

# Assignment-3

## CHAPETR-20

### Vector Algebra and Three Dimensional Geometry

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#### I. D:MCQS WITH ONE OR MORE THAN ONE CORRECT

- 1) Let  $a$  and  $b$  be two non-collinear unit vectors. If  $u = a - (a \cdot b)b$  and  $v = a \times b$ , then  $|v|$  is (1999-3 Marks)
  - a)  $|u|$
  - b)  $|u| + |u \cdot a|$
  - c)  $|u| + |u \cdot b|$
  - d)  $|u| + u \cdot (a + b)$
- 2) Let  $\mathbf{A}$  be vector parallel to line of intersection of planes  $P_1$  and  $P_2$ . Plane  $P_1$  is parallel to the vectors  $2\hat{j} + 3\hat{k}$  and  $4\hat{j} - 3\hat{k}$  and that  $P_2$  is parallel to  $\hat{j} - \hat{k}$  and  $3\hat{i} + 3\hat{j}$ , then the angle between vector  $\mathbf{A}$  and a given vector  $2\hat{i} + \hat{j} - 2\hat{k}$  is (2006-5M,-1)
  - a)  $\frac{\pi}{2}$
  - b)  $\frac{\pi}{4}$
  - c)  $\frac{\pi}{3}$
  - d)  $\frac{3\pi}{4}$
- 3) The vector(s) which is/are coplanar with vectors  $\hat{i} + \hat{j} + 2\hat{k}$  and  $\hat{i} + 2\hat{j} + \hat{k}$ , and perpendicular to the vector  $\hat{i} + \hat{j} + \hat{k}$  is/are (2011)
  - a)  $\hat{j} - \hat{k}$
  - b)  $\hat{i} + \hat{j}$
  - c)  $\hat{i} - \hat{j}$
  - d)  $\hat{j} + \hat{k}$
- 4) If the straight lines  $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$  and  $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$  are coplanar, then the plane(s) containing these two lines is(are) (2012)
  - a)  $y + 2z = -1$
  - b)  $y + z = -1$
  - c)  $y - z = -1$
  - d)  $y - 2z = -1$
- 5) A line  $l$  passing through the origin is perpendicular to the lines
 
$$l_1 : (3 + t)\hat{i} + (1 + 2t)\hat{j} + (4 + 2t)\hat{k}, -\infty < t < \infty$$

$$l_2 : (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}, -\infty < s < \infty$$
 Then, the coordinate(s) of the point(s) on  $l_2$  at a distance of  $\sqrt{17}$  from the point of intersection of  $l$  and  $l_1$  is(are) (JEE Adv.2013)
  - a)  $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$
  - b)  $(-1, -1, 0)$
  - c)  $(1, 1, 1)$
  - d)  $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$
- 6) Two lines  $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$  and  $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$  are coplanar. Then  $\alpha$  can take value(s) (JEE Adv.2013)
  - a) 1
  - b) 2
  - c) 3
  - d) 4
- 7) Let  $\mathbf{x}, \mathbf{y}$  and  $\mathbf{z}$  be three vectors each of magnitude  $\sqrt{2}$  and the angle between each pair of them is  $\frac{\pi}{3}$ . If  $\mathbf{a}$  is a non-zero vector perpendicular to  $\mathbf{x}$  and  $\mathbf{y} \times \mathbf{z}$  and  $\mathbf{b}$  is a non-zero vector perpendicular to  $\mathbf{y}$  and  $\mathbf{z} \times \mathbf{x}$ , then (JEE Adv.2014)
  - a)  $\mathbf{b} = (\mathbf{b} \cdot \mathbf{z})(\mathbf{z} - \mathbf{x})$
  - b)  $\mathbf{a} = (\mathbf{a} \cdot \mathbf{y})(\mathbf{y} - \mathbf{z})$
  - c)  $\mathbf{a} \cdot \mathbf{b} = -(\mathbf{a} \cdot \mathbf{y})(\mathbf{b} \cdot \mathbf{z})$
  - d)  $\mathbf{a} = -(\mathbf{a} \cdot \mathbf{y})(\mathbf{z} - \mathbf{y})$
- 8) From a point  $P(\lambda, \lambda, \lambda)$ , perpendicular PQ and PR are drawn respectively on the lines  $y = x, z = 1$  and  $y = -x, z = -1$ . If P is such that  $\angle QPR$  is a right angle, then the possible value(s) of  $\lambda$  is/(are) (JEE Adv.2014)
  - a)  $\sqrt{2}$
  - b) 1
  - c) -1
  - d)  $-\sqrt{2}$
- 9) In  $R^3$ , consider the planes  $P_1 : y = 0$  and  $P_2 : x + z = 1$ . Let  $P_3$  be the plane different from  $P_1$  and  $P_2$  which passes through the

intersection of  $P_1$  and  $P_2$ . If the distance of the point  $(0, 1, 0)$  from  $P_3$  is 1 and the distance of point  $(\alpha, \beta, \gamma)$  from  $P_3$  is 2, then which of the following relation is(are) true  
(JEE Adv.2015)

- a)  $2\alpha + \beta + 2\gamma + 2 = 0$
- b)  $2\alpha - \beta + 2\gamma + 4 = 0$
- c)  $2\alpha + \beta + 2\gamma - 10 = 0$
- d)  $2\alpha - \beta + 2\gamma - 8 = 0$

- 10) In  $R^3$ , let  $L$  be a straight line passing through the origin. Suppose that all the points on  $L$  are at a constant distance from two planes  $P_1 : x + 2y - z + 1 = 0$  and  $P_2 : 2x - y + z - 1 = 0$ . Let  $M$  be the locus of the foot of the perpendicular drawn from the points on  $L$  to plane  $P_1$ . Which of the following points lie(s) on  $M$ ?  
(JEE Adv.2015)

- a)  $(0, -\frac{5}{6}, -\frac{2}{3})$
- b)  $(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6})$
- c)  $(-\frac{5}{6}, 0, \frac{2}{3})$
- d)  $(-\frac{1}{3}, 0, \frac{2}{3})$

- 11) Let  $\triangle PQR$  be a triangle. Let  $\mathbf{a} = \mathbf{QR}, \mathbf{b} = \mathbf{RP}$  and  $\mathbf{c} = \mathbf{PQ}$ . If  $|\mathbf{a}| = 12$ ,  $|\mathbf{b}| = 4\sqrt{3}$ ,  $\mathbf{b} \cdot \mathbf{c} = 24$ , then which of the following is(are) true?  
(JEE Adv.2015)

- a)  $\frac{|\mathbf{c}|^2}{2} - |\mathbf{a}| = 12$
- b)  $\frac{|\mathbf{c}|^2}{2} + |\mathbf{a}| = 30$
- c)  $|\mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{a}| = 48\sqrt{3}$
- d)  $\mathbf{a} \cdot \mathbf{b} = -72$

- 12) Consider a pyramid  $OPQRS$  located in the first octant ( $x \geq 0, y \geq 0, z \geq 0$ ) with  $O$  as origin, and  $OP$  and  $OR$  along the  $x$ -axis and the  $y$ -axis respectively. The base  $OPQR$  of the pyramid is a square with  $OP=3$ . The point  $S$  is directly above the mid-point,  $T$  of diagonal  $OQ$  such that  $TS=3$ . Then  
(JEE Adv.2016)

- a) the acute angle between  $OQ$  and  $OS$  is  $\frac{\pi}{3}$
- b) the equation of the plane containing the triangle  $OQS$  is  $x - y = 0$
- c) the length of the perpendicular from  $P$  to the plane containing the triangle  $OQS$  is  $\frac{3}{\sqrt{2}}$
- d) the perpendicular distance from  $O$  to the straight line containing  $RS$  is  $\sqrt{\frac{15}{2}}$

- 13) Let  $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$  be a unit vector in  $R^3$  and  $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$ . Given that there exists a vector  $\mathbf{v}$  in  $R^3$  such that  $|\hat{u} \times \mathbf{v}| = 1$

and  $\hat{w}(\hat{u} \times \mathbf{v}) = 1$ . Which of the following statement(s) is(are) correct? (JEE Adv.2016)

- a) there is exactly one choice for such  $\mathbf{v}$
- b) There are infinitely many choices for such  $\mathbf{v}$
- c) If  $\hat{u}$  lies in the  $xy$ -plane then  $|u_1| = |u_2|$
- d) If  $\hat{u}$  lies in the  $xz$ -plane then  $2|u_1| = |u_3|$

- 14) Let  $P_1 : 2x + y - z = 3$  and  $P_2 : x + 2y + z = 2$  be two planes. Then, which of the following statement(s) is(are) TRUE? (JEE Adv.2018)

- a) The line of intersection of  $P_1$  and  $P_2$  has direction ratios  $1, 2, -1$
- b) The line  $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$  is perpendicular to the line of intersection of  $P_1$  and  $P_2$
- c) The acute angle between  $P_1$  and  $P_2$  is  $60^\circ$ .
- d) If  $P_3$  is the plane passing through the point  $(4, 2, -2)$  and perpendicular to the line of intersection of  $P_1$  and  $P_2$ , then the distance of the point  $(2, 1, 1)$  from the plane  $P_3$  is  $\frac{2}{\sqrt{3}}$

- 15) Let  $L_1$  and  $L_2$  denote the lines  
 $\mathbf{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k}), \lambda \in R$  and  
 $\mathbf{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k}), \mu \in R$   
 respectively. If  $L_3$  is a line which is perpendicular to both  $L_1$  and  $L_2$  and cuts both of them, then which of the following option describe(s)  $L_3$ ?  
(JEE Adv.2019)

- a)  $\mathbf{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
- b)  $\mathbf{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
- c)  $\mathbf{r} = t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$
- d)  $\mathbf{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k}), t \in R$