Question-8.1.13

EE24BTECH11048-NITHIN.K

Question:

Area of the region bounded by the curve $y^2 = 4x$, y-axis and the line $y = 3$ is

Theoretical Solution:

The Area bounded by the parabola, the line $y = 3$ and the y-axis is given by

$$= \int_0^3 \frac{y^2}{4} dy \tag{0.1}$$

$$= \left[\frac{y^3}{12}\right]_0^3 \tag{0.2}$$

$$= \frac{27}{12} = \frac{9}{4} = 2.25 \tag{0.3}$$

Computational Solution:

Trapezoid Rule: It is a numerical method used to approximate the value of a definite integral. It is based on approximating the region under the curve by a series of trapezoids and then calculating the area of these trapezoids.

We discretize the range of y-coordinates with uniform step size $h \to 0$, such that the points are $y_0, y_1, y_2, \dots, y_n$ and $y_{n+1} = y_n + h$

Let the sum of all trapezoidal areas upto y_n be A_n and $f(y) = \frac{y^2}{4}$ then the difference equation can be formed as

$$A_n = \frac{h}{2}\left(f(y_0) + f(y_1)\right) + \frac{h}{2}\left(f(y_1) + f(y_2)\right) + \dots + \frac{h}{2}\left(f(y_{n-1}) + f(y_n)\right) \tag{0.4}$$

$$A_n = \frac{h}{2}\left(f(y_0) + f(y_1)\right) + \frac{h}{2}\left(f(y_1) + f(y_2)\right) + \dots + \frac{h}{2}\left(f(y_{n-1}) + f(y_n)\right) \tag{0.5}$$

$$A_{n+1} = A_n + \frac{h}{2}\left(f(y_{n+1}) + f(y_n)\right) \tag{0.6}$$

$$A_{n+1} = A_n + \frac{h}{2}\left(\frac{y_{n+1}^2}{4} + \frac{y_n^2}{4}\right) \tag{0.7}$$

Theoretical Area = 2.25 square units

Computational Area = 2.2500011 square units