Question-6.5.20

EE24BTECH11048-NITHIN.K

Question:
Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

Theoretical Solution:
Total Surface Area of cylinder is

$$S = 2\pi rh + 2\pi r^2 \tag{0.1}$$

Where r is the radius of base of cylinder and h is the height of cylinder.
The Volume of cylinder is

$$V = \pi r^2 h \tag{0.2}$$

For the Given Surface Area writing h in terms of r

$$S = 2\pi rh + 2\pi r^2 \tag{0.3}$$

$$h = \frac{S - 2\pi r^2}{2\pi r} \tag{0.4}$$

Expresssing Volume as a function of r

$$V = \pi r^2 h \tag{0.5}$$

$$V = \frac{r\left(S - 2\pi r^2\right)}{2} \tag{0.6}$$

Differentiate and Find Critical Points Differentiate $V(r)$ with respect to r

$$\frac{dV}{dr} = \frac{S - 6\pi r^2}{2} \tag{0.7}$$

Setting $\frac{dV}{dr} = 0$ to find the critical points:

$$S - 6\pi r^2 = 0 \tag{0.8}$$

$$r = \sqrt{\frac{S}{6\pi}} \tag{0.9}$$

Substituting this r into our expression for h:

$$h = \frac{S - 2\pi r^2}{2\pi r} \tag{0.10}$$

$$h = \frac{6\pi r^2 - 2\pi r^2}{2\pi r} \tag{0.11}$$

$$h = 2r \tag{0.12}$$

Computational Solution:
To Maximize Volume

$$V = \frac{r\left(S - 2\pi r^2\right)}{2} \tag{0.13}$$

$$f(r) = \frac{r\left(S - 2\pi r^2\right)}{2} \tag{0.14}$$

Applying Gradient Ascent

$$r_{n+1} = r_n + \mu f'(r_n) \tag{0.15}$$

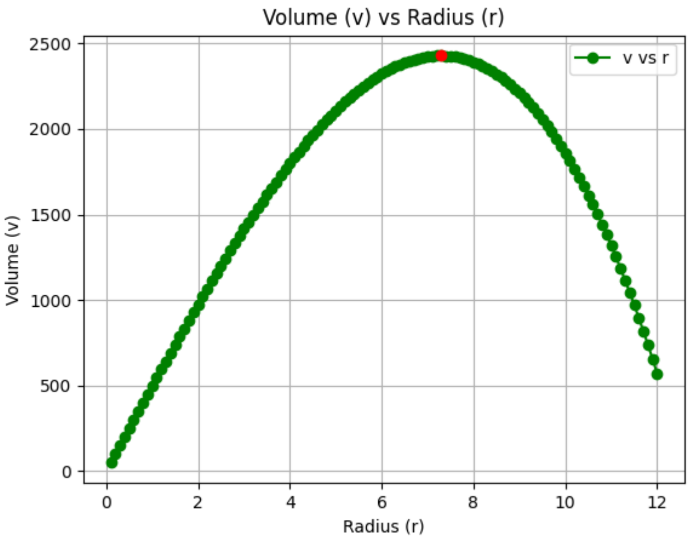
where μ is the step size

$$f'(r_n) = \frac{S - 6\pi r_n{}^2}{2} \tag{0.16}$$

Difference equation is

$$r_{n+1} = r_n + \mu\left(\frac{S - 6\pi r_n{}^2}{2}\right) \tag{0.17}$$

Using $\mu = 0.001$ and S = 1000 the radius for maximum volume was found to be 7.283656 theoretically
And Using Gradient Ascent the radius came out to be 7.28365620


Volume (v) vs Radius (r)
v vs r
Volume (v)
Radius (r)
0
500
1000
1500
2000
2500
0
2
4
6
8
10
12