Transient Response Analysis of an LC Circuit

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1 Objective

To study and analyze the transient response of an LC circuit, determine the natural frequency (Ω_n), and calculate the damping ratio (ξ) using theoretical and experimental methods.

2 Equipment Required

- 472 pF capacitor
- 2.2 mH inductor
- Small-value resistor (if needed)
- DC power supply (5 V)
- Oscilloscope
- Connecting wires

3 Theory

An LC circuit consists of an inductor (L) and a capacitor (C) connected in parallel. When a charged capacitor is connected to an inductor, energy oscillates between the capacitor's electric field and the inductor's magnetic field. The ideal LC circuit follows the second-order differential equation:

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

where q is the charge on the capacitor. The natural frequency of oscillation is given by:

$$\Omega_n = \frac{1}{\sqrt{LC}}$$

However, in real components, the inductor and capacitor have inherent resistance (R), forming an RLC circuit. The presence of resistance introduces damping, and the damping ratio (ξ) is given by:

$$\xi = \frac{R}{2}\sqrt{\frac{C}{L}}$$

The damped frequency (Ω_d) is related to the natural frequency as:

$$\Omega_d = \Omega_n \sqrt{1 - \xi^2}$$

4 Procedure

4.1 Precharging the Capacitor

1. Connect the 100 µF capacitor to a 5 V DC power supply.
2. Once fully charged, disconnect it carefully without discharging.

4.2 Constructing the LC Circuit

1. Connect the charged capacitor in parallel with the 2.2 mH inductor as shown in the figure below.
2. Ensure minimal resistance in wiring.

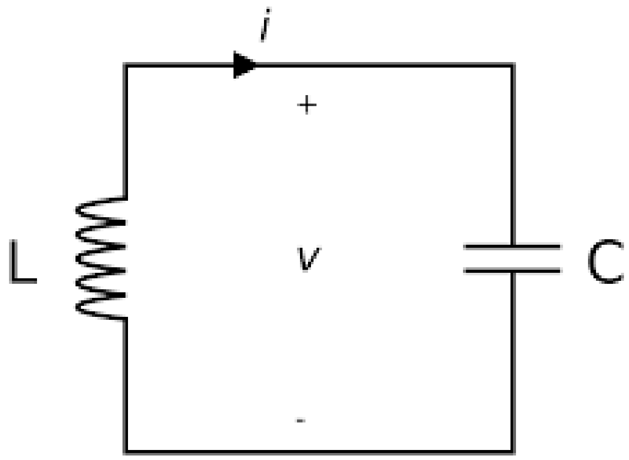

Figure 1: Diagram for LC circuit

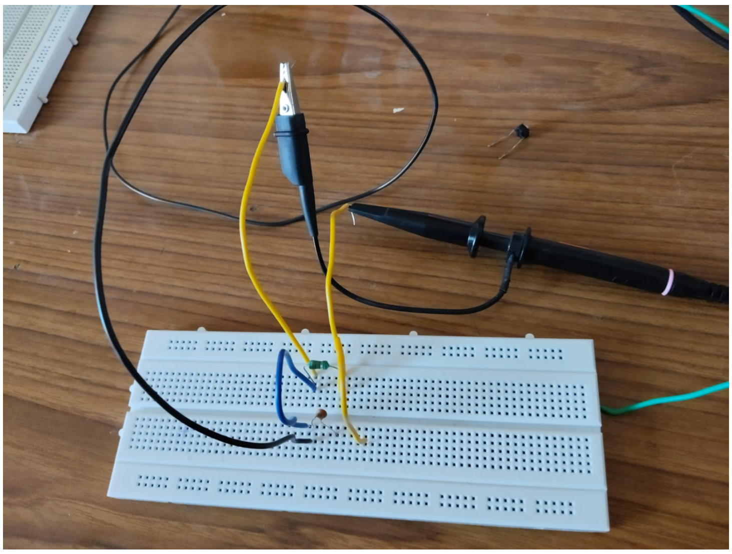

Figure 2: LC Circuit

4.3 Capturing the Transient Response

1. Use an oscilloscope to monitor the voltage across the inductor.

2. Observe the natural oscillations and measure the oscillation period.

4.4 Theoretical Calculations

1. Compute Ω_n using the given values of L and C.

2. Calculate the damping ratio ξ using measured resistance.

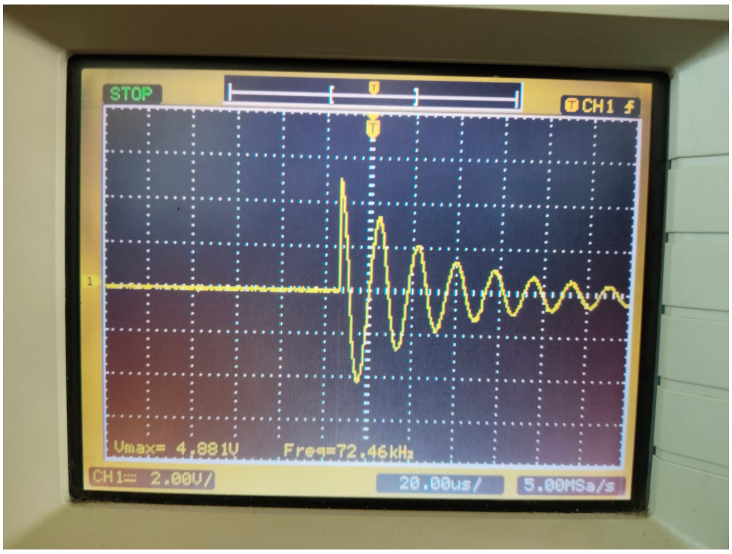

Figure 3: Transient Response of an LC Circuit

4.5 Experimental Data and Observations

- Measured damped frequency: 72.46 kHz
- Theoretical natural frequency: 156.05 kHz
- Calculated damping ratio: $\xi = 0.8855$
- Estimated resistance: 3.83 kΩ (possibly due to additional resistances)

5 Effect of Resistance on Oscillations

In an ideal LC circuit, where the inductor and capacitor have no resistance, energy oscillates continuously between the capacitor's electric field and the inductor's magnetic field. This results in a purely sinusoidal response at the natural frequency Ω_n:

$$V(t) = V_0 \cos(\Omega_n t + \phi)$$

However, real inductors and capacitors have internal resistance, which causes energy loss. The presence of resistance leads to damping, resulting in an exponentially decaying oscillation:

$$V(t) = V_0 e^{-\alpha t} \cos(\Omega_d t + \phi)$$

where $\alpha = \xi \Omega_n$ is the decay rate.

5.1 Illustrative Diagrams

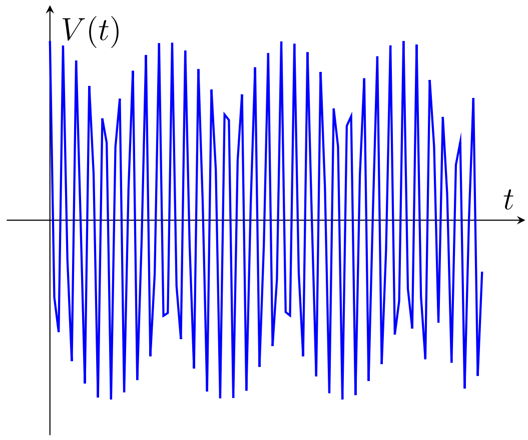

Figure 4: Ideal LC circuit response: Continuous sinusoidal oscillation

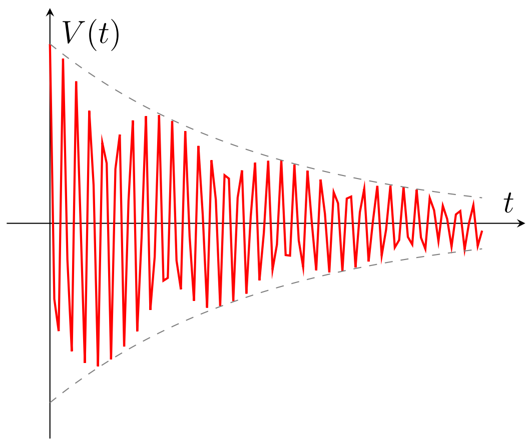

Figure 5: Damped LC circuit response: Exponentially decaying oscillation due to resistance

In the case of strong damping ($\xi > 1$), oscillations may not occur at all, leading to an overdamped response where the system slowly returns to equilibrium. If $\xi = 1$, the system is critically damped and returns to equilibrium in the shortest possible time without oscillating.

5.2 Decay Rate

The decay rate (attenuation coefficient) α determines how quickly oscillations decrease in amplitude due to resistance.
It is also related to the damping ratio as:

$$\alpha = \xi \Omega_n$$

$$\alpha = 0.8855 \times 980.39 \times 1000 = 868.135345 \times 10^3$$

The relation between decay rate and natural and damped frequency is given by:

$$\alpha = \sqrt{\omega_n^2 - \omega_d^2}$$

$$\alpha = \sqrt{(980.39 \times 10^3)^2 - (455.28 \times 10^3)^2}$$

$$\alpha = 868.27 \times 10^3$$

The above experimentally calculated decay rate matches closely with the theoretical value.

The amplitude of the oscillations follows an exponential decay:

$$V(t) = V_0 e^{-\alpha t} \cos(\Omega_d t + \phi)$$

where V_0 is the initial amplitude.

The decay rate can also be experimentally determined using the logarithmic decrement method:

$$\delta = \ln\left(\frac{V_n}{V_{n+1}}\right)$$

where V_n and V_{n+1} are successive peak voltages. The decay rate is then:

$$\alpha = \frac{\delta}{T_d}$$

where T_d is the damped oscillation period.

6 Analysis and Discussion

- The measured damped frequency is significantly lower than the ideal natural frequency, indicating non-negligible resistance in the circuit.
- The calculated resistance of $3.83\,\mathrm{k}\Omega$ seems too high for a typical inductor, suggesting additional resistive losses or measurement artifacts.
- Possible sources of extra resistance include inductor DC resistance, capacitor ESR, wiring resistance, and core losses.

7 Conclusion

The experiment demonstrated transient oscillations in an LC circuit and allowed for a comparison of theoretical and experimental results. The presence of resistance led to damping, reducing the oscillation frequency. Further investigation is needed to accurately determine the resistance components.

8 Further Exploration

- Measure the inductor's DC resistance using a multimeter.
- Use a function generator to study forced oscillations.
- Experiment with different capacitor and inductor values to observe their effects n damping and frequency.

9 Safety Precautions

- Handle charged capacitors carefully to avoid accidental discharges.
- Use components within their rated values to prevent damage.
- Ensure proper oscilloscope grounding to avoid erroneous readings.