EE24BTECH11048-NITHIN.K

Question:

Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2 is

Solution:

FUNCTION	FORMULA
g(x)	$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0$
The points of intersection	$L: \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in \mathbb{R}$
of the line L with the conic	$\kappa_{i} = \frac{1}{\mathbf{m}^{T}\mathbf{V}\mathbf{m}} \left(-\mathbf{m}^{T} \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^{T} \left(\mathbf{V}\mathbf{h} + \mathbf{u} \right)^{2} \right] - g(\mathbf{h}) \left(\mathbf{m}^{T}\mathbf{V}\mathbf{m} \right)} \right)$
section as above are	
given by $\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$	

TABLE 0: Variables Used

On comparing g(x) and $x^2 + y^2 - 4 = 0$ the parameters of the circle are

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{1}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$f = -4 \tag{3}$$

(4)

The area bounded by x = 0, x = 2 and the circle in the first quadrant is

$$\int_0^2 \sqrt{4 - x^2} dx \tag{5}$$

for indefinite integration of above form we get

$$\int \sqrt{4 - x^2} dx = 2\sin^{-1}\frac{x}{2} + x\sqrt{4 - x^2} + c \tag{6}$$

$$2\sin^{-1}\frac{2}{2} + 2\sqrt{4 - 2^2} - 2\sin^{-1}\frac{0}{2} - 0\sqrt{4 - 0^2}$$
 (7)

$$\pi$$
 (8)

Hence the enclosed area is π square units.

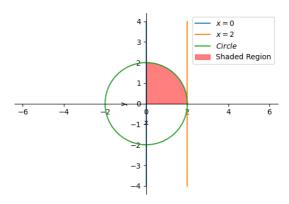


Fig. 0: Enclosed Area