Area Under Graph

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Problem Statement

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines $x = 0$ and $x = 2$ is?

Solution

FUNCTION	FORMULA
$g(x)$	$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0$
The points of intersection of the line L with the conic section as above are given by $\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$	$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in \mathbb{R}$ $\kappa_i = \frac{1}{\mathbf{m}^{\mathbf{T}}\mathbf{V}\mathbf{m}} \left(-\mathbf{m}^{\mathbf{T}} (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^{\mathbf{T}} (\mathbf{V}\mathbf{h} + \mathbf{u})^2 \right] - g(\mathbf{h}) (\mathbf{m}^{\mathbf{T}}\mathbf{V}\mathbf{m})} \right)$

Table: Variables Used

Solution

On comparing $g(x)$ and $x^2 + y^2 - 4 = 0$ the parameters of the circle are

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.1}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{3.2}$$

$$f = -4 \tag{3.3}$$

The area bounded by $x = 0$, $x = 2$ and the circle in the first quadrant is

$$\int_0^2 \sqrt{4 - x^2} dx \tag{3.4}$$

for indefinite integration of above form we get

$$\int \sqrt{4 - x^2} dx = 2 \sin^{-1} \frac{x}{2} + x\sqrt{4 - x^2} + c \tag{3.5}$$

$$\int_0^2 \sqrt{4 - x^2} dx = \pi \tag{3.6}$$

Hence the enclosed area is π square units

C Code for Calculating Area

```
#include <stdio.h>
#include <math.h>
double area(double lower_limit, double upper_limit){
        double sum=0 ;
        for ( double i = lower_limit; i<=upper_limit; i+=1e-7 ){
                sum += sqrt(4-(i*i))*1e-7;
        }
        return sum ;
}
```

Python Code using shared library

```
import ctypes
lib = ctypes.CDLL('./integration.so')
lib.area.argtypes = [ctypes.c_double, ctypes.c_double]
lib.area.restype = ctypes.c_double
print("Area enclosed",lib.area(0,2))
```

Python Code for Plotting

```
import sys
sys.path.insert(0, '/home/nithink/matgeo/codes/CoordGeo')
import numpy as np
import matplotlib.pyplot as plt
from numpy import linalg as LA

# local imports
from line.funcs import *
from triangle.funcs import *
from conics.funcs import *

r = 2
V = np.eye(2)
u = np.array(([0, 0])).reshape(-1, 1)
f = -4
```

Python Code for Plotting

```
# Generating circle
x_circ = circ_gen(-u, r)

n1 = np.array(([1, 0])).reshape(-1,1)
c1 = 0
n2 = np.array(([1, 0])).reshape(-1,1)
c2 = 2
k1 = -4
k2 = 4

#Generating Lines
x_A = line_norm(n1,c1,k1,k2)
x_B = line_norm(n2,c2,k1,k2)
```

Python Code for Plotting

```
# Plotting all lines and circles
plt.plot(x_circ[0, :], x_circ[1, :], label='$Circle$')
plt.plot(x_A[0,:],x_A[1,:],label='$x = 0$')
plt.plot(x_B[0,:],x_B[1,:],label='$x = 2$')

# Adjusting axis spines
ax = plt.gca()
ax.spines['top'].set_color('none')
ax.spines['left'].set_position('zero')
ax.spines['right'].set_color('none')
ax.spines['bottom'].set_position('zero')

# Define the space
x = np.linspace(0,2,100)
y_circle = np.sqrt(4-x**2)
```

Python Code for Plotting

```
# Fill the area between the lines and the circle
plt.fill_between(x,0, y_circle, color='red', alpha=0.5, label='Shaded Region')

# Labels and title
plt.xlabel('x')
plt.ylabel('y')

# Final plot settings
plt.legend(loc='upper right')
plt.axis('equal')

#Display the Plot
plt.show()
```

Diagram

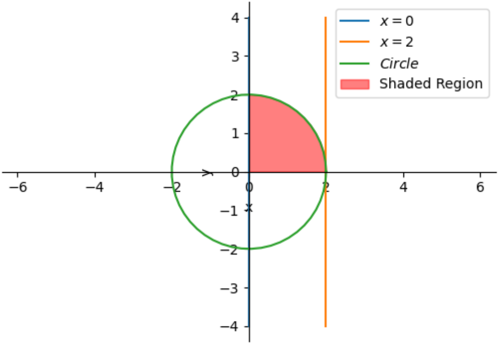

Figure: Enclosed Area