Department of CSE

22CST53 - Theory of Computation

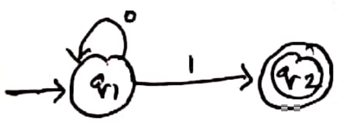
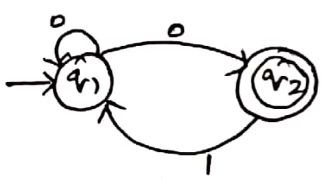
CAT-I Answer Key.

Part-A

1. Steps to be followed to solve problems in Mathematical Induction:

* Basis
* Inductive step (or) Induction

2.

DFA	NFA
* It has a power to move from one state to another State on reading Symbol.	* It has a power of moving to more than one state at a time on reading an input Symbol.
* Occupies more memory.	* Occupies less memory.
* Transistion function $Q \times \Sigma \rightarrow Q$	* Transistion function $Q \times (\Sigma \cup \{\epsilon\}) \rightarrow 2^Q$
* Ex: 	

3. if x is an even number then x^2 is also even:

An even no. can be represented as $2x$. where 'x' is an integer.

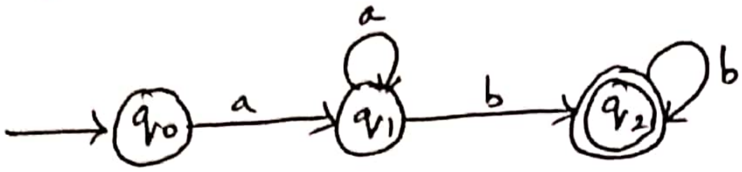
when we square the even number

$$\cancel{K = 2 x^2} \quad K = (2x)^2 = 4x^2$$

$\therefore$ x^2 is multiplied by 4 which is divisible by 2, x^2 is also even number.

4. DFA that accepts strings of the language $L = \{a^m b^n\}$

$L = \{ab, aab, abb, \dots\}$



5. $\Sigma = \{ab, bb\}$

$\Sigma^4 = \{abab, bbab, bbbb, abbb\}$

6. if $a \% b = b \% a$ then $a = b$.

Proving only if part : (using Contradiction)

Statement	Justification
1. $a \neq b$	Negation of Conclusion.
2. when $a < b$, $a \bmod b = a$, $b \bmod a = 0$ to $a-1$	From definition of mod.
3. when $a > b$, $a \bmod b = 0$ to $b-1$, $b \bmod a = b$	From definition of mod.
4. $a \bmod b \neq b \bmod a$	From 2 & 3

Therefore, The negation of hypothesis is came,

$\therefore$ The statement is true.

(or)

7. Language of the given NFA:

Set of all Strings that Starts with a.

$L = \{aa, aab, aaa \ldots\}$

8. Operators of Regular Expression & its priority:

i) * operator → Highest priority

ii) . operator → Next to *

iii) + operator → Next to .

9. Regular Language for the regular expression:

$(0^*, 1^*)^* 000 (0+1)^*$

The Set of strings of 0's and 1's that Should Contain the Substring 000.

10. If $L = \{10, 1\}$

$L^* = \{\epsilon, 10, 1, 1010, 101, 110, 11 \ldots\}$

Part - B

11. i) Mathematical Induction, $2+2^2+2^3+\cdots+2^n = 2^{n+1}-2$

Let us take $S(n) = 2+2^2+2^3+\cdots+2^n = 2^{n+1}-2$

(1 mark)

Basis: Take $n=1$

$LHS = 2$ $\qquad RHS = 2^{n+1}-2$

$= 2^2-2 = 2.$

$LHS = RHS$

$\therefore$ $S(1)$ is true is proved.

(1 mark)

Induction:

Assume $S(n)$ is true.

(ie) $S(n) = 2+2^2+2^3+\cdots+2^n = 2^{n+1}-2$ is true.

Then prove $S(n+1)$ is true.

$$S(n+1) = \underbrace{2+2^2+2^3+\cdots+2^n+2^{n+1}}_{LHS} = \underbrace{2^{(n+1)+1}-2}_{RHS}$$

LHS: $2+2^2+2^3+\cdots+2^n+2^{n+1} = 2^{n+1}-2+2^{n+1}$ (3 marks)

$= 2(2^{n+1})-2$

$= 2^1 \cdot 2^{n+1}-2$

$= 2^{1+n+1}-2$

$= 2^{n+2}-2$

RHS: $2^{(n+1)+1}-2 = 2^{n+2}-2$

$LHS = RHS$

$\therefore$ $S(n+1)$ is true is proved if $S(n)$ is true.

Hence given statement $S(n)$ is proved.

11 ii) Every Tree has one more node than its edges.

Basis: Lets take no. of nodes $n = 1$ (2 marks)

If $n = 1$ then $e = 0$

So $n = e + 1$ is true.

Induction: (3 marks)

No. of nodes in Tree T is written as

$n = n_1 + n_2 + \cdots + n_K + 1$ —(1)

No. of edges in Tree T is written as

$e = k + e_1 + e_2 + \cdots + e_K$ —(2)

$n = 1 + n_1 + n_2 + \cdots + n_K$

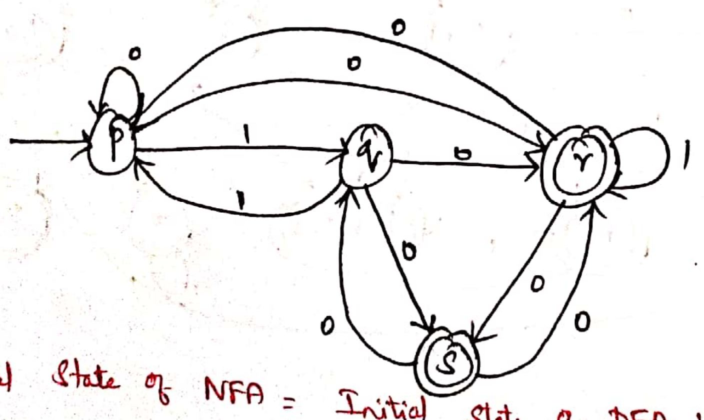
$n = 1 + (e_1 + 1) + (e_2 + 1) + \cdots + (e_K + 1)$

$n = 1 + k + e_1 + e_2 + \cdots + e_K$

$n = 1 + e$

Hence, Proved.

12. NFA to DFA:



Initial State of NFA = Initial state of DFA : [P] —(1 mark)

Transistion of DFA

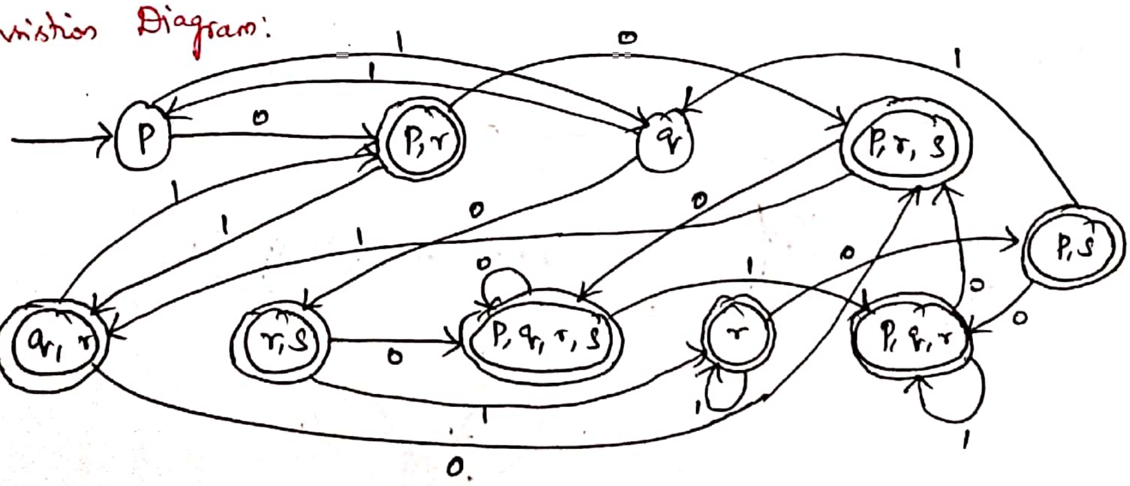
—(4 marks)

Transition Table: —(3 marks)

State	Input 0	Input 1
→{p}	{p, r}	{q}
*{p, r}	{p, r, s}	{q, r}
{q}	{r, s}	{p}
*{p, r, s}	{p, q, r, s}	{q, r}
*{q, r}	{p, r, s}	{p, r}
*{r, s}	{p, q, r, s}	{r}
*{p, q, r, s}	{p, q, r, s}	{p, q, r}
*{r}	{p, s}	{r}
*{p, q, r}	{p, r, s}	{p, q, r}
*{p, s}	{p, q, r}	{q}

—(2 marks)

Transition Diagram:



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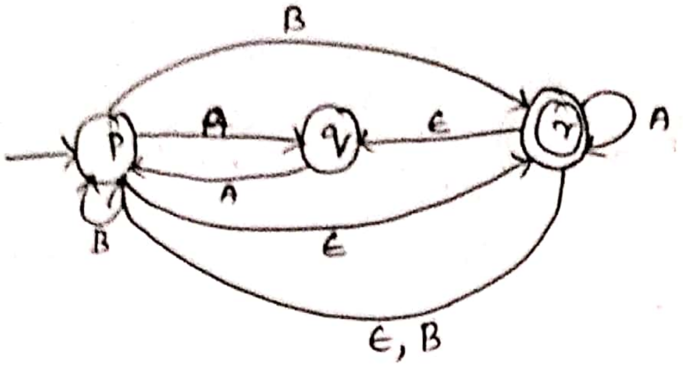


i) ϵ-closure: — (3 marks)

ϵ-closure $\{p\} = \{p, q, r\}$

ϵ-closure $\{q\} = \{q\}$

ϵ-closure $\{r\} = \{p, q, r\}$

ii) ϵ-NFA to DFA:

Initial state of DFA = ϵ-closure {initial state of NFA} = [p, q, r]

↳ (1 mark)

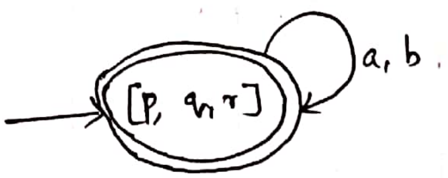
Transitions — (2 marks)

Table: — (1 mark)

State	Input	
	A	B
→*[p, q, r]	[p, q, r]	[p, q, r]

— (1 mark)

Diagram:



iii) Set of strings: — (2 marks)

{a, aa, ab, ba, bb, abb, aab...}

14.

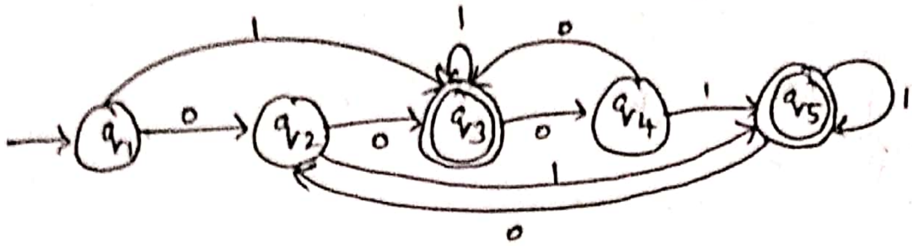


Table:

(5 marks)

q_2	X			
q_3	X	X		
q_4	X		X	
q_5	X	X	X	X
	q_1	q_2	q_3	q_4

Equivalent states: q_2, q_4

Distinguishable states: q_1, q_3, q_5

Diagram:

(5 marks)

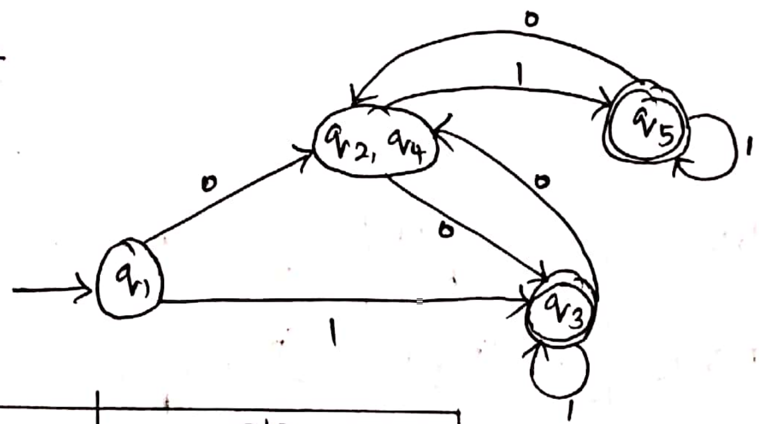


Table:

State	I/P	
	0	1
→ q_1	q_2, q_4	q_3
q_2, q_4	q_3	q_5
* q_3	q_2, q_4	q_3
* q_5	q_2, q_4	q_5

——— X ———

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