Report - Lab 11
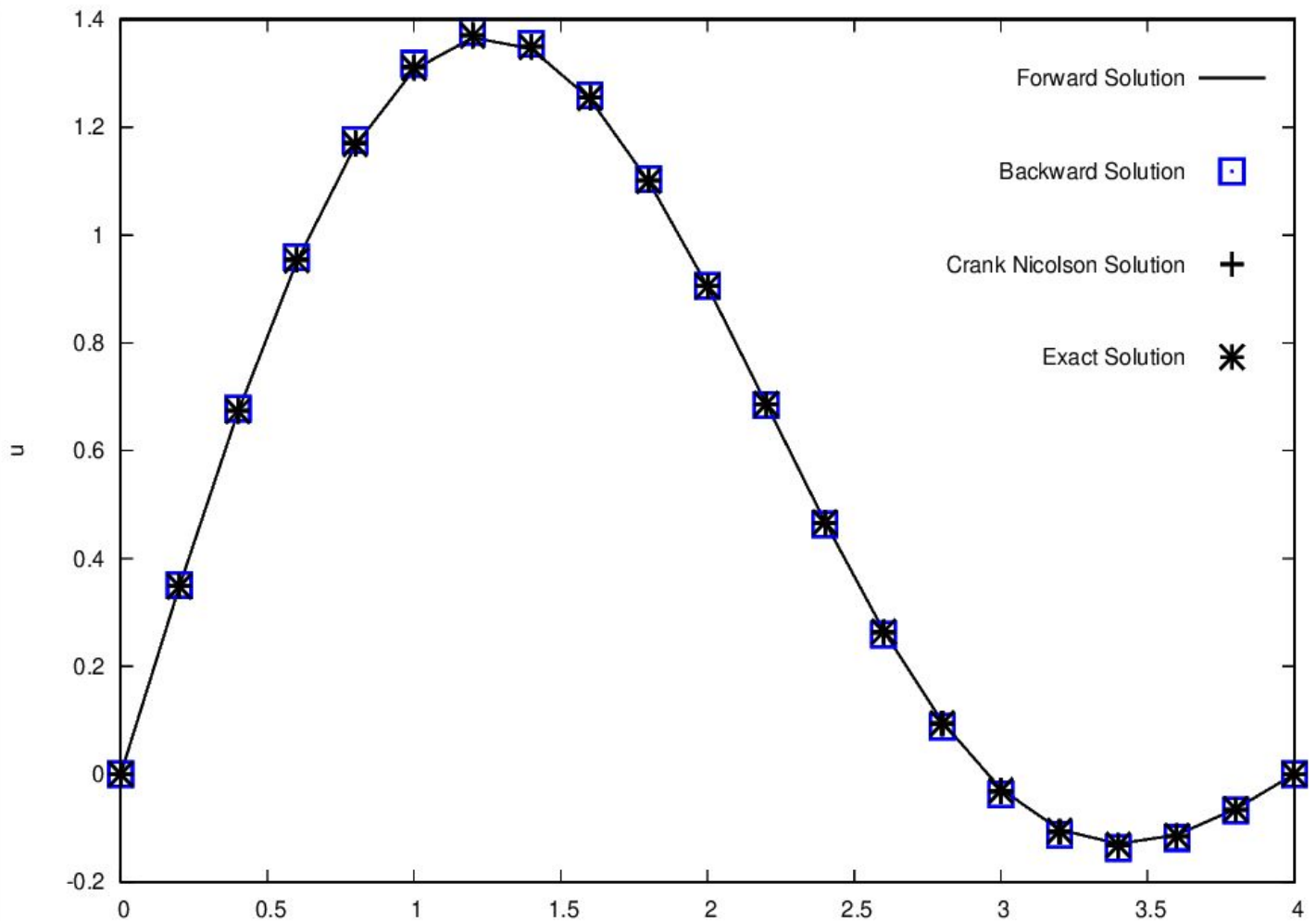
1. Following solution calculated at $t = 0.4 \ and \ \Delta t = 0.04$

x	Exact Solution	Forward Method		Backward Method		Crank-Nicolson Method	
		Solution	Error	Solution	Error	Solution	Error
0	0	0	0	0	0	0	0
0.2	0.3486880433	0.3476532863	0.001034757	0.3510670541	0.0023790108	0.3493865351	0.0006984918
0.4	0.6736143767	0.6716430844	0.0019712923	0.678146787	0.0045324103	0.6749451092	0.0013307325
0.6	0.9530879005	0.9503669774	0.0027209231	0.9593443933	0.0062564928	0.9549248045	0.001836904
0.8	1.169362338	1.166149225	0.0032131132	1.176751562	0.0073892239	1.171531763	0.0021694253
1	1.31013672	1.306734413	0.0034023071	1.317962656	0.0078259356	1.312434282	0.002297562
1.2	1.369541096	1.366268771	0.0032723248	1.377070516	0.0075294195	1.371751486	0.00221039
1.4	1.348516352	1.345678463	0.0028378884	1.355049784	0.006533432	1.350434176	0.0019178246
1.6	1.25455576	1.252412629	0.0021431312	1.259495034	0.0049392739	1.256005377	0.0014496166
1.8	1.100837654	1.099580416	0.0012572383	1.103743434	0.0029057805	1.101690066	0.000852412
2	0.904837418	0.9045697726	0.0002676454	0.905471123	0.000633705	0.9050225844	0.0001851664
2.2	0.686557082	0.6872856198	0.0007285378	0.6849031075	0.0016539745	0.6860704433	0.0004866387
2.4	0.4665472852	0.4681813245	0.0016340393	0.4628133897	0.0037338955	0.4654498759	0.0010974093
2.6	0.2639157339	0.2662766746	0.0023609407	0.2585115723	0.0054041616	0.2623278781	0.0015878558
2.8	0.0945166006	0.097355866	0.0028392654	0.0880125371	0.0065040635	0.092605816	0.0019107846
3	-0.0305033719	-0.0274795727	0.0030237992	-0.0374331134	0.0069297415	-0.0325390692	0.0020356973
3.2	-0.1056621577	-0.1027636807	0.002898477	-0.1123064168	0.0066442591	-0.107613907	0.0019517493
3.4	-0.1315127173	-0.1290348112	0.0024779061	-0.1371938181	0.0056811008	-0.1331814938	0.0016687765
3.6	-0.1143940981	-0.1125882198	0.0018058783	-0.1185348572	0.0041407591	-0.1156103915	0.0012162934
3.8	-0.0655925285	-0.0646415094	0.0009510191	-0.0677732727	0.0021807442	-0.0662330875	0.000640559
4	0	0	0	0	0	0	0

u

1.4

1.2

1

0.8

0.6

0.4

0.2

0

-0.2

Forward Solution ———

Backward Solution ▢

Crank Nicolson Solution ✛

Exact Solution ✳

0   0.5   1   1.5   2   2.5   3   3.5   4

Forward Difference Method becomes unstable when Δt becomes large.

Following solution calculated at $t = 4.0 \; and \; \Delta t = 0.08$

x	Exact Solution	Forward Method		Backward Method		Crank-Nicolson Method	
		Solution	Error	Solution	Error	Solution	Error
0	0	0	0	0	0	0	0
0.2	0.0632088673	-2.935475913	2.998684781	0.0650287444	0.0018198771	0.0635018064	0.0002929391
0.4	0.1244466616	5.963104369	5.838657707	0.1279586737	0.0035120121	0.1250123961	0.0005657345
0.6	0.1818314345	-8.212473212	8.394304646	0.1867913603	0.0049599258	0.1826314384	0.0008000039
0.8	0.2336533179	10.76679174	10.53313842	0.2397219119	0.006068594	0.2346340396	0.0009807217
1	0.2784456864	-11.93058115	12.20902684	0.2852182849	0.0067725985	0.2795431808	0.0010974944
1.2	0.3150399275	13.67137396	13.35633403	0.3220813968	0.0070414693	0.3161853262	0.0011453987
1.4	0.3426006453	-13.67405876	14.01665941	0.3494823663	0.006881721	0.3437259482	0.0011253029
1.6	0.3606398022	14.55443601	14.19379621	0.3669752082	0.006335406	0.3616834477	0.0010436455

1.8	0.3690100785	-13.61503212	13.9840422	0.3744854271	0.0054753486	0.3699217731	0.0009116946
2	0.3678794412	13.79777005	13.42989061	0.3722769804	0.0043975392	0.3686238052	0.000744364
2.2	0.3576903911	-12.27234	12.63003039	0.3609018388	0.0032114477	0.3582490959	0.0005587048
2.4	0.3391084773	11.96603337	11.62692489	0.3411376879	0.0020292106	0.3394806963	0.000372219
2.6	0.3129653191	-10.17022368	10.483189	0.3139200704	0.0009547513	0.3131664826	0.0002011635
2.8	0.2802015121	9.501661503	9.221459991	0.2802754107	0.0000738986	0.2802605197	0.0000590076
3	0.2418144086	-7.622360259	7.864174668	0.2412608697	0.0005535389	0.2417696039	0.0000448047
3.2	0.1988149024	6.619836979	6.421022076	0.1979159258	0.0008989766	0.1987092331	0.0001056693
3.4	0.1521961082	-4.745460851	4.89765696	0.1512290644	0.0009670438	0.1520719727	0.0001241355
3.6	0.1029153368	3.411524452	3.308609115	0.1021211534	0.0007941834	0.1028096446	0.0001056922
3.8	0.0518891799	-1.615541484	1.667430663	0.0514451561	0.0004440238	0.0518291292	0.0000600507
4	0	0	0	0	0	0	0