

Assignment-1

CHAPETR-11

Limits, Continuity and Differentiability

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I. C:MCQs WITH ONE CORRECT ANSWER

1) Let $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the left hand derivative of $|x-1|$ at $x = 1$. If $\lim_{x \rightarrow 1^+} g(x) = p$, then (2008)

- a) $n = 1, m = 1$
- b) $n = 1, m = -1$
- c) $n = 2, m = 2$
- d) $n > 2, m = n$

2) If $\lim_{x \rightarrow 0} [1 + x \ln(1 + b^2)]^{\frac{1}{x}} = 2b \sin^2 \theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is (2011)

- a) $\pm \frac{\pi}{4}$
- b) $\pm \frac{\pi}{3}$
- c) $\pm \frac{\pi}{6}$
- d) $\pm \frac{\pi}{2}$

3) If $\lim_{x \rightarrow \infty} \left(\frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$, then (2012)

- a) $a = 1, b = 4$
- b) $a = 1, b = -4$
- c) $a = 2, b = -3$
- d) $a = 2, b = 3$

4) Let $f(x) = \begin{cases} x^2 \left| \cos \frac{\pi}{x} \right| & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, $x \in \mathbb{R}$ then f is (2012)

- a) differentiable both at $x = 0$ and at $x = 2$
- b) differentiable at $x = 0$ but not differentiable at $x = 2$
- c) not differentiable at $x = 0$ but differentiable at $x = 2$
- d) differentiable neither at $x = 0$ nor at $x = 2$

5) Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $(\sqrt[3]{1+a} - 1)x^2 + (\sqrt[2]{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$ where $a > -1$. then $\lim_{a \rightarrow 0^+} \alpha(a)$ and $\lim_{a \rightarrow 0^+} \beta(a)$ are (2012)

- a) $-\frac{5}{2}$ and 1
- b) $-\frac{1}{2}$ and 1
- c) $-\frac{7}{2}$ and 2
- d) $-\frac{9}{2}$ and 3

II. D:MCQs WITH ONE OR MORE THAN ONE CORRECT

6) If $x + |y| = 2y$, then y as a function of x is (1984-3marks)

- a) defined for all real x
- b) continuous at $x = 0$
- c) differentiable for all x
- d) such that $\frac{dy}{dx} = \frac{1}{3}$ for $x < 0$

7) If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then- (1985-2marks)

- a) $f(x)$ is continuous but not differentiable at $x = 0$
- b) $f(x)$ is differentiable at $x = 0$
- c) $f(x)$ is not differentiable at $x = 0$
- d) none of these

8) The function $f(x) = 1 + |\sin x|$ is (1986-2marks)

- a) continuous nowhere
- b) continuous everywhere
- c) differentiable nowhere
- d) not differentiable at $x = 0$
- e) not differentiable at infinite number of points

9) Let $[x]$ denote the greatest integer less than or equal to x . If $f(x) = [x \sin \pi x]$, then $f(x)$ is (1986-2marks)

- a) continuous at $x = 0$
- b) continuous in $(-1, 0)$
- c) differentiable at $x = 1$

- d) differentiable in $(-1, 1)$
 e) none of these

- b) f is differentiable
 c) f' is continuous
 d) f is continuous

10) The set of all points where the function $f(x) = \frac{x}{(1+|x|)}$ is differentiable, is
 (1987-2marks)

- a) $(-\infty, \infty)$
 b) $[0, \infty)$
 c) $(-\infty, 0) \cup (0, \infty)$
 d) $(0, \infty)$
 e) None

11) The function

$$f(x) = \begin{cases} |x-3| & , x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & , x < 1 \end{cases}, \text{ is}$$

(1988-2marks)

- a) continuous at $x = 1$
 b) differentiable at $x = 1$
 c) continuous at $x = 3$
 d) differentiable at $x = 3$

12) If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$
 (1989-2marks)

- a) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous
 b) $\tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous
 c) $\tan[f(x)]$ and $f^{-1}x$ are both continuous
 d) $\tan[f(x)]$ is continuous but $\frac{1}{f(x)}$ is not

13) The value of $\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$
 (1991-2marks)

- a) 1
 b) -1
 c) 0
 d) none of these

14) The following functions are continuous on $(0, \pi)$
 (1991-2marks)

- a) $\tan x$
 b) $\int_0^x t \sin \frac{1}{t} dt$
 c) $\begin{cases} 1 & , 0 < x \leq \frac{3\pi}{4} \\ 2\sin^2 \frac{x}{9} & , \frac{3\pi}{4} < x < \pi \end{cases}$
 d) $\begin{cases} x \sin x & , 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x) & , \frac{\pi}{2} < x < \pi \end{cases}$

15) Let $f(x) = \begin{cases} 0 & , x < 0 \\ x^2 & , x \geq 0 \end{cases}$ then for all x (1994)

- a) f' is differentiable