

Report - Lab 3

1. Let $f(x) = e^x - x - 1$. Compute the approximate order of convergence of both Newton's and Modified Newton's methods in finding a zero of f in the interval $[-1, 1]$.

| Iteration Number | Newton's Method | | Modified Newton's Method | |
|------------------|-----------------|----------|--------------------------|---------|
| | Error | Alpha | Error | Alpha |
| 1 | -0.418023 | | -0.119955 | |
| 2 | -0.194492 | | -0.00230437 | |
| 3 | -0.0940957 | 0.94894 | -8.84E-07 | 1.9901 |
| 4 | -0.0463101 | 0.976412 | -3.07E-11 | 1.30545 |
| 5 | -0.0229763 | 0.988636 | | |
| 6 | -0.0114442 | 0.99442 | | |
| 7 | -0.00571118 | 0.997234 | | |
| 8 | -0.00285287 | 0.998623 | | |
| 9 | -0.00142576 | 0.999313 | | |
| 10 | -0.000712709 | 0.999657 | | |
| 11 | -0.000356312 | 0.999829 | | |
| 12 | -0.000178146 | 0.999914 | | |
| 13 | -8.91E-05 | 0.999957 | | |

Approximate order of convergence of Newton's Method is ~ 1 .

Approximate order of convergence of Modified Newton's Method is ~ 2 .

2. Compute the approximate order of convergence of the fixed point iteration in finding the fixed point of the function $\cos(x)$ in $[0, \pi/2]$.

| Fixed point's Method | | |
|----------------------|-------------|----------|
| Iteration Number | Error | Alpha |
| 1 | 0.0531378 | |
| 2 | 0.0355771 | |
| 3 | 0.0240524 | 0.975775 |
| 4 | 0.016159 | 1.01606 |
| 5 | 0.0109034 | 0.989071 |
| 6 | 0.00733595 | 1.00731 |
| 7 | 0.00494543 | 0.995053 |
| 8 | 0.00332953 | 1.00332 |
| 9 | 0.00224361 | 0.997758 |
| 10 | 0.00151096 | 1.00151 |
| 11 | 0.00101796 | 0.998983 |
| 12 | 0.000685637 | 1.00068 |
| 13 | 4.62E-04 | 0.999539 |
| 14 | 0.000311117 | 1.00031 |
| 15 | 0.000209579 | 0.999791 |
| 16 | 0.000141172 | 1.00014 |
| 17 | 9.51E-05 | 0.999905 |
| 18 | 6.41E-05 | 1.00006 |
| 19 | 4.32E-05 | 0.999957 |
| 20 | 2.91E-05 | 1.00003 |
| 21 | 1.96E-05 | 0.99998 |
| 22 | 1.32E-05 | 1.00001 |
| 23 | 8.88E-06 | 0.999991 |
| 24 | 5.98E-06 | 1.00001 |
| 25 | 4.03E-06 | 0.999996 |
| 26 | 2.72E-06 | 1 |
| 27 | 1.83E-06 | 0.999998 |
| 28 | 1.23E-06 | 1 |
| 29 | 8.30E-07 | 0.999999 |

Approximate order of convergence of the fixed point iteration is ~ 1 .

3. Let $f(x) = (x-1)(x-6)(x-8)$, compare the approximate order of convergence of both secant and Newton's method in finding the root of f in the interval $[0, 2]$.

| Iteration Number | Newton's Method | | Secant's Method | |
|------------------|-----------------|---------|-----------------|---------|
| | Error | Alpha | Error | Alpha |
| 1 | 0.673684 | | 0.0851064 | |
| 2 | 0.115319 | | 0.0131438 | |
| 3 | 0.00430203 | 1.86318 | 3.94E-04 | 1.87826 |
| 4 | 6.33E-06 | 1.983 | 1.78E-06 | 1.53881 |
| 5 | 1.37E-11 | 1.99966 | 2.40E-10 | 1.65048 |

Approximate order of convergence of Newton's method is ~ 2 .

Approximate order of convergence of Secant's method is ~ 2 .