Assignment-5

EE24BTECH11048-NITHIN.K

1 SECTION-A

1) Let the line passing through the point P$(2,-1,2)$ and Q$(5,3,4)$ meet the plane $x-y+z=4$ at the point T. Then the distance of the point R from the plane $x+2y+3z+2=0$ measured parallel to the line $\frac{x-7}{2}=\frac{y+3}{2}=\frac{z-2}{1}$ is equal to
   a) 3
   b) $\sqrt{61}$
   c) $\sqrt{31}$
   d) $\sqrt{189}$
2) Let the function f : $[0,2] \to$ R be defined as
   $$f(x)=\begin{cases} e^{min\left(x^2,x-[x]\right)} & , x\in [0\ ,\ 1) \\ e^{\left[x-\log_e x\right]} & ,x\in [1\ ,\ 2) \end{cases}$$
   where $[t]$ denotes the greatest integer less than or equal to t. Then the value of the integral $\int_0^2 xf(x)\,dx$ is
   a) $(e-1)\left(e^2+\frac{1}{2}\right)$
   b) $1+\frac{3e}{2}$
   c) $2e-\frac{1}{2}$
   d) 2e - 1
3) For a∈C, let A $=\{z\in C : Re(a+\mathbf{z}) > Im(\mathbf{a}+z)\}$ and B $=\{z\in C : Re(a+\mathbf{z}) < Im(\mathbf{a}+z)\}$. Then among the two statements:
   $(S_1) : If Re(a), Im(a) > 0$, then the set A contains all the real numbers
   $(S_2) : If Re(a), Im(a) < 0$, then the set B contains all the real numbers
   a) only S_1 is true
   b) both are false
   c) only S_2 is true
   d) both are true
4) If $\begin{vmatrix} x+1 & x & x \\ x & x+\lambda & x \\ x & x & x+\lambda^2 \end{vmatrix}=\frac{9}{8}(103x+81)$, then $\lambda, \frac{\lambda}{3}$ are the roots of the equation
   a) $4x^2-24x-27=0$
   b) $4x^2+24x+27=0$
   c) $4x^2-24x+27=0$
   d) $4x^2+24x-27=0$
5) The domain of the function $f(x)=\frac{1}{\sqrt{[x]^2-3[x]-10}}$ is (where $[x]$ denotes the greatest integer less than or equal to x)
   a) $(-\infty\ ,\ -3]\cup[6\ ,\ \infty)$
   b) $(-\infty\ ,\ -2]\cup[5\ ,\ \infty)$

c) $(-\infty\ ,\ -3] \cup [5\ ,\ \infty)$
d) $(-\infty\ ,\ -2] \cup [6\ ,\ \infty)$

2 SECTION-B

1) If A is the area in the first quadrant enclosed by the curve C : $2x^2 - y + 1 = 0$, the tangent to C at the point $(1, 3)$ and the line $x + y = 1$, then the value of 60A is
2) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. Then the number of functions $f : A \to B$ satisfying $f(1) + f(2) = f(4) - 1$ is equal to
3) Let the tangent to the parabola $y^2 = 12x$ at the point $(3, \alpha)$ be perpendicular to the line $2x + 2y = 3$. Then the square of distance of the point $(6, -4)$ from the normal to the hyperbola $\alpha^2 x^2 - 9y^2 = 9\alpha^2$ at its point $(\alpha - 1, \alpha + 2)$ is equal to
4) For $k \in N$, if the sum of the series $1 + \frac{4}{k} + \frac{8}{k^2} + \frac{13}{k^3} + \frac{19}{k^4} + ...$ is 10, then the value of k is
5) Let the line $l : x = \frac{1-y}{-2} = \frac{z-3}{\lambda}, \lambda \in R$ meet the plane $P : x + 2y + 3z = 4$ at the point (α, β, γ). If the angle between the line l and the plane P is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then $\alpha + 2\beta + 6\gamma$ is equal to
6) The number of points where the curve $f(x) = e^{8x} - e^{6x} - 3e^{4x} - e^{2x} + 1, x \in R$ cuts x-axis, is equal to
7) If the line $l_1 : 3y - 2x = 3$ is the angular bisector of the line $l_2 : x - y + 1 = 0$ and $l_3 : \alpha x + \beta y + 17$, then $\alpha^2 + \beta^2 - \alpha - \beta$ is equal to
8) Let the probability of getting head for a biased coin be $\frac{1}{4}$. It is tossed repeatedly until a head appears. Let N be the number of tosses required. If the probability that the equation $64x^2 + 5Nx + 1 = 0$ has no real root is $\frac{p}{q}$, where p and q are co-prime, then $q - p$ is equal to
9) Let $\mathbf{a} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + \mathbf{j} - \mathbf{k}$. If $\mathbf{c}$ is a vector such that $\mathbf{a} \cdot \mathbf{c} = 11$, $\mathbf{b} \cdot (\mathbf{a} \times \mathbf{c}) = 27$ and $\mathbf{b} \cdot \mathbf{c} = -\sqrt{3}|\mathbf{b}|$, then $|\mathbf{a} \times \mathbf{c}|^2$ is equal to
10) Let $S = \left\{z \in C - \{i, 2i\} : \frac{z^2+8iz-15}{z^2-3iz-2} \in R\right\}$. If $\alpha - \frac{13}{11}i \in S$, then $242\alpha^2$ is equal to