

1 Problem 3

1.1 (a) Forward Euler with $Nt = 4$

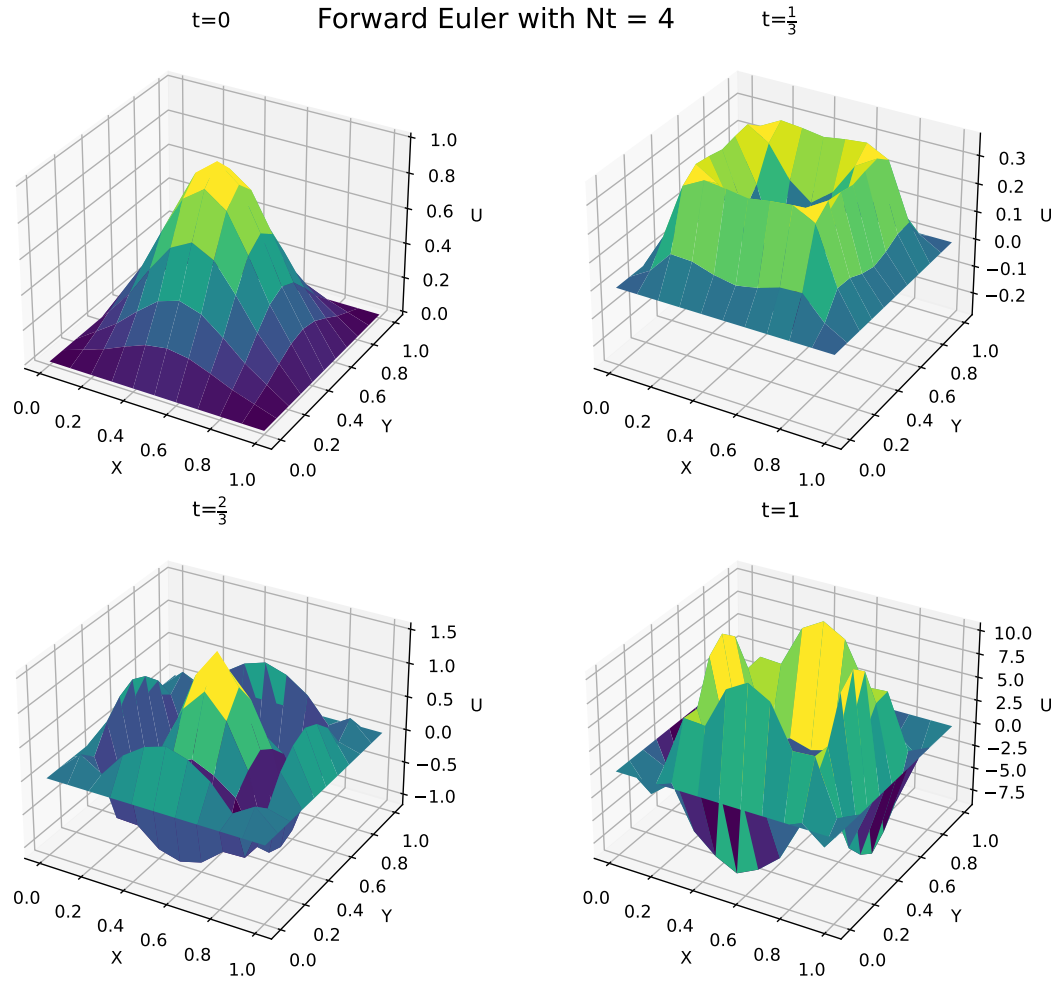


Figure 1: Solution $U(t, x, y)$ using Forward Euler with $Nt = 4$ at $t = 0, \frac{1}{3}, \frac{2}{3}, 1$

1.2 (b) Forward Euler with $Nt = 101$

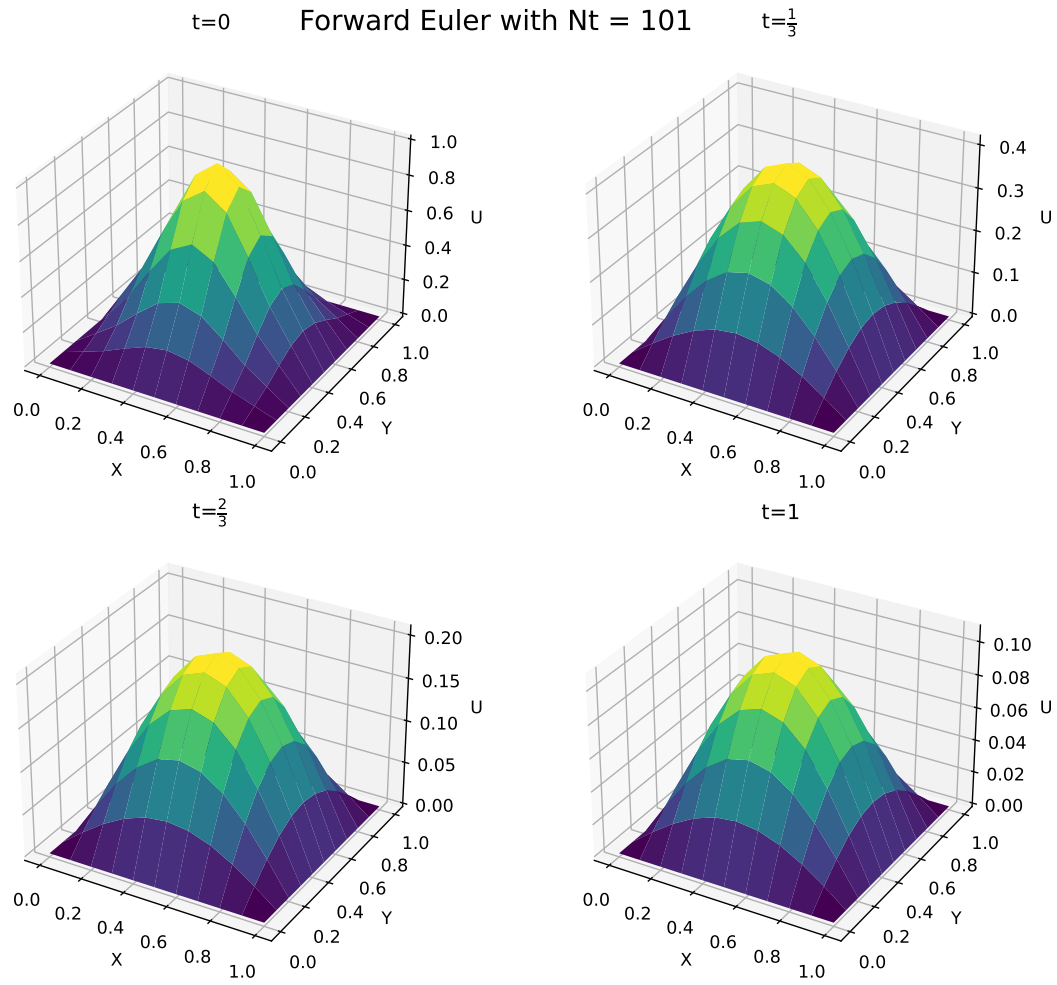


Figure 2: Solution $U(t, x, y)$ using Forward Euler with $Nt = 101$ at $t = 0, \frac{1}{3}, \frac{2}{3}, 1$

1.3 (c) Trapezoidal method with $Nt = 4$

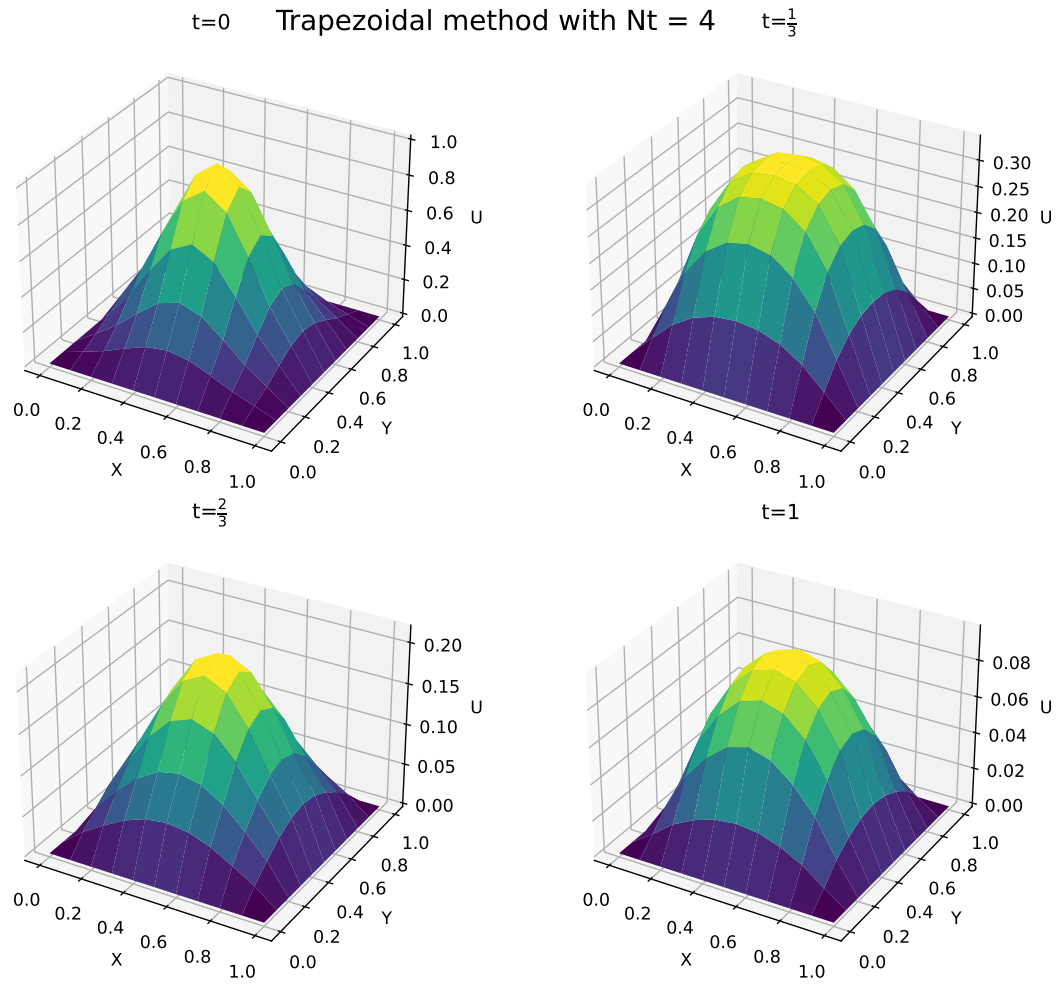


Figure 3: Solution $U(t, x, y)$ using Trapezoidal Method with $Nt = 101$ at $t = 0, \frac{1}{3}, \frac{2}{3}, 1$

1.4 (d) Trapezoidal Method with $Nt = 101$

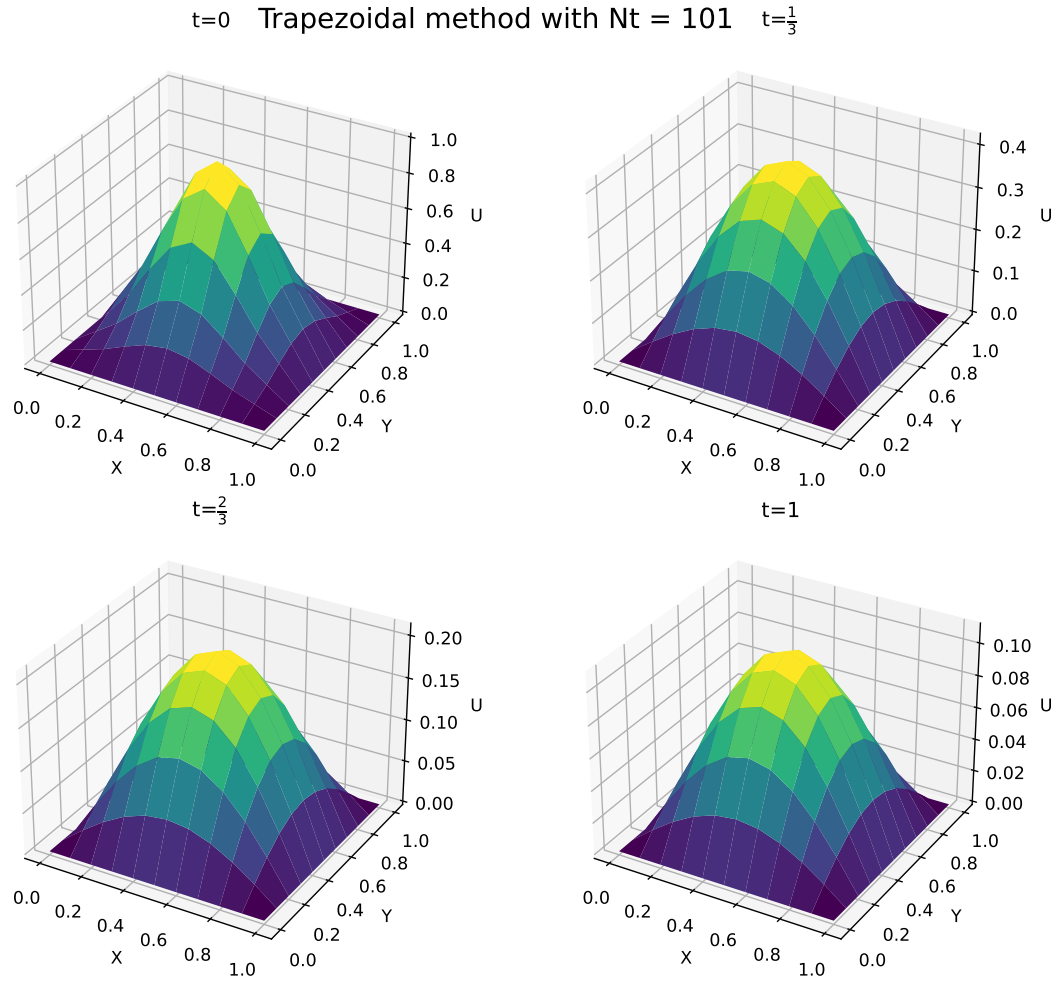


Figure 4: Solution $U(t, x, y)$ using Trapezoidal Method with $Nt = 101$ at $t = 0, \frac{1}{3}, \frac{2}{3}, 1$

1.5 (e) Analysis

- Forward Euler exhibits stability issues, especially with $Nt = 4$
- If I have to get a rough picture of the solution at $t = 0, \frac{1}{3}, \frac{2}{3}, 1$ I'd use Trapezoidal Method if the problem size is small enough that an implicit method is fast enough to compute.
- If the problem size is too big, I am better off using the Forward Euler method, taking care to choose a dt within the stability region. This gives a faster solution irrespective of the problem size as it is an explicit method.
- Forward Euler is explicit method. Hence, it is suitable for large problems which is a pro. But it is unstable which means that extra care should be taken while choosing dt . For some problems, it is always unstable, which is a con.
- Trapezoidal method is always stable for this problem, which is a pro. But it is an implicit method which means that a system of equations are solved at every time step which is a con because the computational time and resources required for a problem increase drastically with its size.