

# Assignment-2

## CHAPETR-15

### Matrices and Determinants

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#### I. SECTION-B

- 1) Let  $A = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$ . The only correct statement about the matrix A is [2004]
- $A^2 = I$
  - $A = (-1)I$ , where I is a unit matrix
  - $A^{-1}$  does not exist
  - A is a zero matrix
- 2) Let  $A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$ , and  $10B = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$ . If B is the inversion of matrix A, then  $\alpha$  is [2004]
- 5
  - 1
  - 2
  - 2
- 3) If  $a_1, a_2, a_3, \dots, a_n, \dots$  are in G.P, then the value of the determinant [2004]
- $$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix},$$
- is
- 2
  - 1
  - 2
  - 0
- 4) If  $A^2 - A + I = 0$ , then the inverse of A is [2005]
- $A+I$
  - A
  - $A-I$
  - $I-A$
- 5) The system of equations
- $$\alpha x + y + z = \alpha - 1$$
- $$x + \alpha y + z = \alpha - 1$$
- $$x + y + \alpha z = \alpha - 1$$
- has infinite solutions, if  $\alpha$  is [2005]
- 2
  - either -2 or 1
  - not -2
  - 1
- 6) If  $a^2 + b^2 + c^2 = -2$  and [2005]
- $$f(x) = \begin{vmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix}$$
- then  $f(x)$  is a polynomial of degree
- 1
  - 0
  - 3
  - 2
- 7) If  $a_1, a_2, a_3, \dots, a_n, \dots$  are in G.P, then the determinant [2005]
- $$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}$$
- is equal to
- 1
  - 0
  - 4
  - 2
- 8) If A and B are square matrices of size n x n such that  $A^2 - B^2 = (A - B)(A + B)$ , then which of the following will be always true? [2006]
- $A = B$
  - $AB = BA$
  - either of A or B is zero matrix
  - either of A or B is identity matrix
- 5) The system of equations
- $$\alpha x + y + z = \alpha - 1$$

- 9) Let  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$  and  $B = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ ,  $a, b \in \mathbb{N}$ . Then [2006]
- there cannot exist any  $B$  such that  $AB = BA$
  - there exist more than one but finite number of  $B$ 's such that  $AB = BA$
  - there exists exactly one  $B$  such that  $AB = BA$
  - there exist infinitely many  $B$ 's such that  $AB = BA$
- 10) If  $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$  for  $x \neq 0, y \neq 0$ , then  $D$  is [2007]
- divisible by  $x$  but not  $y$
  - divisible by  $y$  but not  $x$
  - divisible neither by  $x$  nor  $y$
  - divisible by both  $x$  and  $y$
- 11) Let  $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$ . If  $|A^2| = 25$ , then  $|\alpha|$  equals [2007]
- $\frac{1}{5}$
  - 5
  - $5^2$
  - 1
- 12) Let  $A$  be a  $2 \times 2$  matrix with real entries. Let  $I$  be the  $2 \times 2$  identity matrix. Denote by  $\text{tr}(A)$ , the sum of diagonal entries of  $A$ . Assume that  $A^2 = I$ . [2008]
- Statement-1 : If  $A \neq I$  and  $A \neq -I$ , then  $\det(A) = -1$
- Statement-2 : If  $A \neq I$  and  $A \neq -I$ , then  $\text{tr}(A) \neq 0$ .
- Statement-1 is false, Statement-2 is true
  - Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
  - Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
  - Statement-1 is true, Statement-2 is false
- 13) Let  $a, b, c$  be any real numbers. Suppose that there are real numbers  $x, y, z$  not all zero such that  $x = cy + bz$ ,  $y = az + cx$ , and  $z = bx + ay$ . Then  $a^2 + b^2 + c^2 + 2abc$  is equal to [2008]
- 2
  - 1
  - 0
  - 1
- 14) Let  $A$  be a square matrix all of whose entries are integers. Then which of the following is true? [2008]
- If  $\det(A) \neq \pm 1$ , then  $A^{-1}$  exists but all its entries are not necessarily integers
  - If  $\det(A) \neq \pm 1$ , then  $A^{-1}$  exists and all its entries are non integers
  - If  $\det(A) = \pm 1$ , then  $A^{-1}$  exists but all its entries are integers
  - If  $\det(A) = \pm 1$ , then  $A^{-1}$  need not exist
- 15) Let  $A$  be a  $2 \times 2$  matrix
- Statement-1:  $\text{adj}(\text{adj} A) = A$
- Statement-2:  $|\text{adj} A| = |A|$  [2009]
- Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
  - Statement-1 is true, Statement-2 is false
  - Statement-1 is false, Statement-2 is true
  - Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1