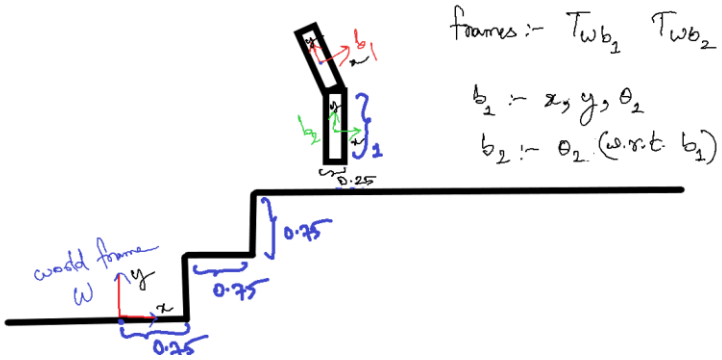
Machine Dynamics Final Project



Description: A double pendulum type object rolling down the stairs as shown above. The configuration variables are x, y, θ_1 for the body b1 θ_2 for the body b2 as shown. The initial conditions are x(0) = 2.25, y(0) = 3.25 with $\dot{x}(0) = 0$. $\theta_1 = 0$, $\theta_2 = 0$, $\theta'_1 = pi/6$. This allows the object to roll down the stairs.

Transformations used:

Twb1 and Twb2 are the frames of centers of the bodies in world frame.

Ta and Tb are the frames of opposite corners of top body w.r.t body frame 1.

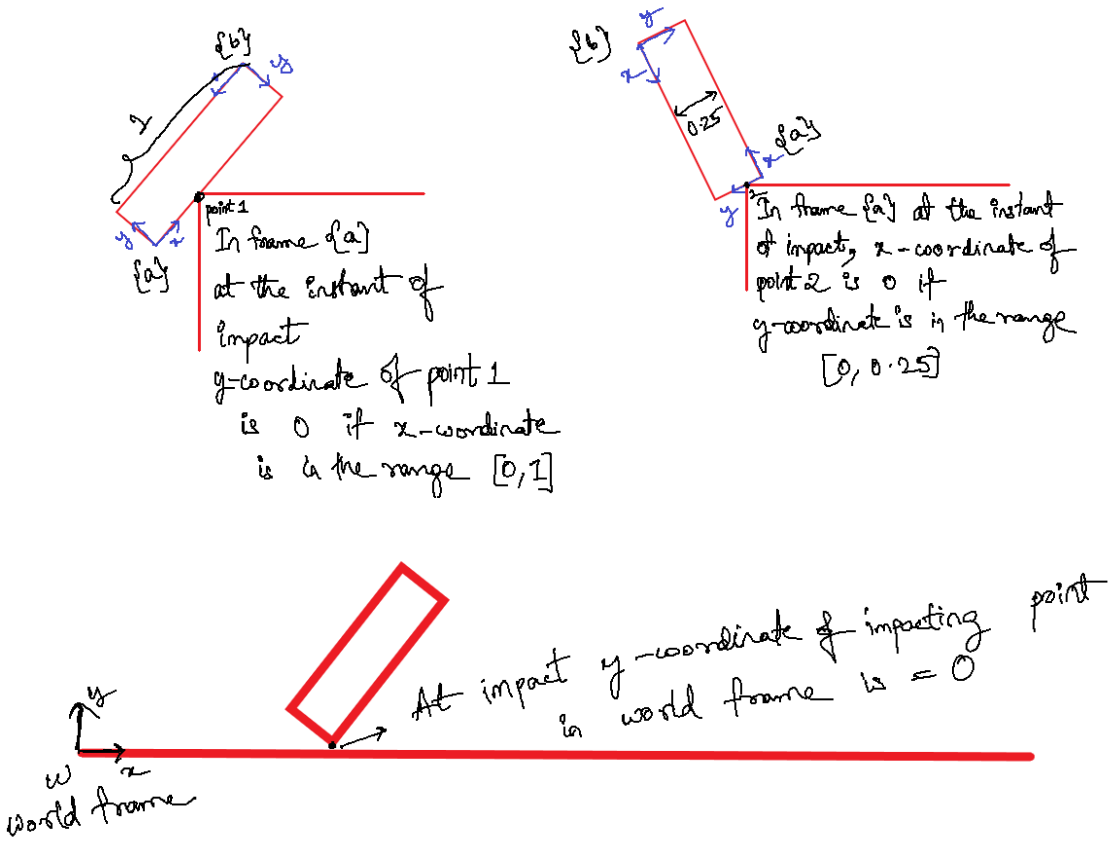
Tc and Td are the frames of opposite corners of bottom body w.r.t body frame 1.

$$T_{wb1}=\begin{bmatrix}\cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 & x(t)\\ \sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 & y(t)\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad T_{wb2}=\begin{bmatrix}\sin(\theta_1(t)) & \cos(\theta_1(t)) & 0 & x(t)-0.5\sin(\theta_1(t))-0.125\cos(\theta_1(t))\\ -\cos(\theta_1(t)) & \sin(\theta_1(t)) & 0 & y(t)-0.125\sin(\theta_1(t))+0.5\cos(\theta_1(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$T_a=\begin{bmatrix}-\sin(\theta_1(t)) & -\cos(\theta_1(t)) & 0 & x(t)+0.5\sin(\theta_1(t))+0.125\cos(\theta_1(t))\\ \cos(\theta_1(t)) & -\sin(\theta_1(t)) & 0 & y(t)+0.125\sin(\theta_1(t))-0.5\cos(\theta_1(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad T_b=\begin{bmatrix}\cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 & x(t)+0.5\sin(\theta_1(t))+0.5\sin(\theta_2(t))\\ \sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 & y(t)-0.5\cos(\theta_1(t))-0.5\cos(\theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

$$T_c=\begin{bmatrix}\sin(\theta_2(t)) & \cos(\theta_2(t)) & 0 & x(t)+0.5\sin(\theta_1(t))-0.125\cos(\theta_2(t))\\ -\cos(\theta_2(t)) & \sin(\theta_2(t)) & 0 & y(t)-0.125\sin(\theta_2(t))-0.5\cos(\theta_1(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix} \quad T_d=\begin{bmatrix}-\sin(\theta_2(t)) & -\cos(\theta_2(t)) & 0 & x(t)+0.5\sin(\theta_1(t))+1.0\sin(\theta_2(t))+0.125\cos(\theta_2(t))\\ \cos(\theta_2(t)) & -\sin(\theta_2(t)) & 0 & y(t)+0.125\sin(\theta_2(t))-0.5\cos(\theta_1(t))-1.0\cos(\theta_2(t))\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Impact conditions:



Impacts are divided into two categories as shown above. side impacting a point, point impacting the floor. For the first case, I chose two frames at opposite corners and estimated the impact condition as shown above.

Using the rigid body transformations, I calculated the kinetic and potential energy of both the centers of mass. For impacts, there are 32 different conditions to check for as shown above. (8 points of the bodies impacting with 5 sides of the floor = 16, 8 sides of the object impacting with 2 points on the stairs = 16. Hence, 32 in total)

Simulation: **The code took around 90 seconds to run**. It works perfectly fine. Initially, I solved for the Langrangian equations and then implemented the 32 impact conditions and impact updates.

Results: For the object to roll down, it needs some nudge or free fall. For testing the code, I used the initial conditions as mentioned above and it results in multiple impacts with the stairs and falls on the ground. I feel the animation resulted is correct as it clearly shows all the impacts possible and update accordingly.