Bode Plot Analysis of Cascaded RC Low-Pass Filters

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1 Introduction

An RC low-pass filter is a fundamental electronic circuit that allows low-frequency signals to pass while attenuating high-frequency components. Cascading multiple RC filters improves the roll-off characteristics, making the filter more effective in suppressing unwanted high-frequency noise.

2 Materials and Apparatus Required

- 3 Resistors (1kΩ used)
- 3 Capacitors (1μF used)
- Breadboard
- Function Generator
- Oscilloscope

3 Procedure

1. Assemble the circuit as shown in the below figure.
2. Apply input V_{in} from a function generator.
3. Measure voltage across each stage.
4. Record phase difference for multiple frequencies.
5. Repeat for 1-stage, 2-stage, and 3-stage filters.
6. Compare theoretical and experimental results.

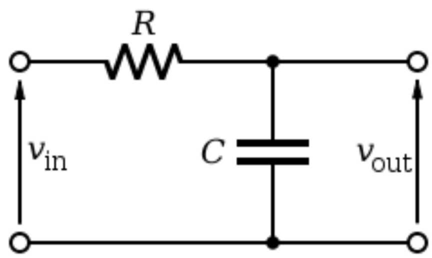

Figure 1: RC circuit

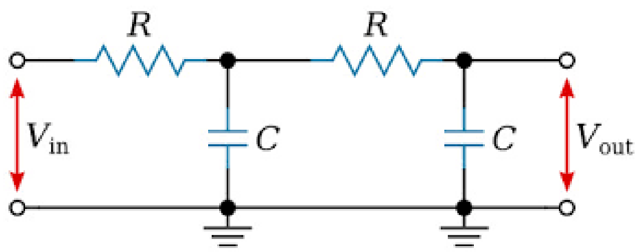

Figure 2: RC cascaded with another RC

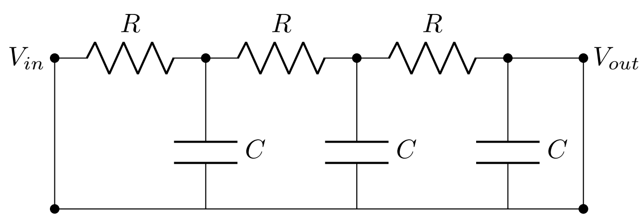

Figure 3: RC cascades with 2 RCs

4 Theory

A single-stage RC low-pass filter consists of a resistor (R) and a capacitor (C) in series, with the output taken across the capacitor. The transfer function is given by:

$$H(s) = \frac{1}{1 + sRC} \tag{1}$$

where $s = j\omega$ and ω is the angular frequency.

The cutoff frequency f_c is determined as:

$$f_c = \frac{1}{2\pi RC} \tag{2}$$

4.1 Cascading RC Filters

Cascading multiple RC filters increases the overall roll-off rate. The roll-off for each stage is approximately -20 dB/decade:

- **1-stage:** -20 dB/decade
- **2-stage:** -40 dB/decade
- **3-stage:** -60 dB/decade

Each stage provides additional attenuation, making cascaded RC filters effective in audio signal processing and communication systems.

5 Theory

The transfer function of a 1-stage RC circuit is given by:

$$H(s) = \frac{1}{1 + sRC}$$

where,

$$s = j\omega.$$

Expanding, we get:

$$H(s) = \frac{1}{\sqrt{1 + (\omega RC)^2}} e^{j\theta}$$

where,

$$\theta = -\tan^{-1}(\omega RC).$$

Applying logarithm on both sides,

$$\begin{aligned}\log H(s) &= \log\left(\frac{1}{\sqrt{1 + (\omega RC)^2}} e^{j(-\tan^{-1}(\omega RC))}\right)\\ &= -\frac{1}{2}\log(1 + (\omega RC)^2) - j\tan^{-1}(\omega RC).\end{aligned}$$

Calculating amplitude gain:

$$\begin{aligned}A &= 20\log|H(s)|\\ &= -10\log(1 + (\omega RC)^2).\end{aligned}$$

This gives the exact equation for the Bode plot of the amplitude gain. For phase difference,

$$\theta = -\tan^{-1}(\omega RC).$$

Similarly, the transfer function of a 2-stage RC circuit is given by:

$$H(s) = \frac{1}{1 + 3sRC + (sRC)^2}.$$

Following this, we get:

$$\log H(s) = -\frac{1}{2}\log\left((1-(\omega RC)^2)^2 + (3\omega RC)^2\right) + j\tan^{-1}\left(\frac{-3\omega RC}{1-(\omega RC)^2}\right).$$

The transfer function for a 3-stage RC circuit is given as:

$$H(s) = \frac{1}{(sRC)^3 + 5(sRC)^2 + 6sRC + 1}.$$

Following that, we get:

$$\log H(s) = -\frac{1}{2}\log\left(\left(1 + 6\omega RC + 5(\omega RC)^2 + (\omega RC)^3\right)^2\right) + j\tan^{-1}\left(\frac{-6\omega RC - (\omega RC)^3}{1 + 5(\omega RC)^2}\right).$$

6 Bode Plot Analysis

The magnitude response for a single-stage RC filter follows:

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}} \tag{3}$$

And the phase response is given by:

$$\theta(\omega) = -\tan^{-1}(\omega RC) \tag{4}$$

For multiple stages, the overall transfer function is:

$$H_n(s) = \left(\frac{1}{1 + sRC}\right)^n \tag{5}$$

where n is the number of stages.

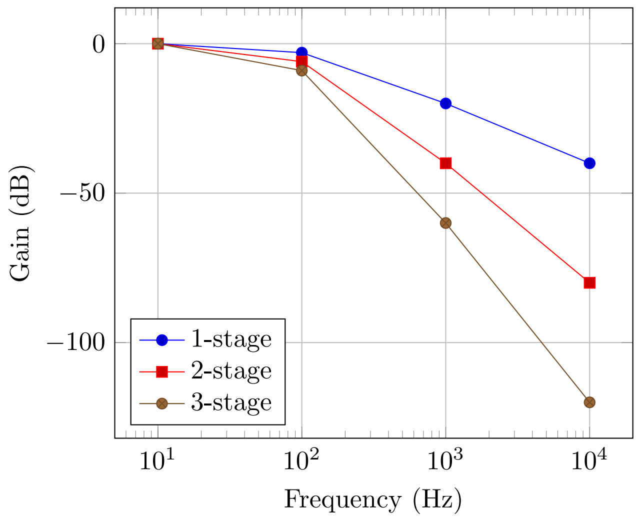

Figure 4: Bode plot for cascaded RC filters

7 Observations

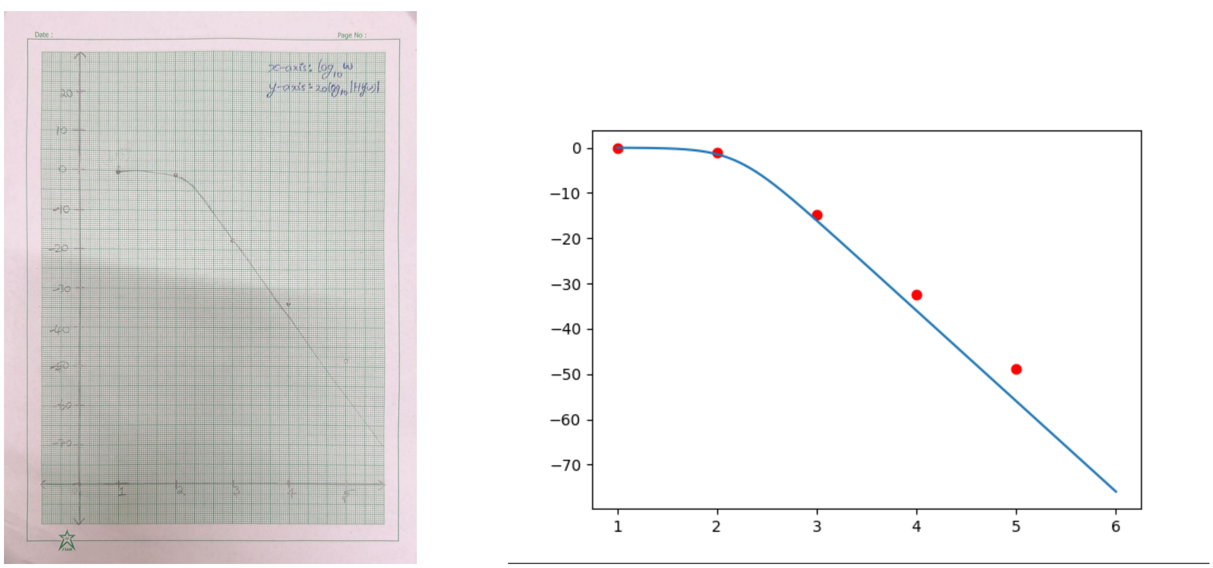

Figure 5: Amplitude Plot for RC circuit

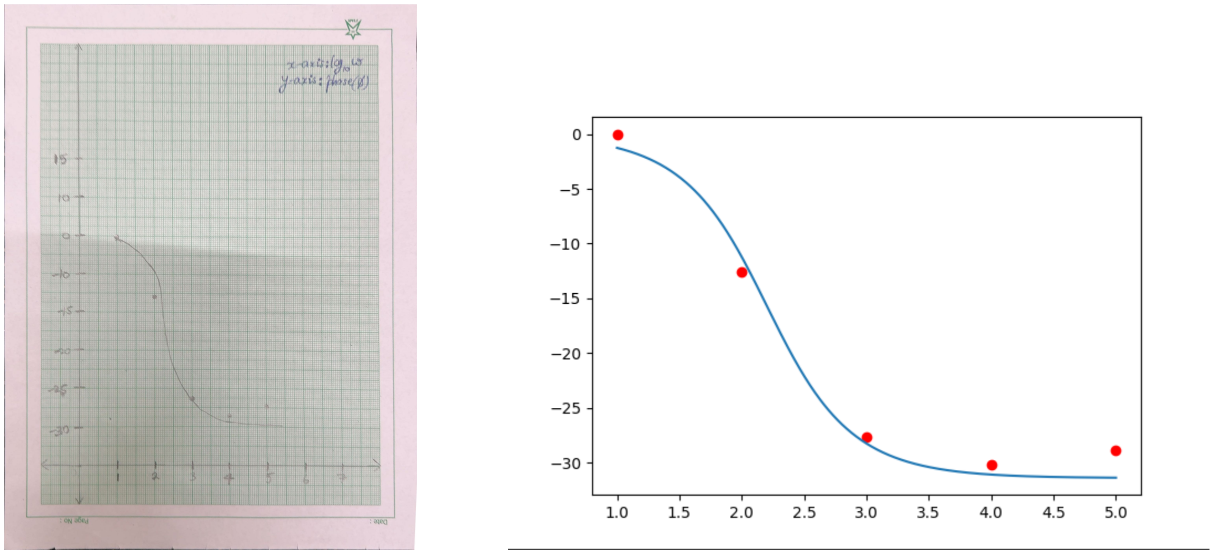

Figure 6: Phase Plot for RC circuit

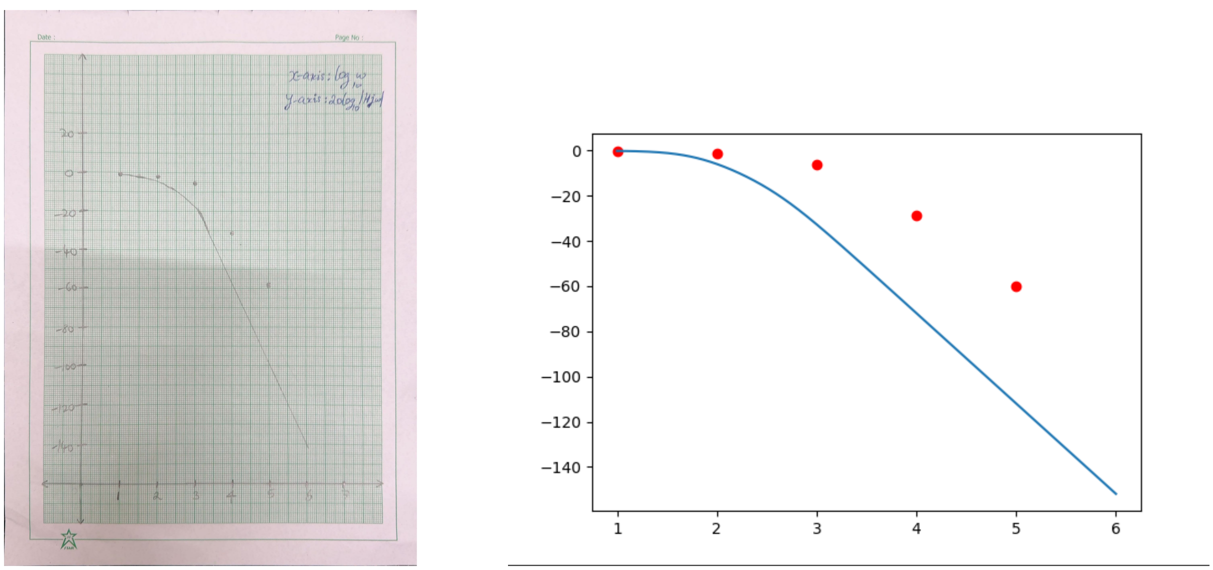

Figure 7: Amplitude Plot for 2 cascaded RC circuit

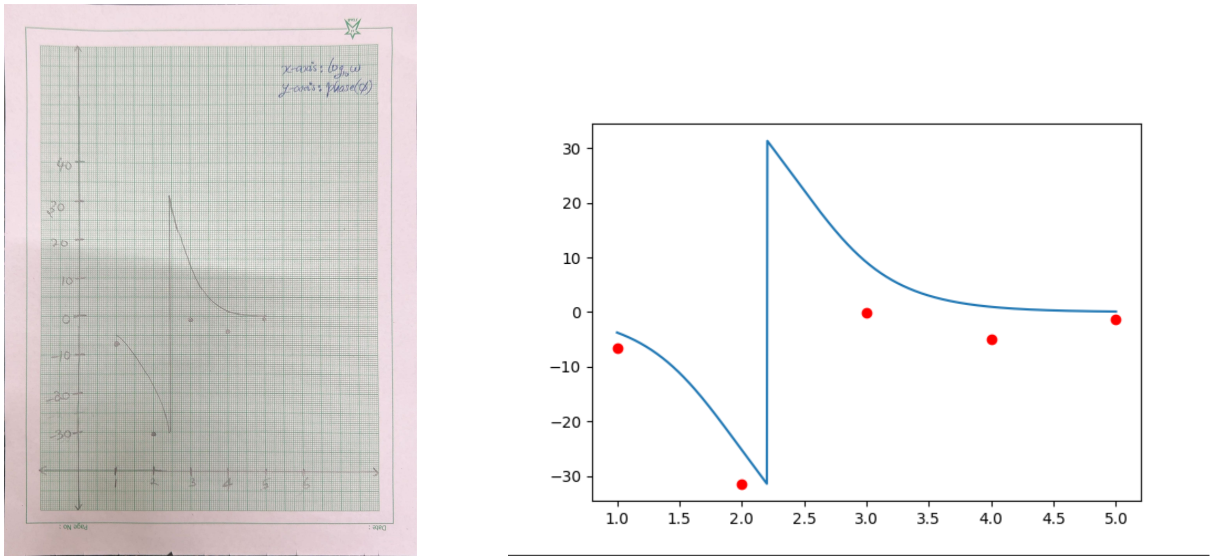

Figure 8: Phase Plot for 2 cascaded RC circuit

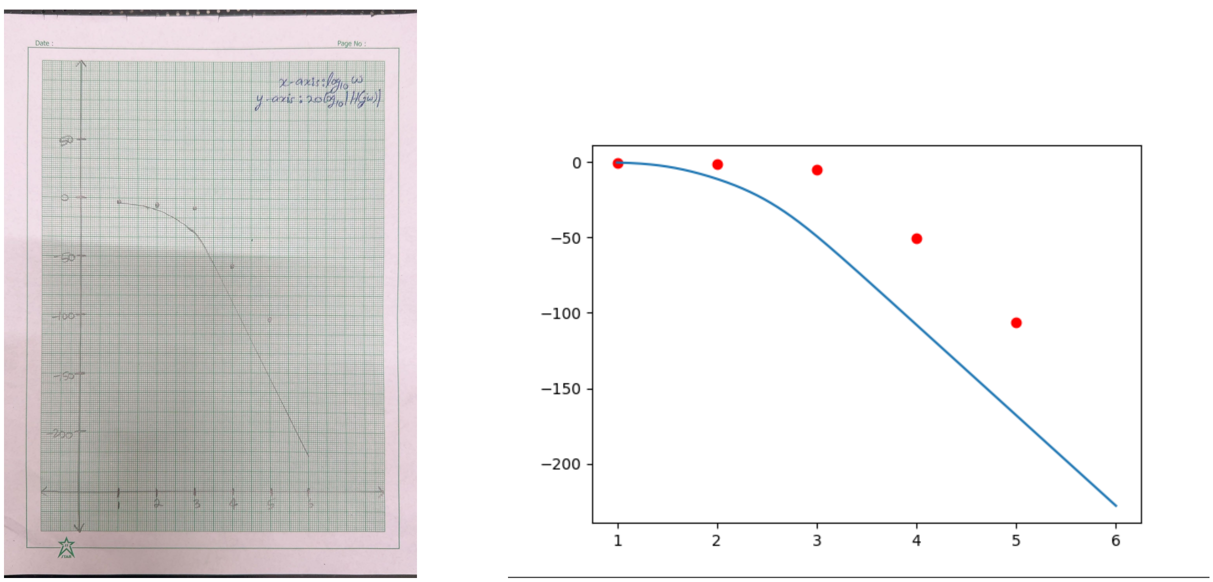

Figure 9: Amplitude Plot for 3 cascaded RC circuit

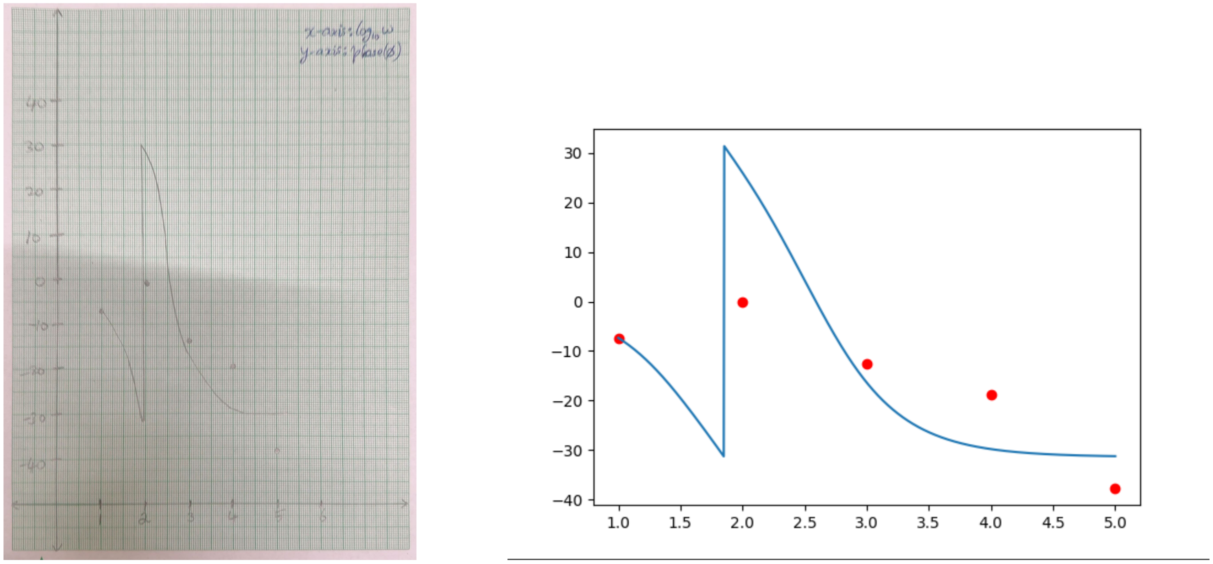

Figure 10: Phase Plot for 3 cascaded RC circuit

8 Additional Points

8.1 Applications of Bode Plots

- Used in circuit design to analyze frequency response.
- Helps in stability analysis of control systems.
- Used in audio and communication system filters.

8.2 Improvements in Experimentation

- Use precision components to reduce error.
- Implement automated data logging for accuracy.
- Consider using a higher-resolution oscilloscope.

9 Conclusion

Cascaded RC low-pass filters provide improved attenuation characteristics by increasing the roll-off rate with each additional stage. These filters are useful in various applications, including audio processing and signal conditioning.