Question-3.4.2.3

EE24BTECH11048 - NITHIN.K

LU Decomposition using Doolittle's Algorithm

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Problem Statement

Question: The sum of the two digits of a two-digit number is 9. Also, nine times this number is twice the number obtained by reversing the order of the digits.

$$x + y = 9 \tag{1}$$
$$9(10x + y) = 2(10y + x) \tag{2}$$
$$8x - y = 0 \tag{3}$$

The system of equations can be written as:

$$A\vec{x} = \vec{b} \tag{4}$$

where

$$A = \begin{bmatrix} 1 & 1 \\ 8 & -1 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}, \quad \vec{b} = \begin{bmatrix} 9 \\ 0 \end{bmatrix} \tag{5}$$

The LU decomposition splits A into:

$$A = LU \tag{6}$$

where **L** is lower triangular and **U** is upper triangular:

$$L = \begin{bmatrix} 1 & 0 \\ L_{21} & 1 \end{bmatrix}, \quad U = \begin{bmatrix} U_{11} & U_{12} \\ 0 & U_{22} \end{bmatrix} \tag{7}$$

LU Decomposition Computation

Elements of **U**:

$$U_{ij} = A_{ij} - \sum_{k=0}^{i-1} L_{ik} U_{kj} \tag{8}$$

Elements of **L**:

$$L_{ij} = \frac{A_{ij} - \sum_{k=0}^{j-1} L_{ik} U_{kj}}{U_{jj}} \tag{9}$$

Performing LU Decomposition, we get:

$$L = \begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix}, \quad U = \begin{bmatrix} 1 & 1 \\ 0 & -9 \end{bmatrix} \tag{10}$$

Solving the System

Forward substitution to solve $L\vec{y} = \vec{b}$:

$$\begin{bmatrix} 1 & 0 \\ 8 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix} \tag{11}$$

$$y_1 = 9, \quad y_2 = -72 \tag{12}$$

Backward substitution to solve $U\vec{x} = \vec{y}$:

$$\begin{bmatrix} 1 & 1 \\ 0 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ -72 \end{bmatrix} \tag{13}$$

$$y = 8, \quad x = 1 \tag{14}$$

Final Answer

Solution: The values of x and y are:

$$x = 1, \quad y = 8 \tag{15}$$