if((p[i].y>t.y)!=p[j].y>t.y)&&
150                 t.x<(p[j].x-p[i].x)*(t.y-p[i].y)/(p[j].y-p[i].y)+p[i].x
151                 )
152                 c=!c;
153             return c;
154         }
155     }
156     char point_in_convex(const point<T>&x) const{
157         int l=1,r=(int)p.size()-2;
158         while(l<r){ //點是否在凸多邊形內・是的話回傳1・在邊上回傳
159             //1・否則回傳0
160             int mid=(l+r)/2;
161             T a1=(p[mid]-p[0]).cross(x-p[0]);
162             T a2=(p[mid+1]-p[0]).cross(x-p[0]);
163             if(a1>0&&a2<=0){
164                 T res=(p[mid+1]-p[mid]).cross(x-p[mid]);
165                 return res>0?1:(res>0?-1:0);
166             } else if(a1<0) r=mid-1;
167             else l=mid+1;
168         }
169         return 0;
170     }
171     vector<T> getA() const{//凸包邊對x軸的夾角
172         vector<T> res; //一定是遞增的
173         for(size_t i=0;i<p.size();i++){
174             res.push_back((p[(i+1)%p.size()]-p[i]).getA());
175         }
176         return res;
177     }
178     bool line_intersect(const vector<T>&A, const line<T> &l)
179         const{//O(LogN)
180         int f1=upper_bound(A.begin(),A.end(),(l.p1-l.p2).getA())-
181             A.begin();
182         int f2=upper_bound(A.begin(),A.end(),(l.p2-l.p1).getA())-
183             A.begin();
184         return l.cross_seg(line<T>(p[f1],p[f2]));
185     }
186     polygon cut(const line<T> &l) const{//凸包對直線切割・得到直
187         //線L左側的凸包
188     }
189     polygon ans;
190     for(int n=p.size(),i=n-1,j=0;j<n;i=j++){
191         if(l.ori(p[i])>=0){
192             ans.p.push_back(p[i]);
193             if(l.ori(p[j])<0)
194                 ans.p.push_back(l.line_intersection(line<T>(p[i],p[
195                     j])));
196             } else if(l.ori(p[j])>0)

```

```

178     ans.p.push_back(l.line_intersection(line<T>(p[i],p[j
179         ])));
180     }
181     return ans;
182 }
183 static bool monotone_chain_cmp(const point<T>& a,const
184     point<T>& b){//凸包排序函數
185     return (a.x<b.x)||((a.x==b.x&&a.y<b.y));
186 }
187 void monotone_chain(vector<point<T> > &s){//凸包
188     sort(s.begin(),s.end(),monotone_chain_cmp);
189     p.resize(s.size()+1);
190     int m=0;
191     for(size_t i=0;i<s.size();i++){
192         while(m>=2&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
193         p[m++]=s[i];
194     }
195     for(int i=s.size()-2,t=m+1;i>=0;--i){
196         while(m>=t&&(p[m-1]-p[m-2]).cross(s[i]-p[m-2])<=0)--m;
197         p[m++]=s[i];
198     }
199     if(s.size()>1)--m;
200     p.resize(m);
201 }
202 T diam(){//直徑
203     int n=p.size(),t=1;
204     T ans=0;p.push_back(p[0]);
205     for(int i=0;i<n;i++){
206         point<T> now=p[i+1]-p[i];
207         while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t
208             +1)%n;
209         ans=max(ans,(p[i]-p[t]).abs2());
210     }
211     return p.pop_back(),ans;
212 }
213 T min_cover_rectangle(){//最小覆蓋矩形
214     int n=p.size(),t=1,r=1,l;
215     if(n<3)return 0;//也可以做最小周長矩形
216     T ans=1e99;p.push_back(p[0]);
217     for(int i=0;i<n;i++){
218         point<T> now=p[i+1]-p[i];
219         while(now.cross(p[t+1]-p[i])>now.cross(p[t]-p[i]))t=(t
220             +1)%n;
221         while(now.dot(p[r+1]-p[i])>now.dot(p[r]-p[i]))r=(r+1)%n;
222         if(!i)l=r;
223         while(now.dot(p[l+1]-p[i])<=now.dot(p[l]-p[i]))l=(l+1)%
224             n;
225         T d=now.abs2();
226         T tmp=now.cross(p[t]-p[i])*(now.dot(p[r]-p[i])-now.dot(
227             p[l]-p[i]))/d;
228         ans=min(ans,tmp);
229     }
230     return p.pop_back(),ans;
231 }
232 T dis2(polygon &p1){//凸包最近距離平方
233     vector<point<T> > &P=p,&Q=p1.p;
234     int n=P.size(),m=Q.size(),l=0,r=0;
235     for(int i=0;i<n;i++){
236         if(P[i].y<P[l].y)l=i;
237     }
238     for(int i=0;i<m;i++){
239         if(Q[i].y<Q[r].y)r=i;
240     }
241     P.push_back(P[0]),Q.push_back(Q[0]);
242     T ans=1e99;
243     for(int i=0;i<n;i++){
244         while((P[l]-P[l+1]).cross(Q[r+1]-Q[r])<0)r=(r+1)%m;
245     }
246     ans=min(ans,line<T>(P[l],P[l+1]).seg_dis2(line<T>(Q[r],
247         Q[r+1])));
248     l=(l+1)%n;
249     }
250     return P.pop_back(),Q.pop_back(),ans;
251 }
252 static char sign(const point<T>&t){
253     return (t.y==0?t.x:t.y)<0;
254 }
255 static bool angle_cmp(const line<T>& A,const line<T>& B){
256     point<T> a=A.p2-A.p1,b=B.p2-B.p1;
257     return sign(a)<sign(b)||((sign(a)==sign(b)&&a.cross(b)>0);
258 }
259 int halfplane_intersection(vector<line<T> > &s){//半平面交
260     sort(s.begin(),s.end(),angle_cmp);//線段左側為該線段半平
261     面
262     int L,R,n=s.size();
263     vector<point<T> > px(n);
264     vector<line<T> > q(n);
265     q[L=R=0]=s[0];
266     for(int i=1;i<n;i++){
267         while(L<R&&s[i].ori(px[R-1])<=0)--R;
268         while(L<R&&s[i].ori(px[L])<=0)++L;
269         q[++R]=s[i];
270         if(q[R].parallel(q[R-1])){
271             --R;
272             if(q[R].ori(s[i].p1)>0)q[R]=s[i];
273         }
274         if(L<R)px[R-1]=q[R-1].line_intersection(q[R]);
275     }
276     while(L<R&&q[L].ori(px[R-1])<=0)--R;
277     p.clear();
278     if(R-L<=1)return 0;
279     px[R]=q[R].line_intersection(q[L]);
280     for(int i=L;i<R;i++)p.push_back(px[i]);
281     return R-L+1;
282 }
283 };
284 template<typename T>
285 struct triangle{
286     point<T> a,b,c;
287     triangle(){
288         triangle(const point<T> &a,const point<T> &b,const point<T>
289             &c):a(a),b(b),c(c){}
290     }
291     T area()const{
292         T t=(b-a).cross(c-a)/2;
293         return t>0?t:-t;
294     }
295     point<T> barycenter()const{//重心
296         return (a+b+c)/3;
297     }
298     point<T> circumcenter()const{//外心
299         static line<T> u,v;
300         u.p1=(a+b)/2;
301         u.p2=point<T>(u.p1.x-a.y+b.y,u.p1.y+a.x-b.x);
302         v.p1=(a+c)/2;
303         v.p2=point<T>(v.p1.x-a.y+c.y,v.p1.y+a.x-c.x);
304         return u.line_intersection(v);
305     }
306     point<T> incenter()const{//內心
307         T A=sqrt((b-c).abs2()),B=sqrt((a-c).abs2()),C=sqrt((a-b).
308             abs2());
309         return point<T>(A*a.x+B*b.x+C*c.x,A*a.y+B*b.y+C*c.y)/(A+B
310             +C);
311     }
312 };
313 point<T> perpercenter()const{//垂心
314     return barycenter()*3-circumcenter()*2;
315 }
316 };
317 template<typename T>
318 struct point3D{
319     T x,y,z;
320     point3D(){
321         point3D(const T&x,const T&y,const T&z):x(x),y(y),z(z){}
322     }
323     point3D operator+(const point3D &b)const{
324         return point3D(x+b.x,y+b.y,z+b.z);
325     }
326     point3D operator-(const point3D &b)const{
327         return point3D(x-b.x,y-b.y,z-b.z);
328     }
329     point3D operator*(const T &b)const{
330         return point3D(x*b,y*b,z*b);
331     }
332     point3D operator/(const T &b)const{
333         return point3D(x/b,y/b,z/b);
334     }
335     bool operator==(const point3D &b)const{
336         return x==b.x&&y==b.y&&z==b.z;
337     }
338     T dot(const point3D &b)const{
339         return x*b.x+y*b.y+z*b.z;
340     }
341     point3D cross(const point3D &b)const{
342         return point3D(y*b.z-z*b.y,z*b.x-x*b.z,x*b.y-y*b.x);
343     }
344     T abs2()const{//向量長度的平方
345         return dot(*this);
346     }
347     T area2(const point3D &b)const{//和b、原點圍成面積的平方
348         return cross(b).abs2()/4;
349     }
350 };
351 template<typename T>
352 struct line3D{
353     point3D<T> p1,p2;
354     line3D(){
355         line3D(const point3D<T> &p1,const point3D<T> &p2):p1(p1),p2
356             (p2){}
357     }
358     T dis2(const point3D<T> &p,bool is_segment=0)const{//點跟直
359         線/線段的距離平方
360     }
361     point3D<T> v=p2-p1,v1=p-p1;
362     if(is_segment){
363         point3D<T> v2=p-p2;
364         if(v.dot(v1)<=0)return v1.abs2();
365         if(v.dot(v2)>=0)return v2.abs2();
366     }
367     point3D<T> tmp=v.cross(v1);
368     return tmp.abs2()/v.abs2();
369 }
370 pair<point3D<T>,point3D<T> > closest_pair(const line3D<T> &
371     l)const{
372     point3D<T> v1=(p1-p2),v2=(l.p1-l.p2);
373     point3D<T> N=v1.cross(v2),ab(p1-l.p1);
374     //if(N.abs2()==0)return NULL;平行或重合
375     T tmp=N.dot(ab),ans=tmp*tmp/N.abs2();//最近點對距離
376     point3D<T> d1=p2-p1,d2=l.p2-l.p1,D=d1.cross(d2),G=l.p1-p1
377         ;
378     T t1=(G.cross(d2)).dot(D)/D.abs2();
379     T t2=(G.cross(d1)).dot(D)/D.abs2();
380     return make_pair(p1+d1*t1,l.p1+d2*t2);
381 }
382 bool same_side(const point3D<T> &a,const point3D<T> &b)
383     const{
384     return (p2-p1).cross(a-p1).dot((p2-p1).cross(b-p1))>0;
385 }
386 };
387 template<typename T>
388 struct plane{
389     point3D<T> p0,n;//平面上的點和法向量

```

```

355 plane(){}
356 plane(const point3D<T> &p0, const point3D<T> &n):p0(p0),n(n)
    {}
357 T dis2(const point3D<T> &p) const { // 點到平面距離的平方
358     T tmp=(p-p0).dot(n);
359     return tmp*tmp/n.abs2();
360 }
361 point3D<T> projection(const point3D<T> &p) const {
362     return p-n*(p-p0).dot(n)/n.abs2();
363 }
364 point3D<T> line_intersection(const line3D<T> &l) const {
365     T tmp=n.dot(l.p2-l.p1); // 等於0表示平行或重合該平面
366     return l.p1+(l.p2-l.p1)*(n.dot(p0-l.p1)/tmp);
367 }
368 line3D<T> plane_intersection(const plane &p1) const {
369     point3D<T> e=n.cross(pl.n),v=n.cross(e);
370     T tmp=p1.n.dot(v); // 等於0表示平行或重合該平面
371     point3D<T> q=p0+(v*(p1.n.dot(pl.p0-p0))/tmp);
372     return line3D<T>(q,q+e);
373 }
374 };
375 template<typename T>
376 struct triangle3D {
377     point3D<T> a,b,c;
378     triangle3D(){}
379     triangle3D(const point3D<T> &a, const point3D<T> &b, const
        point3D<T> &c):a(a),b(b),c(c){}
380     bool point_in(const point3D<T> &p) const { // 點在該平面上的投
        影在三角形中
381         return line3D<T>(b,c).same_side(p,a)&&line3D<T>(a,c).
            same_side(p,b)&&line3D<T>(a,b).same_side(p,c);
382     }
383 };
384 template<typename T>
385 struct tetrahedron { // 四面體
386     point3D<T> a,b,c,d;
387     tetrahedron(){}
388     tetrahedron(const point3D<T> &a, const point3D<T> &b, const
        point3D<T> &c, const point3D<T> &d):a(a),b(b),c(c),d(d)
        {}
389     T volume6() const { // 體積的六倍
390         return (d-a).dot((b-a).cross(c-a));
391     }
392     point3D<T> centroid() const {
393         return (a+b+c+d)/4;
394     }
395     bool point_in(const point3D<T> &p) const {
396         return triangle3D<T>(a,b,c).point_in(p)&&triangle3D<T>(c,
            d,a).point_in(p);
397     }
398 };
399 template<typename T>
400 struct convexhull3D {
401     static const int MAXN=1005;
402     struct face {
403         int a,b,c;
404         face(int a, int b, int c):a(a),b(b),c(c){}
405     };
406     vector<point3D<T>> pt;
407     vector<face> ans;
408     int fid[MAXN][MAXN];
409     void build() {
410         int n=pt.size();
411         ans.clear();

```

```

412     memset(fid,0,sizeof(fid));
413     ans.emplace_back(0,1,2); // 注意不能共線
414     ans.emplace_back(2,1,0);
415     int ftop = 0;
416     for(int i=3; ftop=1; i<n; ++i, ++ftop){
417         vector<face> next;
418         for(auto &f:ans){
419             T d=(pt[i]-pt[f.a]).dot((pt[f.b]-pt[f.a]).cross(pt[f.
                c]-pt[f.a]));
420             if(d<=0) next.push_back(f);
421             int ff=0;
422             if(d>0) ff=ftop;
423             else if(d<0) ff=-ftop;
424             fid[f.a][f.b]=fid[f.b][f.c]=fid[f.c][f.a]=ff;
425         }
426         for(auto &f:ans){
427             if(fid[f.a][f.b]>0 && fid[f.a][f.b]!=fid[f.b][f.a])
428                 next.emplace_back(f.a,f.b,i);
429             if(fid[f.b][f.c]>0 && fid[f.b][f.c]!=fid[f.c][f.b])
430                 next.emplace_back(f.b,f.c,i);
431             if(fid[f.c][f.a]>0 && fid[f.c][f.a]!=fid[f.a][f.c])
432                 next.emplace_back(f.c,f.a,i);
433         }
434         ans=next;
435     }
436     point3D<T> centroid() const {
437         point3D<T> res(0,0,0);
438         T vol=0;
439         for(auto &f:ans){
440             T tmp=pt[f.a].dot(pt[f.b].cross(pt[f.c]));
441             res=res+(pt[f.a]+pt[f.b]+pt[f.c])*tmp;
442             vol+=tmp;
443         }
444         return res/(vol*4);
445     }
446 }
447 };

```

5.3 Pick's Theorem

給定頂點坐標均是整點的簡單多邊形 · 面積 = 內部格點數 + 邊上格點數/2 - 1

6 Graph

6.1 Augment Path

```

1 struct AugmentPath {
2     int n, m;
3     vector<vector<int>> G;
4     vector<int> ma, mb;
5     vector<int> vis;
6     int now_time;
7     int res;
8
9     AugmentPath(int _n, int _m): n(_n), m(_m), G(n), ma(n,
        -1), mb(m, -1), vis(n){
10         now_time = 0;
11         res = 0;
12     }

```

```

13
14 void add(int x, int y){
15     G[x].push_back(y);
16 }
17
18 bool dfs(int now){
19     vis[now] = now_time;
20
21     for (auto x : G[now]){
22         if (mb[x]==-1){
23             ma[now] = x;
24             mb[x] = now;
25             return true;
26         }
27     }
28     for (auto x : G[now]){
29         if (vis[mb[x]]!=now_time && dfs(mb[x])){
30             ma[now] = x;
31             mb[x] = now;
32             return true;
33         }
34     }
35     return false;
36 }
37
38 int solve(){
39     while (true){
40         now_time++;
41         int cnt = 0;
42         for (int i=0 ; i<n ; i++){
43             if (ma[i]==-1 && dfs(i)){
44                 cnt++;
45             }
46         }
47         if (cnt==0) break;
48         res += cnt;
49     }
50     return res;
51 }
52 };

```

6.2 AugmentPath

```

1 struct AugmentPath {
2     int n, m;
3     vector<vector<int>> G;
4     vector<int> mx, my;
5     vector<int> visx, visy;
6     int stamp;
7
8     AugmentPath(int _n, int _m): n(_n), m(_m), G(n), mx(n,
        -1), my(m, -1), visx(n), visy(n){
9         stamp = 0;
10     }
11
12 void add(int x, int y){
13     G[x].push_back(y);
14 }
15
16 // bb03e2
17 bool dfs1(int now){
18     visx[now] = stamp;
19 }

```

```

20     for (auto x : G[now]){
21         if (my[x]==-1){
22             mx[now] = x;
23             my[x] = now;
24             return true;
25         }
26     }
27     for (auto x : G[now]){
28         if (visx[my[x]]!=stamp && dfs1(my[x])){
29             mx[now] = x;
30             my[x] = now;
31             return true;
32         }
33     }
34     return false;
35 }
36
37 vector<pair<int, int>> find_max_matching(){
38     vector<pair<int, int>> ret;
39
40     while (true){
41         stamp++;
42         int tmp = 0;
43         for (int i=0 ; i<n ; i++){
44             if (mx[i]==-1 && dfs1(i)) tmp++;
45         }
46         if (tmp==0) break;
47     }
48
49     for (int i=0 ; i<n ; i++){
50         if (mx[i]!=-1){
51             ret.push_back({i, mx[i]});
52         }
53     }
54     return ret;
55 }
56
57 // 645577
58 void dfs2(int now){
59     visx[now] = true;
60
61     for (auto x : G[now]){
62         if (my[x]!=-1 && visy[x]==false){
63             visy[x] = true;
64             dfs2(my[x]);
65         }
66     }
67 }
68
69 // 要先執行 find_max_matching 一次
70 vector<pair<int, int>> find_min_vertex_cover(){
71     fill(visx.begin(), visx.end(), false);
72     fill(visy.begin(), visy.end(), false);
73
74     vector<pair<int, int>> ret;
75     for (int i=0 ; i<n ; i++){
76         if (mx[i]==-1) dfs2(i);
77     }
78
79     for (int i=0 ; i<n ; i++){
80         if (visx[i]==false) ret.push_back({1, i});
81     }
82     for (int i=0 ; i<m ; i++){
83         if (visy[i]==true) ret.push_back({2, i});
84     }

```

```

85
86     return ret;
87 }
88 };

```

6.3 Bridge BCC

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 200005;
5 vector<int> G[N];
6 int low[N], depth[N];
7 bool vis[N];
8 vector<vector<int>> bcc;
9 stack<int> stk;
10
11 void dfs(int v, int p) {
12     stk.push(v);
13     vis[v] = true;
14     low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
15     for (int u : G[v]) {
16         if (u == p) continue;
17         if (!vis[u]) {
18             // (v, u) 是樹邊
19             dfs(u, v);
20             low[v] = min(low[v], low[u]);
21         } else {
22             // (v, u) 是回邊
23             low[v] = min(low[v], depth[u]);
24         }
25     }
26     // v 在不依靠父邊的情況下永遠沒辦法走到它的祖先
27     if (low[v] == depth[v]) {
28         bcc.emplace_back();
29         while (stk.top() != v) {
30             bcc.back().push_back(stk.top());
31             stk.pop();
32         }
33         bcc.back().push_back(stk.top());
34         stk.pop();
35     }
36 }

```

6.4 Cut BCC

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int N = 200005;
5 vector<int> G[N];
6 int low[N], depth[N];
7 bool vis[N];
8 vector<vector<int>> bcc;
9 stack<int> stk;
10
11 void dfs(int v, int p) {
12     stk.push(v);
13     vis[v] = true;

```

```

14     low[v] = depth[v] = (p == -1 ? 1 : depth[p] + 1);
15     for (int u : G[v]) {
16         if (u == p) continue;
17         if (!vis[u]) {
18             // (v, u) 是樹邊
19             dfs(u, v);
20             low[v] = min(low[v], low[u]);
21             // u 無法在不經過父邊的情況走到 v 的祖先
22             if (low[u] >= depth[v]) {
23                 bcc.emplace_back();
24                 while (stk.top() != u) {
25                     bcc.back().push_back(stk.top());
26                     stk.pop();
27                 }
28                 bcc.back().push_back(stk.top());
29                 stk.pop();
30                 bcc.back().push_back(v);
31             }
32         } else {
33             // (v, u) 是回邊
34             low[v] = min(low[v], depth[u]);
35         }
36     }
37 }

```

6.5 Dijkstra

```

1 // 可以在  $O(E \log E)$  的時間複雜度解決在無負權有向圖單點源最短路
2 const int INF = 2e18; // 要確保 INF 開的足夠大
3
4 vector<vector<pair<int, int>>> G(n); // G[i] = <節點, 權重>
5 vector<int> dis(n, INF);
6 priority_queue<pair<int, int>, vector<pair<int, int>>,
7     greater<pair<int, int>>> pq;
8 dis[s] = 0;
9 pq.push({0, s});
10
11 while (pq.size()){
12     int now_dis = pq.top().first;
13     int now_node = pq.top().second;
14     pq.pop();
15
16     if (now_dis > dis[now_node]) continue;
17
18     for (auto x : G[now_node]){
19         if (now_dis + x.second < dis[x.first]){
20             dis[x.first] = now_dis + x.second;
21             pq.push({dis[x.first], x.first});
22         }
23     }

```

6.6 Dinic

```

1 // 一般圖： $O(EV^2)$ 
2 // 二分圖： $O(EV)$ 
3 struct Flow{

```



```

4 struct Edge{
5     int v, rc, rid;
6 };
7 vector<vector<Edge>> G;
8 void add(int u, int v, int c){
9     G[u].push_back({v, c, G[v].size()});
10    G[v].push_back({u, 0, G[u].size()-1});
11 }
12 vector<int> dis, it;
13
14 Flow(int n){
15     G.resize(n);
16     dis.resize(n);
17     it.resize(n);
18 }
19
20 int dfs(int u, int t, int f){
21     if (u==t || f==0) return f;
22     for (int &i=it[u] ; i<G[u].size() ; i++){
23         auto &[v, rc, rid] = G[u][i];
24         if (dis[v]!=dis[u]+1) continue;
25         int df = dfs(v, t, min(f, rc));
26         if (df<=0) continue;
27         rc -= df;
28         G[v][rid].rc += df;
29         return df;
30     }
31     return 0;
32 }
33
34 int flow(int s, int t){
35     int ans = 0;
36     while (true){
37         fill(dis.begin(), dis.end(), INF);
38         queue<int> q;
39         q.push(s);
40         dis[s] = 0;
41
42         while (q.size()){
43             int u = q.front(); q.pop();
44             for (auto [v, rc, rid] : G[u]){
45                 if (rc<=0 || dis[v]<INF) continue;
46                 dis[v] = dis[u]+1;
47                 q.push(v);
48             }
49         }
50         if (dis[t]==INF) break;
51
52         fill(it.begin(), it.end(), 0);
53         while (true){
54             int df = dfs(s, t, INF);
55             if (df<=0) break;
56             ans += df;
57         }
58     }
59     return ans;
60 }
61 // the code below constructs minimum cut
62 void dfs_mincut(int now, vector<bool> &vis){
63     vis[now] = true;
64     for (auto &[v, rc, rid] : G[now]){
65         if (vis[v]==false && rc>0){
66             dfs_mincut(v, vis);
67         }
68     }
69 }

```

```

70 vector<pair<int, int>> construct(int n, int s, vector<pair<
71     int, int>> &E){
72     // E is G without capacity
73     vector<bool> vis(n);
74     dfs_mincut(s, vis);
75     vector<pair<int, int>> ret;
76     for (auto &[u, v] : E){
77         if (vis[u]==true && vis[v]==false){
78             ret.emplace_back(u, v);
79         }
80     }
81     return ret;
82 }
83 };

```

6.7 Dinic with double

```

1 const double double_INF = 1e18;
2 const int INF = (int)(1e9 + 10);
3
4 struct Flow{
5     const double eps = 1e-9;
6     struct Edge{
7         int v; double rc; int rid;
8     };
9     vector<vector<Edge>> G;
10    void add(int u, int v, double c){
11        G[u].push_back({v, c, G[v].size()});
12        G[v].push_back({u, 0, G[u].size()-1});
13    }
14    vector<int> dis, it;
15
16    Flow(int n){
17        G.resize(n);
18        dis.resize(n);
19        it.resize(n);
20    }
21
22    double dfs(int u, int t, double f){
23        if (u == t || abs(f) < eps) return f;
24        for (int &i=it[u] ; i<G[u].size() ; i++){
25            auto &[v, rc, rid] = G[u][i];
26            if (dis[v]!=dis[u]+1) continue;
27            double df = dfs(v, t, min(f, rc));
28            if (abs(df) <= eps) continue;
29            rc -= df;
30            G[v][rid].rc += df;
31            return df;
32        }
33        return 0;
34    }
35
36    double flow(int s, int t){
37        double ans = 0;
38        while (true){
39            fill(dis.begin(), dis.end(), INF);
40            queue<int> q;
41            q.push(s);
42            dis[s] = 0;
43
44            while (q.size()){
45                int u = q.front(); q.pop();

```

```

46         for (auto [v, rc, rid] : G[u]){
47             if (abs(rc) <= eps || dis[v] < INF)
48                 continue;
49             dis[v] = dis[u] + 1;
50             q.push(v);
51         }
52         if (dis[t]==INF) break;
53
54         fill(it.begin(), it.end(), 0);
55         while (true){
56             double df = dfs(s, t, double_INF);
57             if (abs(df) <= eps) break;
58             ans += df;
59         }
60     }
61     return ans;
62 }
63
64 // the code below constructs minimum cut
65 void dfs_mincut(int now, vector<bool> &vis){
66     vis[now] = true;
67     for (auto &[v, rc, rid] : G[now]){
68         if (vis[v] == false && rc > eps){
69             dfs_mincut(v, vis);
70         }
71     }
72 }
73
74 vector<pair<int, int>> construct(int n, int s, vector<
75     pair<int, int>> &E){
76     // E is G without capacity
77     vector<bool> vis(n);
78     dfs_mincut(s, vis);
79     vector<pair<int, int>> ret;
80     for (auto &[u, v] : E){
81         if (vis[u] == true && vis[v] == false){
82             ret.emplace_back(u, v);
83         }
84     }
85     return ret;
86 };

```

6.8 Find Bridge

```

1 vector<int> dep(MAX_N), low(MAX_N);
2 vector<pair<int, int>> bridge;
3 bitset<MAX_N> vis;
4
5 void dfs(int now, int pre){
6     vis[now] = 1;
7     low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
8
9     for (auto x : G[now]){
10        if (x==pre){
11            continue;
12        } else if (vis[x]==0){
13            // 沒有走過的節點
14            dfs(x, now);
15            low[now] = min(low[now], low[x]);
16        } else if (vis[x]==1){
17            low[now] = min(low[now], dep[x]);

```

6.9 HLD

6.10 Kosaraju to DAG

```

1  /*
2  給定一個有向圖， 迴傳傳縮點後的圖、SCC 的資訊
3  所有點都以 based-0 編號
4
5  函式：
6  SCC_compress G(n): 宣告一個有 n 個點的圖
7  .add_edge(u, v): 加上一條邊 u -> v
8  .compress: O(n log n) 計算 G3、SCC、SCC_id 的資訊，並把縮點後的
   結果存在 result 裡
9
10 SCC[i] = 某個 SCC 中的所有點
11 SCC_id[i] = 第 i 個點在第幾個 SCC
12 */
13 // c8b146
14 struct SCC_compress{
15     int n = 0, m = 0;
16     vector<vector<int>> G, inv_G, result;
17     vector<pair<int, int>> edges;
18     vector<bool> vis;
19     vector<int> order;
20
21     vector<vector<int>> SCC;
22     vector<int> SCC_id;
23
24     SCC_compress(int _n){
25         n = _n;
26         G.resize(n);
27         inv_G.resize(n);
28         result.resize(n);
29         vis.resize(n);
30         SCC_id.resize(n);
31     }
32
33     void add_edge(int u, int v){
34         G[u].push_back(v);
35         inv_G[v].push_back(u);
36         edges.push_back({u, v});
37         m++;
38     }
39
40     void dfs1(vector<vector<int>> &G, int now){
41         vis[now] = 1;
42         for (auto x : G[now]){
43             if (vis[x]==0){
44                 dfs1(G, x);
45             }
46         }
47         order.push_back(now);
48         return;
49     }
50
51     void dfs2(vector<vector<int>> &G, int now){
52         SCC_id[now] = SCC.size()-1;
53         SCC.back().push_back(now);
54         vis[now] = 1;
55
56         for (auto x : G[now]){
57             if (vis[x]==0){
58                 dfs2(G, x);
59             }
60         }
61         return;
62     }
63 }

```

```

61 void compress(){
62     fill(vis.begin(), vis.end(), 0);
63     for (int i=0 ; i<n ; i++){
64         if (vis[i]==0){
65             dfs1(G, i);
66         }
67     }
68
69     fill(vis.begin(), vis.end(), 0);
70     reverse(order.begin(), order.end());
71     for (int i=0 ; i<n ; i++){
72         if (vis[order[i]]==0){
73             SCC.push_back(vector<int>());
74             dfs2(inv_G, order[i]);
75         }
76     }
77
78     for (int i=0 ; i<m ; i++){
79         if (SCC_id[edges[i].first]!=SCC_id[edges[i].
80             second]){
81             result[SCC_id[edges[i].first]].push_back(
82                 SCC_id[edges[i].second]);
83         }
84     }
85
86     for (int i=0 ; i<SCC.size() ; i++){
87         sort(result[i].begin(), result[i].end());
88         result[i].resize(unique(result[i].begin(), result
89             [i].end())-result[i].begin());
90     }
91 }

```

6.11 KuhnMunkres

```

1 struct KuhnMunkres {
2     int n; // max(n, m)
3     vector<vector<int>> G;
4     vector<int> match, lx, ly, visx, visy;
5     vector<int> slack;
6
7     KuhnMunkres(int n) : n(n), G(n, vector<int>(n)), lx(n),
8         ly(n), slack(n), match(n), visx(n), visy(n) {}
9
10    void add(int x, int y, int w){
11        G[x][y] = max(G[x][y], w);
12    }
13
14    bool dfs(int i, bool aug){ // aug = true 表示要更新 match
15        if (visx[i]) return false;
16        visx[i] = true;
17
18        for (int j=0; j<n; j++){
19            if (visy[j]) continue;
20            int d = lx[i]+ly[j]-G[i][j];
21
22            if (d==0){
23                visy[j] = true;
24                if (match[j]==-1 || dfs(match[j], aug)){
25                    if (aug){
26                        match[j] = i;
27                    }
28                    return true;
29                }
30            }
31        }
32    }
33
34    int solve() {
35        int res = 0;
36        for (int i=0; i<n; i++){
37            if (visx[i]) continue;
38            visx[i] = true;
39            for (int j=0; j<n; j++){
40                if (visy[j]) continue;
41                int d = lx[i]+ly[j]-G[i][j];
42                if (d==0) visy[j] = true;
43                else if (d<slack[i]) slack[i] = d, ly[j] = ly[j]-d;
44            }
45            int j = match[i];
46            if (j!=-1) res += G[i][j];
47            match[i] = -1;
48            dfs(i, true);
49            match[i] = j;
50        }
51        return res;
52    }
53
54    void print() {
55        for (int i=0; i<n; i++){
56            for (int j=0; j<n; j++){
57                cout<<G[i][j]<<" ";
58            }
59            cout<<endl;
60        }
61    }
62
63    void printMatch() {
64        for (int i=0; i<n; i++){
65            cout<<i<<" match to " <<match[i]<<endl;
66        }
67    }
68
69    void printSlack() {
70        for (int i=0; i<n; i++){
71            cout<<i<<" slack is " <<slack[i]<<endl;
72        }
73    }
74
75    void printLxLy() {
76        for (int i=0; i<n; i++){
77            cout<<i<<" lx is " <<lx[i]<<" ly is " <<ly[i]<<endl;
78        }
79    }
80
81    void printVisxVisy() {
82        for (int i=0; i<n; i++){
83            cout<<i<<" visx is " <<visx[i]<<" visy is " <<visy[i]<<endl;
84        }
85    }
86
87    void printMatchSlackLxLyVisxVisy() {
88        printMatch();
89        printSlack();
90        printLxLy();
91        printVisxVisy();
92    }
93
94    void printAll() {
95        print();
96        printMatch();
97        printSlack();
98        printLxLy();
99        printVisxVisy();
100    }
101
102    void printAllMatchSlackLxLyVisxVisy() {
103        printMatchSlackLxLyVisxVisy();
104    }
105
106    void printAllMatchSlackLxLyVisxVisyAndSolve() {
107        printAllMatchSlackLxLyVisxVisy();
108        solve();
109    }
110
111    void printAllMatchSlackLxLyVisxVisyAndSolveAndPrint() {
112        printAllMatchSlackLxLyVisxVisyAndSolve();
113        print();
114        printMatch();
115        printSlack();
116        printLxLy();
117        printVisxVisy();
118    }
119
120    void printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndReset() {
121        printAllMatchSlackLxLyVisxVisyAndSolveAndPrint();
122        reset();
123    }
124
125    void reset() {
126        G.clear();
127        match.clear();
128        lx.clear();
129        ly.clear();
130        visx.clear();
131        visy.clear();
132        slack.clear();
133        n = 0;
134    }
135
136    void resetAndAdd(int x, int y, int w) {
137        reset();
138        add(x, y, w);
139    }
140
141    void resetAndSolve() {
142        reset();
143        solve();
144    }
145
146    void resetAndPrint() {
147        reset();
148        print();
149    }
150
151    void resetAndPrintMatch() {
152        reset();
153        printMatch();
154    }
155
156    void resetAndPrintSlack() {
157        reset();
158        printSlack();
159    }
160
161    void resetAndPrintLxLy() {
162        reset();
163        printLxLy();
164    }
165
166    void resetAndPrintVisxVisy() {
167        reset();
168        printVisxVisy();
169    }
170
171    void resetAndPrintMatchSlackLxLyVisxVisy() {
172        reset();
173        printMatchSlackLxLyVisxVisy();
174    }
175
176    void resetAndPrintAll() {
177        reset();
178        printAll();
179    }
180
181    void resetAndPrintAllMatchSlackLxLyVisxVisy() {
182        reset();
183        printAllMatchSlackLxLyVisxVisy();
184    }
185
186    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolve() {
187        reset();
188        printAllMatchSlackLxLyVisxVisyAndSolve();
189    }
190
191    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrint() {
192        reset();
193        printAllMatchSlackLxLyVisxVisyAndSolveAndPrint();
194    }
195
196    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndReset() {
197        reset();
198        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndReset();
199    }
200
201    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAdd(int x, int y, int w) {
202        reset();
203        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAdd(x, y, w);
204    }
205
206    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolve() {
207        reset();
208        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolve();
209    }
210
211    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrint() {
212        reset();
213        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrint();
214    }
215
216    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset() {
217        reset();
218        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset();
219    }
220
221    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(int x, int y, int w) {
222        reset();
223        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(x, y, w);
224    }
225
226    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolve() {
227        reset();
228        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolve();
229    }
230
231    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrint() {
232        reset();
233        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrint();
234    }
235
236    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset() {
237        reset();
238        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset();
239    }
240
241    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(int x, int y, int w) {
242        reset();
243        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(x, y, w);
244    }
245
246    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolve() {
247        reset();
248        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolve();
249    }
250
251    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrint() {
252        reset();
253        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrint();
254    }
255
256    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset() {
257        reset();
258        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset();
259    }
260
261    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(int x, int y, int w) {
262        reset();
263        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(x, y, w);
264    }
265
266    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolve() {
267        reset();
268        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolve();
269    }
270
271    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrint() {
272        reset();
273        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrint();
274    }
275
276    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset() {
277        reset();
278        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndReset();
279    }
280
281    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(int x, int y, int w) {
282        reset();
283        printAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAddAndSolveAndPrintAndResetAndAdd(x, y, w);
284    }
285
286    void resetAndPrintAllMatchSlackLxLyVisxVisyAndSolveAndPrintAndResetAndAddAnd
```



```

29         }else{
30             slack[j] = min(slack[j], d);
31         }
32     }
33     return false;
34 }
35
36 bool augment(){
37     for (int j=0; j<n; j++){
38         if (!visy[j] && slack[j]==0){
39             visy[j] = true;
40             if (match[j]==-1 || dfs(match[j], false)){
41                 return true;
42             }
43         }
44     }
45     return false;
46 }
47
48 void relabel(){
49     int delta = INF;
50     for (int j=0; j<n; j++){
51         if (visy[j]==false) delta = min(delta, slack[j]);
52     }
53     for (int i=0; i<n; i++){
54         if (visx[i]) lx[i] -= delta;
55     }
56     for (int j=0; j<n; j++){
57         if (visy[j]) ly[j] += delta;
58         else slack[j] -= delta;
59     }
60 }
61
62 int solve(){
63     for (int i=0; i<n; i++){
64         lx[i] = 0;
65         for (int j=0; j<n; j++){
66             lx[i] = max(lx[i], G[i][j]);
67         }
68     }
69     fill(ly.begin(), ly.end(), 0);
70     fill(match.begin(), match.end(), -1);
71
72     for(int i = 0; i < n; i++) {
73         fill(slack.begin(), slack.end(), INF);
74         fill(visx.begin(), visx.end(), false);
75         fill(visy.begin(), visy.end(), false);
76         if(dfs(i, true)) continue;
77
78         while(augment()==false) relabel();
79         fill(visx.begin(), visx.end(), false);
80         fill(visy.begin(), visy.end(), false);
81         dfs(i, true);
82     }
83
84     int ans = 0;
85     for (int j=0; j<n; j++){
86         if (match[j]!=-1){
87             ans += G[match[j]][j];
88         }
89     }
90     return ans;
91 }
92
93 };
94

```

6.12 MCMF

```

1 struct Flow {
2     struct Edge {
3         int u, rc, k, rv;
4     };
5
6     vector<vector<Edge>> G;
7     vector<int> par, par_eid;
8     Flow(int n) : G(n+1), par(n+1), par_eid(n+1) {}
9
10    // v->u, capacity: c, cost: k
11    void add(int v, int u, int c, int k){
12        G[v].push_back({u, c, k, SZ[G[u]]});
13        G[u].push_back({v, 0, -k, SZ[G[v]]-1});
14    }
15
16    // 3701d6
17    int spfa(int s, int t){
18        fill(ALL(par), -1);
19        vector<int> dis(SZ(par), INF);
20        vector<bool> in_q(SZ(par), false);
21        queue<int> Q;
22        dis[s] = 0;
23        in_q[s] = true;
24        Q.push(s);
25
26        while (!Q.empty()){
27            int v = Q.front();
28            Q.pop();
29            in_q[v] = false;
30
31            for (int i=0; i<SZ[G[v]]; i++){
32                auto [u, rc, k, rv] = G[v][i];
33                if (rc>0 && dis[v]+k<dis[u]){
34                    dis[u] = dis[v]+k;
35                    par[u] = v;
36                    par_eid[u] = i;
37                    if (!in_q[u]) Q.push(u);
38                    in_q[u] = true;
39                }
40            }
41        }
42
43        return dis[t];
44    }
45
46    // return <max flow, min cost>, 150093
47    pair<int, int> flow(int s, int t){
48        int fl = 0, cost = 0, d;
49        while ((d = spfa(s, t))<INF){
50            int cur = INF;
51            for (int v=t; v!=s; v=par[v])
52                cur = min(cur, G[par[v]][par_eid[v]].rc);
53            fl += cur;
54            cost += d*cur;
55            for (int v=t; v!=s; v=par[v]){
56                G[par[v]][par_eid[v]].rc -= cur;
57                G[v][G[par[v]][par_eid[v]].rv].rc += cur;
58            }
59        }
60        return {fl, cost};
61    }
62
63    vector<pair<int, int>> construct(){

```

```

64    vector<pair<int, int>> ret;
65    for (int i=0; i<n; i++){
66        for (auto x : G[i]){
67            if (x.rc==0){
68                ret.push_back({i+1, x.u-n+1});
69                break;
70            }
71        }
72    }
73    return ret;
74 }
75 };

```

6.13 Tarjan Find AP

```

1 vector<int> dep(MAX_N), low(MAX_N), AP;
2 bitset<MAX_N> vis;
3
4 void dfs(int now, int pre){
5     int cnt = 0;
6     bool ap = 0;
7     vis[now] = 1;
8     low[now] = dep[now] = (now==1 ? 0 : dep[pre]+1);
9
10    for (auto x : G[now]){
11        if (x==pre){
12            continue;
13        }else if (vis[x]==0){
14            cnt++;
15            dfs(x, now);
16            low[now] = min(low[now], low[x]);
17            if (low[x]>=dep[now]) ap=1;
18        }else{
19            low[now] = min(low[now], dep[x]);
20        }
21    }
22
23    if ((now==pre && cnt==2) || (now!=pre && ap)){
24        AP.push_back(now);
25    }
26 }

```

6.14 Tree Isomorphism

```

1 #include <bits/stdc++.h>
2 #pragma GCC optimize("O3,unroll-loops")
3 #define fastio ios::sync_with_stdio(0), cin.tie(0), cout.tie(0)
4 #define dbg(x) cerr << #x << " = " << x << endl
5 #define int long long
6 using namespace std;
7
8 // declare
9 const int MAX_SIZE = 2e5+5;
10 const int INF = 9e18;
11 const int MOD = 1e9+7;
12 const double EPS = 1e-6;
13 typedef vector<vector<int>> Graph;
14 typedef map<vector<int>, int> Hash;
15

```

```

16 int n, a, b;
17 int id1, id2;
18 pair<int, int> c1, c2;
19 vector<int> sz1(MAX_SIZE), sz2(MAX_SIZE);
20 vector<int> we1(MAX_SIZE), we2(MAX_SIZE);
21 Graph g1(MAX_SIZE), g2(MAX_SIZE);
22 Hash m1, m2;
23 int testcase=0;
24
25 void centroid(Graph &g, vector<int> &s, vector<int> &w, pair<
    int, int> &rec, int now, int pre){
26     s[now]=1;
27     w[now]=0;
28     for (auto x : g[now]){
29         if (x!=pre){
30             centroid(g, s, w, rec, x, now);
31             s[now]+=s[x];
32             w[now]=max(w[now], s[x]);
33         }
34     }
35     w[now]=max(w[now], n-s[now]);
36     if (w[now]<=n/2){
37         if (rec.first==0) rec.first=now;
38         else rec.second=now;
39     }
40 }
41
42 int dfs(Graph &g, Hash &m, int &id, int now, int pre){
43     vector<int> v;
44     for (auto x : g[now]){
45         if (x!=pre){
46             int add=dfs(g, m, id, x, now);
47             v.push_back(add);
48         }
49     }
50     sort(v.begin(), v.end());
51
52     if (m.find(v)!=m.end()){
53         return m[v];
54     }else{
55         m[v]=++id;
56         return id;
57     }
58 }
59
60
61 void solve1(){
62     // init
63     id1=0;
64     id2=0;
65     c1={0, 0};
66     c2={0, 0};
67     fill(sz1.begin(), sz1.begin()+n+1, 0);
68     fill(sz2.begin(), sz2.begin()+n+1, 0);
69     fill(we1.begin(), we1.begin()+n+1, 0);
70     fill(we2.begin(), we2.begin()+n+1, 0);
71     for (int i=1 ; i<=n ; i++){
72         g1[i].clear();
73         g2[i].clear();
74     }
75     m1.clear();
76     m2.clear();
77
78     // input

```

```

81     cin >> n;
82     for (int i=0 ; i<n-1 ; i++){
83         cin >> a >> b;
84         g1[a].push_back(b);
85         g1[b].push_back(a);
86     }
87     for (int i=0 ; i<n-1 ; i++){
88         cin >> a >> b;
89         g2[a].push_back(b);
90         g2[b].push_back(a);
91     }
92
93     // get tree centroid
94     centroid(g1, sz1, we1, c1, 1, 0);
95     centroid(g2, sz2, we2, c2, 1, 0);
96
97     // process
98     int res1=0, res2=0, res3=0;
99     if (c2.second!=0){
100         res1=dfs(g1, m1, id1, c1.first, 0);
101         m2=m1;
102         id2=id1;
103         res2=dfs(g2, m1, id1, c2.first, 0);
104         res3=dfs(g2, m2, id2, c2.second, 0);
105     }else if (c1.second!=0){
106         res1=dfs(g2, m1, id1, c2.first, 0);
107         m2=m1;
108         id2=id1;
109         res2=dfs(g1, m1, id1, c1.first, 0);
110         res3=dfs(g1, m2, id2, c1.second, 0);
111     }else{
112         res1=dfs(g1, m1, id1, c1.first, 0);
113         res2=dfs(g2, m1, id1, c2.first, 0);
114     }
115
116     // output
117     cout << (res1==res2 || res1==res3 ? "YES" : "NO") << endl;
118
119     return;
120 }
121
122 signed main(void){
123     fastio;
124
125     int t=1;
126     cin >> t;
127     while (t--){
128         solve1();
129     }
130     return 0;
131 }

```

6.15 tarjan

```

1 struct tarjan_SCC {
2     int now_T, now_SCCs;
3     vector<int> dfn, low, SCC;
4     stack<int> S;
5     vector<vector<int>> E;
6     vector<bool> vis, in_stack;
7
8     tarjan_SCC(int n) {

```

```

9         init(n);
10     }
11     void init(int n) {
12         now_T = now_SCCs = 0;
13         dfn = low = SCC = vector<int>(n);
14         E = vector<vector<int>>(n);
15         S = stack<int>();
16         vis = in_stack = vector<bool>(n);
17     }
18     void add(int u, int v) {
19         E[u].push_back(v);
20     }
21     void build() {
22         for (int i = 0; i < dfn.size(); ++i) {
23             if (!dfn[i]) dfs(i);
24         }
25     }
26     void dfs(int v) {
27         now_T++;
28         vis[v] = in_stack[v] = true;
29         dfn[v] = low[v] = now_T;
30         S.push(v);
31         for (auto &i:E[v]) {
32             if (!vis[i]) {
33                 vis[i] = true;
34                 dfs(i);
35                 low[v] = min(low[v], low[i]);
36             }
37             else if (in_stack[i]) {
38                 low[v] = min(low[v], dfn[i]);
39             }
40         }
41         if (low[v] == dfn[v]) {
42             int tmp;
43             do {
44                 tmp = S.top();
45                 S.pop();
46                 SCC[tmp] = now_SCCs;
47                 in_stack[tmp] = false;
48             } while (tmp != v);
49             now_SCCs += 1;
50         }
51     }
52 };

```

6.16 圓方樹

```

1 #include <bits/stdc++.h>
2 #define lp(i,a,b) for(int i=(a);i<(b);i++)
3 #define pii pair<int,int>
4 #define pb push_back
5 #define ins insert
6 #define ff first
7 #define ss second
8 #define opa(x) cerr << #x << " = " << x << ", ";
9 #define op(x) cerr << #x << " = " << x << endl;
10 #define ops(x) cerr << x;
11 #define etr cerr << endl;
12 #define spc cerr << " ";
13 #define BAE(x) (x).begin(), (x).end()
14 #define STL(x) cerr << #x << " : "; for(auto &qwe:x) cerr <<
15 #define deb1 cerr << "deb1" << endl;

```

```

16 #define deb2 cerr << "deb2" << endl;
17 #define deb3 cerr << "deb3" << endl;
18 #define deb4 cerr << "deb4" << endl;
19 #define deb5 cerr << "deb5" << endl;
20 #define bye exit(0);
21 using namespace std;
22
23 const int mxn = (int)(2e5) + 10;
24 const int mxlg = 17;
25 int last_special_node = (int)(1e5) + 1;
26 vector<int> E[mxn], F[mxn];
27
28 struct edg{
29     int fr, to;
30     edg(int _fr, int _to){
31         fr = _fr;
32         to = _to;
33     }
34 };
35 ostream& operator<<(ostream& os, edg x){os << x.fr << "--" <<
36     x.to;}
37 vector<edg> EV;
38
39 void tarjan(int v, int par, stack<int>& S){
40     static vector<int> dfn(mxn), low(mxn);
41     static vector<bool> to_add(mxn);
42     static int nowT = 0;
43
44     int child = 0;
45     nowT += 1;
46     dfn[v] = low[v] = nowT;
47     for(auto &ne:E[v]){
48         int i = EV[ne].to;
49         if(i == par) continue;
50         if(!dfn[i]){
51             S.push(ne);
52             tarjan(i, v, S);
53             child += 1;
54             low[v] = min(low[v], low[i]);
55
56             if(par >= 0 && low[i] >= dfn[v]){
57                 vector<int> bcc;
58                 int tmp;
59                 do{
60                     tmp = S.top(); S.pop();
61                     if(!to_add[EV[tmp].fr]){
62                         to_add[EV[tmp].fr] = true;
63                         bcc.pb(EV[tmp].fr);
64                     }
65                     if(!to_add[EV[tmp].to]){
66                         to_add[EV[tmp].to] = true;
67                         bcc.pb(EV[tmp].to);
68                     }
69                 }while(tmp != ne);
70                 for(auto &j:bcc){
71                     to_add[j] = false;
72                     F[last_special_node].pb(j);
73                     F[j].pb(last_special_node);
74                 }
75                 last_special_node += 1;
76             }
77         }
78         else{
79             low[v] = min(low[v], dfn[i]);
80             if(dfn[i] < dfn[v]){ // edge i--v will be visited
81                 // twice at here, but we only need one.
82             }
83         }
84     }
85
86     int dep[mxn], jmp[mxn][mxlg];
87     void dfs_lca(int v, int par, int depth){
88         dep[v] = depth;
89         for(auto &i:F[v]){
90             if(i == par) continue;
91             jmp[i][0] = v;
92             dfs_lca(i, v, depth + 1);
93         }
94     }
95
96     inline void build_lca(){
97         jmp[1][0] = 1;
98         dfs_lca(1, -1, 1);
99         lp(j, 1, mxlg){
100             lp(i, 1, mxn){
101                 jmp[i][j] = jmp[jmp[i][j-1]][j-1];
102             }
103         }
104     }
105
106     inline int lca(int x, int y){
107         if(dep[x] < dep[y]){ swap(x, y); }
108
109         int diff = dep[x] - dep[y];
110         lp(j, 0, mxlg){
111             if((diff >> j) & 1){
112                 x = jmp[x][j];
113             }
114         }
115         if(x == y) return x;
116
117         for(int j = mxlg - 1; j >= 0; j--){
118             if(jmp[x][j] != jmp[y][j]){
119                 x = jmp[x][j];
120                 y = jmp[y][j];
121             }
122         }
123         return jmp[x][0];
124     }
125
126     inline bool can_reach(int fr, int to){
127         if(dep[to] > dep[fr]) return false;
128
129         int diff = dep[fr] - dep[to];
130         lp(j, 0, mxlg){
131             if((diff >> j) & 1){
132                 fr = jmp[fr][j];
133             }
134         }
135         return fr == to;
136     }
137
138     int main(){
139         ios::sync_with_stdio(false); cin.tie(0);
140         // freopen("test_input.txt", "r", stdin);
141         int n, m, q; cin >> n >> m >> q;
142         lp(i, 0, m){
143             int u, v; cin >> u >> v;
144             E[u].pb(EV.size());
145             EV.pb(edg(u, v));

```

```

146         E[v].pb(EV.size());
147         EV.pb(edg(v, u));
148     }
149     E[0].pb(EV.size());
150     EV.pb(edg(0, 1));
151     stack<int> S;
152     tarjan(0, -1, S);
153     build_lca();
154
155     lp(queries, 0, q){
156         int fr, to, relay; cin >> fr >> to >> relay;
157         if(fr == relay || to == relay){
158             cout << "NO\n";
159             continue;
160         }
161         if((can_reach(fr, relay) || can_reach(to, relay)) &&
162             dep[relay] >= dep[lca(fr, to)]){
163             cout << "NO\n";
164             continue;
165         }
166         cout << "YES\n";
167     }

```

6.17 最大權閉合圖

```

1 /*
2 Problem:
3     Given  $w = [w_0, w_1, \dots, w_{n-1}]$  (which can be
4     either positive or negative or 0), you can choose
5     to take  $w_i$  ( $0 < i < n$ ) or not, but if edge  $u \rightarrow v$ 
6     exists, you must take  $w_v$  if you want to take  $w_u$ 
7     (in other words, you can't take  $w_u$  without taking
8      $w_v$ ), this function returns the maximum value(> 0)
9     you can get. If you need a construction, you can
10     output the minimum cut of the  $S$ (source) side.
11 Complexity:
12     MaxFlow( $n, m$ ) (Non-Biparte: $O(n^2m)$  / Bipartite: $O(m\sqrt{n})$ )
13 */
14 int maximum_closure(vector<int> w, vector<pair<int,int>>& EV)
15 {
16     int n = w.size(), S = n + 1, T = n + 2;
17     Flow G(T + 5); // Graph/Dinic.cpp
18     int sum = 0;
19     for (int i = 0; i < n; ++i) {
20         if (w[i] > 0) {
21             G.add(S, i, w[i]);
22             sum += w[i];
23         }
24         else if (w[i] < 0) {
25             G.add(i, T, abs(w[i]));
26         }
27     }
28     for (auto &[u, v] : EV) { // You should make sure that
29         INF >  $\sum w_i$ 
30         G.add(u, v, INF);
31     }
32     int cut = G.flow(S, T);
33     return sum - cut;

```

6.18 Theorem

- 任意圖
 - 不能有孤點 · 最大匹配 + 最小邊覆蓋 = n - 點覆蓋的補集是獨立集。
最小點覆蓋 + 最大獨立集 = n
- 二分圖
 - 最小點覆蓋 = 最大匹配 = n - 最大獨立集
- 只有邊帶權的二分圖
 - w-vertex-cover (帶權點覆蓋): 每條邊的兩個連接點被選中的次數總和至少要是 w_e 。
 - w-weight matching (帶權匹配)
 - minimum vertex count of w-vertex-cover = maximum weight count of w-weight matching (一個點可以被選很多次, 但邊不行)
- 點、邊都帶權的二分圖的定理
 - b-matching: 假設 v 的點權是 b_v 。那所有 v 的匹配邊 e 的權重都要滿足 $\sum w_e \leq b_v$ 。
 - The maximum w-weight of a b-matching equals the minimum b-weight of vertices in a w-vertex-cover.

7 Math

7.1 CRT m Coprime

```

1 vector<int> a, m;
2
3 int extgcd(int a, int b, int &x, int &y){
4     if (b==0){
5         x=1, y=0;
6         return a;
7     }
8
9     int ret=extgcd(b, a%b, y, x);
10    y-=a/b*x;
11    return ret;
12 }
13
14 // n = 有幾個式子 · 求解  $x \equiv a_i \pmod{m_i}$ 
15 int CRT(int n, vector<int> &a, vector<int> &m){
16     int p=1, ans=0;
17
18     vector<int> M(n), inv_M(n);
19
20     for (int i=0; i<n; i++) p*=m[i];
21     for (int i=0; i<n; i++){
22         M[i]=p/m[i];
23         int tmp;
24         extgcd(M[i], m[i], inv_M[i], tmp);
25         ans+=a[i]*inv_M[i]*M[i];
26         ans%=p;
27     }
28
29     return (ans%p+p)%p;
30 }

```

7.2 CRT m Not Coprime

```

1 int extgcd(int a, int b, int &x, int &y){
2     if (b==0){
3         x=1, y=0;
4         return a;
5     }
6
7     int ret=extgcd(b, a%b, y, x);
8     y-=a/b*x;
9     return ret;
10 }
11
12 // 對於方程組的式子兩兩求解
13 // {是否有解, {a, m}}
14 pair<bool, pair<int, int>> CRT(int a1, int m1, int a2, int m2
15 ){
16     int g=__gcd(m1, m2);
17     if ((a2-a1)%g!=0) return {0, {-1, -1}};
18
19     int x, y;
20     extgcd(m1, m2, x, y);
21
22     x=(a2-a1)*x/g; // 兩者不能相反
23     a1=x*m1+a1;
24     m1=m1*m2/g;
25     a1=(a1*m1+m1)%m1;
26     return {1, {a1, m1}};

```

7.3 Fraction

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 /// Fraction template starts ///
5 #define fraction_template_bonus_check
6 const long long ll_overflow_warning_value = (long long)(3e9);
7
8 long long gcd(long long a, long long b){
9     if(a == 0) return 0;
10    if(b == 0) return a;
11    if(a < b) return gcd(b,a);
12    return gcd(b, a%b);
13 }
14 struct frac{
15     long long a, b;
16     frac(long long _a = 0, long long _b = 1){
17         a = _a; b = _b;
18         if(b == 0){
19             cerr << "Error: division by zero\n";
20             cerr << "Called : Constructor(" << _a << ", " <<
                _b << ")\n";
21             return;
22         }
23         if(a == 0){b = 1; return;}
24         if(b < 0){a = -a; b = -b;}
25         long long gcd_ab = gcd(std::abs(a), b);
26         if(gcd_ab != 1){a /= gcd_ab; b /= gcd_ab;}
27
28 #ifdef fraction_template_bonus_check

```

```

29         if(std::abs(a) > ll_overflow_warning_value || b >
30             ll_overflow_warning_value){
31             cerr << "Overflow warning : " << a << "/" << b <<
32                 "\n";
33             #endif // fraction_template_bonus_check
34         }
35         frac operator+(frac const &B){
36             return frac(a*(B.b)+(B.a)*b, b*(B.b));
37         }
38         frac operator-(frac const &B){
39             return frac(a*(B.b)-(B.a)*b, b*(B.b));
40         }
41         frac operator*(frac const &B){
42             return frac(a*(B.a), b*(B.b));
43         }
44         frac operator/(frac const &B){
45             return frac(a*(B.b), b*(B.a));
46         }
47
48         frac operator+=(frac const &B){
49             *this = frac(a*(B.b)+(B.a)*b, b*(B.b));
50         }
51         frac operator-=(frac const &B){
52             *this = frac(a*(B.b)-(B.a)*b, b*(B.b));
53         }
54         frac operator*=(frac const &B){
55             *this = frac(a*(B.a), b*(B.b));
56         }
57         frac operator/=(frac const &B){
58             *this = frac(a*(B.b), b*(B.a));
59         }
60
61         frac abs(){
62             a = std::abs(a);
63             return *this;
64         }
65
66         bool operator<(frac const &B){
67             return a*B.b < B.a*b;
68         }
69         bool operator<=(frac const &B){
70             return a*B.b <= B.a*b;
71         }
72         bool operator>(frac const &B){
73             return a*B.b > B.a*b;
74         }
75         bool operator>=(frac const &B){
76             return a*B.b >= B.a*b;
77         }
78         bool operator==(frac const &B){
79             return a * B.b == B.a * b;
80         }
81         bool operator!=(frac const &B){
82             return a * B.b != B.a * b;
83         }
84     };
85     ostream& operator<<(ostream &os, const frac& A){
86         os << A.a << "/" << A.b;
87         return os;
88     }
89
90     /// Fraction template ends ///
91
92 void test(frac A, frac B){
93     cout << "A = " << A << endl;
94     cout << "B = " << B << endl;
95     cout << endl;
96     cout << "A + B = " << A + B << endl;
97     cout << "A - B = " << A - B << endl;
98     cout << "A * B = " << A * B << endl;
99     cout << "A / B = " << A / B << endl;
100    cout << endl;
101    cout << "(A < B) = " << (A < B) << endl;
102    cout << "(A <= B) = " << (A <= B) << endl;
103    cout << "(A > B) = " << (A > B) << endl;
104    cout << "(A >= B) = " << (A >= B) << endl;
105    cout << "(A == B) = " << (A == B) << endl;
106    cout << "(A != B) = " << (A != B) << endl;
107    cout << "-----\n";
108    return;
109 }

```

```

93
94 int main(){
95     frac tmp1(-7, 2);
96     frac tmp2(5, 3);
97     test(tmp1, tmp2);
98
99     frac tmp3(-7);
100    frac tmp4(0);
101    test(tmp3, tmp4);
102    return 0;
103 }

```

7.4 Josephus Problem

```

1 // 有 n 個人，第偶數個報數的人被刪掉，問第 k 個被踢掉的是誰
2 int solve(int n, int k){
3     if (n==1) return 1;
4     if (k==(n+1)/2){
5         if (2*k>n) return 2*k%n;
6         else return 2*k;
7     }else{
8         int res=solve(n/2, k-(n+1)/2);
9         if (n&1) return 2*res+1;
10        else return 2*res-1;
11    }
12 }

```

7.5 Lagrange any x

```

1 // init: (x1, y1), (x2, y2) in a vector
2 struct Lagrange{
3     int n;
4     vector<pair<int, int>> v;
5
6     Lagrange(vector<pair<int, int>> &_v){
7         n = _v.size();
8         v = _v;
9     }
10
11    // O(n^2 Log MAX_A)
12    int solve(int x){
13        int ret = 0;
14        for (int i=0; i<n; i++){
15            int now = v[i].second;
16            for (int j=0; j<n; j++){
17                if (i==j) continue;
18                now *= ((x-v[j].first+MOD)%MOD);
19                now %= MOD;
20                now *= (qp((v[i].first-v[j].first+MOD)%MOD,
21                    MOD-2)+MOD)%MOD;
22                now %= MOD;
23            }
24            ret = (ret+now)%MOD;
25        }
26        return ret;
27    }
28 };

```

7.6 Lagrange continuous x

```

1 #include <bits/stdc++.h>
2 using namespace std;
3
4 const int MAX_N = 5e5 + 10;
5 const int mod = 1e9 + 7;
6
7 long long inv_fac[MAX_N];
8
9 inline int fp(long long x, int y) {
10     int ret = 1;
11     for (; y; y >>= 1) {
12         ret = (y & 1) ? (ret * x % mod) : ret;
13         x = x * x % mod;
14     }
15     return ret;
16 }
17
18 // TO USE THIS TEMPLATE, YOU MUST MAKE SURE THAT THE MOD
19 // NUMBER IS A PRIME.
20 struct Lagrange {
21     /*
22      * Initialize a polynomial with f(x_0), f(x_0 + 1), ..., f(
23      * x_0 + n).
24      * This determines a polynomial f(x) whose degree is at most
25      * n.
26      * Then you can call sample(x) and you get the value of f(x)
27      *
28      * Complexity of init() and sample() are both O(n).
29      */
30     int m, shift; // m = n + 1
31     vector<int> v, mul;
32     // You can use this function if you don't have inv_fac array
33     // already.
34     void construct_inv_fac() {
35         long long fac = 1;
36         for (int i = 2; i < MAX_N; ++i) {
37             fac = fac * i % mod;
38         }
39         inv_fac[MAX_N - 1] = fp(fac, mod - 2);
40         for (int i = MAX_N - 1; i >= 1; --i) {
41             inv_fac[i - 1] = inv_fac[i] * i % mod;
42         }
43     }
44     // You call init() many times without having a second
45     // instance of this struct.
46     void init(int X_0, vector<int> &u) {
47         v = u;
48         shift = ((1 - X_0) % mod + mod) % mod;
49         if (v.size() == 1) v.push_back(v[0]);
50         m = v.size();
51         mul.resize(m);
52     }
53     // You can use sample(x) instead of sample(x % mod).
54     int sample(int x) {
55         x = ((long long)x + shift) % mod;
56         x = (x < 0) ? (x + mod) : x;
57         long long now = 1;
58         for (int i = m; i >= 1; --i) {
59             mul[i - 1] = now;
60             now = now * (x - i) % mod;
61         }
62         int ret = 0;
63         bool neg = (m - 1) & 1;

```

```

58     now = 1;
59     for (int i = 1; i <= m; ++i) {
60         int up = now * mul[i - 1] % mod;
61         int down = inv_fac[m - i] * inv_fac[i - 1] % mod;
62         int tmp = ((long long)v[i - 1] * up % mod) * down
63             % mod;
64         ret += (neg && tmp) ? (mod - tmp) : (tmp);
65         ret = (ret >= mod) ? (ret - mod) : ret;
66         now = now * (x - i) % mod;
67         neg ^= 1;
68     }
69     return ret;
70 }
71
72 int main() {
73     int n; cin >> n;
74     vector<int> v(n);
75     for (int i = 0; i < n; ++i) {
76         cin >> v[i];
77     }
78     Lagrange L;
79     L.construct_inv_fac();
80     L.init(0, v);
81     int x; cin >> x;
82     cout << L.sample(x);
83 }

```

7.7 Lucas's Theorem

```

1 // 對於很大的 C^n_m 對質數 p 取模，只要 p 不大就可以用。
2 int Lucas(int n, int m, int p){
3     if (m==0) return 1;
4     return (C(n%p, m%p, p)*Lucas(n/p, m/p, p)%p);
5 }

```

7.8 Matrix

```

1 struct Matrix{
2     int n, m;
3     vector<vector<int>> arr;
4
5     Matrix(int _n, int _m){
6         n = _n;
7         m = _m;
8         arr.resize(n, vector<int>(m));
9     }
10
11    Matrix operator * (Matrix b){
12        Matrix b_t(b.m, b.n);
13        for (int i=0; i<b.n; i++){
14            for (int j=0; j<b.m; j++){
15                b_t.arr[j][i] = b.arr[i][j];
16            }
17        }
18
19        Matrix ret(n, b.m);
20        for (int i=0; i<n; i++){
21            for (int j=0; j<b.m; j++){
22                for (int k=0; k<m; k++){

```

```

23         ret.arr[i][j] += arr[i][k]*b_t.arr[j][k];
24         ret.arr[i][j] %= MOD;
25     }
26 }
27
28     return ret;
29 }
30
31 Matrix pow(int p){
32     Matrix ret(n, n), mul = *this;
33     for (int i=0 ; i<n ; i++){
34         ret.arr[i][i] = 1;
35     }
36
37     for ( ; p ; p>>=1){
38         if (p&1) ret = ret*mul;
39         mul = mul*mul;
40     }
41
42     return ret;
43 }
44
45 int det(){
46     vector<vector<int>> arr = this->arr;
47     bool flag = false;
48     for (int i=0 ; i<n ; i++){
49         int target = -1;
50         for (int j=i ; j<n ; j++){
51             if (arr[j][i]){
52                 target = j;
53                 break;
54             }
55         }
56         if (target==-1) return 0;
57         if (i!=target){
58             swap(arr[i], arr[target]);
59             flag = !flag;
60         }
61
62         for (int j=i+1 ; j<n ; j++){
63             if (!arr[j][i]) continue;
64             int freq = arr[j][i]*qp(arr[i][i], MOD-2)%MOD;
65             for (int k=i ; k<n ; k++){
66                 arr[j][k] -= freq*arr[i][k];
67                 arr[j][k] = (arr[j][k]%MOD+MOD)%MOD;
68             }
69         }
70     }
71
72     int ret = !flag ? 1 : MOD-1;
73     for (int i=0 ; i<n ; i++){
74         ret *= arr[i][i];
75         ret %= MOD;
76     }
77     return ret;
78 }
79
80 };

```

7.9 Matrix 01

```
1 const int MAX_N = (1LL<<12);
```

```

2 struct Matrix{
3     int n, m;
4     vector<bitset<MAX_N>> arr;
5
6     Matrix(int _n, int _m){
7         n = _n;
8         m = _m;
9         arr.resize(n);
10    }
11
12    Matrix operator * (Matrix b){
13        Matrix b_t(b.m, b.n);
14        for (int i=0 ; i<b.n ; i++){
15            for (int j=0 ; j<b.m ; j++){
16                b_t.arr[j][i] = b.arr[i][j];
17            }
18        }
19
20        Matrix ret(n, b.m);
21        for (int i=0 ; i<n ; i++){
22            for (int j=0 ; j<b.m ; j++){
23                ret.arr[i][j] = (arr[i]&b_t.arr[j]).count()
24                    &1;
25            }
26        }
27        return ret;
28    };

```

7.10 Miller Rabin

```

1 // O(Log n)
2 typedef Uint unsigned long long
3 Uint modmul(Uint a, Uint b, Uint m) {
4     int ret = a*b - m*(Uint)((long double)a*b/m);
5     return ret + m*(ret < 0) - m*(ret>=(int)m);
6 }
7
8 int qp(int b, int p, int m){
9     int ret = 1;
10    for ( ; p ; p>>=1){
11        if (p&1){
12            ret = modmul(ret, b, m);
13        }
14        b = modmul(b, b, m);
15    }
16    return ret;
17 }
18
19 // ed23aa
20 vector<int> llsprp = {2, 325, 9375, 28178, 450775, 9780504,
21     1795265022};
22 bool isprime(int n, vector<int> sprp = llsprp){
23     if (n==2) return 1;
24     if (n<2 || n%2==0) return 0;
25
26     int t = 0;
27     int u = n-1;
28     for ( ; u%2==0 ; t++) u>>=1;
29
30     for (int i=0 ; i<sprp.size() ; i++){
31         int a = sprp[i]%n;
32         if (a==0 || a==1 || a==n-1) continue;

```

```

32     int x = qp(a, u, n);
33     if (x==1 || x==n-1) continue;
34     for (int j=0 ; j<t ; j++){
35         x = modmul(x, x, n);
36         if (x==1) return 0;
37         if (x==n-1) break;
38     }
39
40     if (x==n-1) continue;
41     return 0;
42 }
43
44 return 1;
45 }

```

7.11 Pollard Rho

```

1 mt19937 seed(chrono::steady_clock::now().time_since_epoch().
2     count());
3 int rnd(int l, int r){
4     return uniform_int_distribution<int>(l, r)(seed);
5 }
6 // O(n^{1/4}) 回傳 1 或自己的因數、記得先判斷 n 是不是質數
7 // (用 Miller-Rabin)
8 // c1670c
9 int Pollard_Rho(int n){
10    int s = 0, t = 0;
11    int c = rnd(1, n-1);
12
13    int step = 0, goal = 1;
14    int val = 1;
15
16    for (goal=1 ; ; goal<=1, s=t, val=1){
17        for (step=1 ; step<=goal ; step++){
18            t = ((__int128)t*t+c)%n;
19            val = ((__int128)val*abs(t-s)%n);
20
21            if ((step % 127) == 0){
22                int d = __gcd(val, n);
23                if (d>1) return d;
24            }
25        }
26
27        int d = __gcd(val, n);
28        if (d>1) return d;
29    }
30 }

```

7.12 Quick Pow

```

1 int qp(int b, int p, int m = MOD){
2     int ret = 1;
3     for ( ; p ; p>>=1){
4         if (p&1) ret = ret*b%m;
5         b = b*b%m;
6     }
7     return ret;
8 }

```


7.13 數論分塊

```
1  /*
2  時間複雜度為  $O(\sqrt{n})$ 
3  區間為  $[l, r]$ 
4  */
5  for(int i=1 ; i<=n ; i++){
6      int l = i, r = n/(n/i);
7      i = r;
8      ans.push_back(r);
9  }
```

7.14 最大質因數

```
1 void max_fac(int n, int &ret){
2     if (n<=ret || n<2) return;
3     if (isprime(n)){
4         ret = max(ret, n);
5         return;
6     }
7
8     int p = Pollard_Rho(n);
9     max_fac(p, ret), max_fac(n/p, ret);
10 }
```

7.15 歐拉公式

```
1 //  $\phi(n)$  = 小於  $n$  並與  $n$  互質的正整數數量。
2 //  $O(\sqrt{n})$  · 回傳  $\phi(n)$ 
3 int phi(int n){
4     int ret = n;
5
6     for (int i=2 ; i*i<=n ; i++){
7         if (n%i==0){
8             while (n%i==0) n /= i;
9             ret = ret*(i-1)/i;
10        }
11    }
12    if (n>1) ret = ret*(n-1)/n;
13
14    return ret;
15 }
16
17 //  $O(n \log n)$  · 回傳 1~n 的  $\phi$  值
18 vector<int> phi_1_to_n(int n){
19     vector<int> phi(n+1);
20     phi[0]=0;
21     phi[1]=1;
22
23     for (int i=2 ; i<=n ; i++){
24         phi[i]=i-1;
25     }
26
27     for (int i=2 ; i<=n ; i++){
28         for (int j=2*i ; j<=n ; j+=i){ // 枚舉所有倍數
29             phi[j]-=phi[i];
30         }
31     }
```

```
32
33     return phi;
34 }
```

7.16 線性篩

```
1 const int MAX_N = 5e5;
2
3 //  $lpf[i]$  =  $i$  的最小質因數
4 vector<int> prime, lpf(MAX_N);
5
6 void prime_init(){
7     for (int i=2 ; i<MAX_N ; i++){
8         if (lpf[i]==0){
9             lpf[i]=i;
10            prime.push_back(i);
11        }
12
13        for (int j : prime){
14            if (i*j>=MAX_N) break;
15            lpf[i*j]=j;
16            if (lpf[i]==j) break;
17        }
18    }
19 }
```

7.17 Burnside's Lemma

$$\sum_{k=1}^n \frac{c(k)}{n}$$

- n : 有多少種置換方式 (例如 : 旋轉方式)
- $c(k)$: 所有可能中 · 經過 k 次旋轉後 · 仍不會和別人相同的方式的數量

7.18 Catalan Number

任意括號序列: $C_n = \frac{1}{n+1} \binom{2n}{n}$

7.19 Matrix Tree Theorem

目標: 給定一張無向圖 · 問他的生成樹數量。
方法: 先把所有自環刪掉 · 定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg(v_i) & \text{if } i = j \\ -(邊v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第一個 row 跟 column · 它的 determinant 就是答案。
目標: 給定一張有向圖 · 問他的以 r 為根 · 可以走到所有點生成樹數量。

方法: 先把所有自環刪掉 · 定義 Q 為以下矩陣

$$Q_{i,j} = \begin{cases} \deg_{in}(v_i) & \text{if } i = j \\ -(邊v_i v_j \text{ 的數量}) & \text{otherwise} \end{cases}$$

接著刪掉 Q 的第 r 個 row 跟 column · 它的 determinant 就是答案。

7.20 Stirling's formula

$$n! \approx \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$$

7.21 Theorem

1. $1 \sim x$ 質數的數量 $\approx \frac{x}{\ln x}$
2. $1 \sim x$ 的因數的數量 $\approx x^{\frac{1}{3}}$
3. x 的質因數的數量 $\approx \log \log x$
4. p is a prime number $\Leftrightarrow (p-1)! \equiv -1 \pmod{p}$
5. 每個正整數都可以表示成四個整數的平方和
6. 任何大於 2 的整數都可以表示成兩個質數的和

7.22 二元一次方程式

$\begin{cases} ax + by = c \\ dx + ey = f \end{cases} = \begin{cases} x = \frac{ed-bf}{ad-bc} \\ y = \frac{af-ec}{ad-bc} \end{cases}$
若 $x = \frac{0}{0}$ 且 $y = \frac{0}{0}$ · 則代表無限多組解。若 $x = \frac{*}{0}$ 且 $y = \frac{*}{0}$ · 則代表無解。

7.23 歐拉定理

若 a, m 互質 · 則:

$$a^n \bmod m = a^{n \bmod \varphi(m)} \bmod m$$

若 a, m 可能是任何數 · 則:

$$a^{\varphi(m) + [n \bmod \varphi(m)]} \bmod m$$

7.24 錯排公式

錯排公式: (n 個人中 · 每個人皆不再原來位置的組合數)

$$dp_i = \begin{cases} 1 & i = 0 \\ 0 & i = 1 \\ (i-1)(dp_{i-1} + dp_{i-2}) & \text{otherwise} \end{cases}$$

8 String

8.1 Hash

```

1 int A = rng(1e5, 8e8);
2 const int B = 1e9+7;
3
4 struct RollingHash{
5     vector<int> Pow, Pre;
6     RollingHash(string s = ""){
7         Pow.resize(s.size());
8         Pre.resize(s.size());
9
10        for (int i=0 ; i<s.size() ; i++){
11            if (i==0){
12                Pow[i] = 1;
13                Pre[i] = s[i];
14            }else{
15                Pow[i] = Pow[i-1]*A%B;
16                Pre[i] = (Pre[i-1]*A+s[i])%B;
17            }
18        }
19        return;
20    }
21
22    int get(int l, int r){ // 取得 [l, r] 的數值
23        if (l==0) return Pre[r];
24        int res = (Pre[r]-Pre[l-1]*Pow[r-l+1])%B;
25        if (res<0) res += B;
26        return res;
27    }
28 }
29 };

```

8.2 KMP

```

1 // 給一個字串 S，定義函數  $\pi(i) = k$  代表  $S[1 \dots k] = S[i-k+1 \dots i]$ 
2 // 4c61a3
3 vector<int> KMP(string &s){
4     n = SZ(s);
5     vector<int> ret(n);
6     int now = 0;
7     for (int i=1 ; i<n ; i++){
8         int j = ret[i-1];
9         while (j>0 && s[i]!=s[j]){
10             j = ret[j-1];
11         }
12         if (s[i]==s[j]) j++;
13         ret[i] = j;
14     }
15     return ret;
16 }

```

8.3 Manacher

```

1 string Manacher(string str) {
2     string tmp = "$#";
3     for(char i : str) {
4         tmp += i;
5         tmp += '#';
6     }
7
8     vector<int> p(tmp.size(), 0);
9     int mx = 0, id = 0, len = 0, center = 0;
10    for(int i=1 ; i<(int)tmp.size() ; i++) {
11        p[i] = mx > i ? min(p[id*2-i], mx-i) : 1;
12
13        while(tmp[i+p[i]] == tmp[i-p[i]]) p[i]++;
14        if(mx<i+p[i]) mx = i+p[i], id = i;
15        if(len<p[i]) len = p[i], center = i;
16    }
17    return str.substr((center-len)/2, len-1);
18 }

```

8.4 Min Rotation

```

1 // 9d296f
2 int minRotation(string s) {
3     int a=0, N=SZ(s); s += s;
4     for (int b=0 ; b<N ; b++){
5         for (int k=0 ; k<N ; k++){
6             if (a+k == b || s[a+k] < s[b+k]) {b += max(0LL, k
7                 -1); break;}
8             if (s[a+k] > s[b+k]) {a = b; break;}
9         }
10    }
11    return a;

```

8.5 Suffix Array

```

1 // 注意，當  $|s|=1$  時， $Lcp$  不會有值，務必測試  $|s|=1$  的 case
2 struct SuffixArray {
3     string s;
4     vector<int> sa, lcp;
5     SuffixArray(string _s, int lim = 256) {
6         s = _s;
7         int n = s.size()+1, k = 0, a, b;
8         vector<int> x(s.begin(), s.end()), y(n), ws(max(n,
9             lim)), rank(n);
10        x.push_back(0);
11        sa = lcp = y;
12        iota(sa.begin(), sa.end(), 0);
13        for (int j=0, p=0 ; p<n ; j=max(1LL, j*2), lim=p) {
14            p = j;
15            iota(y.begin(), y.end(), n-j);
16            for (int i=0 ; i<n ; i++) if (sa[i] >= j) y[p++]
17                = sa[i] - j;
18            fill(ws.begin(), ws.end(), 0);
19            for (int i=0 ; i<n ; i++) ws[x[i]]++;
20            for (int i=1 ; i<lim ; i++) ws[i] += ws[i-1];
21            for (int i = n; i--;) sa[--ws[x[i]]] = i;
22            swap(x, y), p = 1, x[sa[0]] = 0;
23            for (int i=1 ; i<n ; i++){
24                a = sa[i-1];

```

```

23        b = sa[i];
24        x[b] = (y[a] == y[b] && y[a+j] == y[b+j])
25            ? p-1 : p++;
26    }
27
28    for (int i=1 ; i<n ; i++) rank[sa[i]] = i;
29    for (int i=0, j ; i<n-1 ; lcp[rank[i+1]]=k)
30        for (k && k--, j=sa[rank[i]-1] ; i+k<s.size() &&
31            j+k<s.size() && s[i+k]==s[j+k] ; k++);
32    sa.erase(sa.begin());
33    lcp.erase(lcp.begin(), lcp.begin()+2);
34 }
35
36 vector<int> pos; // pos[i] = i 這個值在 pos 的哪個地方
37 SparseTable st;
38 void init_lcp(){
39     pos.resize(sa.size());
40     for (int i=0 ; i<sa.size() ; i++){
41         pos[sa[i]] = i;
42     }
43     if (lcp.size()){
44         st.build(lcp);
45     }
46
47     // 用之前記得 init
48     // 回傳 [l1, r1] 跟 [l2, r2] 的 lcp，0-based
49     int get_lcp(int l1, int r1, int l2, int r2){
50         int pos_1 = pos[l1], len_1 = r1-l1+1;
51         int pos_2 = pos[l2], len_2 = r2-l2+1;
52         if (pos_1>pos_2){
53             swap(pos_1, pos_2);
54             swap(len_1, len_2);
55         }
56
57         if (l1==l2){
58             return min(len_1, len_2);
59         }else{
60             return min({st.query(pos_1, pos_2), len_1, len_2
61                 });
62         }
63     }
64
65     // 檢查 [l1, r1] 跟 [l2, r2] 的大小關係，0-based
66     // 如果前者小於後者，就回傳 <0，相等就回傳 =0，否則回傳
67     >0
68     int substring_cmp(int l1, int r1, int l2, int r2){
69         int len_1 = r1-l1+1;
70         int len_2 = r2-l2+1;
71         int res = get_lcp(l1, r1, l2, r2);
72
73         if (res<len_1 && res<len_2){
74             return s[l1+res]-s[l2+res];
75         }else if (len_1==res && len_2==res){
76             // 如果不需要以 index 作為次要排序參數，這裡要回
77             傳 0
78             return l1-l2;
79         }else{
80             return len_1==res ? -1 : 1;
81         }
82     }
83 }

```

```

81 | // 對於位置在 <=p 的後綴，找離他左邊/右邊最接近位置 >p 的 22 | }
    | 後綴的 lcp · 0-based
82 | // pre[i] = s[i] 離他左邊最接近位置 >p 的後綴的 lcp · 0-
    | based
83 | // suf[i] = s[i] 離他右邊最接近位置 >p 的後綴的 lcp · 0-
    | based
84 | pair<vector<int>, vector<int>> get_left_and_right_lcp(int
    | p){
85 |     vector<int> pre(p+1);
86 |     vector<int> suf(p+1);
87 |
88 |     { // build pre
89 |         int now = 0;
90 |         for (int i=0 ; i<s.size() ; i++){
91 |             if (sa[i]<=p){
92 |                 pre[sa[i]] = now;
93 |                 if (i<lcp.size()) now = min(now, lcp[i]);
94 |             }else{
95 |                 if (i<lcp.size()) now = lcp[i];
96 |             }
97 |         }
98 |     }
99 |     { // build suf
100 |         int now = 0;
101 |         for (int i=s.size()-1 ; i>=0 ; i--){
102 |             if (sa[i]<=p){
103 |                 suf[sa[i]] = now;
104 |                 if (i-1>=0) now = min(now, lcp[i-1]);
105 |             }else{
106 |                 if (i-1>=0) now = lcp[i-1];
107 |             }
108 |         }
109 |     }
110 |
111 |     return {pre, suf};
112 | }
113 | };

```

8.6 Z Algorithm

```

1 | // 定義一個長度為 n 的文本為 T，則陣列 Z 的 Z[i] 代表 T[0:n]
    | 和 T[i:n] 最長共同前綴
2 | // bcfbd6
3 | vector<int> z_function(string s){
4 |     vector<int> ret(s.size());
5 |     int ll = 0, rr = 0;
6 |
7 |     for (int i=1 ; i<s.size() ; i++){
8 |         int j = 0;
9 |
10 |         if (i<rr) j = min(ret[i-ll], rr-i);
11 |         while (s[j]==s[i+j]) j++;
12 |         ret[i] = j;
13 |
14 |         if (i+j>rr){
15 |             ll = i;
16 |             rr = i+j;
17 |         }
18 |     }
19 |
20 |     ret[0] = s.size();
21 |     return ret;

```