Optimization of a Right Circular Cylinder

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Problem Statement

Question:
Show that the right circular cylinder of given surface area and maximum volume is such that its height is equal to the diameter of the base.

Surface Area and Volume

Total Surface Area:

$$S = 2\pi rh + 2\pi r^2 \tag{1}$$

Volume:

$$V = \pi r^2 h \tag{2}$$

Expressing h in terms of r:

$$h = \frac{S - 2\pi r^2}{2\pi r} \tag{3}$$

Volume Function

Expressing volume as a function of r:

$$V = \frac{r(S - 2\pi r^2)}{2} \tag{4}$$

Differentiating with respect to r:

$$\frac{dV}{dr} = \frac{S - 6\pi r^2}{2} \tag{5}$$

Setting $\frac{dV}{dr} = 0$:

$$S - 6\pi r^2 = 0 \tag{6}$$

Finding the Optimal Ratio

Solving for r:

$$r = \sqrt{\frac{S}{6\pi}} \tag{7}$$

Substituting in h:

$$h = \frac{S - 2\pi r^2}{2\pi r} = 2r \tag{8}$$

Conclusion: The height of the cylinder is equal to the diameter of the base.

Computational Approach

Using Gradient Ascent:
To Maximize Volume

$$V = \frac{r\left(S - 2\pi r^2\right)}{2} \tag{9}$$

$$f(r) = \frac{r\left(S - 2\pi r^2\right)}{2} \tag{10}$$

$$r_{n+1} = r_n + \mu f'(r_n) \tag{11}$$

$$f'(r_n) = \frac{S - 6\pi r_n^2}{2} \tag{12}$$

$$r_{n+1} = r_n + \mu\left(\frac{S - 6\pi r_n^2}{2}\right) \tag{13}$$

Using $\mu = 0.001$ and $S = 1000$:

- Theoretical radius: 7.283656 units
- Gradient ascent result: 7.28365620 units
- Geometric Programming result: 7.283563 units

Visualization