Question-9.4.19

EE24BTECH11048-NITHIN.K

Question:

The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

Solution:

r_0: Initial radius of the sphere = 3 units.

r_3: Radius of the sphere at 3 seconds = 6 units.

t: The time after which the radius r_t is to be found. It is given that the rate of change in volume is constant hence $\frac{dV}{dt} = K(constant)$

$$\frac{dV}{dt} = \frac{V_3 - V_0}{3 - 0}$$

$$\text{rate of change in volume} = \text{K} = \frac{1}{3} \times \left(\frac{4\pi {r_3}^3}{3} - \frac{4\pi {r_0}^3}{3} \right) = 84\pi$$

$$hence \frac{dV}{dt} = K(constant) = 84\pi$$

$$\frac{V_t - V_0}{t - 0} = 84\pi$$

$$V_t = V_0 + 84\pi t$$

$$\frac{4\pi {r_t}^3}{3} = \frac{4\pi {r_0}^3}{3} + 84\pi t$$

$$r_t = \left({r_0}^3 + 63t \right)^{\frac{1}{3}}$$

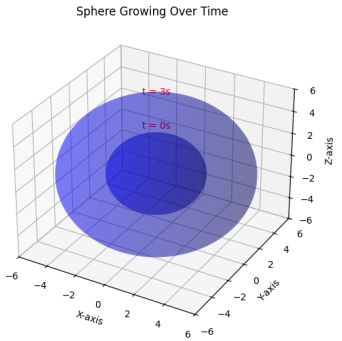

Fig. 0.1: Visualization of the sphere growing over time.

Python Code:

```
import numpy as np
def calculate_radius(r_0, r_3, t):
    t_0 = 3.0 #Time at which r_3 is given (in seconds)
    rate = 4*(np.pi)*(r_3**3 - r_0**3)/(3*t_0) #Calculate the rate of
        change of volume
    r_t = ((r_0**3) + (3*rate*t)/(4*np.pi))**(1/3) #Calculate the
        radius at time t
    return r_t
try:
    r_0 = float(input("Enter initial radius (r_0): "))
    r_3 = float(input("Enter radius at 3 seconds (r_3): "))
    t = float(input("Enter the time (t) after which the radius is to
        be found: "))
    if r_0 <= 0 or r_3 <= 0 or t < 0:
        print("All radii must be positive, and time must be non-
            negative.")
    else:
        r_t = calculate_radius(r_0, r_3, t)
        print(f"The radius of the sphere after {t} seconds is {r_t:.3f
            } units.")
except ValueError:
    print("Invalid input. Please enter numeric values.")
```