Question-9.4.19

EE24BTECH11048-NITHIN.K

Question:
The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Solution:
r_0: Initial radius of the sphere = 3 units.
r_3: Radius of the sphere at 3 seconds = 6 units.
t: The time after which the radius r_t is to be found.
K: rate of change in volume
It is given that the rate of change in volume is constant hence

$$\frac{dV}{dt} = K \tag{0.1}$$

$$\frac{dV}{dt} = \frac{V_3 - V_0}{3 - 0} \tag{0.2}$$

$$K = \frac{1}{3} \times \left(\frac{4\pi {r_3}^3}{3} - \frac{4\pi {r_0}^3}{3} \right) = 84\pi \tag{0.3}$$

$$\frac{dV}{dt} = K = 84\pi \tag{0.4}$$

$$\frac{V_t - V_0}{t - 0} = 84\pi \tag{0.5}$$

$$V_t = V_0 + 84\pi t \tag{0.6}$$

$$\frac{4\pi {r_t}^3}{3} = \frac{4\pi {r_0}^3}{3} + 84\pi t \tag{0.7}$$

$$r_t = \left({r_0}^3 + 63t \right)^{\frac{1}{3}} \tag{0.8}$$

Substituting equation 0.3 in 0.1

$$\frac{dV}{dt} = 84\pi \tag{0.9}$$

$$\frac{4\pi r^2 dr}{dt} = 84\pi \tag{0.10}$$

$$\frac{dr}{dt} = \frac{21}{r^2} \tag{0.11}$$

$$\frac{dy}{dx} = \frac{21}{y^2} \tag{0.12}$$

$$\frac{y_{n+1} - y_n}{dx} = \frac{21}{{y_n}^2} \tag{0.13}$$

$$y_{n+1} = \frac{21dx}{{y_n}^2} + y_n \tag{0.14}$$

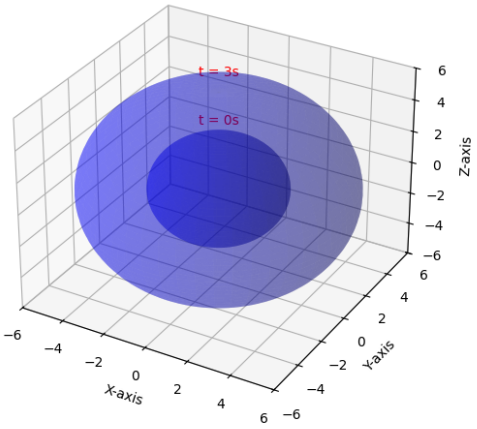

Fig. 0.1: Sphere growing over time.

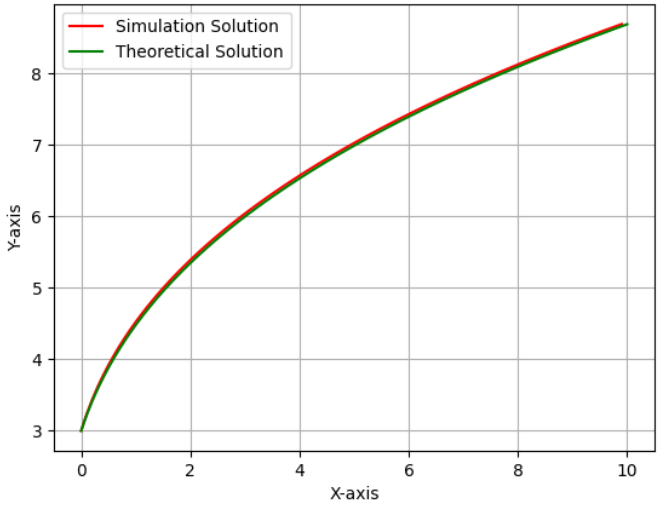

Fig. 0.2: Simulation VS Theoretical Plot.