Agenda

- Time complexity & space complexity
- Asymptotic analysis
- Big O
- Time Limit Exceeded

no of iterations for different loops.

1) sum $1 \longrightarrow N$ $\Rightarrow$ $N*(N+1)/2$

2) $N \xrightarrow{\text{divide by 2}} 1$ $\Rightarrow$ $\log_2 N$

3) $[3, 10]$ $= 8 = 10-3+1$ $b-a$?

$a <= b$ $[a, b]$ $= \boxed{b-a+1}$

$[-1, 7]$ $7-(-1)+1 = 9$

Arithmetic progression

$1 \xleftrightarrow{3} 4 \xleftrightarrow{3} 7 \xleftrightarrow{3} 10 \xleftrightarrow{3} 13 \;.\;.\; 16 \;.\;.\; 19 \quad 22 \;\ldots$

common diff = d

$a \quad a+d \quad a+2d \quad a+3d \quad a+4d \quad a+5d \;\ldots\; a+(n-1)d$

no of terms

$$\Rightarrow \boxed{S_n = \frac{n}{2}\left(2a + (n-1)d\right)} \longrightarrow$$

first — common diff

$1 \;\ldots\ldots\; 10$

$$\boxed{S_n = \frac{n}{2}\left(a + l\right)}$$

first — last i.e.

$1 \quad 3 \quad 5 \quad 7 \quad 9$

$n = 5$

$a = 1$

$l = 9$

$\frac{5}{2} * (1+9) = 25$

Geometric progression

$3 \xrightarrow{*4} 12 \xrightarrow{*4} 48 \xrightarrow{*4} 192 \;\ldots\ldots$

$\frac{12}{3} = 4 \quad \frac{48}{12} = 4 \quad \frac{192}{48} = 4$

common ratio = r

$a \quad ar \quad ar^2 \quad ar^3 \quad ar^4 \;\ldots\; ar^{n-1}$

first term

$$\boxed{S_n = \frac{a(r^n - 1)}{r - 1}}$$

$r > 1$

$$\boxed{S_\infty = \frac{a}{1 - r}} \Longleftarrow 0 < r < 1$$

①

```
void func ( int n )
{
    s=0;
    for( int i=0; i<n; i++)
    {
        s=s+i;
    }
}
```

$0 \longrightarrow n-1$

n

②

```
void func ( int n, int m)
{
    s=0;
    for( int i=0; i<n; i++)
    {     if ( i%2)
            s+=i;
    }
    for( int j=0; j<m; j++)
    {
        if( j%2==0)
            s+=j;
    }
}
```

n

m

$n+m$

③

```
void func ( int n)
{
        s=0;
        for( int i=1 ; i<=n; i+=2)
        {
                s+=i;
        }
}
```

$N=10$

$i=1$
$i=3$
$i=5$
$i=7$
$i=9$
~~$i=11$~~

$\approx N/2$

④

```
void func ()
{
        s=0;
        for( int i=0; i<=100; i++)
        {
                s+=i;
        }
}
```

$[0,\ 100]$ $\equiv 101$

(5)

```
void func (int n)
{
    s=0;
    for(int i=1; i*i<=N; i++)
    {
        s+=i;
    }
}
```

$N = 110$

$i = 1$
2
3
4
5
6
7
8
9
10
11

$\sqrt{N}$

(6)

```
void func (int n)
{
    i=N;
    while( i >= 1)
    {
        i = i/2;
    }
}
```

$i = N$

/2

/2/2

$i = 1/0$

$\log_2 N$

$i = 17$

/2

8

/2

4

/2

2

/2

1

/2

0

⑦

```
void func (int n)
{
    s=0
    for (i=0; i<=N; i*=2)
    {
        s=s+i;
    }
}
```

infinite

$i=0$

$0*2=0$

$i=0$

$0*2=0$

$i=0 + 2$

0

```
void func (int n)
{
    s=0
    for (i=1; i<=N; i*=2)
    {
        s=s+i;
    }
}
```

$N>0$

$i=1$

$i=2$

$i=4$

$i=8$

$i=16$

$i=32$

⋮

$*2 \quad *2$

$1 \quad \quad N \} \log_2 N$

$1 \quad /2 \quad /2 \quad /2 \quad N \} \log_2 N$

(8)

```
void func (int n)
{
    s=0
    for( j=1; j<=10; j++)
    {
        for ( i=1; i<=N; i++)
        {
            s= s + i*j;
        }
    }
}
```

j	i	
j=1	1 → N	N
j=2	1 → N	N
j=3	1 → N	N
⋮		
j=10	1 → N	N
j=11		10*N

10:36

```
void func (int n)
{
    s=0
    for( j=1; j<=N; j++)
    {
        for ( i=1; i<=N; i++)
        {
            s= s + i*j;
        }
    }
}
```

j	i	
j=1	1 → N	N
=2	1 → N	N
=3	1 → N	N
4	1 → N	⋮
5	⋮	⋮
⋮		
N		
		N^2

```
void func (int n)
{
    s=0
    for( i=0; i<n; i++)
    {
        for(j=0; j<=i; j++)
        {
            s=s+i*j;
        }
    }
}
```

i	j	
0	0-0	1
1	0-1	2
2	0-2	3
3	0-3	4
⋮		
n-1	0-n-1	n

$n*(n+1)/2$

$\frac{n^2+n}{2}$

```
void func (int n)
{
    s=0
    for( i=1, i<=n; i++)
    {
        for(j=1; j<=n; j=j*2)
        {
            s=s+i*j;
        }
    }
}
```

i	j	
1	[1→N]	$\log_2 N$
2	[1→N]	$\log_2 N$
3	[1→N]	$\log_2 N$
4	[1–N]	$\log_2 N$
N	[1–N]	$\log_2 N$

$N\log_2 N$

$i = 8$

```
void fun( int ar[], int n)
{                      // N>0              N=8
    for( int i=N; i>0; i = i/2)
    {
        for( int j=1; j<=i; j++)
        {
            // some task
        }
    }
}
```

i	j	
N	1 – N	N
N/2	1 – N/2	N/2
N/4	1 – N/4	N/4
N/8	1 – N/8	N/8
⋮		
1	1 - 1	1

$$N + N/2 + N/4 + N/8 + N/16 \cdots 1$$

$$N\left(1 + 1/2 + 1/4 + 1/8 + 1/16 \cdots\right) + 1$$

2 $\quad r = 1/2$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-1/2}$$

$= 2N$

$$S_\infty = 2$$