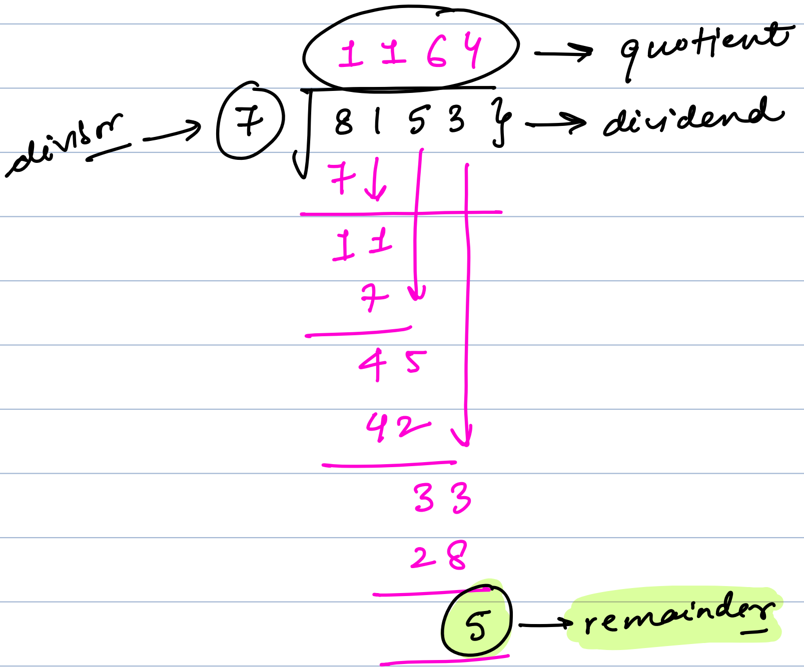


a/b → when a is divided by b

a % b —

dividend = divisor * quoteint + remainder

remainder = dividend − divisor * quotient

↓

largest multiple of divisor <= divid

30 % 4 = 2

2 = 30 − 28

{ c++, java, python } diff -ve number is allo

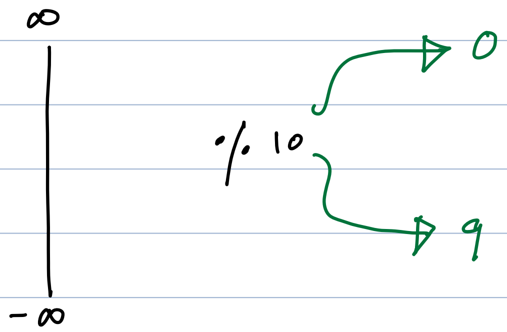
150 % 11 = 150 − 143 = 7

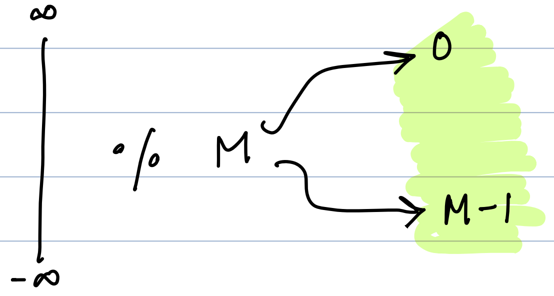
100 % 7 = 100 − 98 = 2

−40 % 7 = (−40) − (−42) = 2

−60 % 9 = (−60) − (−63) = 3

why % ?





limits the range

$\longrightarrow$ $+ , - , * , /$ — advanced curriculum

✓ ✓

Modular Arithmetic

0−M−1   0−M−1   0−M−1

- $(a+b)\%M = ((a\%M) + (b\%M))\%M$

$a = 6,\ b = 8,\ M = 10$

$(6+8)\%10 = (6\%10 + 8\%10)\%10$

$\downarrow$

$\textcircled{4}$

$= (6 + 8)\%10$

$= (14)\%10 = \textcircled{4}$

- $(a*b)\%M = ((a\%M) * (b\%M))\%M$

- $(a\%m)\%m = a\%m$

$17\%7$

$\downarrow$

$(3\%7)\%7$

$(((a\%m)\%m)\%m)\%m \ldots$

- given, a , n , p

Find $a^n \% p$.

$a=2$, $n=5$, $p=7$ 2^5 $32 \% 7 = \boxed{4}$ ⇑

Idea : Multiply a , n times

$a=2, n=5$

```
     for(  i=1;  i<=n;  i++)
       {
X              a = a*a;
       }
```

i	a=2		
1	a=2*2	a=4	2^2
2	a=4*4	a=16	2^4
3	a=16*16	a=256	2^8
4	$a=2^8*2^8$	2^{16}	
5	$a=2^{16}*2^{16}$	$\boxed{2^{32}}$ ↑	

$a=2, n=5$

```
     for ( int  i=1;  i<=n;  i++)
       {
X            ans = a*a;
       }
```

~~a += a;~~

```
         int  ans=0 ;
         for ( int  i=1;  i<= n; i++)
X          {
                  ans = ans*a ;
           }
```

```
long
~~int~~  ans = 1;

for( int i=1; i<=n; i++)
{
        ans = ans * a;                     — 1
        ans = (ans * a) % p;               — 2
  ans = ((ans%p)*(a%p))%p   0-p-1          — 3 ✓
}

return  ans%p;
```

T.C: O(n)
↑
exponent

$p = 10^9$

$1 \le a \le 10^9$

$1 \le n \le 10^5$

$1 \le p \le 10^9$

$1 \le a \le 10^{12}$

$a = 2,\ n = 5,\ p = 10^9$ ✓

$a = 2\ \ n = 10,\ p = 10^9$ ✓

take log → $a = 2,\ n = 40,\ p = 10^9$ ✓

$a = 2,\ n = 1000,\ p = 10^9$

$(a * a * a * a * \ldots n\ \text{times})\ \%\ p$

Before even taking the %, it is already modulo

$(ans * \boxed{a})\ \%\ p$, $\ a \rightarrow 10^{12}$, $\ ans \downarrow \boxed{0\text{-}p\text{-}1} \approx 10^9$

$= 10^9 * 10^{12} = \boxed{10^{21}}$

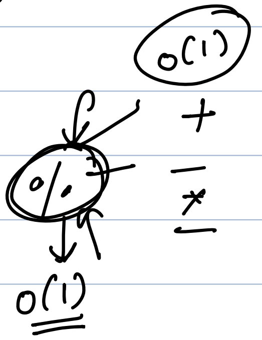
↑ ~~ll~~ ?

$$= \big(\underbrace{(ans\ \%\ p)}_{0\text{-}p\text{-}1} * \underbrace{(a\ \%\ p)}_{0\text{-}p\text{-}1}\big)\ \%\ p$$

%

use large datatypes

distribute your modulo as much as you can



- $\underline{\underline{7326}}$ is divisible by 3

Divisibility rule of 3

$\hookrightarrow$ sum of digits should be divisible by $\underline{\underline{3}}$

$7326 \% 3 = 0$?

$$(3458) \% 3 = (3 * 10^3 + 4 * 10^2 + 5 * 10 + 8 * 1) \% 3$$

$$= ((3 * 10^3) \% 3 + (4 * 10^2) \% 3 + (5 * 10) \% 3 + (8 * 1) \% 3) \% 3$$

$$= ((3 \% 3 * \underset{1}{\cancel{10^3 \% 3}}) \% 3 + (4 \% 3 * \underset{1}{\cancel{10^2 \% 3}}) \% 3$$

$$+ (5 \% 3 * \underset{1}{\cancel{10 \% 3}}) \% 3 + (8 \% 3 * \underset{1}{\cancel{1 \% 3}}) \% 3) \% 3$$

$1 \% 3 = 1$

$10 \% 3 = 1$

$100 \% 3 = 1$

$1000 \% 3 = 1$

$10^x \% 3 = 1$

$$= (3 \% 3 + 4 \% 3 + 5 \% 3 + 8 \% 2) \% 3$$

$$= (\underbrace{\boxed{3 + 4 + 5 + 8}}_{\text{sum of digit}}) \% 3$$

Divisibility rule of 4

→ just check last 2 digits.
if last 2 digits divisible by 4

2 4 (4 2)

$$(4328)\%4 = (4*10^3 + 3*10^2 + 2*10 + 8*1)\%4$$

$$((4*10^3)\%4 + (3*10^2)\%4 + (2*10)\%4 + (8*1)\%4)\%4$$

$$(4\%4 * \cancel{10^3\%4}_{0})\%4 + (3\%4 * \cancel{10^2\%4}_{0})\%4 +$$

$$(2\%4 * 10\%4)\%4 + (8\%4 * 1\%4)\%4$$

$$= ((2*10)\%4 + (8\%4))\%4$$

$$= (28)\%4$$

$10^0 \leftarrow 1\ \%4 = 1$

$10^1 \leftarrow 10\ \%4 = 2$

$10^2 \leftarrow 100\ \%4 = 0$

$10^3 \rightarrow 1000\ \%4 = 0$

$10^4 \rightarrow 10000\ \%4 = 0$

10^5

$$\boxed{10^x \% 4 = 0,\ x >= 2}$$

Google

- Given a number n in the form of array of digits.

Find $n \% p$ → given.

$p = 10^9$

$n = 2134527$

2	1	3	4	5	2	7

100 10 0

{create the number & store it in int}

$$\boxed{1 \le size \le 10^5}$$

$\approx 10^{10^5}$

$2*10^6 + 1*10^5 + 3*10^4 + 4*10^3$
$+ 5*10^2$
$+ 2*10^1 + 7*1 \;\% p$

session

20 min