

## Agenda

- Time complexity & space complexity
- Asymptotic analysis
- <sup>Big O</sup>
- Time limit Exceeded

X

no of iterations for different loops =

1) sum  $1 \rightarrow N \Rightarrow \frac{N * (N+1)}{2}$

2)  $N \rightarrow 1 \Rightarrow \log_2 N$   
divide by 2

3)  $[3, 10] = 8 = 10 - 3 + 1$   $b - a ?$

$a \leq b$   $[a, b] = b - a + 1$   
 $[-1, 7] \rightarrow 7 - (-1) + 1 = 9$

## • Arithmetic progression

1  $\xrightarrow{3}$  4  $\xrightarrow{3}$  7  $\xrightarrow{3}$  10  $\xrightarrow{3}$  13 . . 16 . . 19 22 . . .

common  
diff =  $d$

$a \quad a+d \quad a+2d \quad a+3d \quad a+4d \quad a+5d \dots$

$a+(n-1)d$

$$\Rightarrow S_n = \frac{n}{2} (2a + (n-1)d)$$

1 . . . . . 10  
↑

first

common  
diff

$$S_n = \frac{n}{2} (a + l)$$

first

last term

1 3 5 7 9

$n=5$

$a=1$

$l=9$

$$\frac{5}{2} \times (1+9) = \underline{25}$$

## • Geometric progression

3  $\xrightarrow{\times 4}$  12  $\xrightarrow{\times 4}$  48  $\xrightarrow{\times 4}$  192 . . . . .  
 $\frac{12}{3} = 4 \quad \frac{48}{12} = 4 \quad \frac{192}{48} = 4$

common  
ratio =  $(r)$

$a \quad ar \quad ar^2 \quad ar^3 \quad ar^4 \quad \dots \quad ar^{n-1}$

first  
term

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$r > 1$$

$$S_\infty = \frac{a}{1 - r}$$



$$0 < r < 1$$

① `void func ( int n )`

`s = 0;`

`0 → n-1`

`n`

`for ( int i = 0; i < n; i++ )`

`{`

`s = s + i;`

`}`

`}`

② `void func ( int n, int m )`

`s = 0;`

`for ( int i = 0; i < n; i++ )`

`{`

`if ( i % 2 )`

`s += i;`

`}`

`for ( int j = 0; j < m; j++ )`

`{`

`if ( j % 2 == 0 )`

`s += j;`

`}`

`}`

`n`

`n + m`

`m`

③ `void func ( int n)`

`s=0;`

`for ( int i=1; i<=n; i+=2 )`

`{`

`s+=i;`

`}`

`}`

$i=1$

$N=10$

$i=3$

$i=5$

$i=7$

$i=9$

~~$i=11$~~

$\approx \frac{N}{2}$

④ `void func ()`

`s=0;`

`for ( int i=0; i<=100; i++)`

`{`

`s+=i;`

`}`

`}`

$[0, 100] \approx 101$

⑤ void func (int n)

s = 0;

for (int i = 1; i \* i <= N; i++)

{

s += i;

}

}

N = 110

i = 1

2

3

4

5

6

7

8

9

10

11

$\sqrt{N}$

⑥ void func (int n)

i = N;

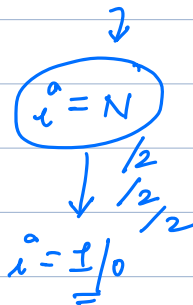
while (i >= 1)

{

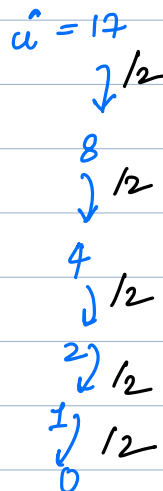
i = i / 2;

}

}



$\log_2 N$



7

void func (int n)

s=0

for (i=0; i <= N; i\*=2)

{

s = s + i;

}

}

infinite

i = 0

0 \* 2 = 0

i = 0

0 \* 2 = 0

i = 0 + 2

0

void func (int n)

s=0

↓  
N > 0

for (i=1; i <= N; i\*=2)

{

s = s + i;

}

}

i = 1

i = 2

i = 4

i = 8

i = 16

i = 32

⋮

1      \*2      \*2  
1

N } log<sub>2</sub> N

1/2   1/2   1/2   N } log<sub>2</sub> N

⑧ void func (int n)

s=0

for ( j=1; j<=10; j++)

{

for ( i=1; i<=N; i++)

{

s = s + i \* j;

}

}

}

j	i	
j=1	1 → N	(N)
j=2	1 → N	(N)
j=3	1 → N	(N)
⋮		
j=10	1 → N	(N)
<u>j=11</u>		<u>10 * N</u>

⇒ 10 \* 26

void func (int n)

s=0

for ( j=1; j<=N; j++)

{

for ( i=1; i<=N; i++)

{

s = s + i \* j;

}

}

}

j	i	
j=1	1 → N	N
= 2	1 → N	N
= 3	1 → N	N
4	1 → N	.
5	.	.
⋮	⋮	⋮
1	1	1
N		
		<u>N<sup>2</sup></u>



```
void func (int n)
```

```
    s = 0
```

```
    for ( i = 0; i < n; i++)
```

```
    {
```

```
        for ( j = 0; j <= i; j++)
```

```
        {
```

```
            s = s + i * j;
```

```
        }
```

```
    }
```

```
}
```

i	j	
0	0-0	1
1	0-1	2
2	0-2	3
3	0-3	4
⋮		
⋮		
⋮		
n-1	0-n-1	n
		$\frac{n \times (n+1)}{2}$

$$\frac{n^2 + n}{2}$$

```
void func (int n)
```

```
    s = 0
```

```
    for ( i = 1; i <= n; i++)
```

```
    {
```

```
        for ( j = 1; j <= n; j = j * 2)
```

```
        {
```

```
            s = s + i * j;
```

```
        }
```

```
    }
```

```
}
```

i	j	
1	[1 → N]	$\log_2 N$
2	[1 → N]	$\log_2 N$
3	[1 → N]	$\log_2 N$
4	[1 → N]	$\log_2 N$
⋮		
N	[1 → N]	$\log_2 N$

$$N \log_2 N$$

$$i^0 = 8$$

```

void fun ( int ar[], int n)
{
    //  $n > 0$ 
    for ( int i = N; i > 0; i = i/2)
    {
        for ( int j = 1; j <= i; j++)
        {
            // some task
        }
    }
}

```

$i^0$	$j$	
$N$	$1-N$	$N$
$N/2$	$1-N/2$	$N/2$
$N/4$	$1-N/4$	$N/4$
$N/8$	$1-N/8$	$N/8$
$\vdots$		
$\vdots$		
$1$	$1-1$	$1$

$$N + N/2 + N/4 + N/8 + N/16 \dots 1$$

$$N \left[ 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} \dots \right] + 1$$

$$r = 1/2$$

$$S_{\infty} = \frac{a}{1-r} = \frac{1}{1-1/2}$$

$$2N$$

$$S_{\infty} = 2$$