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S–247(N)

B. A./B. Sc. (First Semester) EXAMINATION, 2023-24

MATHEMATICS

(Differential Calculus)

(SOS/Maths./CS—I)

Time : Two Hours] [*Maximum Marks : 70*

Note : (i) Attempt any *five* questions from Section A and any *three* questions from Section B.

(ii) Answer each question of Section A within 50 words.

(iii) Limit your answers within the given answer book. Additional answer book (B-Answer book) should not be provided or used.

Section—A

Note : Attempt any *five* questions. Each question carries 5 marks.

1. Show that :

$$\lim_{x \to 4} \sqrt{x} = 2$$

using $\varepsilon - \delta$ definition of limit.

2. Show that :

$$f(x) = |x^2 - 2x|$$

is not differentiable at $x = 0$ and $x = 2$.

3. State the Lagrange's mean value theorem and verify it for the following function :

$$f(x) = x(x-1)(x-2),\ x \in \left[0, \frac{1}{2}\right]$$

4. If $y = (x^2 - 1)^n$, using Lebnitz's theorem prove that :

$$(1 - x^2)y_{n+2} - 2xy_{n+1} + n(n+1)y_n = 0$$

5. If $u = xf\left(\frac{y}{x}\right) + g\left(\frac{y}{x}\right)$, then show that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 0$$

6. Find the radius of curvature at any point of the cycloid :

$$x = a(t + \sin t);\ y = a(1 - \cos t)$$

7. Discuss the maxima and minima of the function :

$$u(x, y) = x^3 + y^3 - 3axy.$$

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Section—B

Note : Attempt any *three* questions. Each question carries 15 marks.

8. (a) Find the value of a, b and c such that :

$$\lim_{x \to 0} \frac{ae^{x} - b\cos x + ce^{-x}}{x \sin x} = 2.$$

(b) If a function is differentiable at a point, then show that it is necessarily continuous at the point.

9. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that :

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x + y + z)^2}.$$

(b) If $x = r\cos\theta$, $y = r\sin\theta$, then show that :

$$\left(\frac{\partial r}{\partial x}\right)^2 + \left(\frac{\partial r}{\partial y}\right)^2 = 1.$$

10. (a) Show that the pedal equation of the parabola $y^2 = 4a(x + a)$ is $p^2 = ar$.

(b) Show that for any curve $\frac{r}{p} = \sin\phi\left(1 + \frac{d\phi}{d\theta}\right)$.

11. (a) Find the asymptotes of the curve :

$$r\cos\theta = a\sin\theta.$$

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(b) Find all the asymptotes of the curve :

$$(x - y)(x + 2y)^2 - 2x + y + 5 = 0$$

12. (a) Trace the curve :

$$y^2(a + x) = x^2(a - x).$$

(b) Find the points of inflection of the curve :

$$y = (x - 1)^4(x - 2)^3.$$

13. (a) Find the Maclaurin series of :

$$f(x) = \log(1 + \tan x).$$

(b) Find all stationary points of :

$$f(x, y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2$$

and determine their nature.

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