Conversion of PDA into CFG:

If $M = \{Q, \Sigma, \Gamma, \delta, q_0, z_0, F\}$ is PDA,

$Q \rightarrow$ Set of states in PDA

$\Sigma \rightarrow$ Input symbols

$\Gamma \rightarrow$ Set of alphabet called stack

$\delta \rightarrow$ Transition Function

$q_0 \rightarrow$ Initial State

$z_0 \rightarrow$ Initial (stack) state of stack

$F \rightarrow$ Set of accepting state

then exist a CFG G such that $L(G) = L(M)$

$G = (\overset{\text{non terminal}}{N}, T, P, S)$ is defined as

$N = \{ ([\underset{\text{initial state}}{q}, \overset{\text{input symbol}}{z}, q'] \mid q, q' \in Q, z \in \Gamma) \cup S \}$ → Top of stack

$T = \Sigma$

$S \rightarrow [q_0, z_0, q]\ q \in Q$

Production defined by two rules:

1. Every move erasing pushdown stack which is of the form $\delta(q, a, z) = (q', \varepsilon)$ is converted as

$$[q, z, q'] \rightarrow a$$

2. The transition that are not erasing push down stack which form $\delta(q, a, z) = (q', z_m, z_{m-1}, \ldots z_2, z_1)$ is covered as

$$[q, z, t] \rightarrow a\,[q', z_m, r][r, z_{m-1}, s] \ldots [\ldots z_1, t]$$

$r, s, t \rightarrow$ States of PDA

a - Input Symbol

z, z_m - Stack symbol

$\delta(q, a, z) = (q', z_m, z_{m-1} \ldots z_2, z_1)$

$[q, z, t] \rightarrow a\,[q', z_m, r][r, z_{m-1}, s] \ldots [\ldots z, t]$

1. Find CFG for the language whose PDA is given as $M = (\overset{Q}{\{q_0, q_1\}}, \overset{\Sigma}{\{0,1\}}, \overset{\Gamma \to \text{stack element}}{\{Z_0, X\}}, \delta, q_0, Z_0, q\}$

with δ as
1. $\delta(q_0, 0, Z_0) = \{(q_0, XZ_0)\}$
2. $\delta(q_1, \varepsilon, X) = \{(q_1, \varepsilon)\}$
3. $\delta(q_0, 1, X) = \{(q_1, \varepsilon)\}$
4. $\delta(q_0, 0, X) = \{(q_0, XX)\}$
5. $\delta(q_1, \varepsilon, Z_0) = \{(q_1, \varepsilon)\}$
6. $\delta(q_1, 1, X) = \{(q_1, \varepsilon)\}$

$G = (N, T, P, S)$

$N = \{ [q, Z, q'] \mid q, q' \in Q, Z \in \Gamma) \cup S\}$

$= \{q_0, q_1\} \{Z_0, X\} \{q_0 q_1\}$

$= \{ [q_0, Z_0, q_0] [q_1, Z_0, q_0]$
$[q_0, Z_0, q_1] [q_1, Z_0, q_1]$
$[q_0, X, q_0] [q_1, X, q_0]$
$[q_0, X, q_1] [q_1, X, q_1], S\}$

$T = \{0, 1\}$

$S = [q_0, Z_0, q]$, $q \in Q$ $\to \{q_0, q_1\}$

$S \to [q_0, Z_0, q_0]$ — A

$S \to [q_0, Z_0, q_1]$ — B

WKT, $\delta(q_0, a, Z) = (q', \varepsilon)$

$[q_0, Z, q'] \to a$

ii) $\delta(q_1, \varepsilon, X) = (q_1, \varepsilon)$

$\underbrace{[q_1, X, q_1]}_{C} \to \varepsilon$ -

iii) $\delta(q_0, 1, X) = (q_1, \varepsilon)$

~~[q0, X, X~~ $\underbrace{[q_0, X, q_1]}_{D} \to 1$

v) $\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)$

$$[\underbrace{q_1, z_0, q_1}_{\varepsilon}] \rightarrow \varepsilon$$

vi) $\delta(q_1, 1, x) = (q_1, \varepsilon)$

$$[\underbrace{q_1, x, q_1}_{C}] \rightarrow 1$$

i) $\delta(q_0, 0, z_0) = (q_0, \underline{x z_0})$

↳ Two => Four

$$[\underbrace{q_0, z_0, q_0}_{A}] \rightarrow 0\,[\underbrace{q_0, x, q_0}_{F}]\,[\underbrace{q_0, z_0, q_0}_{A}]$$

$$[\underbrace{q_0, z_0, q_1}_{B}] \rightarrow 0\,[\underbrace{q_0, x, q_0}_{F}]\,[\underbrace{q_0, z_0, q_1}_{B}]$$

$$[\underbrace{q_0, z_0, q_0}_{A}] \rightarrow 0\,[\underbrace{q_0, x, q_1}_{D}]\,[\underbrace{q_1, z_0, q_0}_{G}]$$

$$[\underbrace{q_0, z_0, q_1}_{B}] \rightarrow 0\,[\underbrace{q_0, x, q_1}_{D}]\,[\underbrace{q_1, z_0, q_1}_{E}]$$

iv) $\delta(q_0, 0, x) = (q_0, x, x)$

1 5 4 2 6 7 3

$$[\underbrace{q_0, x, q_0}_{F}] = 0\,[\underbrace{q_0, x, q_0}_{F}]\,[\underbrace{q_0, x, q_0}_{F}]$$

$$[\underbrace{q_0, x, q_1}_{D}] = 0\,[\underbrace{q_0, x, q_0}_{F}]\,[\underbrace{q_0, x, q_1}_{D}]$$

$$[\underbrace{q_0, x, q_0}_{F}] = 0\,[q_0, x, q_1]\,[q_1, x, q_0]$$

$$[\underbrace{q_0, x, q_1}_{D}] = 0\,[q_0, x, q_1]\,[q_1, x, q_1]$$

8

$S \rightarrow A$ $\quad$ $A \rightarrow 0FA$ ×

$S \rightarrow B$ $\quad$ $B \rightarrow 0FB$ ×

$C \rightarrow \varepsilon$ $\quad$ $A \rightarrow 0DG$ ×

$D \rightarrow 1$ $\quad$ $B \rightarrow 0DE$ ✓

$E \rightarrow \varepsilon$ $\quad$ $F \rightarrow 0FF$ ×

$C \rightarrow 1$ $\quad$ $D \rightarrow 0FD$ ×

$\quad$ $F \rightarrow 0DH$ ×

$\quad$ $D \rightarrow 0DC$ ✓

Final Grammar:

$S \rightarrow B$

$C \rightarrow \varepsilon \mid 1$

$D \rightarrow 1$

$E \rightarrow \varepsilon$

$B \rightarrow 0DE$

$D \rightarrow 0DC$

2. Construct a CFG from PDA $m = \{(q_0, q_1), \{a, b\}, \delta, q_0, z_0, q\}$ and δ defined as

i) $\delta(q_0, a, z_0) = \{(q_0, az_0)\}$

ii) $\delta(q_0, a, a) = \{(q_0, aa)\}$

iii) $\delta(q_0, b, a) = \{(q_0, a)\}$

iv) $\delta(q_0, a, z_0) = \{(q_1, \varepsilon)\}$

v) $\delta(q_1, \varepsilon, z_0) = \{(q_1, \varepsilon)\}$

[Sometime Γ is not given, so take it as $\Gamma = \{a, z_0\}$]

$G = (N, T, P, S)$

$N = \{[q, z, q'] \mid q, q' \in Q, z \in \Gamma) \cup S\}$

$\quad\; \downarrow\; \downarrow\; \downarrow$
$\quad\; Q\;\; \Gamma\;\; Q$

$= \{q_0, q_1\}, \{\underline{a}, z_0\}, \{q_0, q_1\}$

$(\{[q_0, a, q_0] \quad [q_1, a, q_0]$

$[q_0, a, q_1] \quad [q_1, a, q_1]$

$[q_0, z_0, q_0] \quad [q_1, z_0, q_0]$

$[q_0, z_0, q_1] \quad [q_1, z_0, q_1]\} \cup S)$

$T = \Sigma$

$T = \{a, b\}$

$S \to [q_0, z_0, \overset{\to \{q_0, q_1\}}{q}],\ q \in Q$

$S \to [q_0, z_0, q_0] \longrightarrow A$

$S \to [q_0, z_0, q_1] \longrightarrow B$

WKT,

$\delta(q_0, a, z) = (q', \varepsilon)$

$[q_0, z, q'] \to a$

iv) $\delta(q_0, a, a) = (q_1, \varepsilon)$

$\underbrace{[q_0, a, q_1]}_{C} \longrightarrow a$

v) $\delta(q_1, \varepsilon, z_0) = (q_1, \varepsilon)$

$\underbrace{[q_1, z_0, q_1]}_{D} \to \varepsilon$

i) $\delta(q_0, a, z_0) = (q_0, a z_0)$

$\underbrace{[q_0, z_0, q_0]}_{A} \to a \underbrace{[q_0, a, q_0]}_{E} \underbrace{[q_0, z_0, q_0]}_{A}$

$\underbrace{[q_0, z_0, q_1]}_{B} \to a \underbrace{[q_0, a, q_0]}_{E} \underbrace{[q_0, z_0, q_1]}_{B}$

$\underbrace{[q_0, z_0, q_0]}_{A} \to a \underbrace{[q_0, a, q_1]}_{C} \underbrace{[q_1, z_0, q_0]}_{F}$

$\underbrace{[q_0, z_0, q_1]}_{B} \longrightarrow a \underbrace{[q_0, a, q_1]}_{C} \underbrace{[q_1, z_0, q_1]}_{D}$

ii) $\delta(q_0, a, a) = \{q_0, aa\}$

$$\underbrace{[q_0, a, q_0]}_{E} \rightarrow a\,\underbrace{[q_0, a, q_0]}_{E}\,\underbrace{[q_0, a, q_0]}_{E}$$

$$\underbrace{[q_0, a, q_1]}_{C} \rightarrow a\,\underbrace{[q_0, a, q_0]}_{E}\,\underbrace{[q_0, a, q_1]}_{C}$$

$$\underbrace{[q_0, a, q_0]}_{E} \rightarrow a\,\underbrace{[q_0, a, q_1]}_{C}\,\underbrace{[q_1, a, q_0]}_{G}$$

$$\underbrace{[q_0, a, q_1]}_{C} \rightarrow a\,\underbrace{[q_0, a, q_1]}_{C}\,\underbrace{[q_1, a, q_1]}_{H}$$

iii) $\delta(q_0, b, a) = \{(q_0, a)\} \rightarrow$ 1 element $\Downarrow$ 2

$$\underbrace{[q_0, a, q_0]}_{E} \rightarrow b\,\underbrace{[q_0, a, q_0]}_{E}$$

$$\underbrace{[q_0, a, q_1]}_{C} \rightarrow b\,\underbrace{[q_0, a, q_1]}_{C}$$

$S \rightarrow A$ ×
$S \rightarrow B$ ✓
$C \rightarrow a$ ✓
$D \rightarrow \varepsilon$ ✓
$A \rightarrow aEA$ ×
$B \rightarrow aEB$ ×
$A \rightarrow aCF$ ×
$B \rightarrow aCD$ ✓

$E \rightarrow aEEb$ ×
$C \rightarrow aEC$ ×
$E \rightarrow aCG$ ×
$C \rightarrow aCH$ ×
$E \rightarrow bE$ ×
$C \rightarrow bC$ ✓

Final Grammar:

$S \rightarrow B$
$C \rightarrow a$
$D \rightarrow \varepsilon$
$B \rightarrow aCD$
$C \rightarrow bC$

UNIT - 4

Context Free Grammar and Turing Machine

Turing Machine - Advanced PDA

Simplification of CFG:

1. Eliminate Useless Production
2. Elimination of null production (ϵ - Production)
3. Elimination of Unit Production

Useless Production:

* Non terminal and production do not part of derivation of string.
* Findout useless production
    * Generating
    * Reachable

Generating:

A symbol (Non - Terminal) 'A' is said to be generating if there is a production of the form $S \to \alpha AB$, where A is a non - terminal which is able to derive a string.

Reachable:

The non-terminal symbol that are reachable from starting production either directly or indirectly, said to be a reachable symbols.

1. Consider the grammar $E \to CB | e$, $A \to a$ Remove the Useless Production.

$T = \{a, c, e\}$

$N.T = \{E, A, B\}$

Generating

$E \to CB$ ×

$E \to e$

$A \to a$

Generating : $\{A, E\}$

Reachable : $\{E, B\}$

Useful Production = { Generating } and { Reachable }

$= \{A, E\}$ and $\{E, B\}$

$= \{E\}$

Useful Production:

$E \rightarrow e$

Useless production:

$E \rightarrow CB$

$A \rightarrow a$

2. $E \rightarrow AB \mid a$

$A \rightarrow ab \mid C$ Findout useless production.

$C \rightarrow aD \mid B$

$B \rightarrow aB \mid b$

$D \rightarrow dD$

$T = \{a, b, d\}$

$N.T = \{A, B, C, D, E\}$

Generating:

$E \rightarrow AB$ ✗

$E \rightarrow a$ ✓

$A \rightarrow ab$ ✓

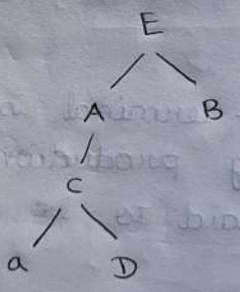
$A \rightarrow C$ ✗

$C \rightarrow aD$ ✗

$C \rightarrow B$ ✗

$B \rightarrow aB$ ✗

$D \rightarrow dD$ ✗



Generating = $\{E, A\}$

Reachable = $\{E, A, B, C, D\}$

Useful Production = $\{E, A\}$

$E \rightarrow a$ $\rightarrow$ Continue to reduce

$A \rightarrow ab$

$T = \{a, b\}$

$N.T = \{E, A\}$

Generating = $\{E, A\}$

Reachable = $\{E\}$

Useful = $\{E\}$

Useful Production

$E \rightarrow a$

Useless production

$A \rightarrow ab$

3. $S \rightarrow aS \mid B$

$A \rightarrow aa$

$B \rightarrow bb$

Findout useless production.

$T = \{a, b\}$

$N.T = \{S, A, B\}$

Generating:

$S \rightarrow aS$

$S \rightarrow B$

$A \rightarrow aa$ ×

$B \rightarrow bb$

Generating = $\{S, A, B\}$

Reachable = $\{S, B\}$

Useful Production = $\{S, B\}$

$S \rightarrow aS$

$S \rightarrow B$

$B \rightarrow bb$

Generating : $\{S, B\}$

Reachable : $\{S, B\}$

Useful Production : $\{S, B\}$

$S \to aS \mid B$

$B \to bb$

4. $S \to aAa \mid aBC$

$A \to aS \mid bD$

$B \to aBa \mid b$ Findout Useless

$C \to abb \mid DD$

$D \to aDa$

$T = \{a, b\}$

$N.T = \{S, A, B, C, D\}$

Generating:

$S \to aAa$ ✗

$S \to aBC$ ✓

$A \to aS$ ✓

$A \to bD$ ✗

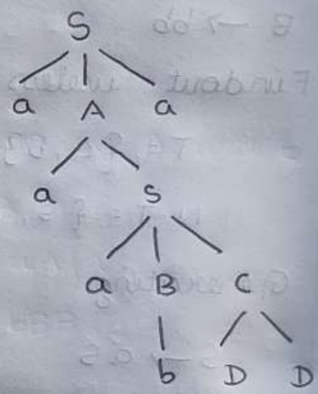
$B \to aBa$ ✓

$B \to b$ ✓

$C \to abb$ ✓

$C \to DD$ ✗

$D \to aDa$ ✗



Generating : $\{S, A, B, C\}$

Reachable : $\{S, A, B, C, D\}$

Useful Production:

$S \to aBC$

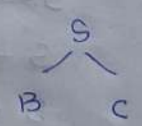
$A \to aS$

$B \to aBa$

$B \to b$

$C \to abb$

$S \to aAa$



Generating : $\{S, A, B, C\}$

Reachable : $\{S, A, B, C\}$

Useful : {S, A, B, C}

Useful Production :

$S \rightarrow aAa$

$S \rightarrow aBC$

$B \rightarrow aBa$

$B \rightarrow b$

$C \rightarrow abb$

$A \rightarrow aS$

$S \rightarrow aAa \mid aBC$

$A \rightarrow aS$

$B \rightarrow aBa \mid b$

$C \rightarrow abb$

Elimination of null production:

1. Find the nullable symbols in grammar G.
2. A variable said to be nullable, if the production of form $A \rightarrow \varepsilon$
3. Replace a nullable variable in the production of G, with all possible combination of non-terminal to generate new set of production 'P'.

1. Eliminate the null production in the grammar:

$S \rightarrow abB$

$B \rightarrow Bb \mid \varepsilon$

Nullable variable

$B \rightarrow \varepsilon$

B is nullable variable, replacing B by ε

$S \rightarrow ab\varepsilon$

$\boxed{S \rightarrow ab}$

$B \rightarrow \varepsilon b$

$\boxed{B \rightarrow b}$

$S \rightarrow abB \mid ab$

$B \rightarrow Bb \mid b$

original production | after removable of ε production

More than one variable associated with null production.

* Generate the new set of production by replacing α_1 with ε.
* Generating the new set of production by replacing α_2 with ε.
* Generating the new set of production by replacing α_1 and α_2 with ε.

Eliminate nullable in the grammar.

$S \rightarrow AB$
$A \rightarrow aAA \mid \varepsilon$
$B \rightarrow bBB \mid \varepsilon$

1. Nullable variable

$A \rightarrow \varepsilon$
$B \rightarrow \varepsilon$

i) Generating new set of production by A with ε

$S \rightarrow B$ $\quad a\overset{①}{A}\overset{②}{A}$ add ε
$A \rightarrow a\varepsilon A \mid a\varepsilon\varepsilon$
$B \rightarrow bBB \mid \varepsilon$

$\Rightarrow$

$S \rightarrow B$
$A \rightarrow aA \mid a$
$B \rightarrow bBB \mid \varepsilon$

$S \rightarrow AB \mid B$
$A \rightarrow aAA \mid aA \mid a$
$B \rightarrow bBB \mid \varepsilon$

Add new production with old production

ii) Generating new set of production by B with ε

$S \rightarrow A \mid \varepsilon$
$A \rightarrow aAA \mid aA \mid a$
$B \rightarrow b\varepsilon B \mid b\varepsilon\varepsilon$

$\Rightarrow$

$S \rightarrow A \mid \varepsilon$
$A \rightarrow aAA \mid aA \mid a$
$B \rightarrow bB \mid b$

$S \rightarrow AB \mid B \mid A \mid \varepsilon$ ← remove ε from 'S'
$A \rightarrow aAA \mid aA \mid a$
$B \rightarrow bBB \mid bB \mid b$

$S \rightarrow AB \mid B \mid A$
$A \rightarrow aAA \mid aA \mid a$
$B \rightarrow bBB \mid bB \mid b$

Elimination of Unit Production:

* The production of CFG from A → B where A and B are non-terminal of grammar.
* A said to be unit production.

Remove unit production

$S \to AaB \mid C$

$A \to a \mid bc \mid B$

$C \to a$

$B \to bb$

N.T → N.T

$A \to aCx$

A → C

Find unit production

$S \to C$

$A \to B$

$S \to AaB \mid C$		$S \to AaB \mid a$
$A \to a \mid bc \mid B$	⇒	$A \to a \mid bc \mid bb$
$C \to a$		$C \to a$
$B \to bb$		$B \to bb$

Eliminate Unit Production

$E \to E + T \mid T$

$T \to T * F \mid F$

$F \to (E) \mid id$

$E \to E + T$

$\boxed{E \to T}$

$T \to T * F$

$\boxed{T \to F}$

$F \to (E)$

$F \to id$

Replace with all T production, it is free from unit production

$E \to E + T$

$E \to T$

$T \to T * F$

$T \to (E) \mid id$

$F \to (E)$

$F \to id$

→

$E \to E + T$

$E \to T * F \mid (E) \mid id$

$T \to T * F$

$T \to (E) \mid id$

$F \to (E)$

$F \to id$

Normal Form CFG:

* Chomsky Normal Form (CNF)
* Gridrack Normal Form

CNF:

* Free form
    * Unit Production
    * Null Production
    * Useless production

$X \to YZ$ $(N.T \to N.T\ N.T)$

$X \to a$ $(N.T \to T)$

1. Convert given CFG into CNF.

$S \to AAC$

$A \to aAb \mid \varepsilon$

$C \to aC \mid a$

$S \to AAC$

$A \to aAb$

$A \to \varepsilon$

$C \to aC$

$C \to a$

① Eliminate Null production

$A \to \varepsilon$

'A' replace with 'ε'

$S \to \varepsilon AC \mid \varepsilon\varepsilon C$ $\to$ $S \to AC \mid C$

$A \to ab$ $\quad$ $A \to ab$

$S \to AAC \mid AC \mid C$

$A \to aAb \mid ab$

$C \to aC$

$C \to a$

② Eliminate Unit Production

Unit Production: $S \to C$

$S \to AAC \mid AC \mid aC \mid a$

$A \to aAb \mid ab$

$C \to aC$

$C \to a$

③ Eliminate Useless production

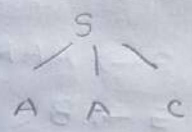
N.T = { S, A, C }

T = { a, b }

Generating : { S, A, C }

Reachable : { S, A, C }

Useful Production : { S, AC }



Finally,

$S \rightarrow AAC \mid AC \mid aC \mid a$

$A \rightarrow aAb \mid ab$

$C \rightarrow aC$

$C \rightarrow a$

CNF:

$$\boxed{\begin{array}{l} N.T \rightarrow N.T\,N.T \\ N.T \rightarrow T \end{array}}$$

$S \rightarrow AAC$

$\boxed{D_1 \rightarrow AA}$

$\boxed{S \rightarrow D_1C}$

$\boxed{S \rightarrow AC}$

$S \rightarrow aC$

$\boxed{D_2 \rightarrow a}$

$\boxed{S \rightarrow D_2C}$

$\boxed{S \rightarrow a}$

$A \rightarrow aAb$

$\boxed{D_3 \rightarrow b}$

$A \rightarrow D_2AD_3$

$D_4 \rightarrow D_2A$

$\boxed{A \rightarrow D_4D_3}$

$A \rightarrow ab$

$\boxed{A \rightarrow D_2D_3}$

$C \rightarrow aC$

$\boxed{C \rightarrow D_2C}$

$\boxed{C \rightarrow a}$

Finally,

$S \rightarrow D_1C \mid AC \mid D_2C \mid a$

$A \rightarrow D_4D_3 \mid D_2D_3$

$C \rightarrow D_2C \mid a$

$D_1 \rightarrow AA$

$D_2 \rightarrow a$

$D_3 \rightarrow b$

$D_4 \rightarrow D_2A$

2. Convert from CFG to CNF

$S \to AAA \mid B$

$A \to aA \mid B$

$B \to \varepsilon$

$S \to AAA$

$S \to B$

$A \to aA$

$A \to B$

$B \to \varepsilon$

① Eliminate Null Production

$B \to \varepsilon$ 'B' replace with ε

$S \to AAA$

$S \to \varepsilon$

$A \to aA$

$A \to \varepsilon$

$\Rightarrow$ $S \to AAA \mid B \mid \varepsilon$

$A \to aA \mid B \mid \varepsilon$

$S \to \varepsilon$

$A \to \varepsilon$

$S \to AA \mid A \mid \varepsilon$

$A \to a$

$\Rightarrow$ $S \to AAA \mid B \mid AA \mid A$

$a \to aA \mid B \mid a$

② Unit Production Elimination

$S \to AAA$ | $S \to AAA$

$\boxed{S \to B}$ → can't replace no production for 'B' | $S \to B$

$S \to AA$ | $S \to AA$

$S \to A$ | $S \to aA \mid B \mid a$

$A \to aA$ | $A \to aA$

$\boxed{A \to B}$ can't replace | $A \to B$

$A \to a$ | $A \to a$

③ Useless Production:

Generating : $\{S, A\}$

Reachable : $\{S, A, B\}$

Useless: $\{B\}$

Useful: $\{S, A\}$

$S \to A$

$S \to B$

$S \to A$

$S \to$

$S \to$

$S \to$

$A \to$

$A \to$

$A \to$

CNF:

N.T

N.T

$S \to AA$

$\boxed{D_1 \to AA}$

$\boxed{S \to D_1}$

$\boxed{S \to AA}$

$S \to aA$

$\boxed{D_2 \to a}$

$\boxed{S \to D_2}$

$\boxed{S \to a}$

$A \to a$

$\boxed{A \to D_2}$

$\boxed{A \to a}$

3. Convert

$S \to b$

$A \to b$

$B \to$

$S \to$

$S \to$

A

A

A

B

S → AAA

S → B ×

S → AA

S → aA

S → B ×

S → a

A → aA

A → B ×

A → a

⇒

S → AAA

S → AA

S → aA

S → a

A → aA

A → a

CNF:

$\boxed{\begin{array}{l} N.T \to N.T\ N.T \\ N.T \to T \end{array}}$

S → AAA

$\boxed{D_1 \to AA}$

$\boxed{S \to D_1 A}$

$\boxed{S \to AA}$

S → aA

$\boxed{D_2 \to a}$

$\boxed{S \to D_2 A}$

$\boxed{S \to a}$

A → aA

$\boxed{A \to D_2 A}$

$\boxed{A \to a}$

⇒

$S \to D_1 A \mid AA \mid D_2 A \mid a$

$A \to D_2 A \mid a$

$D_1 \to AA$

$D_2 \to a$

3. convert CFG to CNF

S → bA | aB

A ~~S~~ → bAA | aS | a

B → aBB | bS | b

S → bA

S → aB

A → bAA

A → aS

A → a

B → aBB

B → bS

$B \to b$

① Eliminate Null Production

② Eliminate Unit Production

No unit production

③ Useless production

Generating : $\{S, A, B\}$

Reachable : $\{S, A, B\}$

Useful : $\{S, A, B\}$

$S \to bA$

$S \to aB$

$A \to bAA$

$A \to aS$

$A \to a$

$B \to aBB$

$B \to bS$

$B \to b$

$A \to bAA$

$A \to bD_2$

CNF:

$$\boxed{\begin{array}{l} N{\cdot}T \to N{\cdot}T\ N{\cdot}T \\ N{\cdot}T \to T \end{array}}$$

$S \to bA$

$\boxed{D_1 \to b}$

$\boxed{S \to D_1 A}$

$S \to aB$

$\boxed{D_2 \to a}$

$\boxed{S \to D_2 B}$

$A \to bAA$

$\boxed{D_3 \to AA}$

$A \to bD_3$

$\boxed{A \to D_1 D_3}$

$A \to aS$

$\boxed{A \to D_2 S}$

$\boxed{A \to a}$

$B \to aBB$

$B \to D_2 BB$

$\boxed{D_4 \to BB}$

$\boxed{B \to D_2 D_4}$

$B \to bS$

$\boxed{D_5 \to b}$

$\boxed{B \to D_5 S}$

$\boxed{B \to b}$

Finally,

$S \to D_1 A \mid D_2 B$

$A \to D_1 D_3 \mid D_2 S \mid a$

$B \to D_2 D_4 \mid D_5 S \mid b$

$D_1 \to b$

$D_2 \to a$

$D_3 \to AA$

$D_4 \to BB$

$D_5 \to b$

Greibach Normal Form (GNF)

$A \to a\alpha$

N.T   T $(N \cup T)^*$

N.T   Terminal

Steps to convert CFG into GNF:

1. Convert the CFG ~~into~~ to Chomsky Normal Form (CNF)
2. Rename the NT in grammer G to $A_1, A_2, A_3 \dots A_n$
3. Modify the production with new NT
4. Examine whether each production in grammer 'G' is of the form of $A_i \to A_j x$ where $i > j$. Apply lemma 1 for these types of production.

Lemma 1:

If there is a production $A \to \alpha_1 B$ in G then the new Grammar is formed by replacing B with its production.

5. Repeat the process until every production in Grammar 'G' is converted in the form of $A_i \to A_j x$, with $i \le j$. After converting, if there is a production in G, $A_i \to A_j x$, with $i = j$ then apply lemma 2.

Lemma 2:

If ~~there~~ production in G is the form $A \to A\alpha_1 \mid A\alpha_2 \mid A\alpha_n \mid \beta_1 \dots \beta_n$ introducing non-terminal say x and form a new set of production as follows

$A \to \beta_1 | \beta_2 \dots \beta_n$

$A \to \beta_1 x | \beta_2 x | \dots \beta_n x$

$x \to \alpha_1 | \alpha_2 \dots | \alpha_n$

$x \to \alpha_1 x | \alpha_2 x \dots | \alpha_n x$

1. Convert the given CFG to GNF

$S \to AA | a$

$A \to SS | b$

Step 1: CFG to CNF

Already in CNF

$S \to AA | a$

$A \to SS | b$

Step 2: Rename all the non-terminal

$N.T = \{S, A\}$

S as A_1

A as A_2

$A_1 \to A_2 A_2 | a$

$A_2 \to A_1 A_1 | b$

Step 3:

$A_1 \to A_2 A_2 | a$

$A_2 \to A_1 A_1 | b$

Step 4: Check the production in the G

$A_1 \to A_2 A_2 | a$ $(i<j)$ (ie) $[1<2]$

$A_2 \to A_1 A_1 | b$ $(i>j)$ (ie) $[2>1]$

↳ lemma 1 $A \to A_2 A_2$ ①

$A_1 \to a$ ②

Step 5:

$A_2 \to A_2 A_2 A_1 | a A_1 | b$ $(i==j)$ $[2==2]$

↳ lemma 2

$A \to A\alpha_1 | A\alpha_2 | A\alpha_n \dots | \beta_1 | \beta_2 \dots \beta_n$

⇓

$A \to \beta_1 | \beta_2 \dots \beta_n$

$A \to \beta_1 x | \beta_2 x \dots \beta_n x$

$x \to \alpha_1 | \alpha_2 \dots \alpha_n$

$x \to \alpha_1 x | \alpha_2 x \dots \alpha_n x$

We have,

$$A_2 \to \underset{A}{\underline{A_2}}\ \underset{\alpha_1}{\underline{A_2 A_1}} \mid \underset{\beta_1}{\underline{a A_1}} \mid \underset{\beta_2}{\underline{b}}$$ [Apply lemma 2]

$A_2 \to a A_1 | b$

$A_2 \to a A_1 x | b x$

$x \to A_2 A_1$

$x \to A_2 A_1 x$

Finally,

$A_1 \to A_2 A_2 | a$

$A_2 \to a A_1 | b | a A_1 x | b x$

$x \to A_2 A_1 | A_2 A_1 x$

WKT,

GNF should be in the form

$A \to a \alpha$ [terminal followed by (Non terminal $\cup$ terminal)]

$\quad \hookrightarrow (N \cup T)^*$

(replace with A_2's production)

$A_1 \to \underline{A_2} A_2 | a$

$A_1 \to a A_1 A_2 | b A_2 | a A_1 x A_2 | b x A_2 | a$

$A_2 \to a A_1 | b | a A_1 x | b x$

$x \to \underline{A_2} A_1 | \underline{A_2} A_1 x$

$x \to a A_1 A_1 | b A_1 | a A_1 x A_1 | b x A_1 | a A_1 A_1 x |$
$\quad b A_1 x | a A_1 x A_1 x | b x A_1 x$

2. Convert CFG into GNF

$A \to B a c$

$B \to b$

$C \to a$

Check for null production, unit production and useless production

No such productions is seen.

WKT in CNF

$N.T \rightarrow N.T\ N.T$

$N.T \rightarrow T$

$A \rightarrow BaC$ $\Rightarrow$ $A \rightarrow BD$

$B \rightarrow b$ $B \rightarrow b$

$C \rightarrow a$ $C \rightarrow a$

$D \rightarrow CC$

Step 2: Rename all N.T

$N.T = \{A, B, C, D\}$

$A \rightarrow A_1$ $C \rightarrow A_3$

$B \rightarrow A_2$ $D \rightarrow A_4$

Step 3:

$A_1 \rightarrow A_2 A_4$

$A_2 \rightarrow b$

$A_3 \rightarrow a$

$A_4 \rightarrow A_3 A_3$

Step 4:

$$\left.\begin{array}{l} A_1 \rightarrow A_2 A_4 \quad (i<j) \\ A_2 \rightarrow b \end{array}\right\} \text{keep as it is}$$

$A_3 \rightarrow a$

$A_4 \rightarrow A_3\ A_3 \quad (i>j)$

$\hookrightarrow$ lemma 1

Step 5:

$A_4 \rightarrow A_3 A_3$

$A_4 \rightarrow aA_3$

Finally,

$A_1 \rightarrow A_2 A_4$

$A_2 \rightarrow b$

$A_3 \rightarrow a$

$A_4 \rightarrow aA_3$

WKT GNF $\rightarrow$ $A \rightarrow a\alpha$

$\therefore\ A_1 \rightarrow A_2 A_4$

$A_1 \to A_2 A_4$

$A_1 \to b A_4$

$A_2 \to b$ Final

$A_3 \to a$

$A_4 \to a A_3$

3. convert CFG to GNF

$S \to AB$

$A \to BS \mid b$

$B \to SA \mid a$

Step 1: CFG into CNF

Already in CNF

Step 2: Rename all N.T

$N.T = \{S, A, B\}$

$S \to A_1$

$A \to A_2$

$B \to A_3$

Step 3:

$A_1 \to A_2 A_3$ $(i<j)$ } Keep as it is

$A_2 \to A_3 A_1 \mid b$ $(i<j)$

$A_3 \to A_1 A_2 \mid a$ $(i>j)$

↳ lemma 1

Turing Machine [Infinite amount of memory]

* Real computer
* Work like human brain
* Model designed to accept pharse structured grammar (Type 0)

Components of Turing Machine:

* Finite control block with read and write head.
* A linear tape divided into no. of cells, it hold one symbol at cell, each cell.

$$M = (Q, \Sigma, \Gamma, \delta, q_0, B, F)$$

Q - Finite set of states

Σ - Set of input symbols

Γ - Set of tape (NUTUB)

$\delta(q, a)$ - (p, b, o)

q_0 - Initial state

B - Blank symbol

F - Set of Final state

1. Design a turing machine for language

$L = \{a^n b^n \mid n \geq 1\}$

$L = \{ab, aabb, \boxed{aaabbb}, \ldots\}$

B	B	B	a	a	a	b	b	b	B	B	B

starting   R/W   ending   [Starting & ending should have blank symbol]

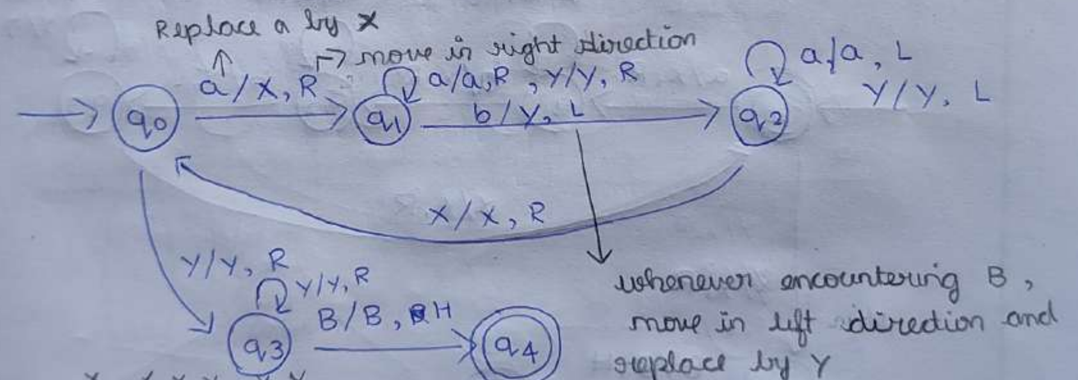
[∵ R/W head moves in bidirection (ie) forward & backward]

If I encounter a, mark x

To check how many a & b exists

Equal no. of a and b should be present

Finally check equal no. of x and Y should present



B	B	B	~~a~~	~~a~~	~~a~~	~~b~~	~~b~~	~~b~~	B	B	B

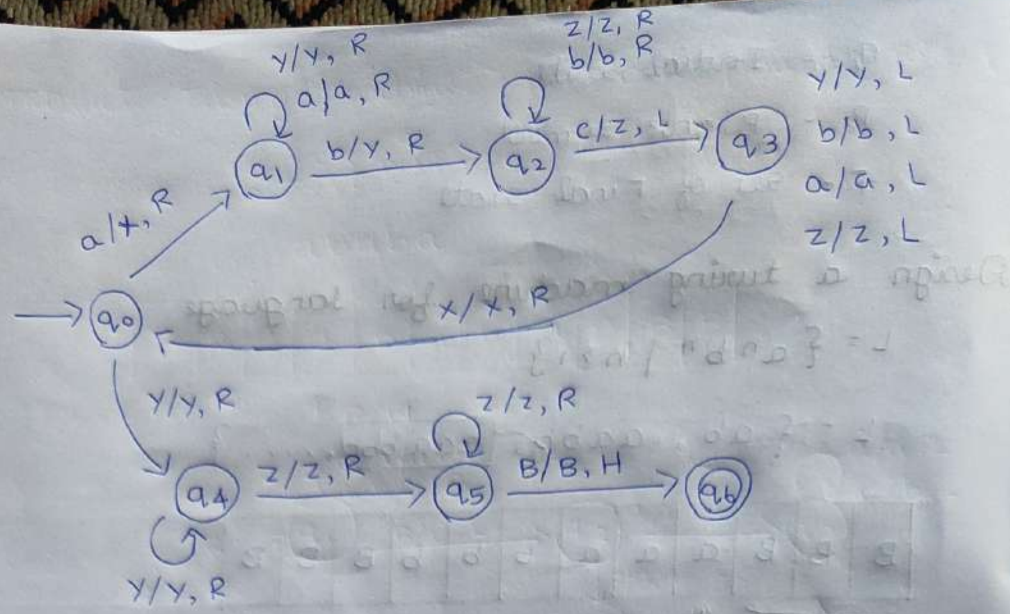
2. Design Turing Machine for language

$L = \{a^n b^n c^n\}$

$L = \{abc, aabbcc, \underline{aaabbbccc}, \ldots\}$

			x			y			z					
B	B	B	~~a~~	a	a	~~b~~	b	b	~~c~~	c	c	B	B	B

R/W



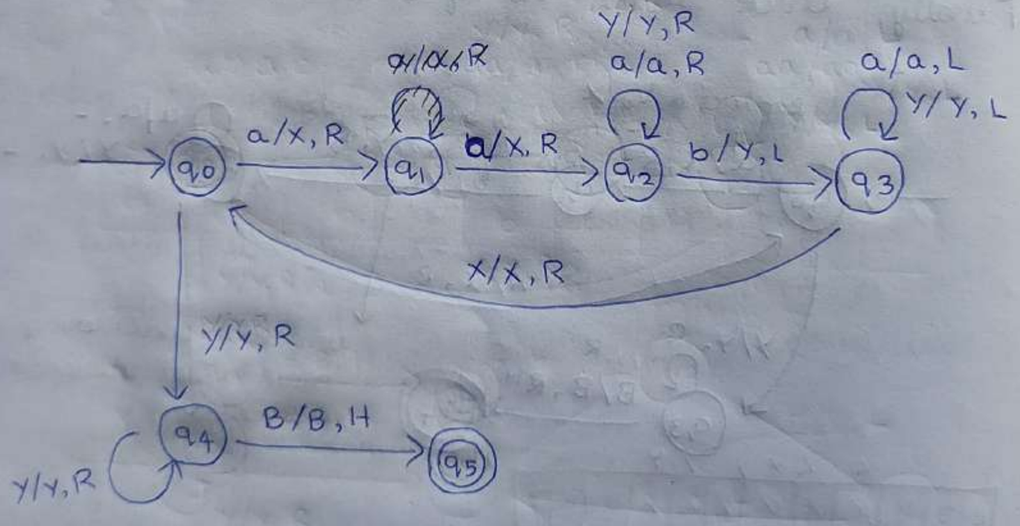
3. Design Turing Machine for the language

$L = \{a^{2n} b^{n} \mid n \geq 1\}$

$L = \{aab, \underline{aaaabb}, \ldots\}$

			X	X	X	X	Y	Y			
B	B	B	a	a	a	a	b	b	B	B	B

↑ R/W



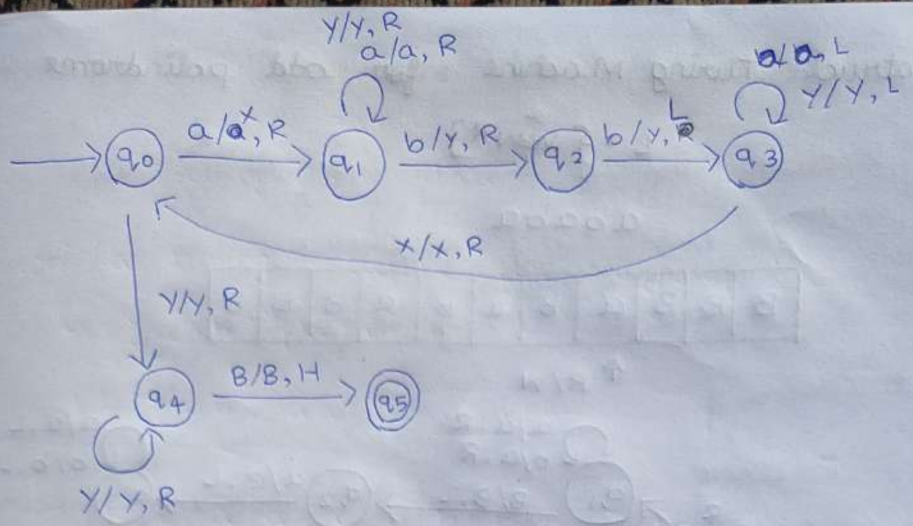
4. Design Turing Machine for the language

$L = \{a^{n} b^{2n} \mid n \geq 1\}$

$L = \{abb, aabbbb, \underline{aaabbbbbb}, \ldots\}$

			X	X	X	Y	Y	Y	Y	Y	Y			
B	B	B	a	a	a	b	b	b	b	b	b	B	B	B

↑ R/W



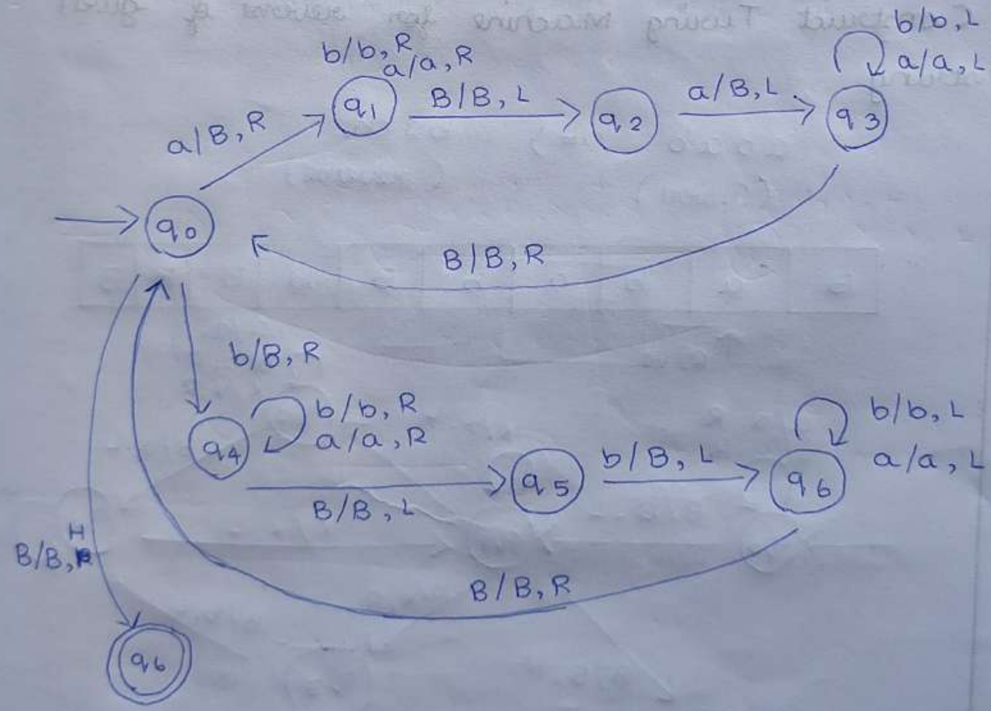
5. Design a Turing Machine for even palindrome

$L = \{WW^R\}$

abba

B	B	~~a~~ B	~~b~~ B	~~b~~ B	~~a~~ B	B	B

↑ R/W



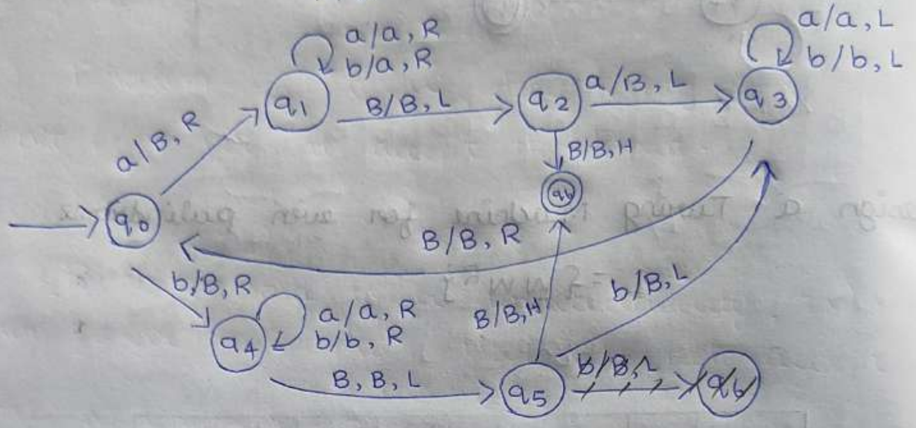
6. Construct Turing Machine for odd palindrome

$L = \{ w a w^R \}$ $\quad w a w^R$

ababa

B	B	B	$\cancel{a}$ B	b B	$\cancel{a}$ B	$\cancel{b}$ B	a B	B	B	B

↑ R/W



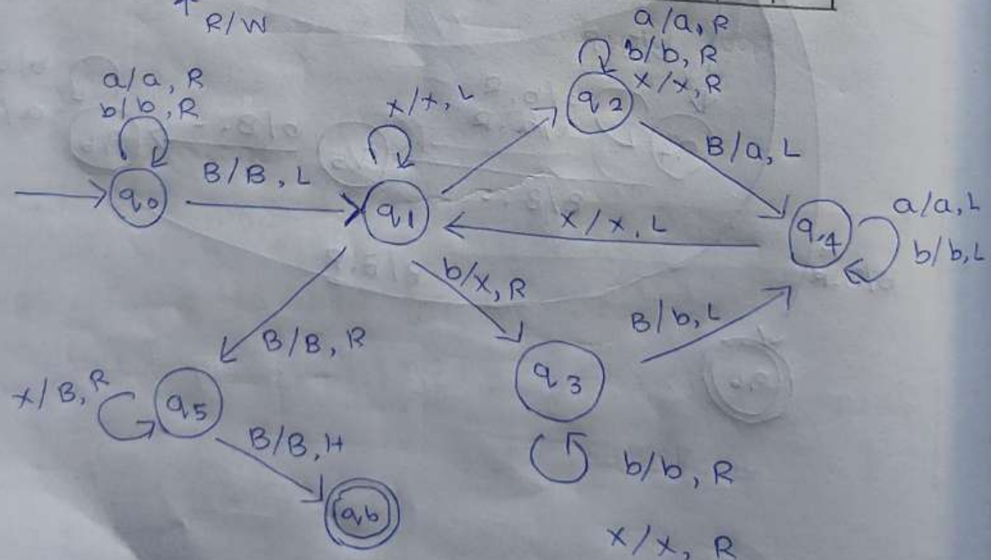
07-10-2025

7. Construct Turing Machine for reverse of given string

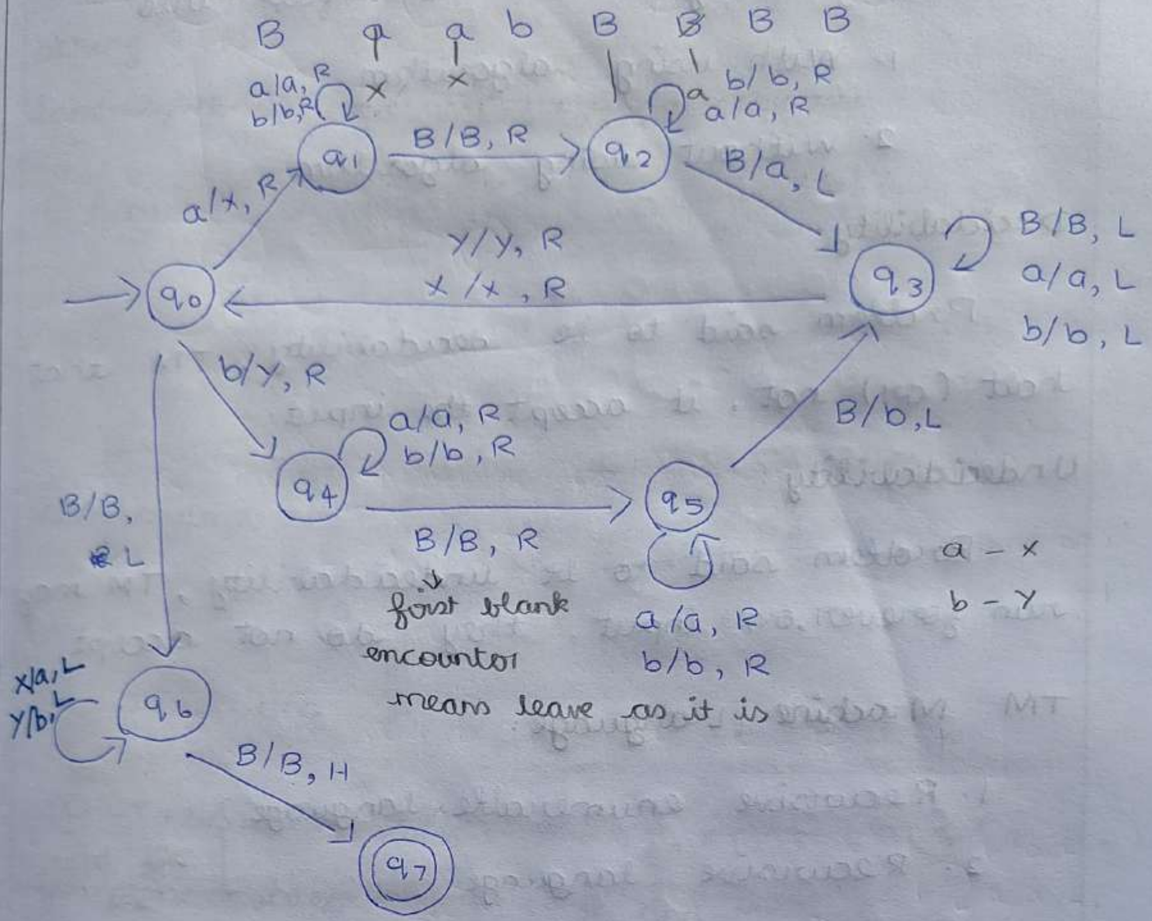
abab ⇒ baba
(Given) (reverse)

B	a	b	a	b	B	B	B	B	B

↑ R/W



8. Construct Turing Machine for copy of the string.



08.10.2025

UNIT-5

Problem Solving is TM classified into

1. With using algorithm
2. Without using algorithm

Decidability:

Problem said to be decidability, TM that halt (or) not, it accept the input.

Undecidability:

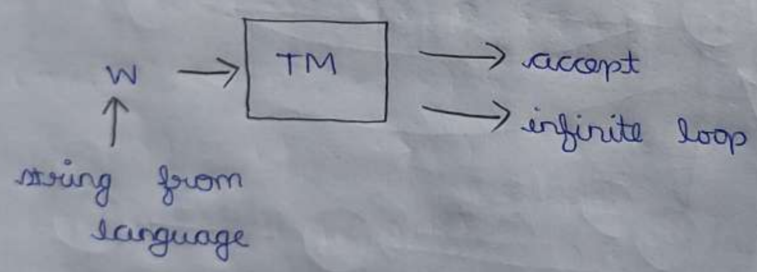
Problem said to be undecidability, TM may run forever, on input, they do not accept.

TM Machine Language:

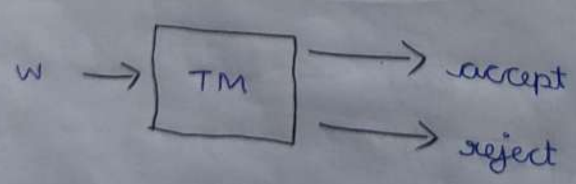
1. Recursive enumerable language
2. Recursive language

} Diff b/w 2M

Recursive enumerable language:



Recursive language:



Code for Turing Machine

$M = (Q, \cancel{\Sigma}\, \{0,1\}, \Gamma, \delta, q, B, F)$ as binary string. First assign integers to the states, tape symbols and directions

① Assume $Q = \{q_1, q_2, \ldots q_K\}$ for some M

start state $= q_1$ $\qquad \{q_0, q_1, q_2\}$

$\Downarrow$

q_1, q_2, q_3

② Assume $\Gamma = \{x_1, x_2, \ldots x_n\}$ for some M

$x_1 \Rightarrow 0 \quad x_2 \Rightarrow 1, \quad x_3 \Rightarrow B$

③ Directions $D_1 \Rightarrow L, \quad D_2 \Rightarrow R$

④ Transition rules: $\delta(q_i, x_j) = (q_K, x_R, D_M)$ for some integer i, j, K, r and m

Code is $\cancel{0^i 1 0^j 1 0^k 1 0^m}$ $\quad C_i = 0^i 1 0^j 1 0^k 1 0^m 1 0^n$

C_i = Code for entire Turing Machine is

$C_1 \, 11 \, C_2 \, 11 \, C_3 \, 11 \, C_4 \, 11 \ldots C_{n-1} \, 11 \, C_n$

1\. Let $M = (\{q_1, q_2, q_3\}, \{0,1\}, \{0,1,B\}, \delta, q_1, B, q_2)$

$\delta(q_1, 1) = (q_3, 0, R)$

$\delta(q_3, 0) = (q_1, 1, R)$

$\delta(q_3, 1) = (q_2, 0, R)$ $\qquad$ TM (0, 1)

$\delta(q_3, B) = (q_3, 1, L)$ $\qquad 0 \rightarrow x_1$

$\qquad 1 \rightarrow x_2$

$\delta(q_{i=1}, x_{j=2}) = (q_{k=3}, x_{m=1}, D_{n=2})$ $\qquad B \rightarrow x_3$

$\delta(q_3, \overset{x_1}{0}) = (q_1, x_2, D_2)$

$\delta(q_3, \overset{x_2}{1}) = (q_2, x_1, D_2)$

$\delta(q_3, \overset{x_3}{B}) = (q_3, x_2, D_1)$

$c_i = 0^i 1 0^j 1 0^k 1 0^m 1 0^n$

$c_1 = 0^1 1 0^2 1 0^3 1 0^1 1 0^2$

$= 0100100010100$

$c_2 = 0^3 1 0^1 1 0^1 1 0^2 1 0^2$

$= 0001010100100$

$c_3 = 0^3 1 0^2 1 0^2 1 0^1 1 0^2$

$= 00010010010100$

$c_4 = 0^3 1 0^3 1 0^3 1 0^2 1 0^1$

$= 0001000100010010$

$c = c_1 \, 11 \, c_2 \, 11 \, c_3 \, 11 \ldots c_n$

$c = 010010001010011 \; 000101010010011$

$0001001001010011 \; 0001000100010010$

2. code for TM $M = (\{q_0, q_1, q_2, q_3, q_4\}, \{0, 1\}, \{\overset{x_1}{0}, \overset{x_2}{1}, \overset{x_3}{X}, \overset{x_4}{Y}, \overset{x_5}{B}\}, \delta, q_0, B, \{q_4\})$

$\delta(q_0, 0) = (q_1, X, R)$

$\delta(q_0, Y) = (q_3, Y, R)$

$\delta(q_1, 0) = (q_1, 0, R)$

$\delta(q_1, 1) = (q_2, Y, L)$

$\delta(q_1, Y) = (q_1, Y, R)$

$\delta(q_2, 0) = (q_2, 0, L)$

$\delta(q_2, X) = (q_0, X, R)$ $\quad 0 \rightarrow x_1$

$\delta(q_2, Y) = (q_2, Y, L)$ $\quad 1 \rightarrow x_2$

$\delta(q_3, Y) = (q_3, Y, R)$ $\quad X \rightarrow x_3$

$\delta(q_3, B) = (q_4, B, R)$ $\quad Y \rightarrow x_4$

$B \rightarrow x_5$

$L \rightarrow D_1$

$R \rightarrow D_2$

$\delta(q_1, x_1) = (q_2, x_3, D_2)$

$\delta(q_1, x_4) = (q_4, x_4, D_2)$

$\delta(q_2, x_1) = (q_2, x_1, D_2)$

$\delta(q_2, x_2) = (q_3, x_4, D_1)$

$\delta(q_2, x_4) = (q_2, x_4, D_2)$

$\delta(q_3, x_1) = (q_3, x_1, D_1)$

$\delta(q_3, x_3) = (q_1, x_3, D_2)$

$\delta(q_3, x_4) = (q_3, x_4, D_1)$

$\delta(q_4, x_4) = (q_4, x_4, D_2)$

$\delta(q_4, x_5) = (q_5, x_5, D_2)$

$c_i = 0^i 1 0^j 1 0^k 1 0^m 1 0^n$

$c_1 = 0^1 1 0^1 1 0^2 1 0^3 1 0^2$

$= 0101001000100$

$c_2 = 0^1 1 0^4 1 0^4 1 0^4 1 0^2$

$= 0100001000010000100$

$c_3 = 0^2 1 0^1 1 0^2 1 0^1 1 0^2$

$= 001010010100$

$c_4 = 0^2 1 0^2 1 0^3 1 0^4 1 0^1$

$= 0010010001000010$

$c_5 = 0^2 1 0^4 1 0^2 1 0^4 1 0^2$

$= 00100000100100000100$

$c_6 = 0^3 1 0^1 1 0^3 1 0^1 1 0^1$

$= 0001010001010$

$c_7 = 0^3 1 0^3 1 0^1 1 0^3 1 0^2$

$= 0001000101000100$

$c_8 = 0^3 1 0^4 1 0^3 1 0^4 1 0^1$

$= 0001000010001000010$

$c_9 = 0^4 1 0^4 1 0^4 1 0^4 1 0^2$

$= 0000100001000010000100$

$c_{10} = 0^4 1 0^5 1 0^5 1 0^5 1 0^2$

$= 0000100000100000100000100$

$c = c_1 \,||\, c_2 \,||\, c_3 \,||\, c_4 \,||\, c_5 \,||\, c_6 \,||\, c_7 \,||\, c_8 \,||$
$c_9 \,||\, c_{10}$

$= 0101001000100110100001000010000100011$
$00101001010011001001000100001011$
$0010000010010000100110001010000101011$
$0001000101000100110001000010001000001011$
$00001000001000010000010011$
$00001000000100000100000100$

Post Correspondent Problem (PCP):

To prove PCP is undesirable goes in infinite loop

Given a sequence of string from some alphabet Σ

$A = w_1, w_2, \ldots w_n$

$B = x_1, x_2, \ldots x_n$

(w_i, x_i) corresponding pair we can say this instance of PCP has solution of there is sequence of integer $i_1, i_2 \ldots i_n$ such that

$w_{i_1}, w_{i_2} \ldots w_{i_n} = x_{i_1}, x_{i_2}, \ldots x_{i_n}$.

$Tm\ (L_u) \longrightarrow$ $\boxed{\text{on algorithm}}$ $\xrightarrow{\text{MPCP}}$ $\boxed{\text{on algorithm}}$ $\rightarrow$ PCP

Ex:

1.

i	list A w_i	list B x_i
1	1	111
2	10111	100
3	10	0

w : 10111 1 1 10

x : 10 111 111 0

=> 2 1 1 3

Take 2nd w_i value and start and stop until w and x

PCP has solution

2.

i	List A w_i	List B x_i
1	10	101
2	1011	11
3	101	011

w: 1011

x: 11

=> 2

This PCP has no solution because it violates the condition

Modified PCP.

$A = w_1, w_2, \ldots w_n$

$B = x_1, x_2, \ldots x_n$

Solution $i = i_1, i_2, \ldots i_m$

$w_{i_1}, w_{i_2} \ldots w_{i_n} = x_{i_1}, x_{i_2}, \ldots x_{i_n}$

(w_1, x_1) is first pair of solution

1-

i	List A w_i	List B x_i
1	1	111
2	10111	10
3	10	0

$w = 111$

$x = 111111111$ $\therefore$ No solution

Conversion of MPCP to PCP:

MPCP Instance $A = w_1, w_2, \dots w_n$

$B = x_1, x_2, \dots x_n$

PCP Instance $A = y_0, y_1, \dots y_{K+1}$

$B = z_0, z_1, \dots z_{K+1}$

C, D is constructed from A, B

1- for $i = 1, 2, 3 \dots K$

y_i be w_i with a * after each symbol of w_i

z_i be x_i with a* before each symbol of x_i

2. $y_0 = * y_1$ and $z_0 = z_1$

3. $y_{K+1} = \$$ and $z_{K+1} = * \$$

MPCP to PCP:

i	List A w_i	List B x_i
1	1	111
2	10111	10
3	10	0

$K = 3$

PCP	List C	List D
i	$y_i \ldots$	$z_i \ldots$
0	*1*	*1*1*1
1	1*	*1*1*1
2	1*0*1*1*1*	*1*0
3	1*0*	*0
4 ↓ k+1	$	*$

MPCP start from 0

Turing Machine convert into PCP.

Given (M, w) Let $\underbrace{M = (Q, \Sigma, \Gamma, \delta, \overset{\text{starting state}}{q_0}, B, F)}_{\text{Definition of TM}}$ and w in Σ^*

Construct (A, B), MPCP instance TM, M accept w if and only if (A, B) has a solution.

1. First pair

List A	List B
#	$\# q_0 w \#$

2. List A   List B

List A	List B
X	X
#	#

X is in Γ except blank

3. To simulate move of M for q in Q - F in Q and X, Y, Z in Γ

list A	list B	
qX	YP	if $\delta(q, X) = (P, Y, R)$
ZqX	PZY	if $\delta(q, X) = (P, Y, L)$
$q\#$	$YP\#$	if $\delta(q, B) = (P, Y, R)$
$Zq\#$	$PZY\#$	if $\delta(q, B) = (P, Y, L)$

4. if q is an accepting state X, Y are in Γ

list A	list B
XqY	q
Xq	q
qY	q

5. Final Pair

list A	list B
$q\#\#$	$\#$

TM into MPCP:

$M = (\{q_1, q_2, q_3\}, \{0, 1\}, \{0, 1, B\}), \delta, q_1, B, q_3)$

$w = 01$

	0	1	B
q_1	$(q_2, 1, R)$	$(q_2, 0, L)$	$(q_2, 1, L)$
q_2	$(q_3, 0, L)$	$(q_1, 0, R)$	$(q_2, 0, R)$
q_3	-	-	-

MPCP Instance:

Rule	List A	List B	Source
1	$\#$	$\#q_1 01\#$	
2	0	0	
	1	1	
	$\#$	$\#$	$\delta(q_1, 0) = (P, Y, R)$
			$(q_1, 0) = (q_2, 1, R)$
3	$q_1 0$	$1q_2$	

List A	List B	Source
$0q_2 1$	$q_2 00$	$\delta(q_1, 1) = (q_2, 0, L)$ P, Y, L
$1q_2 1$	$q_2 10$	
$0q_1 \#$	$q_2 01\#$	$\delta(q_1, B) = (q_2, 1, L)$
$1q_1 \#$	$q_2 11\#$	
$0q_2 0$	$q_2 00$	$\delta(q_2, 0) = (q_3, 0, L)$
$1q_2 1$	$q_3 10$	
$q_2 1$	$0q_1$	$\delta(q_2, 1) = (q_1, 0, R)$
$q_2 \#$	$0q_2\#$	$\delta(q_2, B) = (q_2, 0, R)$

4. $X \quad q \quad Y$

X, Y - I/P
q - final

X — 0, 1
Y — 0, 1

$0q_3 1$	q_3
$0q_3 0$	q_3
$1q_3 1$	q_3
$1q_3 0$	q_3
$0q_3$	q_3
$1q_3$	q_3
$q_3 0$	q_3
$q_3 1$	q_3

5. $q_3\#\#$ $\quad\quad$ $\#$

Rule 2, 3 → Continuously apply rule 2 & 3 until reach final

Rule 4 → Once reached final state means apply rule - 4

TM $\quad w = 01$

MPCP

A: $\# q_1 0 1 \#$

B: $\# q_1 0 1 \#$