

Assignment-2

CHAPETR-15

Matrices and Determinants

EE24BTECH11048-NITHIN.K

1 SECTION-B

1) Let $\mathbf{A} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$. The only correct statement about the matrix A is [2004]

- a) $A^2 = I$
- b) $A = (-1)I$, where I is a unit matrix
- c) A^{-1} does not exist
- d) A is a zero matrix

2) Let $\mathbf{A} = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix}$, and $10\mathbf{B} = \begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & \alpha \\ 1 & -2 & 3 \end{pmatrix}$. If B is the inversion of matrix A, then α is [2004]

- a) 5
- b) -1
- c) 2
- d) -2

3) If $a_1, a_2, a_3, \dots, a_n, \dots$ are in G.P, then the value of the determinant [2004]

$$\begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix}, \text{ is}$$

- a) -2
- b) 1
- c) 2
- d) 0

4) If $A^2 - A + I = 0$, then the inverse of A is [2005]

- a) $A+I$
- b) A
- c) $A-I$
- d) $I-A$

5) The system of equations

$$\begin{aligned} \alpha x + y + z &= \alpha - 1 \\ x + \alpha y + z &= \alpha - 1 \end{aligned}$$

$$x + y + \alpha z = \alpha - 1$$

has infinite solutions, if α is

[2005]

- a) -2
- b) either -2 or 1
- c) not -2
- d) 1

6) If $a^2 + b^2 + c^2 = -2$ and

$$f(x) = \begin{vmatrix} 1 + a^2x & (1 + b^2)x & (1 + c^2)x \\ (1 + a^2)x & 1 + b^2x & (1 + c^2)x \\ (1 + a^2)x & (1 + b^2)x & 1 + c^2x \end{vmatrix} \text{ then } f(x) \text{ is a polynomial of degree [2005]}$$

- a) 1
- b) 0
- c) 3
- d) 2

7) If $a_1, a_2, a_3, \dots, a_n, \dots$ are in G.P, then the determinant

[2005]

$$\Delta = \begin{vmatrix} \log a_n & \log a_{n+1} & \log a_{n+2} \\ \log a_{n+3} & \log a_{n+4} & \log a_{n+5} \\ \log a_{n+6} & \log a_{n+7} & \log a_{n+8} \end{vmatrix} \text{ is equal to}$$

- a) 1
- b) 0
- c) 4
- d) 2

8) If A and B are square matrices of size $n \times n$ such that $A^2 - B^2 = (A - B)(A + B)$, then which of the following will be always true? [2006]

- a) $A = B$
- b) $AB = BA$
- c) either of A or B is zero matrix
- d) either of A or B is identity matrix

9) Let $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$, $a, b \in \mathbb{N}$. Then

[2006]

- a) there cannot exist any B such that $AB = BA$
- b) there exist more than one but finite number of B's such that $AB = BA$
- c) there exists exactly one B such that $AB = BA$
- d) there exist infinitely many B's such that $AB = BA$

10) If $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix}$ for $x \neq 0, y \neq 0$, then D is

[2007]

- a) divisible by x but not y
- b) divisible by y but not x
- c) divisible neither by x nor y
- d) divisible by both x and y

11) Let $A = \begin{vmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{vmatrix}$. If $|A^2| = 25$, then $|\alpha|$ equals [2007]

- a) $\frac{1}{5}$
- b) 5
- c) 5^2
- d) 1

12) Let A be a 2×2 matrix with real entries. Let I be the 2×2 identity matrix. Denote by $\text{tr}(A)$, the sum of diagonal entries of A . Assume that $A^2 = I$. [2008]

Statement-1 : If $A \neq I$ and $A \neq -I$, then $\det(A) = -1$

Statement-2 : If $A \neq I$ and $A \neq -I$, then $\text{tr}(A) \neq 0$.

- a) Statement-1 is false, Statement-2 is true
 - b) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1
 - c) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
 - d) Statement-1 is true, Statement-2 is false
- 13) Let a, b, c be any real numbers. Suppose that there are real numbers x, y, z not all zero such that $x = cy + bz$, $y = az + cx$, and $z = bx + ay$. Then $a^2 + b^2 + c^2 + 2abc$ is equal to [2008]
- a) 2
 - b) -1
 - c) 0
 - d) 1

14) Let A be a square matrix all of whose entries are integers. Then which of the following is true? [2008]

- a) If $\det(A) \neq \pm 1$, then A^{-1} exists but all its entries are not necessarily integers
- b) If $\det(A) \neq \pm 1$, then A^{-1} exists and all its entries are non integers
- c) If $\det(A) = \pm 1$, then A^{-1} exists but all its entries are integers
- d) If $\det(A) = \pm 1$, then A^{-1} need not exist

15) Let A be a 2×2 matrix

Statement-1: $\text{adj}(\text{adj}A) = A$

Statement-2: $|\text{adj}A| = |A|$ [2009]

- a) Statement-1 is true, Statement-2 is true; Statement-2 is not a correct explanation for Statement-1
- b) Statement-1 is true, Statement-2 is false
- c) Statement-1 is false, Statement-2 is true
- d) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1