Assignment-3
CHAPETR-20
Vector Algebra and Three Dimensional Geometry

EE24BTECH11048-NITHIN.K

1 D:MCQs with One or More than One Correct

1) Let a and b be two non-collinear unit vectors. If $u = a - (a \cdot b)\, b$ and $v = a \times b$, then $|v|$ is (1999-3 Marks)
   a) $|u|$
   b) $|u| + |u \cdot a|$
   c) $|u| + |u \cdot b|$
   d) $|u| + u \cdot (a + b)$
2) Let **A** be vector parallel to line of intersection of planes P_1 and P_2. Plane P_1 is parallel to the vectors $2\hat{j} + 3\hat{k}$ and $4\hat{j} - 3\hat{k}$ and that P_2 is parallel to $\hat{j} - \hat{k}$ and $3\hat{i} + 3\hat{j}$, then the angle between vector **A** and a given vector $2\hat{i} + \hat{j} - 2\hat{k}$ is (2006-5M,-1)
   a) $\frac{\pi}{2}$
   b) $\frac{\pi}{4}$
   c) $\frac{\pi}{6}$
   d) $\frac{3\pi}{4}$
3) The vector(s) which is/are coplanar with vectors $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$, and perpendicular to the vector $\hat{i} + \hat{j} + \hat{k}$ is/are (2011)
   a) $\hat{j} - \hat{k}$
   b) $\hat{i} + \hat{j}$
   c) $\hat{i} - \hat{j}$
   d) $\hat{j} + \hat{k}$
4) If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar,then the plane(s) containing these two lines is(are) (2012)
   a) $y + 2z = -1$
   b) $y + z = -1$
   c) $y - z = -1$
   d) $y - 2z = -1$
5) A line L passing through the origin is perpendicular to the lines

$$l_1 : (3 + t)\,\hat{i} + (1 + 2t)\,\hat{j} + (4 + 2t)\,\hat{k}, -\infty < t < \infty$$
$$l_2 : (3 + 2s)\,\hat{i} + (3 + 2s)\,\hat{j} + (2 + s)\,\hat{k}, -\infty < s < \infty$$

Then, the coordinate(s) of the point(s) on l_2 at a distance of $\sqrt{17}$ from the point of intersection of L and l_1 is(are) (JEE Adv.2013)

a) $\left(\frac{7}{3}, \frac{7}{3}, \frac{5}{3}\right)$
b) $(-1, -1, 0)$
c) $(1, 1, 1)$
d) $\left(\frac{7}{9}, \frac{7}{9}, \frac{8}{9}\right)$

6) Two lines $L_1 : x = 5, \frac{y}{3-\alpha} = \frac{z}{-2}$ and $L_2 : x = \alpha, \frac{y}{-1} = \frac{z}{2-\alpha}$ are coplanar. Then α can take value(s) (JEE Adv.2013)

a) 1
b) 2
c) 3
d) 4

7) Let $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ be three vectors each of magnitude $\sqrt{2}$ and the angle between each pair of them is $\frac{\pi}{3}$. If $\mathbf{a}$ is a non-zero vector perpendicular to $\mathbf{x}$ and $\mathbf{y} \times \mathbf{z}$ and $\mathbf{b}$ is a non-zero vector perpendicular to $\mathbf{y}$ and $\mathbf{z} \times \mathbf{x}$, then (JEE Adv.2014)

a) $\mathbf{b} = (\mathbf{b} \cdot \mathbf{z})(\mathbf{z} - \mathbf{x})$
b) $\mathbf{a} = (\mathbf{a} \cdot \mathbf{y})(\mathbf{y} - \mathbf{z})$
c) $\mathbf{a} \cdot \mathbf{b} = -(\mathbf{a} \cdot \mathbf{y})(\mathbf{b} \cdot \mathbf{z})$
d) $\mathbf{a} = -(\mathbf{a} \cdot \mathbf{y})(\mathbf{z} - \mathbf{y})$

8) From a point $P(\lambda, \lambda, \lambda)$, perpendicular PQ and PR are drawn respectively on the lines $y = x, z = 1$ and $y = -x, z = -1$. If P is such that $\angle QPR$ is a right angle, then the possible value(s) of λ is/(are) (JEE Adv.2014)

a) $\sqrt{2}$
b) 1
c) -1
d) -$\sqrt{2}$

9) In R^3, consider the planes $P_1 : y = 0$ and $P_2 : x + z - 1$.Let P_3 be the plane different from P_1 and P_2 which passes through the intersection of P_1 and P_2. If the distance of the point$(0, 1, 0)$ from P_3 is 1 and the distance of point (α, β, γ) from P_3 is 2, then which of the following relation is(are) true (JEE Adv.2015)

a) $2\alpha + \beta + 2\gamma + 2 = 0$
b) $2\alpha - \beta + 2\gamma + 4 = 0$
c) $2\alpha + \beta + 2\gamma - 10 = 0$
d) $2\alpha - \beta + 2\gamma - 8 = 0$

10) In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a costant distance from two planes $P_1 : x + 2y - z + 1 = 0$ and $P_2 : 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendicular drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?
(JEE Adv.2015)

a) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$
b) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$
c) $\left(-\frac{5}{6}, 0, \frac{2}{3}\right)$

d) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

11) Let $\triangle PQR$ be a triangle. Let $\mathbf{a} = \mathbf{QR}$, $\mathbf{b} = \mathbf{RP}$ and $\mathbf{c} = \mathbf{PQ}$. If $|\mathbf{a}| = 12$, $|\mathbf{b}| = 4\sqrt{3}$, $\mathbf{b} \cdot \mathbf{c} = 24$, then which of the following is(are)true? (JEE Adv.2015)
   a) $\frac{|\mathbf{c}|^2}{2} - |\mathbf{a}| = 12$
   b) $\frac{|\mathbf{c}|^2}{2} + |\mathbf{a}| = 30$
   c) $|\mathbf{a} \times \mathbf{b} + \mathbf{c} \times \mathbf{a}| = 48\sqrt{3}$
   d) $\mathbf{a} \cdot \mathbf{b} = -72$

12) Consider a pyramid OPQRS located in the first octant $(x \geq 0, y \geq 0, z \geq 0)$ with O as origin, and OP and OR along the x-axis and the y-axis respectively. The base OPQR of the pyramid is a square with OP=3. The point S is directly above the mid-point, T of diagonal OQ such that TS=3. Then (JEE Adv.2016)
   a) the acute angle between OQ and OS is $\frac{\pi}{3}$
   b) the equation of the plane containg the triangle OQS is $x - y = 0$
   c) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$
   d) the perpendicular distance from O to the staright line containing RS is $\sqrt{\frac{15}{2}}$

13) Let $\hat{u} = u_1\hat{i} + u_2\hat{j} + u_3\hat{k}$ be a unit vector in R^3 and $\hat{w} = \frac{1}{\sqrt{6}}\left(\hat{i} + \hat{j} + 2\hat{k}\right)$. Given that there exists a vector $\mathbf{v}$ in R^3 such that $|\hat{u} \times \mathbf{v}| = 1$ and $\hat{w}\left(\hat{u} \times \mathbf{v}\right) = 1$. Which of the following statement(s) is(are) correct? (JEE Adv.2016)
   a) there is exactly one choice for such $\mathbf{v}$
   b) There are infinitely many choices for such $\mathbf{v}$
   c) If $\hat{u}$ lies in the xy-plane then $|u_1| = |u_2|$
   d) If $\hat{u}$ lies in the xz-plane then $2|u_1| = |u_3|$

14) Let $P_1 : 2x + y - z = 3$ and $P_2 : x + 2y + z = 2$ be two planes. Then,which of the following statement(s) is(are) TRUE? (JEE Adv.2018)
   a) The line of intersection ofP_1 and P_2 has direction ratios 1,2,-1
   b) The line $\frac{3x-4}{9} = \frac{1-3y}{9} = \frac{z}{3}$
   is perpendicular to the line of intersection of P_1 and P_2
   c) The acute angle between P_1 and P_2 is $60°$.
   d) If P_3 is the plane passing through the point $(4, 2, -2)$ and perpendicular to the line of intersection of P_1 and P_2,then the distance of the point $(2, 1, 1)$ from the plane P_3 is $\frac{2}{\sqrt{3}}$

15) Let L_1 and L_2 denote the lines
   $\mathbf{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right), \lambda \in R$ and
   $\mathbf{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right), \mu \in R$
   respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following option describe(s) L_3? (JEE Adv.2019)
   a) $\mathbf{r} = \frac{2}{9}\left(4\hat{i} + \hat{j} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$
   b) $\mathbf{r} = \frac{2}{9}\left(2\hat{i} - \hat{j} + 2\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$
   c) $\mathbf{r} = t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$

d) $\mathbf{r} = \frac{1}{3}\left(2\hat{i} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in R$