for $x = 6$

$$f(6) = \frac{x^2 - 9}{x + 3} = \frac{36 - 9}{6 + 3} = \frac{27}{9} = 3$$

2) $\lim_{x \to 6^+} \frac{x^2 - 9}{x + 3}$

$$\lim \frac{(x-3)\cancel{(x+3)}}{\cancel{(x+3)}}$$

$$\lim_{x \to 6^+} (x-3) = 6-3 = 3$$

$$\lim_{x \to 6} x + 3 = 3 + 6 = 9$$

$\therefore$ L.H.L $\neq$ R.H.L.

function is not continous

6. 1) $f(x) = \begin{cases} \frac{1 - \cos 4x}{x^2} & x < 0 \\ k & x = 0 \end{cases}$ at $x = 0$

Soln - f is continous at $x = 0$

$$\lim_{x \to 0} f(x) = f(0)$$

$$\lim_{x \to 0} \frac{1 - \cos 4x}{x^2} = k$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \dfrac{\tan\frac{\pi}{3} + \tan h}{1 - \tan\frac{\pi}{3}\cdot \tan h}}{\cancel{\pi} - \cancel{\pi} - 3h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}\left(1 - \tan\frac{\pi}{3}\cdot \tan h\right) - \left(\tan\frac{\pi}{3} + \tan h\right)}{1 - \tan\frac{\pi}{3}\cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3}\times\sqrt{3}\cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \tan \pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - 3\cdot \tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - 3\tan h - \sqrt{3} - \tan h)}{1 - \sqrt{3}\cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)}$$

$$\frac{4}{3} \lim_{h\to 0} \frac{\tan h}{h} \quad \lim_{h\to 0} \frac{1}{(1 - \sqrt{3}\tan h)} \qquad \frac{\tan h}{h} = 1$$

$= \frac{4}{3} \frac{1}{(1-3x)(x)}$

$= \frac{4}{3}\left(\frac{1}{1}\right) = 4/3$

7.(i) $f(x) = \frac{1-\cos 3x}{x \sin 3x}$, $x \neq 0$

$= 9$, $x = 0$ } at $x = 0$

$f(x) = \frac{1-\cos 3x}{x \sin 3x}$

$\lim_{x \to 0} \frac{2 \sin^2 3/2\, x}{x \sin 3x}$

$\lim_{x \to 0} = \frac{\frac{2 \sin^2 \frac{3x}{2}}{x^2} \times x^2}{\frac{\sin 3x}{x^2} \times x^2}$

$\therefore f$ is not continous at $x=0$

Redefine function

$$f(x) = \begin{cases} \dfrac{1-\cos 3x}{x \sin x} & x \neq 0 \\ \dfrac{9}{2} & x = 0 \end{cases}$$

Now $\lim\limits_{x \to 0} f(x) = f(0)$

f has removable discontinuity at $x=0$

2 ii) $f(x) = \begin{cases} \dfrac{(e^{3x}-1)\sin x^{\circ}}{x^2} & x \neq 0 \\ = \pi/6 & x = 0 \end{cases}$ at $x=0$

$$\lim_{x \to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$$

$$\lim_{x \to 0} \frac{e^{3x}-1}{x} \lim_{x \to 0} \frac{\sin\left(\frac{\pi x}{180}\right)}{x}$$

$$\lim_{x \to 0} 3 \cdot \frac{e^{3x}-1}{3x} \lim_{x \to 0} \sin\left(\frac{\pi x}{180}\right)$$

$$3 \lim_{x \to 0} \frac{e^{3x}-1}{} \lim_{x \to 0} \sin\left(\frac{\pi x}{180}\right)$$

$\log e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

f is continous at $x=0$

8. $f(x) = \frac{e^{x^2} - \cos x}{x^2} \quad x = 0$

is continous at $x = 0$

$\therefore$ Given,

f is continous at $x=0$

$\lim_{x \to 0} f(x) = f(0)$

$\therefore \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$\lim_{x \to 0} \frac{(e^{x^2} - 1) + (1 - \cos x)}{x^2}$

$\lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$

$\log e + 2 \lim_{x\to 0}\left(\frac{\sin x/2}{x}\right)^2$

Multiply with 2 on Num & Denominator

$= 1 + 2 \times \frac{1}{4^2} = \frac{3}{2} = f(0)$

$f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}, \; x \neq \pi/2$

$f(0)$ is continous at $x = \pi/2$

$\lim_{x\to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim_{x\to\pi/2} \frac{2 - 1 + \sin x}{\cos^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{1 + \sin x}{1 - \sin^2 x\,(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{\cancel{1+\sin x}}{(1-\sin x)\cancel{(1-\sin x)}(\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x\to\pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2} + \sqrt{2})}$

$= \frac{1}{2(2\sqrt{2})}$

$= \frac{1}{4\sqrt{2}}$

$\therefore f(\pi/2) = \frac{1}{4\sqrt{2}}$

1:A

Practical - 02

Derivative

Q.1) Show that the following function defined from $\mathbb{R}$ to $\mathbb{R}$ are differentiable.

i) $\cot x$

$f(x) = \cot x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$$

$$= \lim_{x \to a} \frac{\tan a - \tan x}{(x - a) \tan x \tan a}$$

put $x - a = h$

$x = a + h$

~~as~~ $x \to a,\ h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{(a + h - a) \tan(a+h) \tan a}$$

$$= \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$$

formula: $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$\tan A - \tan B = \tan(A-B)(1 - \tan A \tan B)$

$= \lim_{h\to 0} \dfrac{\tan(a - \alpha - h) - (1 + \tan a \tan(a+h))}{h \times \tan(a+h) \tan a}$

$= \lim_{h\to 0} -\dfrac{\tan h}{h} \times \dfrac{1 + \tan a \tan(a+h)}{\tan(a+h) \tan a}$

$= -1 \times \dfrac{1 + \tan^2 a}{\tan^2 a}$

$= \dfrac{-\sec^2 a}{\tan^2 a}$

$= \dfrac{-1}{\cos^2 a} \times \dfrac{\cos^2 a}{\sin^2 a}$

$= -\mathrm{cosec}^2 a$

$\therefore Df(a) = -\cos^2 a$

$\therefore$ f is differentiate $\forall a \in R$

$\sec x$

$f(x) = \sec x$

$$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x - a}$$

$$= \lim_{x \to a} \frac{\frac{1}{\cos x} - \frac{1}{\cos a}}{x - a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

put $x - a = h$

$x = a + h$

as $x \to a,\ h \to 0$

$$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h + \cos a \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h - \cos a \cos(a+h)}$$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin -\frac{h}{2}}{\cos a \cos(a+h) \times -\frac{h}{2}} \times \frac{-1}{2}$$

$$= \frac{-1}{2} \times \frac{-2\sin\left(\frac{2a+a}{2}\right)}{\cos a \cos(a+0)}$$

12

i) cosec x

$f(x) = \text{cosec}\, x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\text{cosec}\, x - \text{cosec}\, a}{x - a}$

$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$

$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a) \sin a \sin x}$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{(a+h-a) \sin a \sin(a+h)}$

formula:

$\sin C - \sin D = 2 \cos\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$= \lim_{h \to 0} \frac{2 \cos\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{h \times \sin a \sin(a+h)}$

$= \lim_{h \to 0} \frac{-\sin \frac{h}{2}}{\frac{h}{2}} \times \frac{1}{2} \times \frac{2 \cos\left(\frac{2a+h}{2}\right)}{\sin a \sin(a+h)}$

$= -\frac{1}{2} \times \frac{2 \cos\left(\frac{2a+0}{2}\right)}{\sin(a+0)}$ $= -\frac{\cos a}{\sin}$ $= -\cot a\, \text{cosec}\, a$

$= -\frac{1}{2} \times -2 \frac{\sin a}{\cos a \times \cos a}$

$= \tan a \sec a$

Q7) If $f(x) = 4x+1, x \le 2$
$= x^2+5 \quad x > 0$ at $x=2$, then find function is differentiable or not.

solution -

LHD:

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x)-f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$

$= \lim_{x \to 2^-} \frac{4x-8}{x-2}$

$= \lim_{x \to 2^-} \frac{4(x-2)}{x-2} = 4$

$Df(2^-) = 4$

RHD:

$Df(2^+) = \lim_{x \to 2^+} \frac{x^2+5-9}{x-2}$

$= \lim_{x \to 2^+} \frac{x^2-4}{x-2}$

$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{x-2}$

$= 2+2 = 4$

$Df(2^+) = 4$

RHD = LHD

f is differentiable

$Df(3^+) = 9$

$LHD = Df(3^-)$

$$= \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x + 7 - 19}{x-3}$$

$$= \lim_{x \to 3^-} \frac{4x - 12}{x-3}$$

$$= \lim_{x \to 3} \frac{4(x-3)}{(x-3)}$$

$Df(3^+) = 4$

$RHD \neq LHD$

f is not differentiable at $x = 3$

Q.4) If $f(x) = 8x - 5$, $x \leq 2$

$= 3x^2 - 4x + 7$, $x > 2$ at $x = 2$, Then

find f is differentiable or not.

Solution:

$f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

RHD:

$$Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$$

$$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x - 2}$$

$$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$$

$$= \lim_{x \to 2^+} \frac{(3x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$

$$= 3 \times 2 + 2 = 8$$

$Df(2^+) = 8$

LHD:

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{8x - 5 - 11}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8x - 16}{x - 2}$$

$$= \lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$$

$$= 8$$

$Df(2^-) = 8$

LHD = LHD

f is differentiable at $x = 3$

11/12/19

If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$, then find f is differentiable or not!

Solution:
RHD:

$$Df(3^+) = \lim_{x\to 3^+} \frac{f(x)-f(3)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-(3^2+3\cdot3+1)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x+1-19}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x^2+6x-3x-18}{x-3}$$

$$= \lim_{x\to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$$

$$= \lim_{x\to 3^+} \frac{(x+6)(x-3)}{(x-3)}$$

$$= 3+6$$

$$= 9$$

c] $f(x) = 2x^3 + x^2 - 20x + 4$

Solution: f is increasing if & only if

$f'(x) > 0$

$\because f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - \cancel{x} - 20 > 0$

$\therefore 6x^2 + 2x - 20 > 0$

$\therefore 6x^2 + 12x - 10x - 20 > 0$

$\therefore 6x(x+2) - 10(x+2) > 0$

$\therefore (x+2)(6x-10) > 0$

$\therefore x = -2, \, 5/3$

$\therefore x \in (-\infty, -2) \cup (5/3, \infty)$

Now f is decreasing if & only if

$f'(x) < 0$

$\therefore 6x^2 + 2x - 20 < 0$

$\therefore (x+2)(6x-10) < 0$

$\therefore x = -2, \, 5/3$

$x \in (-2, 5/3)$

d) $f(x) = x^3 - 27x + 5$

Solution: f is increasing if & only if

$f'(x) > 0$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore 3x^2 - 27 > 0$

$\therefore 3(x^2 - 9) > 0$

$\therefore x^2 - 9 > 0$

$\therefore x = 3, -3$

$\therefore x \in (-\infty, -3) \cup (3, \infty)$

Now f is decreasing if & only if

$f'(x) < 0$

$\therefore 3x^2 - 27 < 0$

$\therefore 3(x^2 - 9) < 0$

$\therefore x^2 - 9 < 0$

$\therefore x = 3, -3$

$\therefore x \in (-3, 3)$

c] $f(x) = 69 - 24x - 9x^2 + 2x^3$

Solution : f is increasing if & only if $f'(x) > 0$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore -24 - 18x + 6x^2 > 0$

$\therefore -6(-4 - 3x + x^2) > 0$

$\therefore x^2 - 3x - 4 > 0$

$\therefore x^2 - 4x + x - 4 > 0$

$\therefore (x-4)(x+1) > 0$

$\therefore x = 4, -1$

$x \in (-\infty, -1) \cup (4, \infty)$

Now f is decreasing if & only if $f'(x) < 0$

$\therefore -24 - 18x + 6x^2 < 0$

$\therefore 6(-4 - 3x + x^2) < 0$

$\therefore (x-4)(x-1) < 0$

$\therefore x = 4, -1$

$x \in (-1, 4)$

(1) Find the intervals in which function is concave upwards & concave downwards.

a] $y = 3x^2 - 2x^3$

Solution - $\therefore f(x) = 3x^3 - 2x^3$

$\therefore f'(x) = 6x - 6x^2$

$\therefore f''(x) = 6 - 12x$

$\therefore f$ is concave upward if & only if $f''(x) > 0$

$\therefore 6 - 12x > 0$

$\therefore 6(1 - 12x) > 0$

$\therefore (1 - 12x) > 0$

$\therefore -(2x - 1) > 0$

$x \in (-\infty, 1/2)$

f is concave downwards if & only if $f''(x) < 0$

$\therefore 6(1 - 12x < 0$

$\therefore -(2x - 1) < 0$

$x \in (1/2, \infty)$

Practical 3

Title Application of derivatives

Q.1] Find the internals in which function is increasing or decreasing.

a] $f(x) = x^3 - 5x - 11$

Solution : f is increasing if & only $f'(x) > 0$

$\because f(x) = x^3 - 5x - 11$

$\therefore f(x) = 3x^3 - 5$

$\therefore 3x^2 - 5 > 0$

$\therefore x = \pm\sqrt{5/3}$

$\therefore x \in \left(-\infty, -\sqrt{\frac{5}{3}}\right) \cup \left(\sqrt{\frac{5}{3}}, \infty\right)$

Now f is decreasing if & only if $f'(x) < 0$

$\therefore 3x^2 - 5 < 0$

$\therefore x = \pm\sqrt{\frac{5}{3}}$

$\therefore x \in \left(-\sqrt{\frac{5}{3}}, \sqrt{\frac{5}{3}}\right)$

b] $f(x) = x^2 - 4x$

Solution: f is increasing if & only if $f'(x) > 0$

$\therefore f(x) = x^2 - 4x$

$\therefore f'(x) = 2x - 4$

$\therefore 2x - 4 > 0$

$\therefore x - 2 > 0$

$\therefore x = 2$

$\therefore x \in (2, \infty)$

Now f is decreasing if & only if $f'(x) < 0$

$\therefore 2x - 4 < 0$

$\therefore 2(x-2) < 0$

$\therefore x - 2 < 0$

$\therefore x = 2$

$\therefore x \in (-\infty, 2)$

b] $y = x^4 - 6x^3 + 12x^2 + 5x + 7$

Solution: $\therefore y = f(x)$

$\therefore f(x) = x^4 - 6x^3 + 12x^2 + 5x$

$\therefore f'(x) = 4x^3 - 18x^2 + 12x + 5$

$\therefore f''(x) = 12x^2 - 36x + 24$

$\therefore f$ is concave upwards if & only if

$f''(x) > 0$

$\therefore 12x^2 - 36x + 24 > 0$

$\therefore 12(x^2 - 3x + 2) > 0$

$\therefore x^2 - 3x + 2 > 0$

$\therefore (x-2)(x-1) > 0$

$\therefore x = 2, 1$

$x \in (-\infty, 1) \cup (2, \infty)$

$\therefore f$ is concave downward if & only if

$f''(x) < 0$

$\therefore 12x^2 - 36x + 24 < 0$

$\therefore 12(x^2 - 3x + 2) < 0$

$\therefore x^2 - 3x + 2 < 0$

$\therefore (x-2)(x-1) < 0$

$\therefore x = 2, 1$

$x \in (1, 2)$

$y = x^3 - 27x + 5$

Solution: $\therefore y = f(x)$

$\therefore f(x) = x^3 - 27x + 5$

$\therefore f'(x) = 3x^2 - 27$

$\therefore f''(x) = 6x$

$\therefore f$ is concave upward if & only if

$f''(x) > 0$

$\therefore 6x > 0$

$\therefore x > 0$

$\therefore x = 0$

$\therefore x \in (0, \infty)$

$\therefore f$ is concave downward if & only if

$f''(x) < 0$

$\therefore 6x < 0$

$\therefore x < 0$

$\therefore x = 0$

$x \in (-\infty, 0)$

d) $y = 69 - 24x - 9x^2 + 2x^3$

Solution: $\therefore y = f(x)$

$\therefore f(x) = 69 - 24x - 9x^2 + 2x^3$

$\therefore f'(x) = -24 - 18x + 6x^2$

$\therefore f''(x) = -18 + 12x$

$\therefore f$ is concave upward if & only if $f''(x) > 0$

$\therefore -18 + 12x > 0$

$\therefore 6(2x - 3) > 0$

$\therefore 2x - 3 > 0$

$\therefore x = 3/2$

$x \in \left(\frac{3}{2}, \infty\right)$

$\therefore f$ is concave downwards if & only if $f''(x) < 0$

$\therefore -18 + 12x < 0$

$\therefore 6(2x - 3) < 0$

$\therefore 2x - 3 < 0$

$\therefore x = 3/2$

$x \in \left(-\infty, \frac{3}{2}\right)$

e) $y = 2x^3 + x^2 - 20x + 4$

Solution: $\therefore y = f(x)$

$\therefore f(x) = 2x^3 + x^2 - 20x + 4$

$\therefore f'(x) = 6x^2 + 2x - 20$

$\therefore f''(x) = 12x + 2$

$\therefore f$ is concave upwards if & only if $f''(x) > 0$

$\therefore 12x + 2 > 0$

$\therefore 2(6x + 1) > 0$

$\therefore 6x + 1 > 0$

$\therefore x = -1/6$

$x \in (-1/6, \infty)$

$\therefore f$ is concave downwards if & only if $f''(x) < 0$

$\therefore 12x + 2 < 0$

$\therefore 2(6x + 1) < 0$

$\therefore 6x + 1 < 0$

$\therefore x = -1/6$

18/12/19

$x \in (-\infty, -1/6)$

$$f(x) = x^2 + \frac{16}{x^2}$$

$$f'(x) = 2x - \frac{32}{x^3}$$

for maxima / minima

$$f'(x) = 0$$

$$2x - \frac{32}{x^3} = 0$$

$$2x = \frac{32}{x^3}$$

$$x^4 = 16$$

$$x = \pm 2$$

$$f''(x) = 2 + \frac{96}{x^4}$$

$$f''(2) = f''(-2) = 2 + \frac{96}{(\pm 2)^4} = 2 + \frac{96}{16} = 8 > 0$$

$\therefore f(x)$ is minimum at $x = \pm 2$

$\therefore f(2) = 8$ is the maximum value.

ii) $f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = 15x^4 - 15x^2$

for maxima / minima

$f'(x) = 15x^4 - 15x^2 = 0$

$= x^4 - x^2 = 0$

$x^2(x^2-1) = 0$

$x = 0, -1, 1$

$f''(x) = 60x^3 - 30x$

$f''(0) = 0$

$f''(-1) = -60 + 30 = -30 < 0$

$f''(1) = 60 - 30 = 30 > 0$

$f(x)$ is maximum at -1

$f(-1) = 3 + 5 - 3 = 5$

$f(1) = 3 - 5 + 3 = 1$

$f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

for maximum/minima

$f'(x) = 0$

$3x^2 - 6x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, 2$

$f''(x) = 6x - 6$

$f''(0) = -6 < 0$

$f''(2) = 6 > 0$

$f(x)$ is maximum at $x = 0$ and minimum $x = 2$

$f(0) = 1$

$f(2) = -3$

1.3

iv) Solution:-

$f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

for maximum / minima

$f'(x) = 0$

$\therefore 6x^2 - 6x - 12 = 0$

$\therefore x^2 - x - 2 = 0$

$\therefore x^2 - 2x + x - 2 = 0$

$\therefore x(x-2) + 1(x-2) = 0$

$\therefore (x+1)(x-2) = 0$

$\therefore x = -1, 2$

$f''(x) = 12x - 6$

$f''(-1) = -12 - 6 = -18 < 0$

$f''(2) = 24 - 6 = 18 > 0$

$f(x)$ is maximum at $x = -1$ & minimum at $x = 2$

$\therefore f(-1) = 8$

$\therefore f(2) = -19$

$f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

$x_0 = 0$

$f(x_0) = 9.5$

$f'(x_0) = -55$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 0 - \frac{9.5}{-55}$

$x_1 = 0.1727$

$f(x_1) = -0.0828$

$f'(x_1) = -55.9467$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{-0.0828}{-55.9467}$

$x_2 = 0.1712$

$f(x_2) = 0.0011$

$f'(x_2) = -55.1393$

$\therefore x_3 = 0.1712$

$\therefore x = 0.1712$ is the root of the equation

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(1) = -9$

$f(3) = 6$

$\therefore$ 6 is closer to 0 in the number

$x_0 = 3$

$f(x_0) = 6$

$f(x_0) = 23$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_1)}$

$x_1 = 2.7391$

$f(x_1) = 0.5912$

$f'(x_1) = 18.508$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 7391 - 0.5912/18.508$

$x_2 = 2.707$

$f(x_2) = 0.0085$

$f'(x_2) = 17.9835$

$= x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.707 - \frac{0.0085}{17.9835}$

$x_3 = 2.7065$

$f(x_3) = -0.0005$

$f'(x_3) = 17.9757$

$\therefore x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 2.7065 - \frac{0.0005}{17.9757}$

$x_4 = 2.7065$

$\therefore$ 2.7065 is the root of the given ladu

$f(x) = x^3 - 18x^2 - 10x + 17$

$f'(x) = 3x^2 - 3.6x - 10$

$f(1) = 1 - 1.8 - 10 + 17 = 6.2$

$f(2) = -2.2$

$\therefore -2.2$ is closer to 0 as the number is

$x_0 = 2$

$f'(x_0) = -2.2$

$f'(x_0) = -5.2$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$= 2 - \frac{-2.2}{-5.2}$

$x_1 = 1.5769$

$f(x_1) = (1.5769)^3 - 1.8(1.5769)^2 - 10(1.5769) + 17$

$= 0.6762$

$f'(x_2) = -8.217$

$x_2 = x_1 - \frac{f(x_1)}{f'(x)}$

$= 1.5769 - \frac{0.6762}{-8.217}$

$x_2 = 1.6592$

$\therefore f(x_2) = 0.0204$

$\therefore f'(x_2) = -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$\therefore 1.6592 - \frac{0.0204}{-7.7143}$

$x_3 = 1.6618$

$\therefore f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 16(1.6698) + 17$

$\therefore f(x_3) = 0$

$\therefore 1.6618$ is the root of the function.

Proc. 5

i) $I = \int \frac{1}{\sqrt{x^2+2x-3}}\, dx$

$= \int \frac{1}{\sqrt{x^2-2x+1-4}}\, dx$

$= \int \frac{1}{\sqrt{(x+1)-2^2}}\, dx$

$I = \ln| x+1 + \sqrt{x^2+2x-3}| + c$

ii) $I = \int (4e^{3x}+1)\, dx$

$= 4\int e^{3x} dx + \int 1\, dx$

$I = \frac{4e^{3x}}{3} + x + c$

iii) $I = \int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= 2\int x^2 dx - 3\int \sin x dx + 5\int x^{1/2} dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{5\times 2}{3} x^{3/2} + c$

$I = \frac{2x^3}{3} + 3\cos x + \frac{10}{3} x \sqrt{x} + c$

vii) $\int \frac{1}{x^3} \sin\left(\frac{1}{x^2}\right) dx$

$\frac{1}{x^2} = t$

$\therefore \frac{-2}{x^3} = \frac{dt}{dx}$

$\therefore \frac{dx}{x^3} = \frac{-1}{2} dt$

$\therefore I = \frac{-1}{2} \int \sin t \, dt$

$= \frac{1}{2} (-\cos t + c$

$= \frac{\cos t}{2} + c$

$I = \frac{\cos(1/x^2)}{2} + c$

$I = \int \frac{\cos x}{\sqrt[3]{\sin^2 x}} dx$

let $\sin x = t$

$\therefore \cos x \, dx = dt$

$\therefore I = \int \frac{1}{t^{2/3}} dt$

$= \int t^{-2/3} dt$

$= \frac{t^{-2/3+1}}{-2/3+1} + c$

$= 3t^{1/3} + c$

$I = 3\sqrt[3]{\sin x} + c$

ix) $I = \int e^{\cos^2 x} \cdot \sin 2x \, dx$

Put $\cos^2 x = t$

$\therefore -2 \cos x \sin x = \frac{dt}{dx}$

$\therefore \sin 2x \, dx = -dt$

$\therefore I = -\int e^t dt$

$= -e^t + c$

$I = -e^{\cos^2 x} + c$

x) $I = \int \left(\frac{x^2 - 2x}{x^3 - 3x^2 + 1}\right) dx$

$x^3 - 3x^2 + 1 = t$

$\therefore 3x^2 - 6x = \frac{dt}{dx}$

$(x^2 - 2x) dx = \frac{dt}{3}$

$\therefore I = \frac{1}{3} \int \frac{1}{t} dt$

$= \frac{1}{3} \cdot \log |t| + c$

$I = \frac{1}{3} \log |x^2 - 3x^2 + 1| + c$

iv) $I = \int \frac{x^3 + 3x + 4}{\sqrt{x}} \, dx$

put $\sqrt{x} = t$

$\therefore \frac{1}{2\sqrt{x}} = \frac{dt}{dx} \quad \frac{dx}{\sqrt{x}} = 2\,dt$

$I = \int \frac{(\sqrt{x})^6 + 3(\sqrt{x})^2 + 4}{\sqrt{x}} \, dx$

$= 2\int t^6 + 3t^2 + 4 \, dt$

$= 2\left[\frac{t^7}{7} + \frac{3t^3}{3} + 4t\right] + c$

$2\left[\frac{x^{7/2}}{7} - x^{3/2} + 4x^{1/2}\right] + c$

v) $I = \int t^7 \sin(2t) \, dt$

$\int t^4 \sin(6t^4) \times t^3 \, dt$

Let $t^4 = x$

$4t^3 = \frac{dx}{dt}$

$t^3 \, dt = \frac{1}{4} dx$

$I = \frac{1}{4} \int x \sin(2x) \, dx$

$I = \frac{1}{4}\left[x \int \sin 2x \, dx - \int \left(\frac{d}{dx} x\right) \int \sin 2x \, dx \, dx\right]$

$\frac{1}{4}\left[-x \cdot \frac{\cos 2x}{2} + \frac{1}{2}\int \cos 2x \, dx\right]$

$= \frac{1}{16} \sin 2x - \frac{x \cdot \cos 2x}{8} + c$

$= \frac{1}{16} \sin 2t^4 - \frac{t^4 \cdot \cos 2t^4}{8} + c$

vi) $I = \int \sqrt{x}\,(x^2 - 1) \, dx$

$\therefore I = \int x^2 \sqrt{x} \, dx - \int \sqrt{x} \, dx$

$= \int x^{5/2} \, dx - \int x^{1/2} \, dx$

$= \frac{x^{7/2}}{7/2} - \frac{x^{3/2}}{3/2} + c$

$= \frac{2}{7} x^3 \sqrt{x} - \frac{2}{3} x \sqrt{x} + c$

Practical - 6

$y = \sqrt{4 - x^2}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \times (-2x)$$

$$= \frac{-x}{\sqrt{4-x^2}}$$

$$I = \int_{-2}^{2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_{-2}^{2} \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$$

$$= \int_{-2}^{2} \sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$$

$$= 2\int_{-2}^{2} \frac{1}{\sqrt{2^2-x^2}}\, dx$$

$$= 2\left[\sin\left(\frac{x}{2}\right)\right]_{-2}^{2}$$

$$= 2\left[\sin^{-1}(1) - \sin^{-1}(-1)\right]$$

$$= 2\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$I = 2\pi$$

3. $y = x^{3/2} \qquad x \in [0,4]$

$$\frac{dy}{dx} = \frac{3}{2} x^{1/2}$$

$$\therefore I = \int_0^1 \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_0^1 \sqrt{1 + \frac{9}{4}x}\, dx$$

$$= \frac{1}{2}\int_0^4 \sqrt{4+9x}\, dx$$

$$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2} \times \frac{1}{9}\right]_0^4$$

$$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$$

$$= \frac{-1}{27}\left[(4+0)^{3/2} - (4+36)^{3/2}\right]$$

$$= \frac{1}{27}\left[40^{3/2} - 8\right) \text{ units}$$

4. $x = 3\sin t \qquad y = 3\cos t$

$$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$$

$$I = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (3\sin t)^2}\, dt$$

$$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dx$$

$$= \int_0^{2\pi} 3\sqrt{1}\, dt$$

$$= 3\int_0^{2\pi} dt$$

$$= 3[x]_0^{2\pi}$$

$$= 3[2\pi - 0)$$

$$L = 6\pi \text{ units}$$

$$x = \frac{1}{6}y^3 + \frac{1}{2y}$$

$$\therefore \frac{dx}{dy} = \frac{y^2}{2} = \frac{1}{2y^2}$$

$$\frac{dx}{dy} = \frac{y^4 - 1}{2y^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

$$= \int_1^2 \sqrt{\frac{1 + (y^4-1)^2}{4y^4}}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4-1) + 4 \times y^3 \times 1}{4y^4}}\, dy$$

$$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$$

$$= \frac{1}{2}\int_1^2 y^2\, dy + \frac{1}{2}\int_1^2 y^{-2}\, dy$$

$$= \frac{1}{2}\left\{\frac{y^3}{3} - \frac{y^{-1}}{1}\right]_1^2$$

$$= \frac{1}{2}\left[\frac{8}{3} - \frac{1}{2} - \frac{1}{3} + 1\right]$$

$$= \frac{1}{2}\left\{\frac{7}{3} + \frac{1}{2}\right\}$$

$$= \frac{1}{2}\left[\frac{17}{6}\right]$$

$$L = \frac{17}{12} \text{ units}$$

Q.2.
i) $\int_0^2 e^{x^2} dx$ with $n = 4$

$$h = \frac{b-a}{n} = \frac{2-0}{4} = 0.5$$

x	0	0.5	1	1.5	2
y	1	1.284	2.7183	9.4877	54.5982
	y	y_1	y_2	y_3	y_3

$$\int_0^2 e^{x^2} dx = \frac{\cdot 5}{3}\left[(y_0 + y_6) + 4(y_1 + y_3) + 2(y_e)\right]$$

$$= \frac{0.5}{3}\left[(1 + 54.5982) + 5(1.285 + 9.587) + 2.27885\right]$$

$$= \frac{0.5}{3}\left[55.5982 + 43.0868 + 5.436\right]$$

$$\int_0^2 e^{x^2} dx = 17.3535$$

ii) $\int_0^4 x^2 dx$ $n = 4$

$$L = \frac{4-0}{4} = 1$$

x	0	1	2	3	4
y	0	1	4	9	16
	y_0	y_1	y_2	y_3	y_4

$$\int_0^1 x^2 dx = \frac{1}{3}\left[(y_0 + y_4) + 4(y_1 + y_3) + 2y_2\right)$$

$$= \frac{1}{3}\left[0 + 16 + 4(1+9) + 2 \times 4\right]$$

$$= \frac{1}{3}\left[16 + 4(10) + 8\right]$$

$$= \frac{64}{3}$$

$$\int_0^4 x^2 dx = 21.3333$$

iii) $\int_0^{\pi/3} \sqrt{\sin x}\, dx \quad n = 6$

$h = \frac{\pi/3 - 0}{6} = \frac{\pi}{18}$

$0 \quad \frac{\pi}{18} \quad \frac{2\pi}{18} \quad \frac{3\pi}{18} \quad \frac{4\pi}{18} \quad \frac{5\pi}{18} \quad \frac{6\pi}{18}$

$0 \quad 4.167 \quad 0.5848 \quad 0.7071 \quad 0.8017 \quad 0.8752 \quad 0.9306$

$y_0 \quad y_1 \quad y_2 \quad y_3 \quad y_4 \quad y_5 \quad y_6$

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[y_0 + y_6 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$

$= \frac{\pi/18}{13}\{0.4167 + 0.9306 + 4(0.4167 + 0.7071 + 0.8752) + 2(0.5848 + 0.8471)$

$= \frac{\pi}{54}[1.3473 + 4(1.999) + 2(1.3865)]$

$= \frac{\pi}{54}[1.3473 + 7.996 + 2.773]$

$= \frac{\pi}{54} \times 12.1163$

$\int_0^{\pi/3} \sqrt{\sin}\, dx = 0.7049$

Practical 7

Topic - Differential equation

$$x\frac{dy}{dx} + y = e^x$$

$$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$$

$$P(x) = \frac{1}{x} \qquad Q(x) = \frac{e^x}{x}$$

$$I.F = e^{\int P(x)\,dx}$$

$$= e^{\int 1/x\,dx}$$

$$= e^{\log x}$$

$$I.F = x$$

$$y(I.F) = \int Q(x)\,(I.F)\,dx + c$$

$$= \int \frac{e^x}{x} \cdot x \cdot dx + c$$

$$= \int e^x\,dx + c$$

$$xy = e^x + c$$

7) $\frac{dy}{dx} = \sin^2(x-y+1)$

put $x-y+1=v$

Differentiating on both sides

$x-y+1=v$

$1-\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{1-dv}{dx} = \frac{dy}{dx}$

$1-\frac{dv}{dx} = \sin^2 v$

$\frac{dv}{dx} = 1-\sin^2 v$

$\frac{dv}{dx} = \cos^2 v$

$\frac{dv}{\cos^2 v} = dx$

$\int \sec^2 v\, dv = \int dx$

$\tan v = x+c$

$\tan(x-y+1) = x+c$

8 $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

put $2x+3y=v$

$2+\frac{3dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx}-2\right)$

$\frac{1}{3}\left(\frac{dv}{dx}-2\right) = \frac{1}{3}\frac{(v-1)}{(v+2)}$

$\frac{dv}{dx} = \frac{v-1}{v+2}+2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

F.S.

Q.2) $e^{x}\frac{dy}{dx} + 2e^{x}y = 1$

$\frac{dy}{dx} + \frac{2e^{x}}{e^{x}}y = \frac{1}{e^{x}}$

$\frac{dy}{dx} + 2y = \frac{1}{e^{x}}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \qquad Q(x) = e^{-x}$

$\int p(x)\,dx$

$I.F = e^{\int 2\,dx}$
$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)\,dx + c$

$y \cdot e^{2x} = \int e^{-x} \cdot e^{2x}\,dx + c$
$= \int e^{x}\,dx + c$
$ye^{2x} = e^{x} + c$

Q.3) $x = 3\sin t \qquad y = 3\cos t$

$\frac{dx}{dt} = 3\cos t \qquad \frac{dy}{dt} = -3\sin t$

$l = \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\,dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\,dt$

$= \int_0^{2\pi} 3\,dt$

$= 3\int_0^{2\pi} dt$

$= 3[t]_0^{2\pi}$

$= 3[2\pi - 0]$

$= l = 6\pi$ units

71

(c) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2/x \quad Q(x) = \frac{\cos x}{x^2}$

$I.F = e^{\int P(x)\,dx}$
$= e^{\int 2/x\,dx}$
$= e^{2\ln x}$
$= \ln x^2$

$I.F = x^2$

$y(I.F) = \int Q(x)(I.F)\,dx + c$
$= \int \frac{\cos x}{x^2} \cdot x^2\,dx + c$
$= \int \cos x + c$

$x^2 y = \sin x + c$

(f) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$I.F = e^{\int P(x)\,dx}$
$= e^{\int 2\,dx}$
$= e^{2x}$

$y(I.F) = \int Q(x)(I.F)\,dx + c$
$= \int 2xe^{-2x} e^{2x}\,dx + c$
$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$

(d) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on both side)

$P(x) = 3/x \quad Q(x) = \sin x/x^3$

$= e^{\int P(x)\,dx}$
$= e^{\int 3/x\,dx}$
$= e^{3\ln x}$
$= e^{\ln x^3}$

$I.F = x^3$

$y(I.F) = \int Q(x)(I.F)\,dx + c$
$= \int \frac{\sin x}{x^3} x^3\,dx + c$
$= \int \sin x\,dx + c$

$x^3 y = -\cos x + c$

(g) $\sec^2 x \cdot \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \cdot \tan x\,dy$

$\frac{\sec^2 x\,dx}{\tan x} = \frac{-\sec^2 y\,dy}{\tan y}$

$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$

$\therefore \log|\tan x| = -\log|\tan y| + c$

$\log|\tan x \cdot \tan y| = c$

$\tan x \cdot \tan y = e^{c}$

35

Q3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$ find $y(1) = ?$

$x_0 = 0$ $y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.0894
2	0.4	1.0894	0.6059	1.2105
3	0.6	1.2105	0.7040	1.3513
4	0.8	1.3513	0.7696	1.5051
5	1	1.5051		

$y(1) = \underline{1.5051}$

Q4) $\frac{dy}{dx} = 3x^2+1$ $y(1) = 2$ find $y(2)$ $h = 0.5$

$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	4	7.75	7.875
2	2	7.875		

$y(2) = 7.875$

$y_0 = 2$ $x_0 = 1$ $h = 0.25$

76

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	59.6569	19.3360
3	1.75	19.3360	1122.8426	299.9960
4	2	299.9960		

$y(2) = 299.9960$

Q5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $y(1) = 1$ $h = 0.2$

$x_0 = 1$ $y_0 = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1	3	3.6
1	1.2	3.6		

$y(1.2) = \underline{\underline{3.6}}$

Practical - 8

Euler's Method

1) $\frac{dy}{dx} = y + e^{x} - 2$, $y(0) = 2$, $h = 0.5$ find $y(2) = ?$

Sol:

$f(x) = y + e^{x} - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.1487	3.5743
2	1	3.5743	4.2925	5.7205
3	1.5	5.7205	8.2021	9.8215
4	2	9.8215		

$\therefore y(2) = 9.8215$

2) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 1$ $h = 0.2$ find $(y(1)) ?$

$y_0 = 0$, $y_0 = 0$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
				0.2
0	0	0	1.04	0.408
1	0.2	0.2	1.1664	0.6412
2	0.4	0.408	1.4111	0.9234
3	0.6	0.6412	1.896	1.2939
4	0.8	0.9234		
5	1	1.2939		

$\boxed{y(1) = 1.2939}$

85 $f_x = \dfrac{x^2(0, y) - (y^2 - xy)2x}{x^4}$

$= \dfrac{x^2y - 2xy^2 + 2x^2y}{x^4}$

$f_x = \dfrac{x^2y - 2xy^2}{x^4}$

$f_{xx} = \dfrac{x^4(2xy - 2y^2) - (x^2y - 2xy^2)(4x^3)}{x^8}$

$= \dfrac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$

$= \dfrac{2x^5y - 2x^4y^2 - (4x^5y - 8x^4y^2)}{x^8}$

$= \dfrac{2x^5y - 2x^4y^2 - 4x^5y + 8x^4y^2}{x^8}$

$= \dfrac{-2x^5y + 6x^4y^2}{x^8}$

$= \dfrac{6x^4y^2 - 2x^5y}{x^8}$

$f_{xx} = \dfrac{6y^2 - 2xy}{x^4}$

$f_y = \dfrac{1}{x^2}(2y - x) \quad \therefore f_y = \dfrac{2y - x}{x^2}$

$f_{yy} = \dfrac{1}{x^2} \cdot 2 = \dfrac{2}{x^2}$

$f_{xy} = \dfrac{2y - x}{x^2}$

$= \dfrac{x^2(-1) - (2y - x)(2x)}{x^4}$

$= \dfrac{-x^2 - 4xy + 2x^2}{x^4}$

$= \dfrac{x^2 - 4xy}{x^4}$

$= \dfrac{x - 4y}{x^3}$

$f_{xy} = \dfrac{x - 4y}{x^3}$

$\therefore f_{yx} = \dfrac{x^2y - 2xy}{x^4}$

$= \dfrac{x^2 - 4xy}{x^4}$

$= \dfrac{x - 4y}{x^3}$

$= f_{xy}$

$f_{yx} = f_{xy}$

Hence, verified.

Q.5) Find the linearization of $f(x, y)$ at given points

$f(x, y) = 1 - x + y \sin x$ at $\frac{\pi}{2}$

$f(\pi/2, 0) = 1 - \frac{\pi}{2} + 0 + \sin\frac{\pi}{2}$

$f\left(\frac{\pi}{2}, 0\right) = \frac{2 - \pi}{2}$

3) $\lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2-z^2}{x^3-x^2y^2}$

$= \lim_{(x,y,z)\to(1,1,1)} \frac{x^2-y^2z^2}{x^3-x^2y^2}$

Apply limit

$= \frac{(1)^2-(1)^2(1)^2}{(1)^3-(1)^2(1)(1)} = \frac{1-1}{1-1} = \frac{0}{0}$: limits does not exist

Q2) Find f_x, f_y for each of the following of

i) $f(x,y) = xye^{x^2} + y^2$

$f_x = y(1\, e^{x^2}+y^2) + 2xy\,(e^{x^2}+y^2 \cdot 2x$

$= y\, e^{x^2}+y^2 + 2x^2ye^{x^2+y^2}$

$f_y = x(1\, e^{x^2}+y^2) + xy(e^{x^2}+y^2 \cdot 2y)$

$= x\, e^{x^2}+y^2 + 2xy^2 e^{x^2}+y^2$

ii) $f(x,y) = e^x\cos y$

$f_x = \cos y e^x$

$f_y = e^x - \sin y$

$f_y = -\sin y e^x$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = y^2 3x^2 - 3y^2 x + 0 + 0$

$= 3x^2y^2 - 6xy$

$f_y = x^3 2y - 3x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

Q3) Using definition find value of f_x, f_y at $(0,0)$ for

$f(x,y) = \frac{2x}{1+y^2}$

$f_x(a,b) = \lim_{h\to 0} \frac{f(a+h,b) - f(a,b)}{h}$

$f_y(a,b) = \lim_{h\to 0} \frac{f(a,b+h) - f(a,b)}{h}$

According to given $(a,b) = (0,0)$

$f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h} = 2$

$f_y(0,0) = \lim_{h\to 0} f(0,h) - f(0,0)$

$= \lim_{h\to 0} \frac{0-0}{h} = 0$

$f_x = 2, \; f_y = 0$

for $f_x(0,0) = \lim_{h\to 0} \frac{f(h,0) - f(0,0)}{h}$

$= \lim_{h\to 0} \frac{2h-0}{h} = 2$

$f_x = 2, \; f_y = 0$

Q4) Find all second order partial derivatives of f. Also verify whether $f_{xy} = f_{yx}$

i) $f(x,y) = \frac{y^2-xy}{x^2}$

$\therefore f_{xx} = \frac{d^2f}{dx^2}$. $f_{yy} = \frac{d^2f}{dy^2}$

Applying $\frac{x}{y}$ rule.

15

$f_x = -1 + y\cos x \qquad f_y = \sin x$

$f_x\left(\frac{\pi}{2}, 0\right) = -1 + 0 \cdot \cos\frac{\pi}{2}$

$= -1$

$f_y\left(\frac{\pi}{2}, 0\right) = \sin\frac{\pi}{2}$

$= 1$

$L(x,y) = f\left(\frac{\pi}{2}, 0\right) + f_x\left(\frac{\pi}{2}, 0\right)(x - \pi/2) + f_y(\pi/2, 0)(y-0)$

$= \frac{2-\pi}{2} + (-1)\left(x - \frac{\pi}{2}\right) + 1(y)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$L(x,y) = 1 - x + y$

c) $f(x,y) = \log x + \log y$

$f(1,1) = \log 1 + \log 1$

$= 0$

$f_x = \frac{1}{x} \qquad f_y = \frac{1}{y}$

$f_x(1,1) = 1 \qquad f_y(1,1) = 1$

$L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= 0 + 1(x-1) + 1(y-1)$

$L(x,y) = x + y - 2$

80

d) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{1+1} = \sqrt{2}$

$f_x = \frac{1}{2\sqrt{x^2+y^2}} \times 2x \qquad f_y = \frac{2y}{2\sqrt{x^2+y^2}}$

$\therefore f_x = \frac{x}{\sqrt{x^2+y^2}} \qquad f_y = \frac{y}{\sqrt{x^2+y^2}}$

$f_x(1,1) = \frac{1}{\sqrt{2}} \qquad f_y(1,1) = \frac{1}{\sqrt{2}}$

$\therefore L(x,y) = f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1)$

$= \sqrt{2} + \frac{x-1}{\sqrt{2}} + \frac{y-1}{\sqrt{2}}$

$= \sqrt{2} + \frac{x+y-2}{\sqrt{2}}$

$= \frac{2+x+y-2}{\sqrt{2}}$

$= \frac{2+y+x-2}{\sqrt{2}}$

$\boxed{L(x,y) = \frac{x+y}{\sqrt{2}}}$

Practical - 9

Limits & Partial Order Derivative

Evaluate the following limits

$$\lim_{(x,y)\to(-4,-1)} \frac{x^3-3y+y^2-1}{xy+5}$$

Applying limits

$$\frac{(-4)^3-3(-1)+(-1)^2-1}{(-4)(-1)+5}$$

$$\frac{-64+3+1-1}{4+5} = \frac{-52}{9}$$

2) $$\lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

$$= \lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

$$= \lim_{(x,y)\to(2,0)} \frac{(y+1)(x^2+y^2-4x)}{x+3y}$$

Apply limit

$$= \frac{(0+1)((2)^2+(0)^2-4(2))}{2+3(0)} = \frac{1(4+0-8)}{2} = \frac{4-8}{2} = \frac{-4}{2} = -2$$

Practical-10

Q1] Find the directional derivative of the following function at given points & in the direction of given vector.

i] $f(x,y) = x+2y-3$, $a = (1,-1)$, $u = 3i-j$

$\to u = 3\hat{i}-\hat{j}$

$\therefore \hat{u} = \frac{\bar{u}}{|u|} = \frac{1}{\sqrt{3^2+(-1)^2}} (3\hat{i}-\hat{j})$

$\therefore \hat{u} = \frac{1}{\sqrt{10}} (3\hat{i}-\hat{j})$

$\hat{u} = \left(\frac{3}{\sqrt{10}}, -\frac{1}{\sqrt{10}}\right)$

$a = (1,-1)$

$f(a) = 1+2(-1)-3$

$= 1+(-2)-3$

$= -4$

$f(a+hu) = f\left((1,-1)+h\left(\frac{1}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)\right)$

$= f\left(\left(1+\frac{3}{\sqrt{10}}h\right), \left(-1-\frac{h}{\sqrt{10}}\right)\right)$

$= 1+\frac{3}{\sqrt{10}}h+2\left(-1-\frac{h}{\sqrt{10}}\right)-3$

$= 1+\frac{3}{\sqrt{10}}h-2-\frac{2h}{\sqrt{10}}$

$\therefore f(a+hu) = \frac{h}{\sqrt{10}}-4$

$\therefore D_u f = \lim_{h\to 0} \frac{f(a+hu)-f(a)}{h}$

$= \lim_{h\to 0} \frac{\frac{h}{\sqrt{10}}-4-(-4)}{h}$

$= \frac{1}{\sqrt{10}} \lim_{h\to 0} \frac{h}{h}$

$= \frac{1}{\sqrt{10}}$

ii) $f(x,y) = y^2-4y+1$, $a = (3,4)$; $u = \hat{i}+5\hat{j}$

$\to f(x,y) = y^2-4y+1$

$\bar{u} = \hat{i}+5\hat{j}$

$\therefore \hat{u} = \frac{\bar{u}}{|u|} = \frac{\hat{i}+5\hat{j}}{\sqrt{(1)^2+5^2}} = \frac{1}{\sqrt{26}}(\hat{i}+5\hat{j})$

$\therefore \hat{u} = \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = (4)^2-4(3)+1$

$= 16-12+1$

$f(a) = 5$

$f(a+hu) = f\left[(3,4)+h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)\right]$

$= f\left(\left(3+\frac{h}{\sqrt{26}}\right), \left(4+\frac{5h}{\sqrt{26}}\right)\right)$

$$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{26}}\right) + 1$$

$$= 16 + \frac{40h}{\sqrt{26}} + \frac{25h^2}{26} - 12 - \frac{4h}{\sqrt{26}}$$

$$= \frac{25h^2}{26} - \frac{36h}{\sqrt{26}} + 5$$

$$D \cdot f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{25h^2}{26} - \frac{36h}{\sqrt{26}} + 5 - 5}{h}$$

$$= \lim_{h \to 0} \frac{h\left(\frac{25h}{26} - \frac{36}{\sqrt{26}}\right)}{h}$$

$$= \lim_{h \to 0} \frac{25h}{26} - \frac{36}{\sqrt{26}}$$

$$D \cdot f(a) = \frac{36}{\sqrt{26}}$$

(iii) $f(x,y) = 2x + 3y$, $a = (1,2)$ $u = 3i + 4j$

$\rightarrow f(x,y) = 2x + 3y$ $a(1,2)$

$$\bar{u} = 3\hat{i} + 4\hat{j}$$

$$\hat{u} = \frac{\bar{u}}{|u|} = \frac{1}{\sqrt{3^2+4^2}}(3\hat{i} + 4\hat{j})$$

$$= \frac{1}{\sqrt{25}}(3\hat{i} + 4\hat{j})$$

$$\hat{u} = ((3/5), (4/5))$$

$$f(a) = 2(1) + 3(2))$$

$$= 2 + 6$$

$$f(a) = 8$$

$$f(a+hu) = f(1,2) + h(3/5, 4/5))$$

$$= f((1 + 3h/5), (2 + 4h/5))$$

$$= 2(1 + 3h/5) + 3(2 + 4h/5))$$

$$= 2 + 6h/5 + 6 + 12h/5$$

$$= 18h/5 + 8$$

$$D \cdot f(a) = \lim_{h \to 0} \frac{f(a+hu)(f(a))}{h}$$

$$\rightarrow Df(a) = \lim_{h \to 0} \frac{18h/5 + 8 - 8}{5}$$

$$= \lim_{h \to 0} 18h/h$$

$$D \cdot f(a) = 18/5$$

Q.3) Find the equation of tangent & normal to each of the following using values of given point.

(1) $x^2 \cos y + e^{xy} = 2$ at $(1, 0)$

$\rightarrow$ $x^2 \cos y + e^{xy} - 2 = 0$

$f_x = 2x \cos y + y \cdot e^{xy}$

$f_y = -x \sin y + x e^{xy}$

Tangent

$f_x (x - x_0) + f_y (y - y_0) = 0$

$(2x \cos y + y e^{xy})(x-1) + (-x^2 \sin y + x e^{xy})(y-0) = 0$

$2x^2 \cos y + x y e^{xy} y - 2x \cos y - y e^{xy} - (x^2 \sin y - x e^{xy}) y = 0$

$2x^2 \cos y + x y e^{xy} - 2x \cos y - y e^{xy} - (x^2 \sin y - x e^{xy}) y = 0$

(2) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2, -2)$

$\rightarrow$

$f(x,y) = x^2 + y^2 - 2x + 3y + 2 = 0$

$f_x = 2x - 2$ $\qquad f_x(2,-2) = 2$

$f_y = 2y + 3$ $\qquad f_y(2,-2) = 1$

Tangent: $x - 2y + d = 0$

$2 - 2(-2) + d = 0$

$\therefore d = 2$

$\therefore x - 2y + 2 = 0$

Q.4) Find the equation of tangent & normal line to each of the following surfaces

(1) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$\rightarrow$ $f(x, y, z) = x^2 - 2yz + 3y + xz - 7$

$f_x = 2x + z$

$f_y = -2z + 3$

$f_z = -2y + x$

$f_x [x_0, y_0, z_0] = 2(2) + 0 = 4$

$f_y [x_0, y_0, z_0] = -2(0) + 3 = 2$

$f_z (x_0, y_0, z_0) = -2(1) + 2 = 0$

Normal

$$\frac{x - x_0}{f_x(x_0, y_0, z_0)} = \frac{y - y_0}{f_y(x_0, y_0, z_0)} = \frac{z - z_0}{f_z(x_0, y_0, z_0)}$$

$$\frac{x - 2}{4} = \frac{y - 1}{1} = \frac{z - 0}{0}$$

Q2) Find Gradient vector for the following function at given point.

① $f(x,y) = x^y + y^x, a = (1,1)$

$\rightarrow f(x,y) = x^y + y^x$

$f_x = \frac{\partial}{\partial x}(x^y + y^x)$

$f_x = yx^{y-1} + y^x \cdot \log x$

$\nabla f(x,y) = (f_x, f_y)$

$\nabla f(x,y) = (yx^{x-1} + y^x \cdot \log y, xy^2 + x^y \log x)$

$\nabla f(1,1) = (1(1)^{1-1} + 1^1 \log 1, 1(1)^{1-1} + 1^1 \log 1)$

$\nabla f(1,1) = (1,1)$

② $f(x,y) = (\tan^{-1} x) \cdot y^2, a = (1,-1)$

$\rightarrow f(x,y) = (\tan^{-1} x) \cdot y^2$

$f_x = \frac{\partial}{\partial x} (\tan^{-1} x) y^2$

$f_x = \frac{y^2}{1+x^2}$

$f_y = \frac{\partial}{\partial y} (\tan^{-1} x) y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = f(f_x, f_y)$

$= f\left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$\nabla f(1,-1) = \left(\frac{1^2}{1+(-1)^2}, 2(-1)\tan^{-1}(1)\right)$

$\nabla = \left(\frac{1}{1+1}, -2 \times \pi/4\right)$

$= \left(1/_{1+1}, -2 \times \pi/4\right)$

$\nabla f(1,-1) = \left(1/2, -\pi/2\right)$

③ $f(x,y,z) = xyz - e^{x+y}$, at $a = (1,-1,0)$

$f(x,y,z) = xyz - e^{x+y+z}$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$f_z = xy - e^{x+y+z}$

$\nabla f(x,y,z) = (f_x, f_y, f_z)$

$= (yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z})$

$\nabla f(1,-1,-2)$

$$\lim_{h\to 0} \frac{-\sin\frac{4h}{2}}{-6h}$$

$$\lim_{h\to 0} \frac{\sin \cancel{4} h}{\cancel{12} h}$$

$$\frac{1}{3}\lim_{h\to 0} \frac{\sin h}{h} = \frac{1}{3}\times 1 = \frac{1}{3}//$$

4) $$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

By rationalizing Numerator & Denominator both

$$\lim_{x\to\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\times\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{x\to\infty}\left[(x^2+5-x^2+3)\frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2-3})}\right]$$

$$\lim_{x\to\infty}\frac{8(\sqrt{x^2+3}+\sqrt{x^2+1})}{2(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty}\frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+\frac{1}{x^2})}}{\sqrt{x^2(1+\frac{5}{x^2})}+\sqrt{x^2(1-3/x^2)}}$$

After applying limit we get,

$= 4//$

$$\lim_{x \to \pi/6} \frac{\cos x - \sqrt{3} \sin x}{\pi - 6x}$$

By substituting $x - \frac{\pi}{6} = h$

$$x = h + \frac{\pi}{6}$$

where $h \to 0$

$$\lim_{h \to 0} \frac{\cos\left(h + \frac{\pi}{6}\right) - \sqrt{3} \sin\left(h + \frac{\pi}{6}\right)}{\pi - 6\left(h - \frac{\pi}{6}\right)}$$

using
$\cos(A+B) = \cos A \cdot \cos B - \sin A \cdot \sin B$
$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$$\lim_{h \to 0} \frac{\cos h \cdot \cos\frac{\pi}{6} - \sin h \sin\frac{\pi}{6} - \sqrt{3}\left(\sin h \cos\frac{\pi}{6} + \cos h \sin\frac{\pi}{6}\right)}{\pi - \cancel{6}\left(\frac{6h + \pi}{\cancel{6}}\right)}$$

$\cos\frac{\pi}{6} = \cos 30° = \sqrt{3}/2$
$\sin\frac{\pi}{6} = \sin 30° = 1/\sqrt{2}$

$$\lim_{h \to 0} \frac{\cos h \frac{\sqrt{3}}{2} - \sin h \frac{1}{2} - \sqrt{3}\left(\sin h \frac{\sqrt{3}}{2} + \cos h \frac{1}{2}\right)}{\cancel{\pi} - 6h + \cancel{\pi}}$$

$$\lim_{h \to 0} \frac{\cos \frac{\sqrt{3}}{2} h - \sin \frac{h}{2} - \sin \frac{3h}{2} - \cos \frac{\sqrt{3}h}{2}}{-6h}$$

Practical - 1

1) $\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\times\dfrac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}}\times\dfrac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right]$

$\lim_{x\to a}\dfrac{(a+2x-3x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{(3a+x-4x)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

$\lim_{x\to a}\dfrac{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{(3a-3x)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

$\dfrac{1}{3}\lim_{x\to a}\dfrac{(a-x)\left(\sqrt{3a+x}+2\sqrt{x}\right)}{(a-x)\left(\sqrt{a+2x}+\sqrt{3x}\right)}$

$\dfrac{1}{3}\lim_{x\to a}\dfrac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{a+2a}+\sqrt{3a}}$

$\dfrac{1}{3}\times\dfrac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$

$\dfrac{1}{3}\times\dfrac{\cancel{4}^{2}\cancel{\sqrt{a}}}{\cancel{2}\sqrt{3}\cancel{\sqrt{a}}}$

$=\dfrac{2}{3\sqrt{3}}$

2) $\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \right]$

$\lim_{y \to 0} \left[\frac{\sqrt{a+y} - \sqrt{a}}{y\sqrt{a+y}} \times \frac{\sqrt{a+y} + \sqrt{a}}{\sqrt{a+y} + \sqrt{a}} \right)$

$\lim_{y \to 0} \frac{a+y-a}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$\lim_{y \to 0} \frac{y}{y\sqrt{a+y}(\sqrt{a+y} + \sqrt{a})}$

$\frac{1}{\sqrt{a+0}(\sqrt{a+0} + \sqrt{a}}$

$\frac{1}{\sqrt{a}(\sqrt{a} + \sqrt{a})}$

$\frac{1}{a(2\sqrt{a})} = \frac{1}{2a}$

5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$

$= \dfrac{\cos x}{\pi - 2x}$, for $\dfrac{\pi}{2} < x < \pi$ } at $x = \pi/2$

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\sqrt{1-\cos 2(\pi/2)}}$ $\therefore f(\pi/2) = 0$

f at $x = \pi/2$ define.

ii) $\lim\limits_{x \to \pi/2} f(x) = \lim\limits_{x \to \pi/2} \dfrac{\cos x}{\pi - 2x}$

By substituting method

$x - \dfrac{\pi}{2} = h$

$x = h + \pi/2$

where $h \to 0$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - 2(h + \pi/2)}$

$\lim\limits_{h \to 0} \dfrac{\cos(h + \pi/2)}{\pi - \cancel{2}\left(\dfrac{2h + \pi}{\cancel{2}}\right)}$

$\lim\limits_{h \to 0} \dfrac{\cos\left(h + \dfrac{\pi}{2}\right)}{-2h}$ Using $\cos(A+B) = \cos A \cos B - \sin A \sin B$

$$f(x) = \frac{x^2-9}{x-3} \qquad 0 < x < 3$$
$$= x+3 \qquad 3 \leq x \leq 6$$
$$= \frac{x^2-9}{x+3} \qquad 6 \leq x < 9$$

at $x=3$ & $x=6$

at $x=3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

f at $x=3$ define

ii) $\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim\limits_{x \to 3^+} f(x) = \lim\limits_{x \to 3^+} (x+3) = 6$.

$\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{\cancel{(x-3)}}$

$=$ LHL $=$ RHL

f is continoust at $x=3$

112

$$\lim_{h\to 0} \frac{\cos h \cdot \cos\frac{\pi}{2} - \sin h \cdot \sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

$$\frac{1}{2} \lim_{h\to 0} \frac{\sin h}{h}$$

$$= \frac{1}{2} //$$

b. $\lim_{x\to\pi/2^-} f(x) = \lim_{x\to\pi/2^-} \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

using
$\sin 2x = 2\sin x\cos x$

$$\lim_{x\to\pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x\to\pi/2^-} \frac{2\sin x \cdot \cos x}{\sqrt{2}\sin x}$$

$$\lim_{x\to\pi/2^-} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x\to\pi/2^-} \cos x$$

Ex.

$$\lim_{x\to 0}(1+\tan^2 x)\frac{1}{\tan^2 x}$$

we know that

$$\lim_{x\to 0}(1+px)\frac{1}{px}=e$$

$\therefore\ =e$

$k=e$

iii) $f(x)=\dfrac{\sqrt{3}-\tan x}{\pi-3x}$ $x\neq \pi/3$ $\Big\}$ at $x=\dfrac{\pi}{3}$

$=k$ $x=\pi/3$

$x-\dfrac{\pi}{3}=h$

$x=h+\dfrac{\pi}{3}$

where $h\to 0$

$$f(\pi/3+h)=\frac{\sqrt{3}-\tan(\pi/3+h)}{\pi-3(\pi/3+h)}$$

$$\lim_{h\to 0}\frac{\sqrt{3}-\tan(\pi/3+h)}{\pi-3(\pi/3+h)}$$

$$\lim_{x \to 0} \frac{2\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \frac{\sin^2 2x}{x^2} = k$$

$$2 \lim_{x \to 0} \left(\frac{\sin . 2x}{x}\right)^2 = k$$

$$2(2)^2 = k$$

$$\therefore \quad k = 8$$

ii) $f(x) = (\sec^2 x) \cot^2 x \qquad x \neq 0$

$= k \qquad x = 0 \quad \}$ at $x = 0$

Solution -

$$f(x) = (\sec^2 x) \cot^2 x$$

Using

$\tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1 + \tan^2 x$

&

$$\cot^2 x = \frac{1}{\tan^2 x}$$

$$\therefore \lim_{x \to 0} (\sec^2 x) \cot^2 x$$