

Question-9.6.13

EE24BTECH11048-NITHIN.K

Question:

Find a particular solution for the differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$; $y = 0$ when $x = \frac{\pi}{3}$

Solution:

The original Differential Equation is

$$\frac{dy}{dx} + 2y \tan x = \sin x \quad (0.1)$$

$$\frac{dy}{dx} = \sin x - 2y \tan x \quad (0.2)$$

By Euler's approximation

$$\frac{y_{n+1} - y_n}{dx} = \sin x_n - 2y_n \tan x_n \quad (0.3)$$

Hence the difference equation is

$$y_{n+1} = y_n + dx (\sin x_n - 2y_n \tan x_n) \quad (0.4)$$

Solution for the First Order Differential Equation:

$$\frac{dy}{dx} + 2y \tan x = \sin x \quad (0.5)$$

$$\frac{dy}{dx} + P(x)y = Q(x) \quad (0.6)$$

$$P(x) = 2 \tan x, Q(x) = \sin x \quad (0.7)$$

$$IF = e^{\int 2 \tan x dx} = e^{2 \ln |\sec x|} = (\sec x)^2 \quad (0.8)$$

By multiplying the Integrating Factor on both sides

$$(\sec x)^2 \frac{dy}{dx} + 2y (\sec x)^2 \tan(x) = (\sec x)^2 \sin x \quad (0.9)$$

$$\frac{d}{dx} [(\sec x)^2 y] = (\sec x)^2 \sin x \quad (0.10)$$

and Integrating on both sides

$$(\sec x)^2 y = \int (\sec x)^2 \sin(x) dx \quad (0.11)$$

$$y = \cos x - 2\cos x^2 \quad (0.12)$$

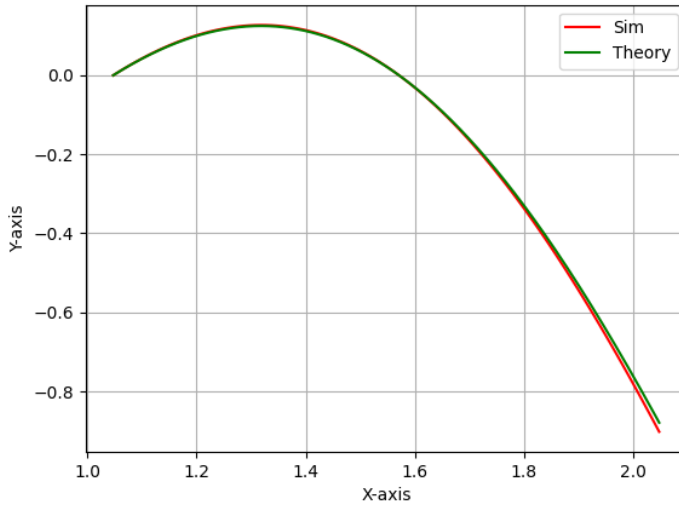


Fig. 0.1: Simulation VS Theoretical Plot.