Assignment-4

EE24BTECH11048-NITHIN.K

1 SECTION-A

1) If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to
   a) $3 + 4\sqrt{2}$
   b) $-5 + 6\sqrt{2}$
   c) $-4 + 3\sqrt{2}$
   d) $7 + 6\sqrt{2}$
2) Let Q be the mirror image of the point $P(1, 0, 1)$ with respect to the plane S : $x + y + z = 5$. If a line L passing through $(1, -1, -1)$, parallel to the line PQ meets the plane S at R, then QR^2 is equal to:
   a) 2
   b) 5
   c) 7
   d) 11
3) If the solution curve $y = y(x)$ of the differential equation $y^2dx + \left(x^2 - xy + y^2\right)dy = 0$, which passes through the point $(1, 1)$ and intersects the line $y = \sqrt{3}x$ at the point $\left(\alpha, \sqrt{3}\alpha\right)$, then the value of $\ln\left(\sqrt{3}\alpha\right)$ is equal to
   a) $\frac{\pi}{3}$
   b) $\frac{\pi}{2}$
   c) $\frac{\pi}{12}$
   d) $\frac{\pi}{6}$
4) Let $x = 2t$, $y = \frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that $SA \perp BA$, where A is any point on the conic. If k is the ordinate of the centroid of $\triangle SAB$, then $\lim_{t \to 1} k$ is equal to
   a) $\frac{17}{18}$
   b) $\frac{19}{18}$
   c) $\frac{11}{18}$
   d) $\frac{13}{18}$
5) Let a circle C in complex plane pass through the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z (\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then $\arg(z)$ is equal to :
   a) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right) - \pi$
   b) $\tan^{-1}\left(\frac{24}{7}\right) - \pi$
   c) $\tan^{-1}(3) - \pi$
   d) $\tan^{-1}\left(\frac{3}{4}\right) - \pi$

2 SECTION-B

1) Let C_r denote the binomial coefficient of x^r in the expansion of $(1+x)^{10}$. If $\alpha, \beta \in R$. $C_1 + 3 \cdot 2C_2 + 5 \cdot 3C_3 + ...$ upto 10 terms $= \frac{\alpha \times 2^{11}}{2^{\beta}-1}\left(C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ...upto 10 terms\right)$ then the value of $\alpha + \beta$ is equal to
2) The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is
3) Let θ be the angle between the vectors $\mathbf{a}$ and $\mathbf{b}$, where $|\mathbf{a}| = 4, |\mathbf{b}| = 3$, $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then $|(\mathbf{a} - \mathbf{b})\, x\, (\mathbf{a} + \mathbf{b})|^2 + 4(\mathbf{a} \cdot \mathbf{b})^2$ is equal to
4) Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2\left(x^2 + y^2\right)$ - 11x -14y - 22 =0, then 2r + s - 2q + p is equal to
5) The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14 \operatorname{cosec}^2 x - 2\sin^2 x = 21 - 4\cos^2 x$ holds, is
6) For a natural number n, let $a_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9 - \alpha_1 0}{57\alpha_8}$ is
7) Let f : $R \to R$ be a function defined by $f(x) = \left(2\left(1 - \frac{x^{25}}{2}\right)\left(2 + x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x) = f(f(f(x))) + f(f(x))$, then the greatest integer less than or equal to g(1) is
8) Let the lines
   $L_1 : \mathbf{r} = \lambda(\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}),\ \lambda \in R$
   $L_2 : \mathbf{r} = (\mathbf{i} + 3\mathbf{j} + \mathbf{k}) + \mu(\mathbf{i} + \mathbf{j} + 5\mathbf{k})\ ;\ \mu \in R$
   intersect at the point S. If a plane $ax + by - z + d = 0$ passes through S and is parallel to both the lines $L_1 and L_2$, then the value of a+b+d is equal to
9) Let A be a 3 x 3 matrix having entries from the set $\{-1, 0, 1\}$. The number of all such matrices A having sum of all entries equal to 5, is
10) The greatest integer less than or equal to the sum of first 100 terms of the sequence $\frac{1}{3}, \frac{5}{9}, \frac{19}{27}, \frac{65}{81}, ...$ is equal to