9-9.2-24

EE24BTECH11048-NITHIN.K

Question:
Area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0 and x = 2 is
Solution:

FUNCTION	FORMULA
$g(x)$	$\mathbf{x}^{\mathbf{T}}\mathbf{V}\mathbf{x} + 2\mathbf{u}^{\mathbf{T}}\mathbf{x} + f = 0$
The points of intersection of the line L with the conic section as above are given by $\mathbf{x}_i = \mathbf{h} + \kappa_i \mathbf{m}$	$L : \mathbf{x} = \mathbf{h} + \kappa \mathbf{m}, \kappa \in \mathbb{R}$ $\kappa_i = \frac{1}{\mathbf{m}^{\mathbf{T}}\mathbf{V}\mathbf{m}} \left(-\mathbf{m}^{\mathbf{T}} (\mathbf{V}\mathbf{h} + \mathbf{u}) \pm \sqrt{\left[\mathbf{m}^{\mathbf{T}} (\mathbf{V}\mathbf{h} + \mathbf{u})^2\right] - g(\mathbf{h}) (\mathbf{m}^{\mathbf{T}}\mathbf{V}\mathbf{m})} \right)$

TABLE 0: Variables Used

On comparing $g(x)$ and $x^2 + y^2 - 4 = 0$ the parameters of the circle are

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{1}$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2}$$

$$f = -4 \tag{3}$$

$$\tag{4}$$

The area bounded by $x = 0$, $x = 2$ and the circle in the first quadrant is

$$\int_0^2 \sqrt{4 - x^2} dx \tag{5}$$

for indefinite integration of above form we get

$$\int \sqrt{4 - x^2} dx = 2 \sin^{-1} \frac{x}{2} + x \sqrt{4 - x^2} + c \tag{6}$$

$$\int_0^2 \sqrt{4 - x^2} dx = 2 \sin^{-1} \frac{2}{2} + 2 \sqrt{4 - 2^2} - 2 \sin^{-1} \frac{0}{2} - 0 \sqrt{4 - 0^2} \tag{7}$$

$$\int_0^2 \sqrt{4 - x^2} dx = \pi \tag{8}$$

Hence the enclosed area is π square units.

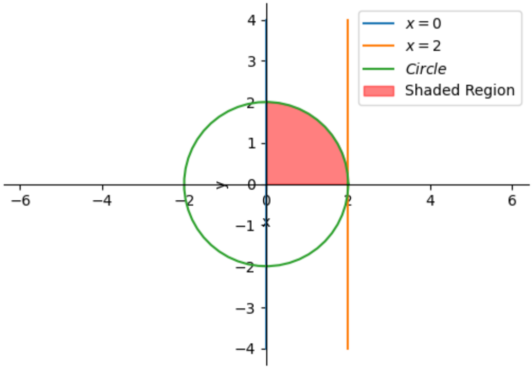

Fig. 1: Enclosed Area