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UNIT-3

CONTEXT FREE GRAMMAR AND PUSH DOWN AUTOMATA (PDA)

Formal Definition of CFG:

Context Free Grammar is a type-2 grammar which is represented by $G = (N, T, P, S)$

Where N is finite non-empty set of Non-terminals (which is uppercase alphabets A-Z)

N can also be represented as V.

T is a finite non-empty set of terminal symbols (lowercase alphabets a-z, numbers 0-9 and operators)

P is set of productions (rules)

S is start symbol.

* A grammar G is said to be CFG if every production in G is of the form $A \rightarrow \alpha$ Where
* A is a single non-terminal and lenght of α is greater than or equal to A i.e, $|\alpha| \geq |A|$

Context Free Language (CFL)

A language L is said to CFL if there exist a CFG ~~grammar~~ $L = L(G)$ i.e, set of all strings derived from the CFG forms the CFL.

Sentential form:

It is any derivative of a unique non-terminal say 'S' i.e, $S \xrightarrow{*} \alpha$ (It is derived in one or more steps) Where α is the sentential form of S

L(G)

Means that the language (set of all strings) generated by the CFG which can be written as

$L(G) = \{w \mid w \in T^* \text{ and } S \xrightarrow{*} w\}$

A string is in L(G) if (i) string consist only of terminal symbols

(iii) string must be derived from the starting state S using the productions of G.

Problems:

1) Let G be a CFG with $N = \{S\}$, $T = \{a, b\}$.

P, $S \rightarrow aSbb$. Find $L(G)$

$S \rightarrow abb$

$S = S$

$S \rightarrow aSbb$
abb

sol:

$L = \{abb, aabbbb, aaabbbbbb, \ldots\}$

$S \rightarrow aSbb$
$aSbb$
abb

$L = \{a^n b^{2n} \mid n > 0\}$

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2) Find $L(G)$ for $G = (\{S\}, \{a, b\}, P, S)$ with

P : $S \rightarrow aSa$
$S \rightarrow bSb$
$S \rightarrow \epsilon$

$S \rightarrow aSa$
$a\epsilon a$
aa

$S \rightarrow bSb$
$b\epsilon b$
bb

$S \rightarrow aSa$
$aaSaa$
$aa\epsilon aa$
$aaaa$

$S \rightarrow aSa \rightarrow abSba \rightarrow ab\epsilon ba \rightarrow abba$

sol:

$L = \{\epsilon, aa, bb, aaaa, abba, bbbb, baab, \ldots\}$

$L = \{WW^R \mid w \in \{a, b\}^*\}$

3) Find $L(G)$ for $G = (\{S\}, \{a, b\}, P, S)$ with

P : $S \rightarrow aSa$
$S \rightarrow bSb$
$S \rightarrow a \mid b \rightarrow$ $S \rightarrow a$, $S \rightarrow b$

~~$L = \{aa|ba, ba|bb, a|b,$~~

$L = \{a, b, aaa, aba, bab, bbb, aabaa, abbba, \ldots\}$

$L = \{wcw^R \mid w \in \{a, b\}^*$ & $c \in \{a, b\}\}$

4) Find $L(G)$ for $G = (\{S\}, \{a\}, P, S\}$ with P: $S \rightarrow SS$
$S \rightarrow aS$
$S \rightarrow a$

$L = \{a, aa, aaa, aaaa, aaaaa, \ldots\}$

$L = \{a^n \mid n > 0\}$

5) Find $L(G)$ for $S \to SaS$ SaS

$S \to b$ SaSab
babab

$L = \{b, bab, babab, \dots\}$

$L = \{b(ab)^n \mid n \geq 0\}$

---

6) Find $L(G)$ for $S \to A$

$A \to aS \mid B$

$B \to b$

$L = \{b, ab, aab, aaab, \dots\}$

$L = \{(a)^n b \mid n \geq 0\}$

---

7) Find $L(G)$ for $S \to aSbS \mid bSaS \mid \epsilon$

$L = \{\epsilon, ab, ba, aabb, abab, baba, \dots\}$

$L = \{$ set of all strings with equal no. of a's and b's over $\{a, b\}\}$

For the language given find the context Free Grammar (CFG)

i) Given $L = \{0^m \mid m > 0\}$

$L = \{0, 00, 000, 000, \dots\}$

$S \to 0$

$S \to 0S$

CFG:

$G = (\{S\}, \{0\}, P, S)$

$P.\ S \to 0$

$S \to 0S$

---

2) $L = \{a^n b^n \mid n > 0\}$

$L = \{ab, aabb, aaabbb, \dots\}$

$S \to ab$

$S \to aSb$

$G = (\{S\}, \{a, b\}, P, S)$

$P.\ S \to ab$

$S \to aSb$

---

3) $L = \{wcw^R \mid w \in \{a, b\}^*\}$

terminal

$L = \{c, aca, bcb, aacaa, abcba, \ldots\}$

$S \to c$

$S \to aSa$

$S \to bSb$

$G = (\{S\}, \{a, b, c\}, P, S)$

$P.\ S \to c$

$S \to aSa$

$S \to bSb.$

---

4) $L = \{b^n a b^n \mid n \geq 0\}$

$L = \{a, bab, bbabb, \ldots\}$

$S \to a$

$S \to bSb$

$G = (\{S\}, \{a, b\}, P, S)$

---

5) $L = \{0^m 1^n \mid m \neq n\}$

$L = \{0, 1, 00, 11, 001, 110, \ldots\}$

$S \to 0 \mid 1$

$S \to 0A \mid 1A$

$S \to 0 \mid 0S \mid 1S \mid 1$

$G = (\{S\}, \{0, 1\}, P, S)$

---

6) $L = \{a^i b^j \mid j > i\}$

$L = \{b, abb, bb, aabbb, bbbb, \ldots\}$

$S \to b$

$S \to Sb \mid aSb$

$G = (\{S\}, \{a, b\}, P, S)$

P $S \to b$

$S \to Sb \mid aSb$

7) set of all strings with twice as many 0's as 1's

8) set of all strings with even number of b's

9) set of all strings with alternate 0's & 1's.

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Derivation Trees

It is a rooted tree in which every node represents the non-terminal of the grammar i.e, LHS of the production in G and the leaf node represents the yield of the tree which will the terminal symbols of the grammar.

Note:

The yield of the tree represents the string derived from the grammar G.

eg: For the given productions, derive the string "abab" and draw the Parse tree/ Derivation tree for the string

$S \rightarrow aB \mid bA$

$A \rightarrow a \mid aS \mid bAA$

$B \rightarrow b \mid bS \mid aBB$

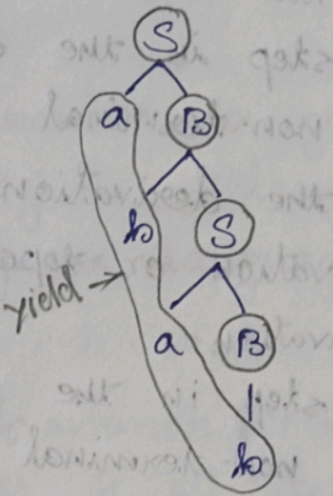
Sol:

$S \rightarrow aB$, $B \rightarrow bS$

$S \rightarrow abS$ $S \rightarrow aB$

$S \rightarrow abaB$ $B \rightarrow b$

$S \rightarrow abab$



2) abaabb

$S \rightarrow aB$

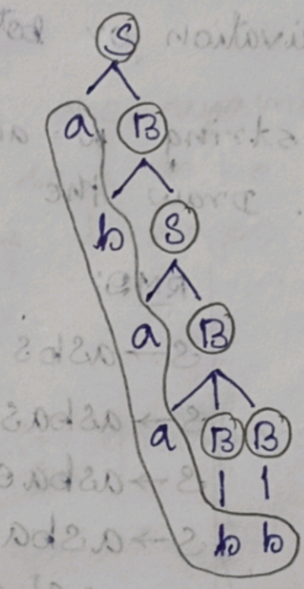
$S \rightarrow abS$

$S \rightarrow abaB$

$S \rightarrow abaaBB$

$S \rightarrow abaabB$

$S \rightarrow abaabb$



3) abbaba

$S \rightarrow aB$

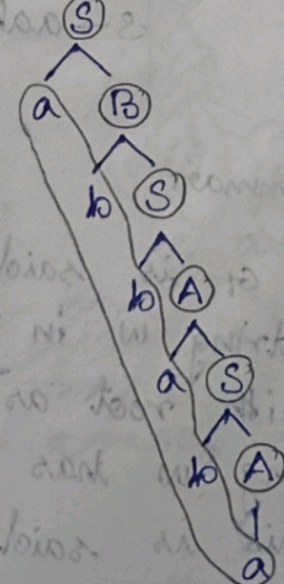
$S \rightarrow abS$

$S \rightarrow abbA$

$S \rightarrow abbaS$

$S \rightarrow abbabA$

$S \rightarrow abbaba$



Types of Derivation:

1) Left Most Derivation (LMD)

2) Right Most Derivation (RMD)

Left Most Derivation:

If at each step in the derivation of a string, the left most non-terminal of the production is replaced than the derivation is said to be left most derivation or topdown parsing.

Right Most Derivation:

If at each step in the derivation of a string, the right most non-terminal of production is replaced then the derivation is said to be Right most derivation or bottomup parsing

Eg: Derive a string $w = aaba$ from the production $S \rightarrow aSbS \mid aS \mid \epsilon$. Draw the LMD and RMD of the tree

LMD:

$S \rightarrow aSbS$

$S \rightarrow aaSbS$

$S \rightarrow aa\epsilon bS$

$S \rightarrow aabS$

$S \rightarrow aabaS$

$S \rightarrow aabaaS$

$S \rightarrow aabaa\epsilon$

$S \rightarrow aaba$

RMD:

$S \rightarrow aSbS$

$S \rightarrow aSbaS$

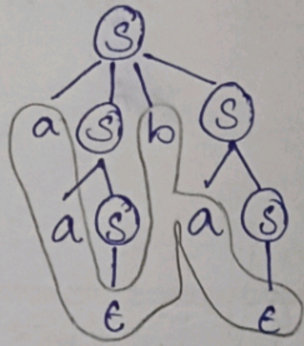
$S \rightarrow aSba\epsilon$

$S \rightarrow aSba$

$S \rightarrow aaSba$

$S \rightarrow aa\epsilon ba$

$S \rightarrow aaba$



Ambiguous Grammar:

A grammar G is said to be ambiguous if atleast one string w in T^* has two different parse trees with root as S and yield w.

If a string w has more than one LMD or RMD then this is said to be ambiguous.

Unambiguous Grammar:

If every string generated by the grammar has unique derivation tree or parse tree then it is said to be unambiguous.

Ambiguous Language:

If there are no unambiguous grammar generated for a string in a language then it is said to be ambiguous language.

Inherently Ambiguous:

The context Free Grammar (CFG) is said to be inherently ambiguous if all its grammar are ambiguous grammar.

UnAmbiguous ~~gr~~ Language:

If there is atleast one unambiguous grammar then it is said to be unambiguous for a string in a language language.

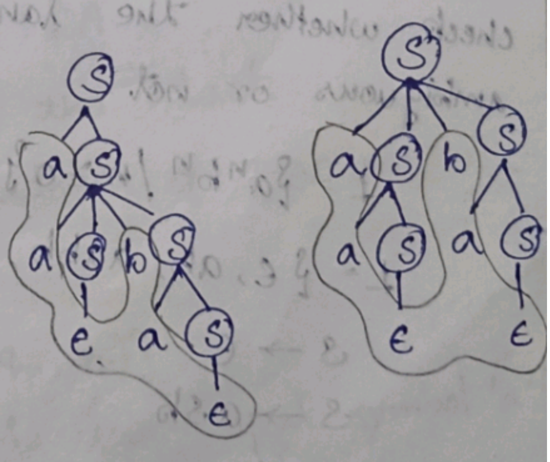
Problems:

1) Show that the following grammar is ~~un~~ambiguous

(i) $S \to aS / aSbS / \epsilon$

aaba

$S \to aS$	$S \to aSbS$
$S \to aaSbS$	$S \to aaSbS$
$S \to aa\epsilon bS$	$S \to aa\epsilon bS$
$S \to aabaS$	$S \to aabaS$
$S \to aaba\epsilon$	$S \to aaba\epsilon$
$S \to aaba$	$S \to aaba$



∴ A string has two parse tree, it is ambiguous

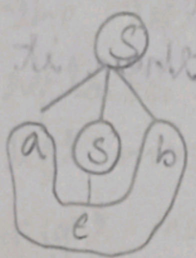
2) Check whether the given grammar is ambiguous.

$S \rightarrow aSb \mid SS \mid \epsilon$

$S \rightarrow aSb$

$S \rightarrow a\epsilon b$

$S \rightarrow ab$



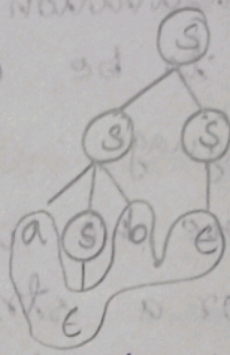
$S \rightarrow SS$

$S \rightarrow aSbS$

$S \rightarrow a\epsilon bS$

$S \rightarrow ab\epsilon$

$S \rightarrow ab$



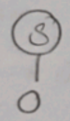
∴ It has two different parse tree for a string, it is ambiguous.

3) check whether the language $L = \{0^n / n > 0\}$ is ambiguous.

$L = \{0, 00, 000, \ldots\}$

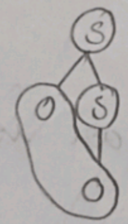
CFG - $S \rightarrow 0S \mid 0$

$S \rightarrow 0$



$S \rightarrow 0S$

$S \rightarrow 00$



∴ A string has unique parse tree, it is unambiguous.

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check whether the language $L = \{a^n b^n / n \geq 0\}$ is ambiguous or not.

$L = \{a^n b^n / n \geq 0\}$

$L = \{\epsilon, ab, aabb, \ldots\}$

$S \rightarrow \epsilon$

$S \rightarrow aSb$

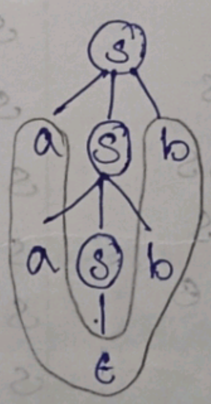
$w = aabb$

$S \rightarrow aSb$

$S \rightarrow aaSbb$

$S \rightarrow aa\epsilon bb$

$S \rightarrow aabb$



∴ A string has unique parse tree, it is unambiguous.

Pushdown Automata

A model designed to accept the string derived from context free grammar is called pushdown model.

Components of PDA:

* Input tape
* control unit
* Read write head
* Memory or stack unit

Types of PDA:

* NPDA - Non-Deterministic PushDown Automata
* DPDA - Deterministic Push Down Automata

Formal Definition PDA:

PDA is defined as $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Where Q is the ~~sta~~ set of states of PDA

Σ - set of input symbols.

Γ - set of alphabets called stack symbol or Push Down symbol

δ is a transition ~~symbol~~ function which is represented as $\delta \rightarrow Q \times (\Sigma \cup \epsilon \cup \Gamma)$, $\delta \rightarrow Q \times \Gamma^*$

q_0 - initial state of PDA

Z_0 - initial element of the stack

F - set of accepting states of PDA

Notations and Meaning:

1) $\delta(q, a, Z) = \{(q_1, r_1), (q_2, r_2), \ldots (q_n, r_n)\}$ means when the state 'q' reads the input symbol 'a' with the topmost stack symbol as 'Z' it moves to the states $q_1, q_2, \ldots q_n$ replacing Z by $r_1, r_2, \ldots r_n$

2) $\delta(q, \epsilon, Z) = \{(q_1, r_1), (q_2, r_2), \ldots (q_n, r_n)\}$

it means that 'q' has the transition to $q_1, q_2, \ldots q_n$ without reading any input symbol and replaces the 'Z' with $r_1, r_2, \ldots r_n$

Instantaneous Description (ID):

If ~~with~~ $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ then ID is refered as (q, x, Z) where $q \in Q$, $x \in \Sigma$, $Z \in \Gamma$

Initial Instantaneous Description (IID):

It is denoted as (q_0, x, Z_0) where

q_0 is the initial state of PDA

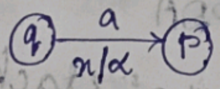
x is the string to be processed

Z_0 is the initial stack symbol

Diagramatic Representation of PDA:

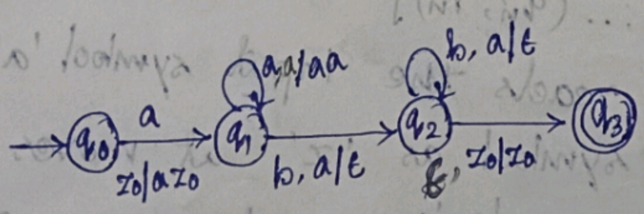
* Nodes of the PDA represents the states of PDA
* start state is indicated with arrow
* Double circle in the node represents final or accepting states of PDA.
* Arc from one node to another represents the transition of PDA.

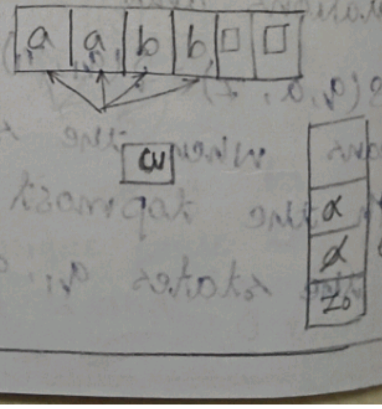
Eg: $\delta(q, a, x) \vdash (P, \alpha)$



Problem:

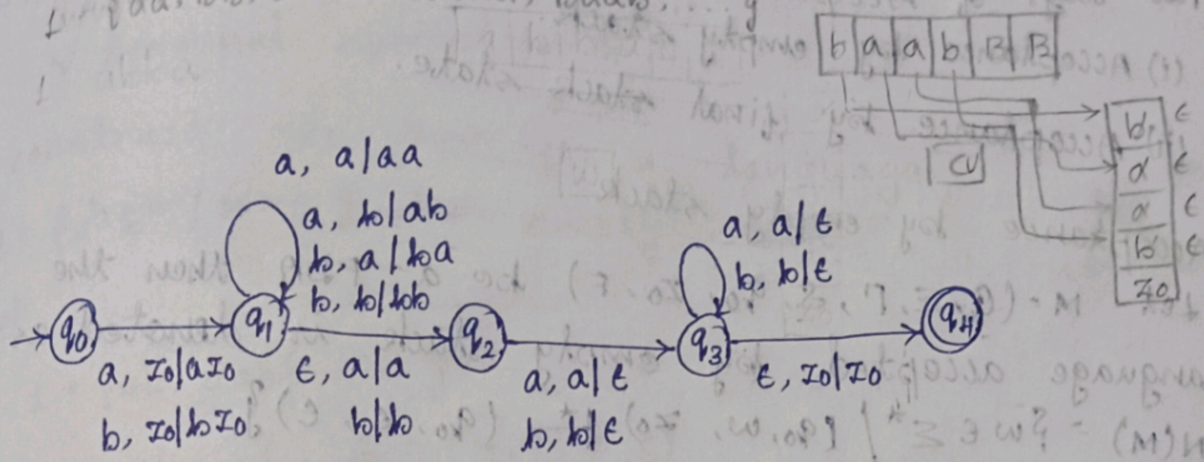
i) Design a PDA for a language $L = \{a^n b^n \mid n > 0\}$ (Diagramatic representation)





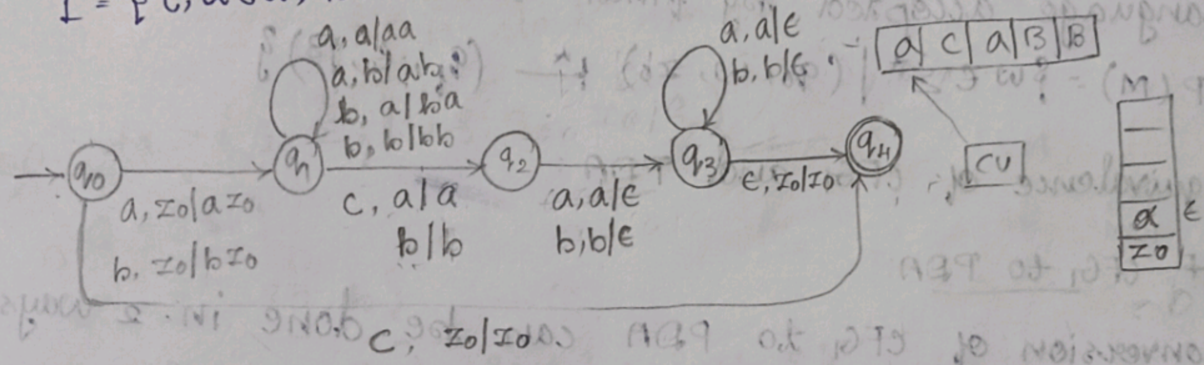
2) $L = \{ww^R \mid w \in \{a,b\}^+\}$

$L = \{aa, bb, abba, aaaa, baab, \ldots\}$



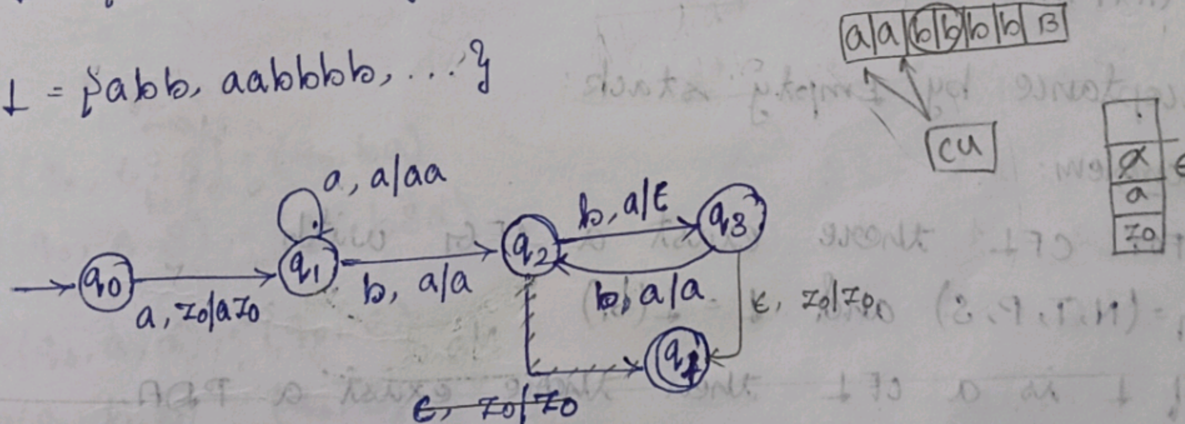
3) $L = \{wcw^R \mid w \in \{a,b\}^*\}$

$L = \{c, aca, bcb, abcba, aacaa, \ldots\}$



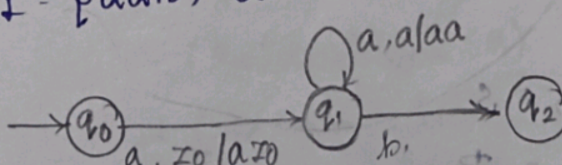
4) $L = \{a^n b^{2n} \mid n > 0\}$

$L = \{abb, aabbbb, \ldots\}$



5) $L = \{a^{2n} b^n \mid n > 0\}$

$L = \{aab, aaaabb, \ldots\}$



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Two ways of Acceptance by PDA:

(i) Acceptance by empty stack.

(ii) Acceptance by final ~~stack~~ state.

Acceptance by empty stack:

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ be a PDA then the language accepted by empty stack is denoted as

$N(M) = \{w \in \Sigma^* / (q_0, w, Z_0) \vdash^* (q_0, \epsilon, \epsilon)\}$

Acceptance by final state.

Let $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ be a PDA then the language accepted by final state is denoted as

$P(M) = \{w \in \Sigma^* | (q_0, w, Z_0) \vdash^* (q_f, \epsilon, \beta)\}$

Equivalence of CFG and PDA:

10m

~~GF~~ CFG to PDA:

conversion of CFG to PDA can be done in 2 ways

(i) Empty stack

(ii) Final state.

10m

Acceptance by Empty stack:

Problem:

For CFL there exist a CFG with

$G = (N, T, P, S)$ and $L = L(G)$

If L is a CFL then there exist a PDA.

$L(M) = L(P_N)$ or $L(P_F)$

construct the PDA from the grammar G with

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

Where $Q = q_0$

$\Sigma = T$

$\Gamma = N \cup T$

$q_0 = q_0$

$Z_0 = S$

$F = \phi$

and δ is defined as

(i) $\delta(q_0, \epsilon, A) = (q_0, \alpha^R)$

✓ all productions in G of form $A \to \alpha$

(iii) $\delta(q_0, a, a) = (q_0, \epsilon)$

✓ terminal symbol in G

i) construct the PDA for a language $L = \{a^n b^n \mid n > 0\}$ by empty stack.

sol:

$L = \{ab, aabb, \ldots\}$

P: $S \to ab$

$S \to aSb$

$G = (N, T, P, S)$

$N = \{S\}$

$T = \{a, b\}$

S is start symbol.

PDA, $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0\}$

$\Sigma = T = \{a, b\}$

$\Gamma = N \cup T = \{S\} \cup \{a, b\} = \{a, b, S\}$

$q_0 = q_0$

$Z_0 = S$

$F = P$

(i) $\delta(q_0, \epsilon, S) = (q_0, ba)$

$\delta(q_0, \epsilon, S) = (q_0, bSa)$

(ii) $\delta(q_0, a, a) = (q_0, \epsilon)$

$\delta(q_0, b, b) = (q_0, \epsilon)$

$\delta(q_0, w) = (q_0, w, Z_0)$

$= (q_0, aabb, S)$

$= (q_0, \cancel{a}abb, bS\cancel{a})$

$= (q_0, abb, bS)$

$= (q_0, \cancel{a}bb, bb\cancel{a})$

$= (q_0, \cancel{b}b, b\cancel{b})$

$= (q_0, b, b)$

$= (q_0, \epsilon, \epsilon)$

2) $L = \{wcw^R / w \in \{a,b\}^*\}$

sol: $L = \{c, aca, bca, abcba, \ldots\}$

$P: S \to c$

$S \to aSa / bSb$

$G = (N, T, P, S)$

$N = \{S\}$

$T = \{a, b, c\}$

S is start symbol

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0\}$

$\Sigma = T = \{a, b, c\}$

$\Gamma = N \cup T = \{S, a, b, c\}$

$q_0 = q_0$

$Z_0 = S$

$F = \phi$

(i) $\delta(q_0, \epsilon, S) = (q_0, c)$

$\delta(q_0, \epsilon, S) = (q_0, aSa)$

$\delta(q_0, \epsilon, S) = (q_0, bSb)$

(ii) $\delta(q_0, a, a) = (q_0, \epsilon)$

$\delta(q_0, b, b) = (q_0, \epsilon)$

$\delta(q_0, c, c) = (q_0, \epsilon)$

if $w = bacab$

$\delta(q_0, w) = (q_0, w, Z_0)$

$= (q_0, bacab, S)$

$= (q_0, \cancel{b}acab, bSb) = (q_0, acab, bS)$

$= (q_0, \cancel{a}cab, baS\cancel{a}) = (q_0, cab, baS)$

$= (q_0, \cancel{c}ab, ba\cancel{c})$

$= (q_0, ab, ba)$

$= (q_0, b, b) = (q_0, \epsilon, \epsilon)$

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Acceptance by Final State:

It is similar to acceptance by empty stack with change in δ and F for $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ we have $Q = \{q_0', q_0, q_f\}$

$\Sigma = T$

$\Gamma = N \cup T$

$q_0 = q_0'$

$Z_0 = S$

$F = q_f$

δ is defined as (i) $\delta'(q_0', \epsilon, Z_0') = (q_0, Z_0', Z_0)$

(ii) $\delta(q_0, \epsilon, A) = (q_0, \alpha^R)$

(iii) $\delta(q_0, a, a) = (q_0, \epsilon)$

(iv) $\delta(q_0, \epsilon, Z_0') = (q_f, \epsilon)$

1) Design a PDA for the language $L = \{a^n b^n \mid n > 0\}$ by final state.

$L = \{ab, aabb, aaabbb, \ldots\}$

P: $S \to ab$

$S \to aSb$

$G = (N, T, P, S)$

$N = \{S\}$

$T = \{a, b\}$

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0', q_0, q_f\}$

$\Sigma = \{a, b\}$

$\Gamma = N \cup T = \{S, a, b\}$

$q_0 = q_0'$

$Z_0 = S$

$F = q_f$

(i) $\delta(q_0', \epsilon, Z_0') = (q_0; Z_0'; Z_0)$

(ii) $\delta(q_0, \epsilon, S) = (q_0, ba)$

$\delta(q_0, \epsilon, S) = (q_0, bSa)$

(iii) $\delta(q_0, \overset{i/p}{a}, \overset{\text{stack symbol}}{a}) = (q_0, \epsilon)$

$\delta(q_0, b, b) = (q_0, \epsilon)$

(iv) $\delta(q_0, \epsilon, Z_0) = (q_f, \epsilon)$

if $w = ab$

$\delta(q_0', w, Z_0') \vdash (q_0, Z_0', Z_0)$

$= (q_0, w, SZ_0)$

$= (q_0, ab, SZ_0)$

$= (q_0, ab, ba\,Z_0)$

$= (q_0, b, b\,Z_0)$

$= (q_0, \epsilon, \epsilon\,Z_0)$

$= (q_f, \epsilon)$

---

2) $L = \{wcw^R / w \in \{a,b\}^*\}$

$L = \{c, aca, bcb, abcba, \ldots\}$

$P: S \rightarrow c$

$S \rightarrow aSa \mid bSb$

$G = (N, T, P, S)$

$N = \{S\}$

$T = \{a, b, c\}$

$M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

$Q = \{q_0', q_0, q_f\}$

$\Sigma = T = \{a, b, c\}$

$\Gamma = N \cup T = \{S, a, b, c\}$

$q_0 = q_0'$

$Z_0 = S$

$F = q_f$

(i) $\delta(q_0', \epsilon, Z_0') = (q_0, Z_0', Z_0)$

(ii) $\delta(q_0, \epsilon, S) = (q_0, c)$

$\delta(q_0, \epsilon, S) = (q_0, aSa)$

$\delta(q_0, \epsilon, S) = (q_0, bSb)$

(iii) $\delta(q_0, a, a) = (q_0, \epsilon)$

$\delta(q_0, b, b) = (q_0, \epsilon)$

$\delta(q_0, c, c) = (q_0, \epsilon)$

(iv) $\delta(q_0, \epsilon, Z_0) = (q_f, \epsilon)$

if $w = bacab$

$\delta(q_0', w, Z_0') = (q_0, Z_0', Z_0)$

$= (q_0, w, S\,Z_0)$

$= (q_0, bacab, S\,Z_0)$

$= (q_0, \cancel{b}acab, bSb\,Z_0)$

$= (q_0, \cancel{a}cab, baSa\,Z_0)$

$= (q_0, \cancel{c}ab, bac\,Z_0)$

$= (q_0, \cancel{a}b, ba\,Z_0)$

$= (q_0, b, b\,Z_0)$

$= (q_0, \epsilon, \epsilon\,Z_0)$

$= (q_f, \epsilon)$

3) $L = \{a^{2n} b^n \mid n > 0\}$

4) $L = \{ww^R \mid w \in \{a,b\}^+\}$

5) $L = \{a^n b^{2n} \mid n > 0\}$

6) $L = \{a^n b^m c^n \mid m > n\}$

7) $L = \{a^n b c^n \mid n > 0\}$

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conversion of PDA to CFG:

If $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ is a PDA then there exist a CFG G such that $L(G) = L(M)$

$G = (N, T, P, S)$ is defined as

$N = \{([q, Z, q'] \mid q, q' \in Q, Z \in \Gamma) \cup S\}$

$T = \Sigma$

$S \rightarrow [q_0, Z_0, q], q \in Q$

P is defined by two rules

(i) Every move erasing the pushdown stack which is of the form $\boxed{\begin{array}{l}\delta(q, a, Z) = (q', \epsilon) \\ [q, Z, q'] \rightarrow a\end{array}}$ is converted as

(ii) The transitions that are not erasing the pushdown stack which is of the form $\delta(q, a, Z) = (q', Z_m, Z_{m-1}, \ldots Z_2 Z_1)$ is converted as $\delta(q, a, Z) = (q', Z_m Z_{m-1} \ldots Z_2 Z_1)$

$[q, Z, t] \rightarrow a[q', Z_m, r][r, Z_{m-1}, s] \ldots [\ldots Z_1, t]$

where r, s, t - states of PDA

a - input symbol

Z, Z_m - stack symbols.

Note:

If the no. of states of PDA is 'K' with the no. of stack symbols (Γ) as 'm' then 'N' of CFG will hav $K^2 \cdot m + 1$ Non-terminals.

16M

i) Find the CFG for the language whose PDA is given as $M = (\{q_0, q_1\}, \{0,1\}, \{I_0, x\}, \delta, q_0, I_0, q)$ with δ as

(i) $\delta(q_0, 0, I_0) = \{(q_0, xI_0)\}$

(ii) $\delta(q_1, \epsilon, x) = \{(q_1, \epsilon)\}$

(iii) $\delta(q_0, 1, x) = \{(q_1, \epsilon)\}$

(iv) $\delta(q_0, 0, x) = \{(q_0, xx)\}$

(v) $\delta(q_1, \epsilon, I_0) = \{(q_1, \epsilon)\}$

(vi) $\delta(q_1, 1, x) = \{(q_1, \epsilon)\}$

$N = ([q, I, q'] \cup S)$

$\{q_0, q_1\}$ $\{I_0, x\}$ $\{q_0, q_1\}$

$\cup S$

union

k^{m+1}

$2^2 \cdot 2 + 1 = 8 + 1$

$= 9$

CFG

$S \to B \mid 2$

$C \to \epsilon$

$C \to 1$

$D \to$

$B \to ODE$

$D \to ODC$

$E \to \epsilon$

$[q_0, I_0, q_0]$	$[q_1, I_0, q_0]$
$[q_0, I_0, q_1]$	$[q_1, I_0, q_1]$
$[q_0, x, q_0]$	$[q_1, x, q_0]$
$[q_0, x, q_1]$	$[q_1, x, q_1]$

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(i) $T = \Sigma = \{0, 1\}$

$S = [q_0, I_0, q], \; q \in Q$

$S \to [q_0, I_0, q_0] \to A$

$S \to [q_0, I_0, q_1] \to B$

P $\delta(q, a, I) = (q', \epsilon)$

$[q, I, q'] \to a$

(ii) $\delta(q_1, \epsilon, x) = (q_1, \epsilon)$

C

$[q_1, x, q_1] \to \epsilon$

~~$S \to A$~~

$S \to B$

$C \to \epsilon$

$D \to 1$

$E \to \epsilon$

$C \to 1$

~~$B \to FB$~~

~~$A \to DG$~~

~~$D \to FD$~~

~~$F \to DH$~~

~~$A \to 0FA$~~

~~$B \to 0FB$~~

~~$A \to 0DG$~~

$B \to 0DE$

~~$F \to 0FF$~~

~~$D \to 0FD$~~

~~$F \to 0DH$~~

$D \to 0DC$

~~$A \to FA$~~

~~$B \to FB$~~

$A \to 0DG$

$B \to DC$

$F \to {}^{2}FF$

$D \to FD$

$F \to {}^{q}DG$

$D \to DC$

(iii) $\delta(q_0, 1, x) = (q_1, \epsilon)$

$\overset{D}{[q_0, x, q_1]} \rightarrow 1$

(v) $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$\overset{E}{[q_1, Z_0, q_1]} \rightarrow \epsilon$

(vi) $\delta(q_1, 1, x) = (q_1, \epsilon)$

$\overset{C}{[q_1, x, q_1]} \rightarrow 1$

(i) $\delta(q_0, 0, Z_0) = (q_0, x Z_0)$

q_0, q_1

$\overset{B}{[q_0, Z_0, q_1]} \rightarrow 0 \overset{F}{[q_0, x, q_0]} \overset{B}{[q_0, Z_0, q_1]}$

$\overset{A}{[q_0, Z_0, q_0]} \rightarrow 0 \overset{D}{[q_0, x, q_1]} \overset{G}{[q_1, Z_0, q_0]}$

(iv) $\delta(q_0, 0, x) = (q_0, xx)$

$\overset{D}{[q_0, x, q_1]} \rightarrow 0 \overset{F}{[q_0, x, q_0]} \overset{D}{[q_0, x, q_1]}$

$\overset{F}{[q_0, x, q_0]} \rightarrow 0 \overset{D}{[q_0, x, q_1]} \overset{H}{[q_1, x, q_0]}$

$\overset{A}{[q_0, Z_0, q_0]} \rightarrow 0 \overset{F}{[q_0, x, q_0]} \overset{A}{[q_0, Z_0, q_0]}$

$\overset{B}{[q_0, Z_0, q_1]} \rightarrow 0 \overset{F}{[q_0, x, q_0]} \overset{B}{[q_0, Z_0, q_1]}$

$\overset{A}{[q_0, Z_0, q_0]} \rightarrow 0 \overset{D}{[q_0, x, q_1]} \overset{G}{[q_1, Z_0, q_0]}$

$\overset{B}{[q_0, Z_0, q_1]} \rightarrow 0 \overset{D}{[q_0, x, q_1]} \overset{E}{[q_1, Z_0, q_1]}$

$\overset{F}{[q_0, x, q_0]} \rightarrow 0 \overset{F}{[q_0, x, q_0]} \overset{F}{[q_0, x, q_0]}$

$\overset{D}{[q_0, x, q_1]} \rightarrow 0 \overset{F}{[q_0, x, q_0]} \overset{D}{[q_0, x, q_1]}$

$\overset{F}{[q_0, x, q_0]} \rightarrow 0 \overset{D}{[q_0, x, q_1]} \overset{H}{[q_1, x, q_0]}$

$\overset{D}{[q_0, x, q_1]} \rightarrow 0 \overset{D}{[q_0, x, q_1]} \overset{C}{[q_1, x, q_1]}$

2) construct a CFG for

$M = \{(q_0, q_1), \{a, b\}, \delta, q_0, Z_0, \emptyset\}$ and δ is defined as

(i) $\delta(q_0, a, Z_0) = \{(q_0, aZ_0)\}$

(ii) $\delta(q_0, a, a) = \{(q_0, aa)\}$

(iii) $\delta(q_0, b, a) = \{(q_0, a)\}$

(iv) $\delta(q_0, a, a) = \{(q_1, \epsilon)\}$

(v) $\delta(q_1, \epsilon, Z_0) = \{(q_1, \epsilon)\}$

sometimes Γ is not given so take as $\Gamma = \{a, Z_0\}$

sol:

$N = ([q, Z, q'] \cup S)$

$k^2 \cdot m^{-1}$

$2^2 \cdot 2^{-1} = 2^{+1}$
$= 9$

$\cup S$

$[q_0, a, q_0]$ $[q_1, a, q_0]$

$[q_0, a, q_1]$ $[q_1, a, q_1]$

$[q_0, Z_0, q_0]$ $[q_1, Z_0, q_0]$

$[q_0, Z_0, q_1]$ $[q_1, Z_0, q_1]$

$T = \Sigma = \{a, b\}$

$S = [q_0, Z_0, q], q \in Q$

$S \rightarrow [q_0, \overset{A}{Z_0}, q_0]$

$S \rightarrow [q_0, \overset{B}{Z_0}, q_1]$

P $\delta(q, a, Z) = (q', \epsilon)$

$[q, Z, q'] \rightarrow a$

(iv) $\delta(q_0, a, a) = (q_1, \epsilon)$

$[q_0, \overset{C}{a}, q_1] \rightarrow a$

(v) $\delta(q_1, \epsilon, Z_0) = (q_1, \epsilon)$

$[q_1, \overset{D}{Z_0}, q_1] \rightarrow \epsilon$

(i) $\delta(q_0, a, Z_0) = (q_0, a Z_0)$

$$\overset{A}{[q_0, Z_0, q_0]} \rightarrow a\,\overset{E}{[q_0, a, q_0]}\,\overset{A}{[q_0, Z_0, q_0]}$$

$$\overset{B}{[q_0, Z_0, q_1]} \rightarrow a\,\overset{E}{[q_0, a, q_0]}\,\overset{B}{[q_0, Z_0, q_1]}$$

$$\overset{A}{[q_0, Z_0, q_0]} \rightarrow a\,\overset{C}{[q_0, a, q_1]}\,\overset{F}{[q_1, Z_0, q_0]}$$

$$\overset{B}{[q_0, Z_0, q_1]} \rightarrow a\,\overset{C}{[q_0, a, q_1]}\,\overset{D}{[q_1, Z_0, q_1]}$$

(ii) $\delta(q_0, a, a) \Rightarrow (q_0, aa)$

$$\overset{E}{[q_0, a, q_0]} \rightarrow a\,\overset{E}{[q_0, a, q_0]}\,\overset{E}{[q_0, a, q_0]}$$

$$\overset{C}{[q_0, a, q_1]} \rightarrow a\,\overset{E}{[q_0, a, q_0]}\,\overset{C}{[q_0, a, q_1]}$$

$$\overset{E}{[q_0, a, q_0]} \rightarrow a\,\overset{C}{[q_0, a, q_1]}\,\overset{G}{[q_1, a, q_0]}$$

$$\overset{C}{[q_0, a, q_1]} \rightarrow a\,\overset{C}{[q_0, a, q_1]}\,\overset{H}{[q_1, a, q_1]}$$

(iii) $\delta(q_0, b, a) = (q_0, a)$

$$\overset{E}{[q_0, a, q_0]} \rightarrow b\,\overset{E}{[q_0, a, q_0]}$$

$$\overset{C}{[q_0, a, q_1]} \rightarrow b\,\overset{C}{[q_0, a, q_1]}$$

$S \rightarrow A$

$S \rightarrow B$

$C \rightarrow a$

$D \rightarrow \epsilon$

$A \rightarrow aEA$

$B \rightarrow aEB$

$A \rightarrow aCF$

$B \rightarrow aCD$

$E \rightarrow aEE$

$C \rightarrow aEC$

$E \rightarrow aCG$

$C \rightarrow aCH$

$E \rightarrow bE$

$C \rightarrow bC$

final Grammar

$S \rightarrow B$

$C \rightarrow a$

$D \rightarrow \epsilon$

$B \rightarrow aCD$

$C \rightarrow bC$

Pumping lemma for context Free Language:

Let L be any context Free Language then there is constant 'n' depending only on L such that 'z' is in L with $|z| \geq n$

split z into uvwxy such that

(i) $|vx| \geq 1$

(ii) $|vwx| \leq n$

(iii) $\forall i \geq 0$, show that $uv^i w x^i y$ is in L

1) show that the language $L = \{a^n b^n c^n / n \geq 0\}$ is not CFL.

step-1: Let n be a pumping lemma constant where $z = a^n b^n c^n$ with $|z| = 3n > n$

step-2: split z into uvwxy

$$z = a^n b^n c^n$$

$$= \underset{u}{a^{n-i}} \cdot \underset{v}{a^i} \; \underset{w}{b^{n-i}} \; \underset{x}{b^i} \; \underset{y}{c^n}$$

Pump vx for 2 times

$$uv^2wx^2y = uvvwxxy$$

$$= a^{n-i} a^i a^i b^{n-i} b^i b^i c^n$$

$$= a^n a^i b^n b^i c^n$$

which is not equal to $a^n b^n c^n$

$\therefore L = \{a^n b^n c^n | n \geq 0\}$ is not CFL.