Question-9.3.11.3

EE24BTECH11048-NITHIN.K

Question:
Solve the differential equation $\frac{d^2y}{dx^2} + 1 = 0$ with initial conditions $y(0) = 1$ and $y'(0) = 0$
Theoritical Solution:
Laplace Transform :

$$\mathcal{L}(f(t)) = \int_0^\infty e^{-st} f(t)\, dt \tag{0.1}$$

Properties of Laplace tranform

$$\mathcal{L}(y'') = s^2\mathcal{L}(y) - sy(0) - y'(0) \tag{0.2}$$

$$\mathcal{L}(1) = \frac{1}{s} \tag{0.3}$$

$$\mathcal{L}^{-1}\left(\frac{2}{s^3}\right) = x^2 u(x) \tag{0.4}$$

$$\mathcal{L}(cf(t)) = c\mathcal{L}(f(t)) \tag{0.5}$$

Applying the properties to the given equation

$$y'' + 1 = 0 \tag{0.6}$$

$$\mathcal{L}(y'') + \mathcal{L}(1) = 0 \tag{0.7}$$

$$s^2\mathcal{L}(y) - sy(0) - y'(0) + \frac{1}{s} = 0 \tag{0.8}$$

Substituting the initial conditions gives

$$s^3\mathcal{L}(y) + 1 = 0 \tag{0.9}$$

$$\mathcal{L}(y) = \frac{-1}{s^3} \tag{0.10}$$

$$y = \mathcal{L}^{-1}\left(\frac{-1}{s^3}\right) \tag{0.11}$$

$$y = \frac{-1}{2}\mathcal{L}^{-1}\left(\frac{2}{s^3}\right) \tag{0.12}$$

$$y = \frac{-1}{2} x^2 u(x) \tag{0.13}$$

Computational Solution:

Finite Difference Approximation

The second derivative $\frac{d^2y}{dx^2}$is replaced by:

$$\frac{d^2y}{dx^2} \approx \frac{y_{n+1} - 2y_n + y_{n-1}}{\Delta x^2} \tag{0.14}$$

Substitute this into the differential equation:

$$\frac{y_{n+1} - 2y_n + y_{n-1}}{h^2} + 1 = 0 \tag{0.15}$$

$$y_{n+1} - 2y_n + y_{n-1} + h^2 = 0 \tag{0.16}$$

$$y_{n+1} = 2y_n - y_{n-1} - h^2 \tag{0.17}$$

If $y(0) = 0$, then $y_0 = 0$.

To compute y_1, use the derivative approximation:

$$y'(0) \approx \frac{y_1 - y_0}{h} \tag{0.18}$$

$$y_1 = 0 \tag{0.19}$$

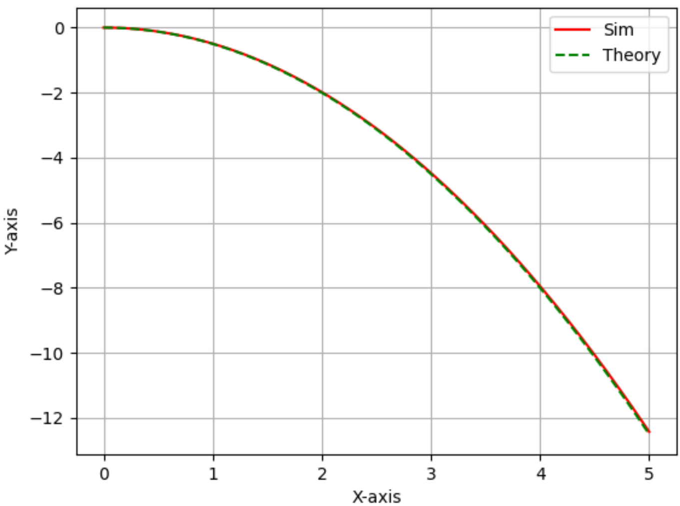

Fig. 0.1: Plot