

Assignment-3

CHAPETR-20

Vector Algebra

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I. SECTION-D

- 1) Let \mathbf{a} and \mathbf{b} be two non-collinear unit vectors. If $\mathbf{u} = \mathbf{a} - (\mathbf{a} \cdot \mathbf{b})\mathbf{b}$ and $\mathbf{v} = \mathbf{a} \times \mathbf{b}$, then $|\mathbf{v}|$ is (1999-3 Marks)
 - a) $|\mathbf{u}|$
 - b) $|\mathbf{u}| + |\mathbf{u} \cdot \mathbf{a}|$
 - c) $|\mathbf{u}| + |\mathbf{u} \cdot \mathbf{b}|$
 - d) $|\mathbf{u}| + \mathbf{u} \cdot (\mathbf{a} + \mathbf{b})$
- 2) Let \mathbf{A} be vector parallel to line of intersection of planes P_1 and P_2 . Plane P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and that P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector \mathbf{A} and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is (2006-5M,-1)
 - a) $\frac{\pi}{2}$
 - b) $\frac{\pi}{4}$
 - c) $\frac{\pi}{6}$
 - d) $\frac{3\pi}{4}$
- 3) The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is/are (2011)
 - a) $\hat{j} - \hat{k}$
 - b) $\hat{i} + \hat{j}$
 - c) $\hat{i} - \hat{j}$
 - d) $\hat{j} + \hat{k}$
- 4) If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are) (2012)
 - a) $y + 2z = -1$
 - b) $y + z = -1$
 - c) $y - z = -1$
 - d) $y - 2z = -1$
- 5) A line l passing through the origin is perpendicular to the lines

$$l_1 : (3+t)\hat{i} + (1+2t)\hat{j} + (4+2t)\hat{k}, -\infty < t < \infty$$

$$l_2 : (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}, -\infty < s < \infty$$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of l and l_1 is(are) (JEE Adv.2013)

 - a) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$
 - b) $(-1, -1, 0)$
 - c) $(1, 1, 1)$
 - d) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$
- 6) Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) (JEE Adv.2013)
 - a) 1
 - b) 2
 - c) 3
 - d) 4
- 7) Let \mathbf{x}, \mathbf{y} and \mathbf{z} be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If \mathbf{a} is a non-zero vector perpendicular to \mathbf{x} and $\mathbf{y} \times \mathbf{z}$ and \mathbf{b} is a non-zero vector perpendicular to \mathbf{y} and $\mathbf{z} \times \mathbf{x}$, then (JEE Adv.2014)
 - a) $\mathbf{b} = (\mathbf{b} \cdot \mathbf{z})(\mathbf{z} - \mathbf{x})$
 - b) $\mathbf{a} = (\mathbf{a} \cdot \mathbf{y})(\mathbf{y} - \mathbf{z})$
 - c) $\mathbf{a} \cdot \mathbf{b} = -(\mathbf{a} \cdot \mathbf{y})(\mathbf{b} \cdot \mathbf{z})$
 - d) $\mathbf{a} = -(\mathbf{a} \cdot \mathbf{y})(\mathbf{z} - \mathbf{y})$
- 8) From a point $P(\lambda, \lambda, \lambda)$, perpendicular PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/(are) (JEE Adv.2014)
 - a) $\sqrt{2}$
 - b) 1
 - c) -1
 - d) $-\sqrt{2}$
- 9) In R^3 , consider the planes $P_1 : y = 0$ and $P_2 : x + z = 1$. Let P_3 be the plane different from P_1 and P_2 which passes through the intersection of P_1 and P_2 . If the distance of

the point $(0, 1, 0)$ from P_3 is 1 and the distance of point (α, β, γ) from P_3 is 2, then which of the following relation is(are) true (JEE Adv.2015)

- a) $2\alpha + \beta + 2\gamma + 2 = 0$
- b) $2\alpha - \beta + 2\gamma + 4 = 0$
- c) $2\alpha + \beta + 2\gamma - 10 = 0$
- d) $2\alpha - \beta + 2\gamma - 8 = 0$

- 10) In R^3 , let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the foot of the perpendicular drawn from the points on L to plane P_1 . Which of the following points lie(s) on M ? (JEE Adv.2015)

- a) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$
- b) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$
- c) $\left(-\frac{5}{6}, 0, \frac{2}{3}\right)$
- d) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

- 11) Let $\triangle PQR$ be a triangle. Let $\mathbf{a} = \overrightarrow{QR}$, $\mathbf{b} = \overrightarrow{RP}$ and $\mathbf{c} = \overrightarrow{PQ}$. If $|\mathbf{a}| = 12$, $|\mathbf{b}| = 4\sqrt{3}$, $\mathbf{b} \cdot \mathbf{c} = 24$, then which of the following is(are) true? (JEE Adv.2015)

- a) $\frac{|\mathbf{c}|}{2} - |\mathbf{a}| = 2$
- b) $\frac{|\mathbf{c}|}{2} + |\mathbf{a}| = 30$
- c) $|\mathbf{a} \times \mathbf{b}| = \sqrt[48]{3}$
- d) $\mathbf{a} \cdot \mathbf{b} = -42$

- 12) Consider a pyramid $OPQRS$ located in the first octant ($x \geq 0, y \geq 0, z \geq 0$) with O as origin, and OP and OR along the x -axis and the y -axis respectively. The base $OPQR$ of the pyramid is a square with $OP=3$. The point S is directly above the mid-point, T of diagonal OQ such that $TS=3$. Then (JEE Adv.2016)

- a) the acute angle between OQ and OS is $\frac{\pi}{3}$
- b) the equation of the plane containing the triangle OQS is $x - y = 0$
- c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
- d) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

- 13) Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in R^3 and $\hat{w} = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$. Given that there exists a vector \mathbf{v} in R^3 such that $|\hat{u} \times \mathbf{v}| = 1$ and $\hat{w}(\hat{u} \times \mathbf{v}) = 1$. Which of the following

statement(s) is(are) correct? (JEE Adv.2016)

- a) there is exactly one choice for such \mathbf{v}
- b) There are infinitely many choices for such \mathbf{v}
- c) If \hat{u} lies in the xy -plane then $|u_1| = |u_2|$
- d) If \hat{u} lies in the xz -plane then $2|u_1| = |u_3|$

- 14) Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then, which of the following statement(s) is(are) TRUE? (JEE Adv.2018)

- a) The line of intersection of P_1 and P_2 has direction ratios 1, 2, -1
- b) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$ is perpendicular to the line of intersection of P_1 and P_2
- c) The acute angle between P_1 and P_2 is 60° .
- d) If P_3 is the plane passing through the point $(4, 2, -2)$ and perpendicular to the line of intersection of P_1 and P_2 , then the distance of the point $(2, 1, 1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$

- 15) Let L_1 and L_2 denote the lines $\mathbf{r} = \hat{i} + \lambda(-\hat{i} + 2\hat{j} + 2\hat{k})$, $\lambda \in R$ and $\mathbf{r} = \mu(2\hat{i} - \hat{j} + 2\hat{k})$, $\mu \in R$ respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following option describe(s) L_3 ? (JEE Adv.2019)

- a) $\mathbf{r} = \frac{2}{9}(4\hat{i} + \hat{j} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$
- b) $\mathbf{r} = \frac{2}{9}(2\hat{i} - \hat{j} + 2\hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$
- c) $\mathbf{r} = t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$
- d) $\mathbf{r} = \frac{1}{3}(2\hat{i} + \hat{k}) + t(2\hat{i} + 2\hat{j} - \hat{k})$, $t \in R$