

# Assignment-1

## CHAPETR-11

### Limits, Continuity and Differentiability

EE24BTECH11048-NITHIN.K

#### I. C:MCQs WITH ONE CORRECT ANSWER

1) Let  $g(x) = \frac{(x-1)^n}{\log \cos^m(x-1)}$ ;  $0 < x < 2$ ,  $m$  and  $n$  are integers,  $m \neq 0$ ,  $n > 0$ , and let  $p$  be the left hand derivative of  $|x-1|$  at  $x = 1$ . If  $\lim_{x \rightarrow 1^+} g(x) = p$ , then (2008)

- a)  $n = 1, m = 1$
- b)  $n = 1, m = -1$
- c)  $n = 2, m = 2$
- d)  $n > 2, m = n$

2) If  $\lim_{x \rightarrow 0} [1 + x \ln(1 + b^2)]^{\frac{1}{x}} = 2b \sin^2 \theta$ ,  $b > 0$  and  $\theta \in (-\pi, \pi]$ , then the value of  $\theta$  is (2011)

- a)  $\pm \frac{\pi}{4}$
- b)  $\pm \frac{\pi}{3}$
- c)  $\pm \frac{\pi}{6}$
- d)  $\pm \frac{\pi}{2}$

3) If  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + x + 1}{x + 1} - ax - b \right) = 4$ , then (2012)

- a)  $a = 1, b = 4$
- b)  $a = 1, b = -4$
- c)  $a = 2, b = -3$
- d)  $a = 2, b = 3$

4) Let  $f(x) = \begin{cases} x^2 |\cos \frac{\pi}{x}| & , x \neq 0 \\ 0 & , x = 0 \end{cases}$ ,  $x \in \mathbb{R}$  then  $f$  is (2012)

- a) differentiable both at  $x = 0$  and at  $x = 2$
- b) differentiable at  $x = 0$  but not differentiable at  $x = 2$
- c) not differentiable at  $x = 0$  but differentiable at  $x = 2$
- d) differentiable neither at  $x = 0$  nor at  $x = 2$

5) Let  $\alpha(a)$  and  $\beta(a)$  be the roots of the equation  $(\sqrt[3]{1+a} - 1)x^2 + (\sqrt[2]{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$  where  $a > -1$ . then  $\lim_{a \rightarrow 0^+} \alpha(a)$  and  $\lim_{a \rightarrow 0^+} \beta(a)$  are (2012)

- a)  $-\frac{5}{2}$  and 1
- b)  $-\frac{1}{2}$  and 1
- c)  $-\frac{7}{2}$  and 2
- d)  $-\frac{9}{2}$  and 3

#### II. D:MCQs WITH ONE OR MORE THAN ONE CORRECT

6) If  $x + |y| = 2y$ , then  $y$  as a function of  $x$  is (1984-3marks)

- a) defined for all real  $x$
- b) continuous at  $x = 0$
- c) differentiable for all  $x$
- d) such that  $\frac{dy}{dx} = \frac{1}{3}$  for  $x < 0$

7) If  $f(x) = x(\sqrt{x} - \sqrt{x+1})$ , then- (1985-2marks)

- a)  $f(x)$  is continuous but not differentiable at  $x = 0$
- b)  $f(x)$  is differentiable at  $x = 0$
- c)  $f(x)$  is not differentiable at  $x = 0$
- d) none of these

8) The function  $f(x) = 1 + |\sin x|$  is (1986-2marks)

- a) continuous nowhere
- b) continuous everywhere
- c) differentiable nowhere
- d) not differentiable at  $x = 0$
- e) not differentiable at infinite number of points

9) Let  $[x]$  denote the greatest integer less than or equal to  $x$ . If  $f(x) = [x \sin \pi x]$ , then  $f(x)$  is (1986-2marks)

- a) continuous at  $x = 0$
- b) continuous in  $(-1, 0)$
- c) differentiable at  $x = 1$

- d) differentiable in  $(-1,1)$   
 e) none of these

- a)  $f'$  is differentiable  
 b)  $f$  is differentiable  
 c)  $f'$  is continuous  
 d)  $f$  is continuous

10) The set of all points where the function  $f(x) = \frac{x}{(1+|x|)}$  is differentiable, is  
 (1987-2marks)

- a)  $(-\infty, \infty)$   
 b)  $[0, \infty)$   
 c)  $(-\infty, 0) \cup (0, \infty)$   
 d)  $(0, \infty)$   
 e) None

11) The function

$$f(x) = \begin{cases} |x-3| & , x \geq 1 \\ \frac{x^2}{4} - \frac{3x}{2} + \frac{13}{4} & , x < 1 \end{cases}, \text{ is}$$

(1988-2marks)

- a) continuous at  $x=1$   
 b) differentiable at  $x=1$   
 c) continuous at  $x=3$   
 d) differentiable at  $x=3$

12) If  $f(x) = \frac{1}{2}x - 1$ , then on the interval  $[0, \pi]$   
 (1989-2marks)

- a)  $\tan[f(x)]$  and  $\frac{1}{f(x)}$  are both continuous  
 b)  $\tan[f(x)]$  and  $\frac{1}{f(x)}$  are both discontinuous  
 c)  $\tan[f(x)]$  and  $f^{-1}x$  are both continuous  
 d)  $\tan[f(x)]$  is continuous but  $\frac{1}{f(x)}$  is not

13) The value of  $\lim_{x \rightarrow 0} \frac{\sqrt{\frac{1}{2}(1-\cos 2x)}}{x}$   
 (1991-2marks)

- a) 1  
 b) -1  
 c) 0  
 d) none of these

14) The following functions are continuous on  $(0, \pi)$   
 (1991-2marks)

- a)  $\tan x$   
 b)  $\int_0^x t \sin \frac{1}{t} dt$   
 c)  $\begin{cases} 1 & , 0 < x \leq \frac{3\pi}{4} \\ 2\sin^2 \frac{x}{9} & , \frac{3\pi}{4} < x < \pi \end{cases}$   
 d)  $\begin{cases} x \sin x & , 0 < x \leq \frac{\pi}{2} \\ \frac{\pi}{2} \sin(\pi + x) & , \frac{\pi}{2} < x < \pi \end{cases}$

15) Let  $f(x) = \begin{cases} 0 & , x < 0 \\ x^2 & , x \geq 0 \end{cases}$  then for all  $x$  (1994)