G – 16801

Register No. | | | | | | | | |

BE Degree Examination November 2024

Fifth Semester

Computer Science and Engineering

22CST53 – THEORY OF COMPUTATION

(Regulations 2022)

Time: Three hours   Maximum: 100 marks

Answer all Questions

Part – A (10 × 2 = 20 marks)

1. Differentiate between NFA and DFA. [CO1,K2]
2. Construct the DFA that accepting all strings in $\{0,1\}^*$ having even number of 0's. [CO1,K3]
3. Write the RE for the set of all strings that begin with 110. [CO2,K3]
4. Prove that $L = \{0^n1^n / n>=1\}$ is not a Regular. [CO2,K3]
5. Outline the formal definition of CFG. [CO3,K2]
6. Classify the different ways of language that accepted by PDA. [CO3,K2]
7. How will you identify whether the given grammar is CNF or not? [CO4,K2]
8. Indicate any four closure properties of CFL. [CO4,K2]
9. Infer the properties of recursive and recursive enumerable language. [CO5,K2]
10. Let $\Sigma = \{0,1\}$ and A and B be the list as [CO5,K3]

	List A	List B
i	w_i	x_i
1	10	101
2	011	11
3	101	011

Find the instance of PCP.

Part – B (5 × 16 = 80 marks)

11. a. Construct DFA equivalent to NFA ({p, q, r, s}, {0, 1}, δ, p, {s}) where δ is defined as (16) [CO1,K3]

δ	0	1
→ p	{p, q}	{p}
q	{r}	{r}
r	{s}	-
*s	{s}	{s}

(OR)

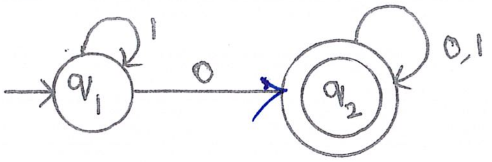
b. i) Prove that every tree has one more node than it has edges. (8) [CO1,K3]

ii) Prove the following by induction for all $n \geq 0$ (8) [CO1,K3]

$$S(n) = 1^2+2^2+.....+n^2 = \frac{n(n+1)(2n+1)}{6}$$

MB 210, 211, MB201, 202, SES 112, SES 113, SES 204, SES 207, SES 104, 105, CH - 203 (DH)

12. a. Convert the following automata to Regular expression. (16) [CO2, K3]



(OR)

b. i) Show that the closure under union and intersection of a Regular language is Regular. (8) [CO2, K3]

ii) Prove that $L=\{a^n \mid n=i^2, i\geq 1\}$ is not Regular. (8) [CO2, K3]

13. a. Consider the following grammar, $S \rightarrow aS/aSbS/\epsilon$ show that this grammar is ambiguous for the string 'aab' that has two parse trees, ~~two~~ left most derivation and ~~two~~ right most derivation trees. (16) [CO3,K3]

(OR)

b. Design the PDA to accept the language $L=\{0^n1^n/n\geq 1\}$ accepting by final state and empty stack. (16) [CO3,K3]

$S \rightarrow 0S1$

$S \rightarrow 01$

14. a. Simplify the following grammar and Find its equivalent CNF. (16) [CO4,K3]

$S \rightarrow ASB/\epsilon$

$A \rightarrow AaS/A$

$B \rightarrow SbS/A/bb$

(OR)

b. Design a TM to perform the multiplication f (m,n) = m*n using subroutine. Also construct transition table and transition functions of the above function. (16) [CO4,K3]

15. a. Explain the steps to find the codes of TM with the help of given transition functions. (16) [CO5,K2]

$M=(\{q_1,q_2,q_3\}, \{0, 1\}, \{0, 1, B\}, \delta, q_1, B, \{q_1\})$

Where δ consist of the following rules

$\delta(q_1,1) = (q_3, 0, R)$

$\delta(q_3,0) = (q_1, 1, R)$

$\delta(q_3,1) = (q_2, 0, R)$

$\delta(q_3,B) = (q_3, 0, L)$

(OR)

b. Explain in detail about MPCP with suitable example. (16) [CO5,K2]

Bloom's Taxonomy Level	Remembering (K1)	Understanding (K2)	Applying (K3)	Analysing (K4)	Evaluating (K5)	Creating (K6)
Percentage	–	24	76	–	–	–

22CST53 - Theory of Computation

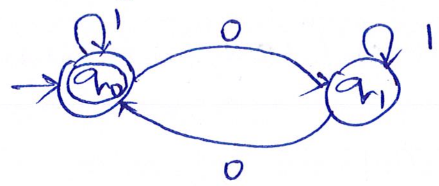
Part - A (10 x 2 = 20 marks)

[Any 4]

1.

NFA	DFA
(i) There is more than one possible transition from one state on the same input symbol.	(i) For each input symbol there is only one transition from a state.
(ii) NFA requires less space than DFA.	(ii) DFA requires more space
(iii) ε-transition is allowed in NFA.	(iii) ε-transition is not allowed in DFA.
(iv) Transition function is $\delta : Q \times \Sigma \rightarrow 2^Q$.	(iv) Transition function is $\delta : Q \times \Sigma \rightarrow Q$.
(v) Not all NFA are DFA	(v) All DFA are NFA.
(vi) Total time required to run any input string in NFA is longer.	(vi) Total time it takes to run any input string is less than NFA.
(vii) Any Example.	(vii) Any Example.

2. DFA : even no. of 0's over $\Sigma = \{0,1\}^*$



3. Regular Expression for set of all strings begin with 110.

$$RE = 110(0|1)^*$$

(or)

$$RE = 110(0+1)^*$$

4. $L = \{0^n 1^n \mid n >= 1\}$ is not Regular.

(i) Assume given L is a regular

$L = \{01, 0011, 000111, \dots\}$

(ii) choose 'n' & 'w' such that $|w| \geq n$

$n = 2 \Rightarrow w = 0011$ such that $|w| \geq n$

$4 \geq 2 \Rightarrow T$

(iii) $w = 0011$, split into 3 parts

$$w = \underline{0}\,\underline{0}\,\underline{11}$$
$x \quad y \quad z$

(a) $|y| > 0 \Rightarrow 1 > 0 \Rightarrow T$

(b) $|xy| \leq n \Rightarrow 2 \leq 2 \Rightarrow T$

(c) xy^kz

(i) $k = 0 \Rightarrow xy^0z \Rightarrow 011 \notin L$

We are getting a string not belongs to L. so given Language L is not regular. Hence proved

5. Definition of CFG:

→ A Grammar is a Context - free Grammar (CFG) if every production is of the form $\alpha \to \beta$ where length of β must be greater than (or) equal to α and α is a single non-terminals.

→ CFG is denoted by,

$$G = (N, T, P, S)$$

→ In CFG, the start symbol is used to derive the string.

6. Different ways of language acceptance by PDA:

→ PDA can accept the language in 2 ways.

1. Acceptance by Empty stack.
2. Acceptance by final state.

7. Grammar in CNF (or) not:

→ If all productions in Grammar is of the form $A \to BC$ and $A \to a$ then the grammar is said to be in CNF. Here A, B, C are non-terminals and a is the terminal symbol.

8. Any 4 closure properties of CFL:
(any 4)
The CFL are closed under the following operations:

1. Union.
2. Concatenation
3. Closure (*) and positive closure (+)
4. Homomorphism.
5. Reversal.
6. Inverse Homomorphism.

9. Properties of Recursive and Recursive Enumerable language:

Recursive Language

→ A language L is said to be recursive language, if there exists a Turing machine which will accept all the strings in L and halts

→ It rejects all the strings not in L and halts.

→ There exist a TM 'M' such that

1. M accepts all strings $w \in L$ and halts.
2. M rejects all strings $w \notin L$ and halts.

Recursively Enumerable Language

→ The language L is said to be recursively enumerable language, if there exists a TM which accepts all the strings in L and halts. It rejects or enter into infinite loop for strings not in L.

→ There exist a TM 'M' such that $w \in L$ and halts.

1. M accepts all strings
2. M rejects (or) enter into infinite loop for strings $w \notin L$.

10. Instance of PCP:

List A: 1010110l ....

List B: 101011011 ...

There is no solution for this particular instance of PCP.

Part - B (5 x 16 = 80 marks)

11. a. Equivalent DFA for the given NFA:

Given,

δ	0	1
$\rightarrow p$	$\{p, q\}$	$\{p\}$
q	$\{r\}$	$\{r\}$
r	$\{s\}$	—
$* s$	$\{s\}$	$\{s\}$

Solution: — (16 marks)

$\delta(p, 0) = \{p, q\} \rightarrow B$

$\delta(p, 1) = \{p\} \rightarrow A$

$\delta(\{p, q\}, 0) = \delta(p, 0) \cup \delta(q, 0) = \{p, q\} \cup \{r\}$

$= \{p, q, r\} \rightarrow C$

$\delta(\{p, q\}, 1) = \delta(p, 1) \cup \delta(q, 1) = \{p\} \cup \{r\}$

$= \{p, r\} \rightarrow D$

$\delta(\{p, q, r\}, 0) = \{p, q, r, s\} \rightarrow E$

$\delta(\{p, q, r\}, 1) = \{p, r\} \rightarrow D$

$\delta(\{p, r\}, 0) = \{p, q, s\} \rightarrow F$

$\delta(\{p, r\}, 1) = \{p\} \rightarrow A$

$\delta(\{p,q,r,s\},0) = \{p,q,r,s\} \rightarrow E$

$\delta(\{p,q,r,s\},1) = \{p,r,s\} \rightarrow G$

$\delta(\{p,q,s\},0) = \{p,q,r,s\} \rightarrow E$

$\delta(\{p,q,s\},1) = \{p,r,s\} \rightarrow G$

$\delta(\{p,r,s\},0) = \{p,q,s\} \rightarrow F$

$\delta(\{p,r,s\},1) = \{p,s\} \rightarrow H$

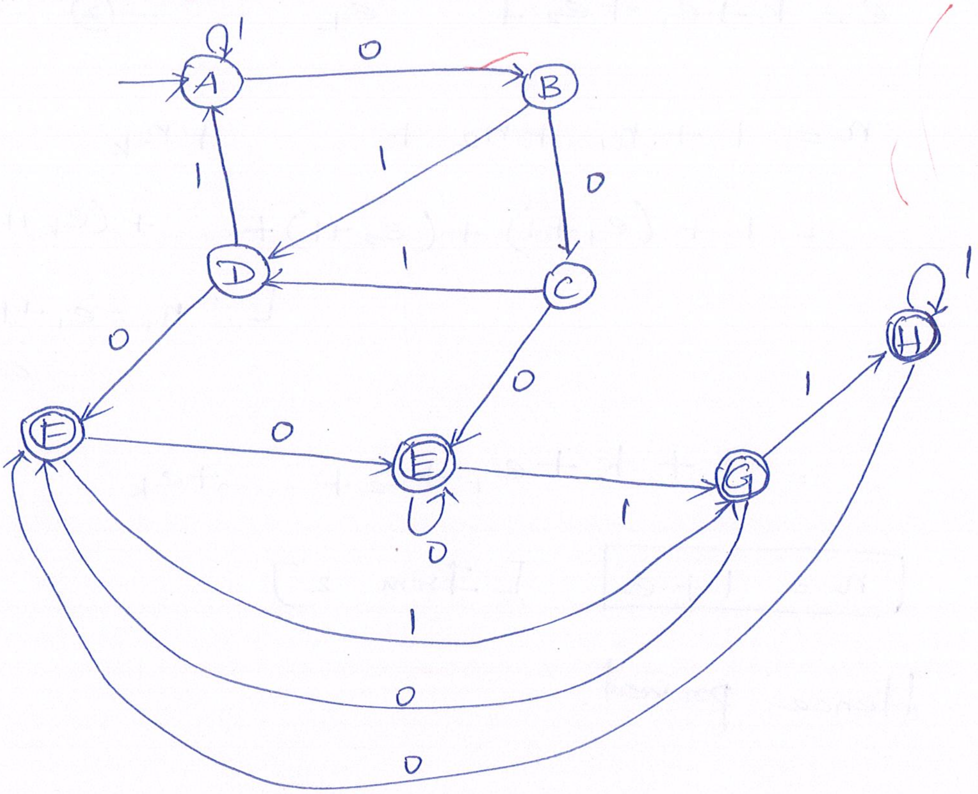
$\delta(\{p,s\},0) = \{p,q,s\} \rightarrow F$

$\delta(\{p,s\},1) = \{p,s\} \rightarrow H$

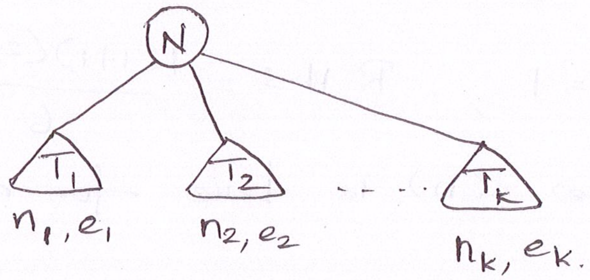
Transition table: — (5 marks)

	0	1
$\rightarrow$ A $\{p\}$	B $\{p,q\}$	A $\{p\}$
B $\{p,q\}$	C $\{p,q,r\}$	D $\{p,r\}$
C $\{p,q,r\}$	E $\{p,q,r,s\}$	D $\{p,r\}$
D $\{p,r\}$	F $\{p,q,s\}$	A $\{p\}$
* E $\{p,q,r,s\}$	E $\{p,q,r,s\}$	G $\{p,r,s\}$
* F $\{p,q,s\}$	E $\{p,q,r,s\}$	G $\{p,r,s\}$
* G $\{p,r,s\}$	F $\{p,q,s\}$	H $\{p,s\}$
* H $\{p,s\}$	F $\{p,q,s\}$	H $\{p,s\}$

Transition Diagram (DFA): — (~~5~~4 marks)



11. b. (i) Every tree has one more node than its edges



Basis: Lets take no. of nodes $n=1$ — (2 marks)

If $n=1$ then $e=0$

So $n=e+1$ is true.

Induction:

(5 marks)

Assumption: Every sub tree has one more node than its edges is true for 'k' subtrees. — (1 mark)

(ie) $n_1 = e_1 + 1$, $n_2 = e_2 + 1$ .... $n_k = e_k + 1$

Number of nodes in Tree T is written as,

$$n = n_1 + n_2 + \dots n_k + 1 \quad — (1)$$

No. of edges in Tree T is written as,

$$e = k + e_1 + e_2 + \dots e_k \quad -②$$

$$n = 1 + n_1 + n_2 + \dots + n_k$$

$$= 1 + (e_1+1) + (e_2+1) + \dots + (e_k+1)$$

[$\because n_1 = e_1 + 1$ as per assumption].

$$= 1 + k + e_1 + e_2 + \dots + e_k.$$

$$\boxed{n = 1 + e} \quad [\text{from } 2].$$

Hence proved.

11. b ii) $S(n) = 1^2 + 2^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$.

Basis: Let $n=1$ — (2 marks)

$L.H.S = 1$, $R.H.S = \frac{1(1+1)(2+1)}{6} = \frac{6}{6} = 1$

(ie) $S(n)$ is true for $n=1$ is proved.

Induction:

Assume $S(n)$ is true and prove $S(n+1)$ is also true. — (1 mark)

(5 marks)

$$\underbrace{S(n+1) = 1^2 + 2^2 + \dots + n^2 + (n+1)^2}_{L.H.S} = \underbrace{\frac{(n+1)(n+1+1)(2(n+1)+1)}{6}}_{R.H.S}$$

$$L.H.S \Rightarrow \underbrace{1^2 + 2^2 + \dots + n^2}_{} + (n+1)^2 = \frac{n(n+1)(2n+1)}{6} + (n+1)^2$$

$$= \frac{(n^2+n)(2n+1)}{6} + (n^2+2n+1)$$

$$= \frac{(2n^3 + n^2 + 2n^2 + n) + 6(n^2+2n+1)}{6}$$

$$= \frac{2n^3 + 3n^2 + n + 6n^2 + 12n + 6}{6}$$

$$L.H.S = \frac{2n^3 + 9n^2 + 13n + 6}{6}$$

$$R.H.S \Rightarrow \frac{(n+1)(n+1+1)(2(n+1)+1)}{6} = \frac{(n+1)(n+2)(2n+2+1)}{6}$$

$$= \frac{(n^2 + 2n + n + 2)(2n+3)}{6}$$

$$R.H.S = \frac{2n^3 + 9n^2 + 13n + 6}{6}$$

$$L.H.S = R.H.S$$

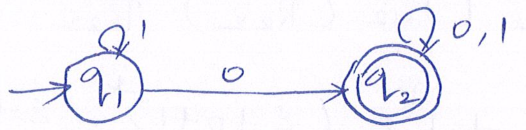
So, $S(n+1)$ is True

(ie) True of $S(n)$ makes $S(n+1)$ also True

So given statement is True for all values of n.

Hence Proved.

12. a. Automata to Regular Expression:



Basis: $k=0$ (3 marks)

$$R_{11}^{0} = \varepsilon + 1$$

$$R_{12}^{0} = 0$$

$$R_{21}^{0} = \phi$$

$$R_{22}^{0} = \varepsilon + 0 + 1$$

The RE for the given DFA is, (2 marks)

$$R_{ij}^{k} = R_{12}^{2}$$

$$R_{12}^{2} = R_{12}^{1} + R_{12}^{1} (R_{22}^{1})^{*} R_{22}^{1}$$

$$R_{12}^{1} = R_{12}^{0} + R_{11}^{0} (R_{11}^{0})^{*} R_{12}^{0}$$

$$R_{22}^{1} = R_{22}^{0} + R_{21}^{0} (R_{11}^{0})^{*} R_{12}^{0}$$

Induction:

k=1: – (5 marks)

$$R^{1}_{12} = R^{0}_{12} + R^{0}_{11}\,(R^{0}_{11})^{*}\,R^{0}_{12}$$

$$= 0 + (\varepsilon+1)(\varepsilon+1)^{*}0$$

$$= 0 + (\varepsilon+1)\,1^{*}0 \qquad [\because (R+\varepsilon)^{*} = R^{*}]$$

$$= 0 + 1^{*}0 \qquad [\because (R+\varepsilon)R^{*} = R^{*}]$$

$$= 1^{*}0 \qquad [\because S + R^{*}S = R^{*}S]$$

$$R^{1}_{22} = R^{0}_{22} + R^{0}_{21}\,(R^{0}_{11})^{*}\,R^{0}_{12}$$

$$= (\varepsilon+0+1) + \phi\,(\varepsilon+1)^{*}0$$

$$= (\varepsilon+0+1) + \phi \qquad [\because \phi R = \phi]$$

$$= \varepsilon+0+1 \qquad [\because \phi + R = R]$$

k=2: – (5 marks)

$$R^{2}_{12} = R^{1}_{12} + R^{1}_{12}\,(R^{1}_{22})^{*}\,R^{1}_{22}$$

$$= 1^{*}0 + 1^{*}0\,(\varepsilon+0+1)^{*}(\varepsilon+0+1)$$

$$= 1^{*}0 + 1^{*}0\,(0+1)^{*}(\varepsilon+0+1) \qquad [\because (R+\varepsilon)^{*} = R^{*}]$$

$$= 1^{*}0 + 1^{*}0\,(0+1)^{*} \qquad [\because (R+\varepsilon)R^{*} = R^{*}]$$

$$= 1^{*}0\,(0+1)^{*} \qquad [\because S + SR^{*} = SR^{*}]$$

∴ The final Regular Expression for given DFA is,

$$\boxed{RE = 1^{*}0\,(0+1)^{*}}$$ – (1 mark)

12.b.(i) **Closure under Union and Intersection of a Regular Language is Regular.**

(i) Union of two Regular Languages is also Regular — (2 marks)

→ Consider there are two regular languages L and M.

→ Since L and M are regular, they have regular expressions $L = L(R)$ and $M = L(S)$. Then $L \cup M = L(R+S)$ by the definition of the + operator of regular expressions.

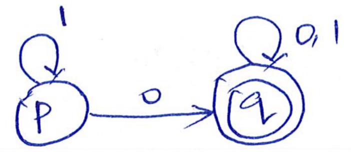
(ii) Intersection of two Regular languages is also Regular: — (6 marks)

Let $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ accept the language L_1 and L_2. We can construct the finite automata M that accepts $L_1 \cap L_2$ using following steps.

$M = (Q, \Sigma, \delta, q_0, F)$ where

1. $Q = Q_1 \times Q_2$
2. $\Sigma = \Sigma$
3. $\delta(pq, a) = \delta_1(p, a)\delta_2(q, a)$
4. $q_0 = q_1 q_2$
5. $F = \{pq \mid p \in F_1 \text{ and } q \in F_2\}$

M_1 :



M_2 :

1. $Q = Q_1 \times Q_2 = (p, q) \times (r, s) = (pr, ps, qr, qs)$

2. $\Sigma = \{0, 1\}$

3. $\delta(pr, 0) = qr$

   $\delta(pr, 1) = ps$

   $\delta(ps, 0) = qs$

   $\delta(ps, 1) = ps$

   $\delta(qr, 0) = qr$

   $\delta(qr, 1) = qs$

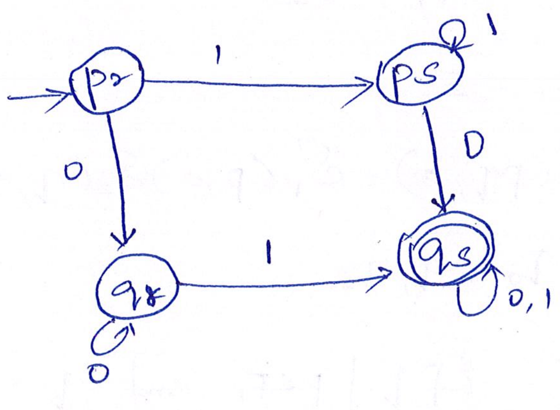
   $\delta(qs, 0) = qs$

   $\delta(qs, 1) = qs$.

4. $q_0 = q_1, q_2 = pr$

5. $F = qs$.

Final automata for $L_1 \cap L_2$ is,



12.b (ii) $L = \{a^n \mid n = i^2, i \geq 1\}$ is not Regular.

Solution:

(1 mark)

(i) Assume L as regular

$L = \{a, aaaa, aaaaaaaaa, \ldots\}$

(ii) choose 'n' & 'w' such that $|w| \geq n$

$n = 2 \Rightarrow w = aaaa$ such that $|w| \geq n$

$4 \geq 2 \Rightarrow$ True

(iii) split w into 3 parts (2 marks)

$w = \underbrace{a}_{x}\underbrace{a}_{y}\underbrace{aa}_{z}$

(a) $|y| > 0 \Rightarrow 1 > 0 \Rightarrow$ True.

(b) $|xy| \leq n \Rightarrow 2 \leq 2 \Rightarrow$ True.

(c) xy^kz

(i) $k=1 \Rightarrow xy^1z \Rightarrow aaaa \in L$

(ii) $k=2 \Rightarrow xy^2z \Rightarrow aaaaa \notin L$. (5 marks)

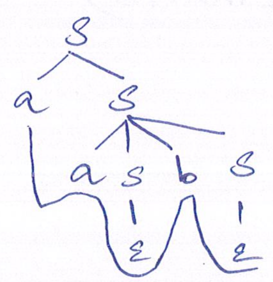
We are getting a string $\notin L$. So given L is not Regular. Hence proved.

13. a. Solution:

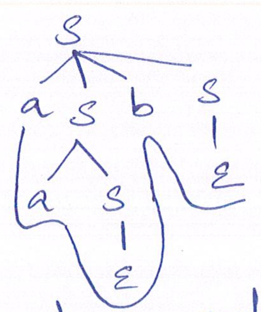
Given Grammar productions are,

$S \rightarrow aS \mid aSbS \mid \varepsilon$

Parse Tree: string: aab. — (4 marks)



string: aab



string: aab

Left-most derivation: — (8 marks)

$S \Rightarrow aS$

$\Rightarrow aaSbS$

$\Rightarrow aa\varepsilon bS$

$\Rightarrow aa\varepsilon b\varepsilon$

$\Rightarrow aab$

$S \Rightarrow aSbS$

$\Rightarrow aaSbS$

$\Rightarrow aa\varepsilon bS$

$\Rightarrow aa\varepsilon b\varepsilon$

$\Rightarrow aab$

(or)

Right-most derivation:

$S \Rightarrow aS$

$\Rightarrow aaSbS$

$\Rightarrow aaSb\varepsilon$

$\Rightarrow aa\varepsilon b\varepsilon$

$\Rightarrow aab$

$S \Rightarrow aSbS$

$\Rightarrow aSb\varepsilon$

$\Rightarrow aaSb\varepsilon$

$\Rightarrow aa\varepsilon b\varepsilon$

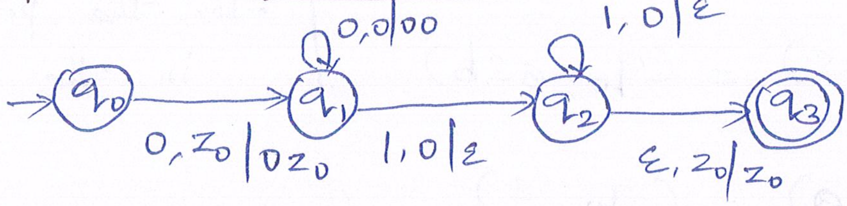
$\Rightarrow aab$

Since for the string "aab" there is more than one parse tree, more than one left-most derivation and more than one right-most derivation, given grammar is a ambiguous grammar.

(2 marks)

13. b. PDA for language $L = \{0^n 1^n \mid n \geq 1\}$

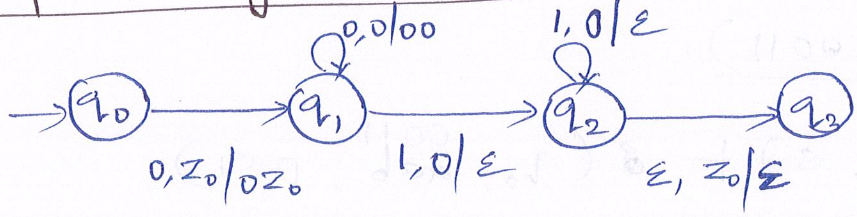
Acceptance by Final state:



— (6 marks)

Transition — (2 marks)

Acceptance by Empty stack:



— (6 marks)

Transition — (2 marks)

(or)

$L = \{0^n 1^n \mid n \geq 1\}$

$L = \{01, 0011, 000111, \ldots\}$

The grammar productions for the above language is,

$S \rightarrow 01, \quad S \rightarrow 0S1.$

$G = (\{S\}, \{0, 1\}, P, S)$ — (2 marks)

Acceptance by Empty stack:

The PDA, M for the above grammar is, — (2 marks)

$Q = q_0$

$\Sigma = \{0, 1\}$

$\Gamma = \{q_0, 0, 1\}$

$q_0 = q_0$

$Z_0 = S$

$F = \phi$

— (4 marks)

δ is written as,

$\delta(q_0, \varepsilon, S) = (q_0, 01)$
$\delta(q_0, \varepsilon, S) = (q_0, 0S1)$ } For the productions in the Grammar.

$\delta(q_0, 0, 0) = (q_0, \varepsilon)$
$\delta(q_0, 1, 1) = (q_0, \varepsilon)$ } For terminals in the Grammar.

Acceptance (0011):

$(q_0, 0011, S) \vdash (q_0, 0011, 0S1)$

$\vdash (q_0, 011, S1)$

$\vdash (q_0, 011, 011)$

$\vdash (q_0, 11, 11)$

$\vdash (q_0, 1, 1)$

$\vdash (q_0, \varepsilon, \varepsilon)$.

string is accepted.

Acceptance by Final state:

The PDA for the given grammar is,

— (3 marks)

$Q = \{q_0', q_0, q_f\}$

$\Sigma = \{0, 1\}$

$\Gamma = N \cup T \cup Z_0'$

$q_0 = q_0'$

$Z_0 = Z_0'$

$F = q_f$

δ is written as, — (5 marks)

i) $\delta(q_0', \varepsilon, Z_0') = (q_0, SZ_0')$

ii) $\delta(q_0, \varepsilon, S) = (q_0, 01)$

$\delta(q_0, \varepsilon, S) = (q_0, 0S1)$

iii) $\delta(q_0, 0, 0) = (q_0, \varepsilon)$

$\delta(q_0, 1, 1) = (q_0, \varepsilon)$

iv) $\delta(q_0, \varepsilon, Z_0') = (q_f, Z_0')$.

Acceptance (0011):

$\delta(q_0', 0011, Z_0') = (q_0, 0011, SZ_0')$

$= (q_0, 0011, 0S1Z_0')$

$= (q_0, 011, S1Z_0')$

$= (q_0, 011, 011Z_0')$

$= (q_0, \varepsilon, Z_0')$

$= (q_f, \varepsilon, Z_0')$

→ Final state is reached.

String is accepted.

14. a. Simplify the Grammar and find equivalent CNF:

Simplification of Grammar:

1. Elimination of NULL production:– (3 marks)

$S \rightarrow \varepsilon$

Substitute $S \rightarrow \varepsilon$ in the given productions,

$S \rightarrow AB$

$A \rightarrow Aa$

$B \rightarrow Sb \mid bS \mid b.$

Append with Grammar,

$S \rightarrow ASB \mid AB.$

$A \rightarrow AaS \mid A \mid Aa$

$B \rightarrow SbS \mid A \mid bb \mid Sb \mid bS \mid b.$

2. Elimination of Unit production: – (3 marks)

Unit productions in the above Grammar are,

$A \rightarrow A$ [remove it].

$B \rightarrow A$

Substitute 'A's' productions in above one

$B \rightarrow AaS \mid Aa.$

Append with Grammar,

$S \rightarrow ASB \mid AB$

$A \rightarrow AaS \mid Aa$

$B \rightarrow SbS \mid bb \mid Sb \mid bS \mid b \mid AaS \mid Aa$

3: Elimination of Useless symbols:

Generating = {B}

Reachable = {S, A, B}

Useful symbols = {B}

Useless symbols = {S, A}

(or)

$S \to ASB \mid AB$

$A \to AaS \mid Aa$

$B \to SbS \mid bb \mid Sb \mid bS \mid b \mid AaS \mid Aa$

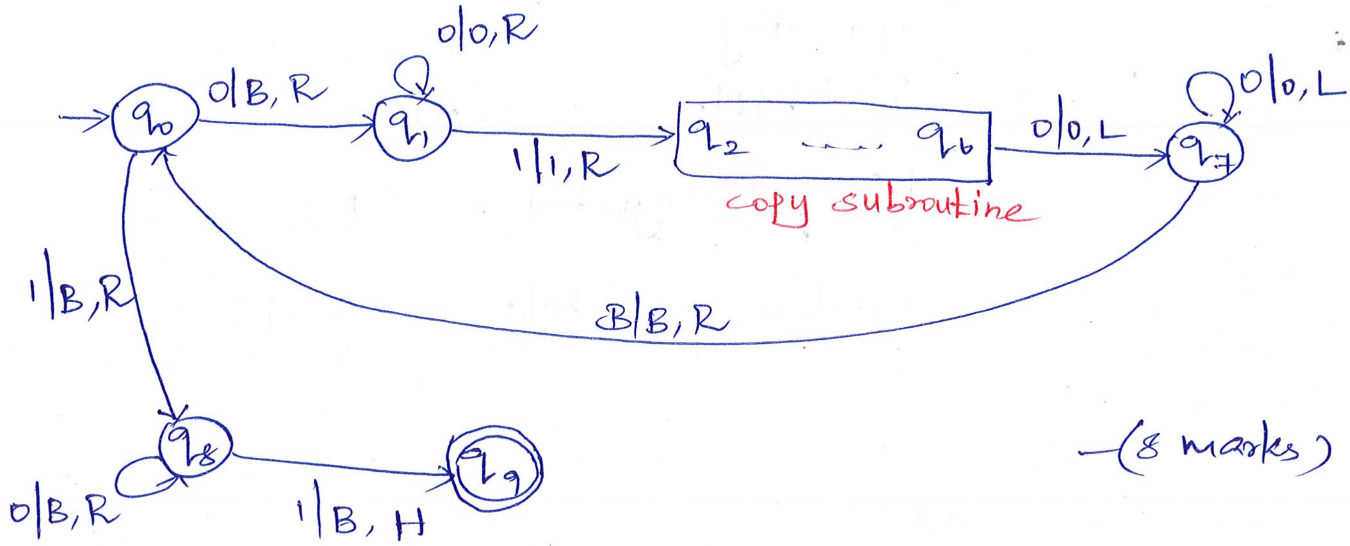
Convert to CNF:

$S \to ASB$	$S \to AB$	$A \to AaS$	$A \to Aa$
$D_1 \to AS$		$D_2 \to a$	$A \to AD_2$
$S \to D_1B$		$D_3 \to D_2S$	
		$A \to AD_3$	

$B \to SbS$	$B \to bb$	$B \to Sb$	$B \to bS$	$B \to b$
$D_4 \to b$	$B \to D_4D_4$	$B \to SD_4$	$B \to D_4S$	
$D_5 \to D_4S$				
$B \to SD_5$				

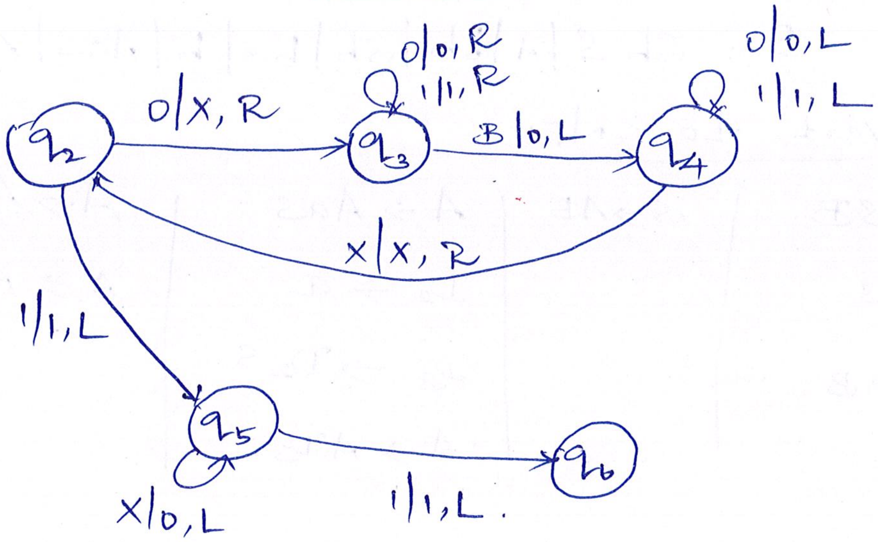
$B \to AaS$	$B \to Aa$
$B \to AD_3$	$B \to AD_2$.

14.b. Turing Machine for multiplication using subroutine:



-(8 marks)

copy subroutine. — (6 marks)



Input tape initially, concept - (2marks)

0	0	0	1	0	0	1	B	B	B	B

15.a. **Codes for Turing Machine:**

Solution:

1. States are q_1, q_2, q_3.

2. Tape symbols: — (2 marks)

$0 \rightarrow X_1, \quad 1 \rightarrow X_2, \quad B \rightarrow X_3$.

3. Direction: — (2 marks)

$L \rightarrow D_1, \quad R \rightarrow D_2$

4. Transition function: — (8 marks)

$$\delta(q_1, 1) = (q_3, 0, R) \Rightarrow \delta(q_1, X_2) = (q_3, X_1, D_2)$$

$$C_1 \Rightarrow 0^1 1 0^2 1 0^3 1 0^1 1 0^2$$

$$\delta(q_3, 0) = (q_1, 1, R) \Rightarrow \delta(q_3, X_1) = (q_1, X_2, D_2)$$

$$C_2 \Rightarrow 0^3 1 0^1 1 0^1 1 0^2 1 0^2$$

$$\delta(q_3, 1) = (q_2, 0, R) \Rightarrow \delta(q_3, X_2) = (q_2, X_1, D_2)$$

$$C_3 \Rightarrow 0^3 1 0^2 1 0^2 1 0^1 1 0^2$$

$$\delta(q_3, B) = (q_3, 0, L) \Rightarrow \delta(q_3, X_3) = (q_3, X_1, D_1)$$

$$C_4 \Rightarrow 0^3 1 0^3 1 0^3 1 0^1 1 0^1$$

Finally the code for TM is, — (4 marks)

$$C_1 11 C_2 11 C_3 11 C_4$$

$$0^1 1 0^2 1 0^3 1 0^1 1 0^2 11 0^3 1 0^1 1 0^1 1 0^2 1 0^2 11 0^3 1 0^2 1 0^2 1 0^1 1 0^2 11 0^3 1 0^3 1 0^3 1 0^1 1 0$$

15.b. Modified Post Correspondence Problem (MPCP):

(5 marks)

→ Given two sequence of strings of some alphabet Σ.

$$A = w_1 w_2 \ldots w_n \qquad B = x_1, x_2 \ldots x_m.$$

→ For particular sequence of integers $i_1, i_2, \ldots i_m$ if we can concatenate strings from A and B and getting the same string means then we can say the sequence is the solution for the particular instance MPCP.

→ The sequence should start from first string in both the lists.

Example: (any one example)

i	List A w_i	List B x_i
1	1	10
2	110	0
3	0	11

List A: 10110

List B: 10110

Sequence is 1 3 2.

(11 marks)

Given a pair (M, w) construct an instance (A, B) of MPCP such that TM M accepts input w if and only if (A, B) has a solution

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$ be a TM and w is the input string. An instance of MPCP is constructed as:

1. The first pair is

List A	List B
#	$\# q_0 w \#$

2. Tape symbols & separator # can be appended to both lists.

List A	List B	
X	X	for each X in Γ
#	#	

3. For each q in $Q-F$, p in Q, and x, y, z in Γ

List A	List B	
qX	Yp	if $\delta(q, X) = (p, Y, R)$
ZqX	pZY	if $\delta(q, X) = (p, Y, L)$
$q\#$	$Yp\#$	if $\delta(q, B) = (p, Y, R)$
$Zq\#$	pZY	if $\delta(q, B) = (p, Y, L)$

4. For each q in F and X in Γ

List A	List B
XqY	q
Xq	q
qY	q.

5. For each q in F

List A	List B
q##	#

Any Example