MA311 (Scientific computing)-IITG

15-11-18

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1. Consider the following Laplace's equation with Dirichlet boundary condition:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad \text{in } \Omega = (0,10) \times (0,15),$$

and on the boundary

$$u(x,y) = \begin{cases} 0 & \text{if} \quad x = 0, \quad 0 \le y \le 15 \\ 0 & \text{if} \quad y = 0, \quad 0 \le x \le 10 \\ 0 & \text{if} \quad x = 10, \quad 0 \le y \le 15 \\ 100\sin(\pi x/10) & \text{if} \quad y = 15, \quad 0 \le x \le 10. \end{cases}$$

Using the method of separation of variables, the exact solution of this problem is obtained as

$$u(x,y) = 100\frac{\sinh(\pi y/10)\sin(\pi x/10)}{\sinh(15\pi/10)}.$$

(a) Find the numerical approximated solution of the above problem using a five-point finite difference scheme on a 5×7 and 9×13 grids. Do a 2D contour plot and compare it with the exact solution.

(b) Compare the errors in the numerical solution computed on these grids.

(c) Compute numerically the order of accuracy of this method.

2. Consider the following advection equation

$$u_t + cu_x = 0$$

with the initial condition

$u(x,0) = \sin(2\pi x),$ for $0 \le x \le 1$ and with periodic boundary condition. The exact solution of this problem is given by

$$u(x,t) = \sin(2\pi(x - ct)).$$

(a) For $c = -1/8$, use the first order upwind method to compute the numerical solution, and compare it with the exact solution at time $t = 2.0$.

(b) For $c = 1/8$, use the Lax-Wendroff and BTCS methods to compute the numerical solution at $t = 2.0$ and compare it with the exact solution.

(c) For $c = 1/8$, compute the order or of accuracy of Upwind, BTCS and Lax-Wendroff methods numerically, by computing the errors at time $t = 2.0$.

(d) Repeat all the above questions (a) to (c) for a continuous but non smooth initial data in the domain $[0, 6]$

$$u(x,t) = \begin{cases} 0 & \text{if} \quad 0 \leq x \leq 2 \\ 3(x-2) & \text{if} \quad 2 \leq x \leq 3 \\ 3(4-x) & \text{if} \quad 3 \leq x \leq 4 \\ 0 & \text{if} \quad 4 \leq x \leq 6, \end{cases}$$

and zero boundary conditions and with $c = 1/2$, and final time $t = 2.0$