Riddler Express

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1 Solution

Let $\mathcal{I}\%$ be the specific interest rate that guarantees that the Rule of 72 is exactly correct. Let t be the actual doubling time for this interest rate, and let t' be the doubling time predicted by the Rule of 72. We can then write:

$$\left(1 + \frac{\mathcal{I}}{100}\right)^t = 2, \tag{1}$$

$$t' = \frac{72}{\mathcal{I}}, \tag{2}$$

and

$$t = t'. \tag{3}$$

Combining these equations and with some rearrangement of terms, we get

$$\frac{\ln 2}{\ln\left(1 + \frac{\mathcal{I}}{100}\right)} = \frac{72}{\mathcal{I}}, \tag{4}$$

where ln is the natural logarithm. (4) can be solved numerically via any number of root-finding algorithms to finally yield the required value of interest rate:

$$\mathcal{I} = 7.847, \text{ or the interest rate is } 7.847\%.$$

2 Where does the Rule of 72 come from?

As a side exercise, I was curious how one arrives at the Rule of 72. It's easy to see by looking at the right-hand side of (4) and using the approximation $\ln(1 + x) \approx x$ for small x:

$$\begin{aligned}\frac{\ln 2}{\ln\left(1 + \frac{\mathcal{I}}{100}\right)} &\approx \frac{100 \ln 2}{\mathcal{I}} \\ &= \frac{69.31}{\mathcal{I}}.\end{aligned}$$

Picking 72 as the numerator instead of 69.31 makes sense because 72 is divisible by most whole numbers less than 10 (which is the range interest rates tend to fall in): $1, 2, 3, 4, 6, 8$ and 9. 5 and 7 can be handled by assuming that $69.31 \approx 70$.