Assignment-1
CHAPETR-11
Limits, Continuity and Differentiability

EE24BTECH11048-NITHIN.K

1 C:MCQs with One Correct Answer

1) Let $g(x) = \frac{(x-1)^n}{logcos^m(x-1)}$; $0 < x < 2$, m and n are integers, $m \neq 0$, $n > 0$, and let p be the left hand derivative of $|x-1|$ at $x = 1$. If $\lim_{x \to 1^+} g(x) = p$, then (2008)
   a) $n = 1, m = 1$
   b) $n = 1, m = -1$
   c) $n = 2, m = 2$
   d) $n > 2, m = n$
2) If $\lim_{x \to 0}[1 + xln\left(1 + b^2\right)]^{\frac{1}{x}} = 2bsin^2\theta$, $b > 0$ and $\theta \in (-\pi\ ,\ \pi]$, then the value of θ is (2011)
   a) $\pm\frac{\pi}{4}$
   b) $\pm\frac{\pi}{3}$
   c) $\pm\frac{\pi}{6}$
   d) $\pm\frac{\pi}{2}$
3) If $\lim_{x \to \infty}(\frac{x^2+x+1}{x+1} - ax - b) = 4$, then (2012)
   a) $a = 1, b = 4$
   b) $a = 1, b = -4$
   c) $a = 2, b = -3$
   d) $a = 2, b = 3$
4) Let $f(x) = \begin{cases} x^2 \left|cos\frac{\pi}{x}\right| & , x \neq 0 \\ 0 & , x = 0 \end{cases}$, x $\in$ R then f is (2012)
   a) differentiable both at $x = 0$ and at $x = 2$
   b) differentiable at $x = 0$ but not differentiable at $x = 2$
   c) not differentiable at $x = 0$ but differentiable at $x = 2$
   d) differentiable neither at $x = 0$ nor at $x = 2$
5) Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation $(\sqrt[3]{1+a} - 1)x^2 + (\sqrt[2]{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$ where $a > -1$. then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$ are (2012)
   a) $-\frac{5}{2}$ and 1
   b) $-\frac{1}{2}$ and 1
   c) $-\frac{7}{2}$ and 2
   d) $-\frac{9}{2}$ and 3

2 D:MCQs with One or More than One Correct

6) If $x + |y| = 2y$, then y as a function of x is (1984-3marks)
   a) defined for all real x
   b) continuous at $x = 0$
   c) differentiable for all x
   d) such that $\frac{dy}{dx} = \frac{1}{3}$ for$x < 0$
7) If $f(x) = x\left(\sqrt{x} - \sqrt{x+1}\right)$, then- (1985-2marks)
   a) $f(x)$ is continuous but not differentiable at $x = 0$
   b) $f(x)$ is differentiable at $x = 0$
   c) $f(x)$ is not differentiable at $x = 0$
   d) none of these
8) The function $f(x) = 1 + |sinx|$ is (1986-2marks)
   a) continuous nowhere
   b) continuous everywhere
   c) differentiable nowhere
   d) not differentiable at $x = 0$
   e) not differentiable at infinite number of points
9) Let $[x]$ denote the greatest integer less than or equal to x. If $f(x) = [xsin\pi x]$, then $f(x)$ is (1986-2marks)
   a) continuous at $x = 0$
   b) continuous in $(-1, 0)$
   c) differentiable at $x = 1$
   d) differentiable in $(-1, 1)$
   e) none of these
10) The set of all points where the function $f(x) = \frac{x}{(1+|x|)}$ is differentiable, is (1987-2marks)
    a) $(-\infty, \infty)$
    b) $[0 , \infty)$
    c) $(-\infty, 0) \bigcup (0, \infty)$
    d) $(0, \infty)$
    e) None
11) The function
    $f(x) = \begin{cases} |x-3| & ,x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4}, & ,x < 1 \end{cases}$,is (1988-2marks)
    a) continuous at $x = 1$
    b) differentiable at $x = 1$
    c) continuous at $x = 3$
    d) differentiable at $x = 3$
12) If $f(x) = \frac{1}{2}x - 1$, then on the interval $[0, \pi]$ (1989-2marks)
    a) $tan[f(x)]$ and $\frac{1}{f(x)}$ are both continuous
    b) $tan[f(x)]$ and $\frac{1}{f(x)}$ are both discontinuous
    c) $tan[f(x)]$ and $f^{-1}x$ are both continuous

d) $tan\left[f(x)\right]$ is continuous but $\frac{1}{f(x)}$ is not

13) The value of $\lim_{x\to 0}\frac{\sqrt{\frac{1}{2}(1-cos2x)}}{x}$ (1991-2marks)

a) 1
b) -1
c) 0
d) none of these

14) The following functions are continuous on $(0,\pi)$ (1991-2marks)

a) $tanx$
b) $\int_0^x tsin\frac{1}{t}dt$
c) $\begin{cases} 1 & ,0 < x \le \frac{3\pi}{4} \\ 2sin\frac{2}{9}x & ,\frac{3\pi}{4} < x < \pi \end{cases}$
d) $\begin{cases} xsinx & ,0 < x \le \frac{\pi}{2} \\ \frac{\pi}{2}sin(\pi + x) & ,\frac{\pi}{2} < x < \pi \end{cases}$

15) Let $f(x) = \begin{cases} 0 & ,x < 0 \\ x^2 & ,x \le 0 \end{cases}$ then for all x (1994)

a) f' is differentiable
b) f is differentiable
c) f' is continuous
d) f is continuous