

1 Problem 4

1.1 (a) Laplacian using 5-point Laplacian

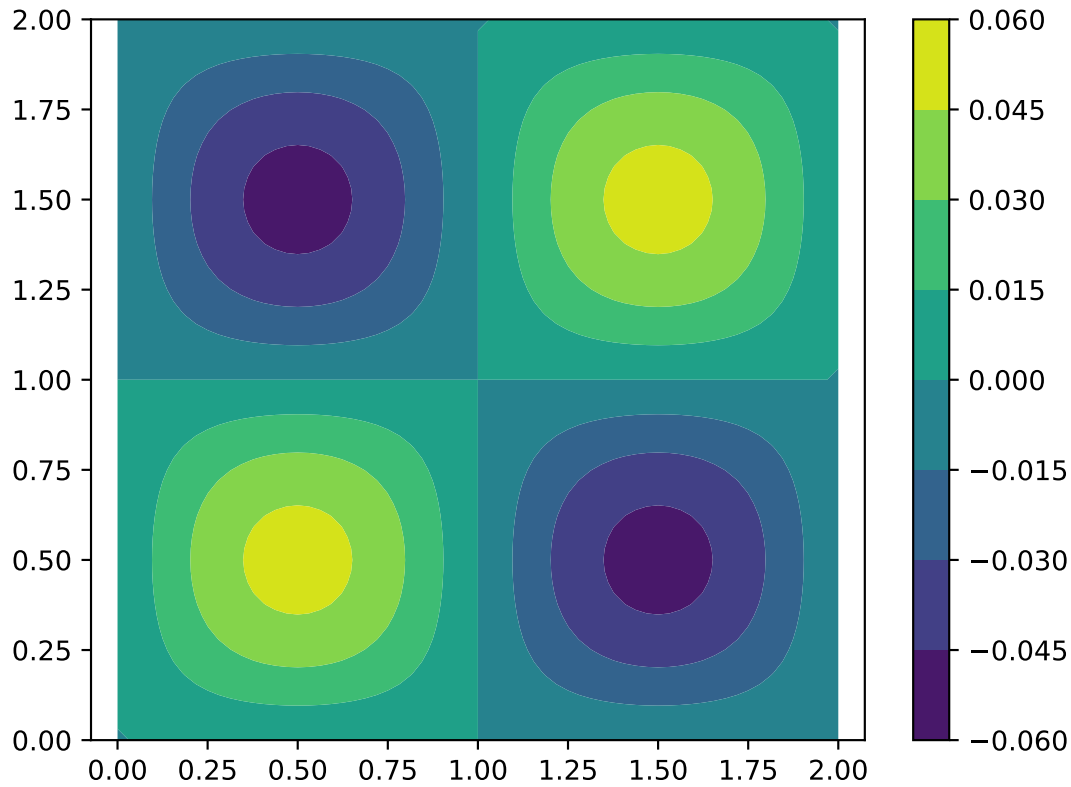


Figure 1: Solution $U(x, y)$ using 5-point Laplacian with $dx = 2^{-5}$

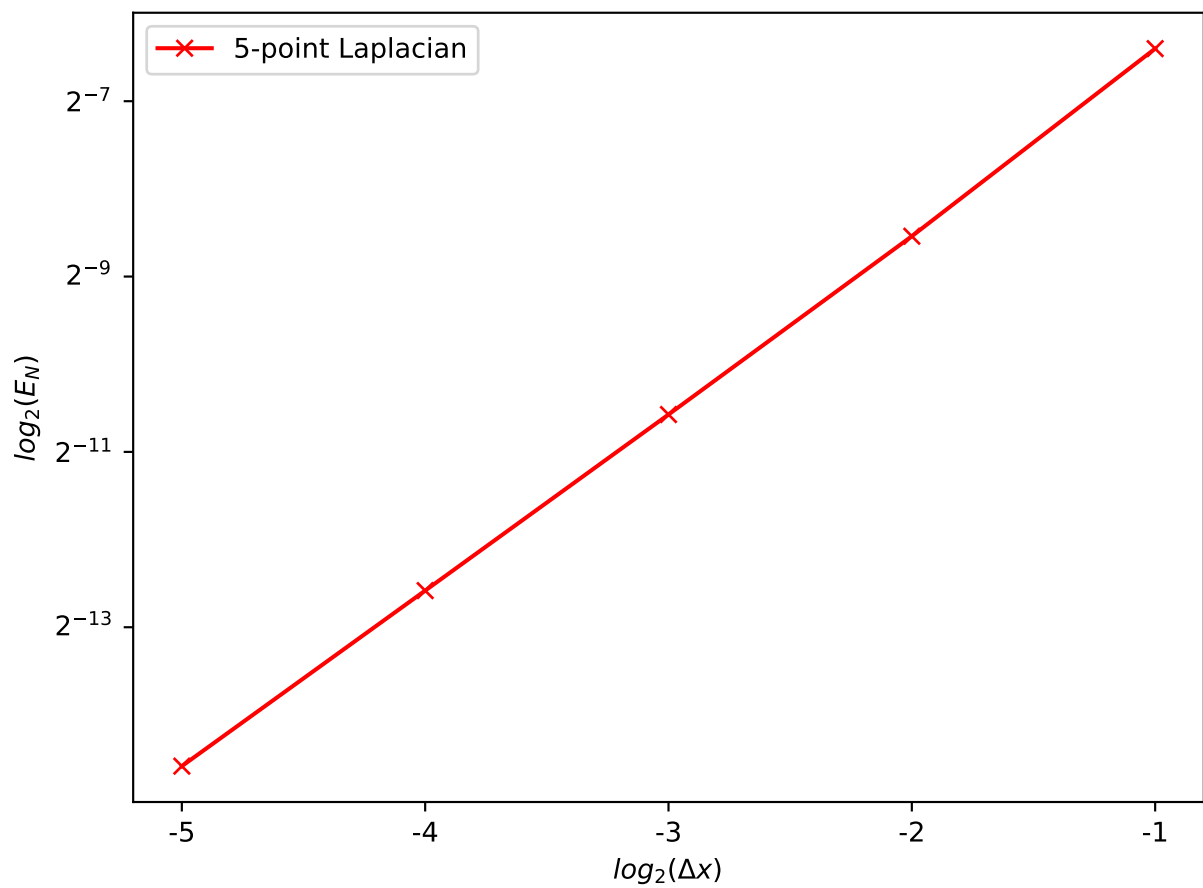


Figure 2: Order of accuracy using 5-point Laplacian

From the plot, it can be clearly seen that the order of accuracy = 2

1.2 (b) Laplacian using 9-point Laplacian

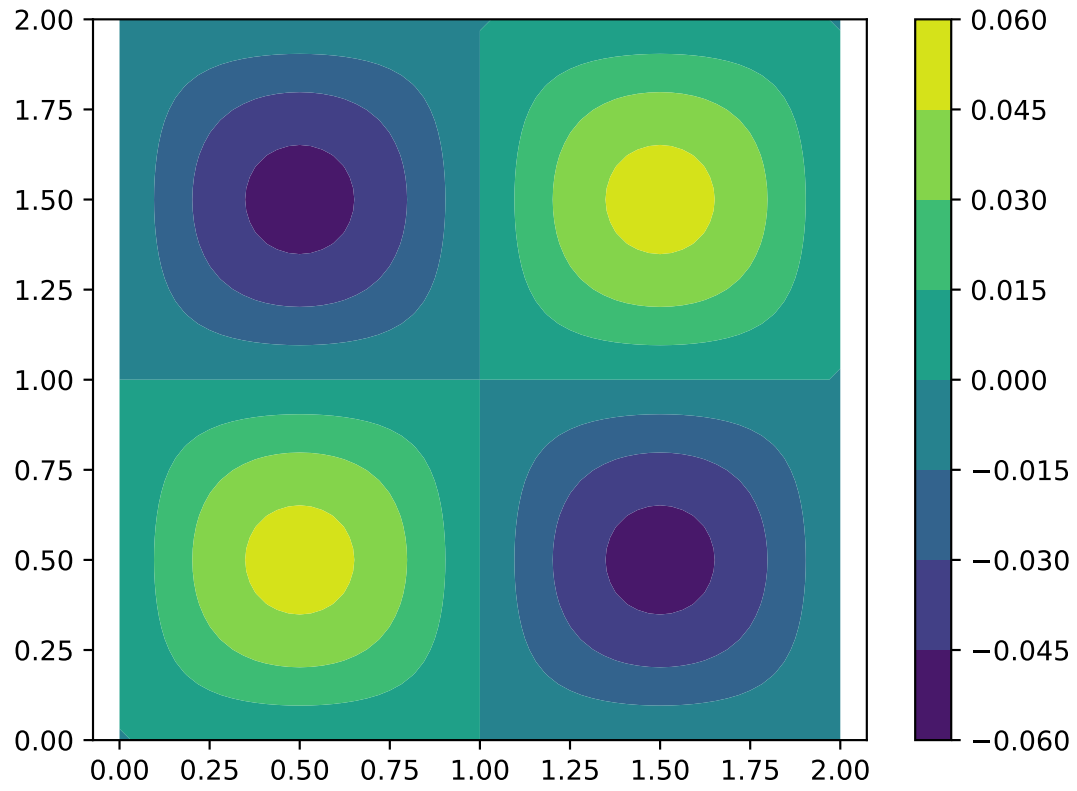


Figure 3: Solution $U(x, y)$ using 9-point Laplacian with $dx = 2^{-5}$

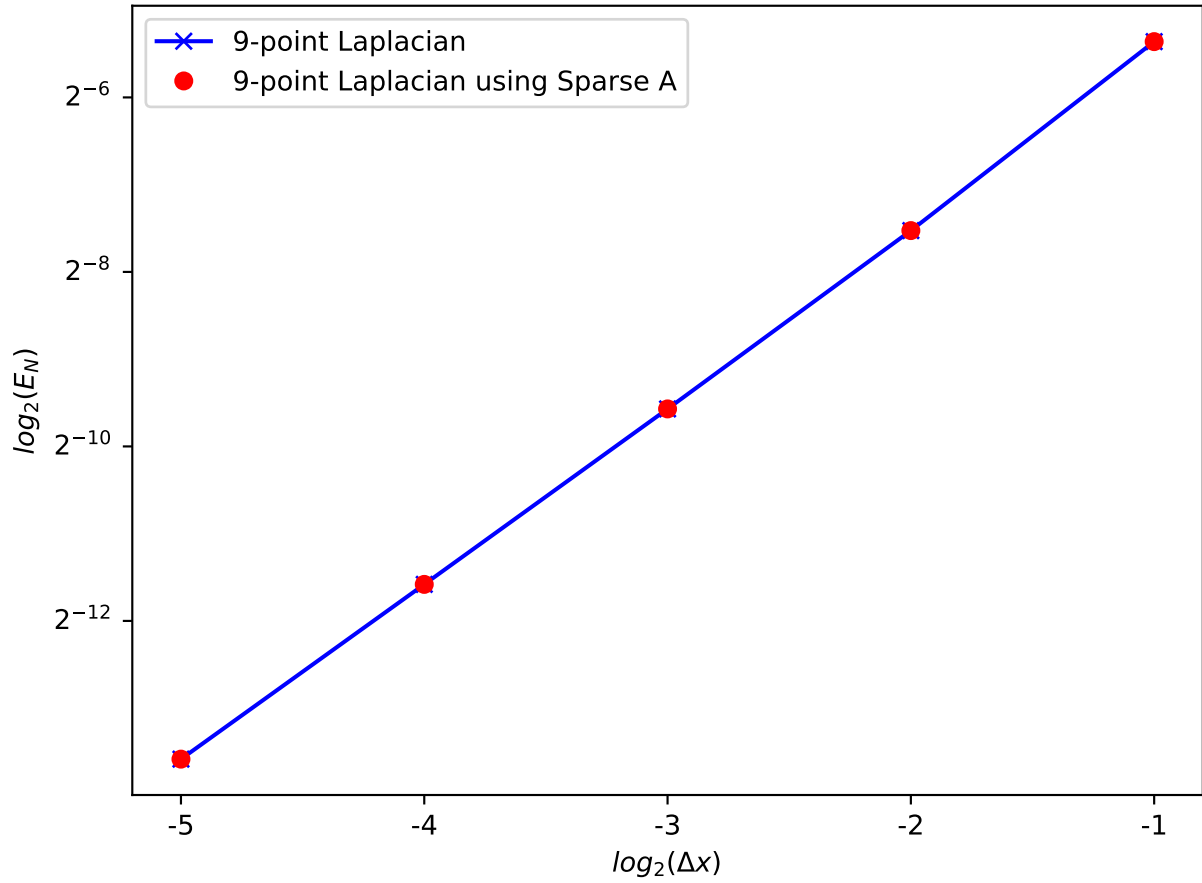
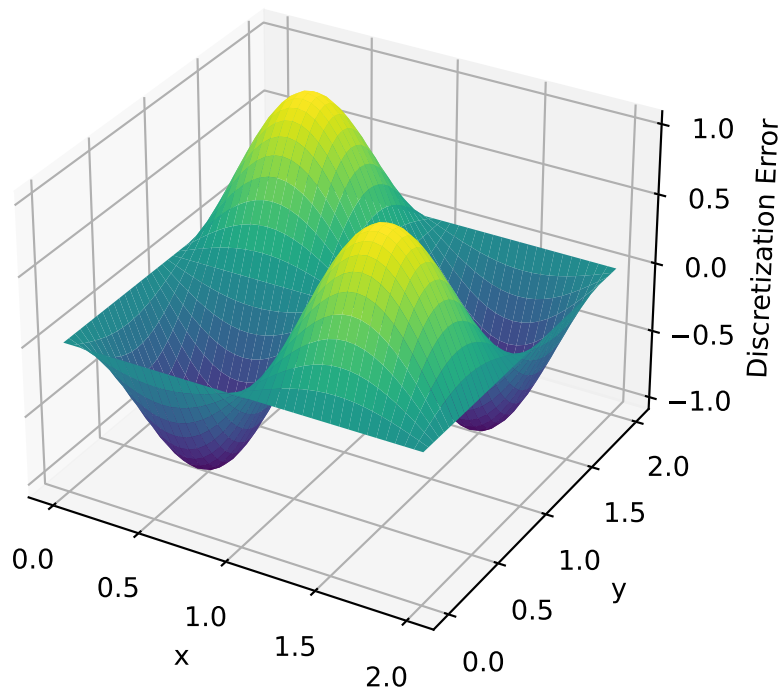


Figure 4: Order of accuracy using 9-point Laplacian

The order of accuracy is expected to improve for 9-point Laplacian. I expected it to be 4^{th} order accurate. I have implemented the 9-point stencil using both square and sparse matrices. Both resulted in a 2^{nd} order accuracy.

Discretization Error of 9-point Laplacian

Figure 5: $\nabla_9^2 u - f$

Analytical Laplacian

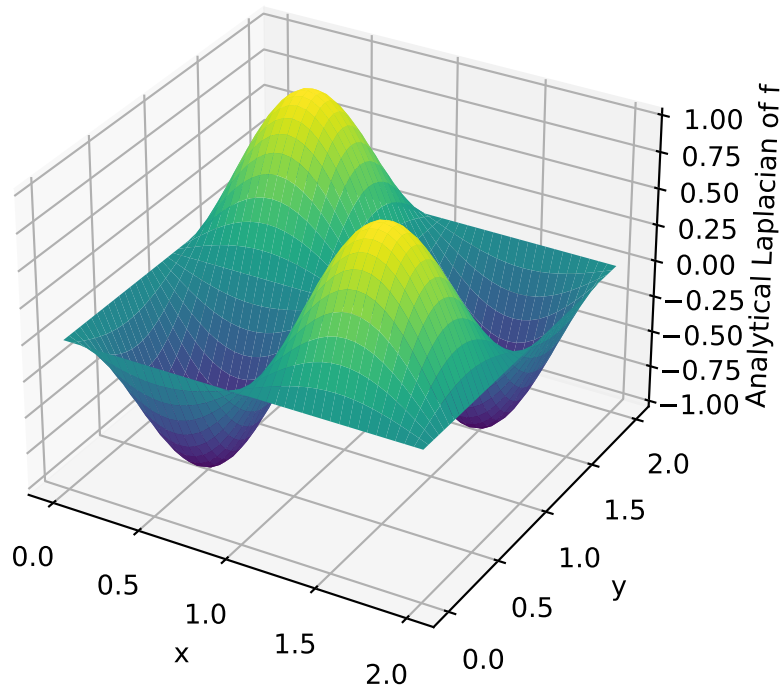


Figure 6: $f_{xx} + f_{yy}$ evaluated analytically

Using the fact that the error term is directly proportional to $f_{xx} + f_{yy}$, we could incorporate schemes that could forcibly iterate until the error is less than the RHS term by maybe incorporating sub-grid scaling. What I mean by that is that, laplacian solvers or poisson solvers are usually used to solve for pressure in fluid flow equations. And we could increase local gridding thus minimizing the local error where it is maximum. And we know where it is maximum because it is proportional to $f_{xx} + f_{yy}$.