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AMATH 581
Report - Homework 1

UNIVERSITY of WASHINGTON

1 Problem 1

1.1 Forward-Euler Method

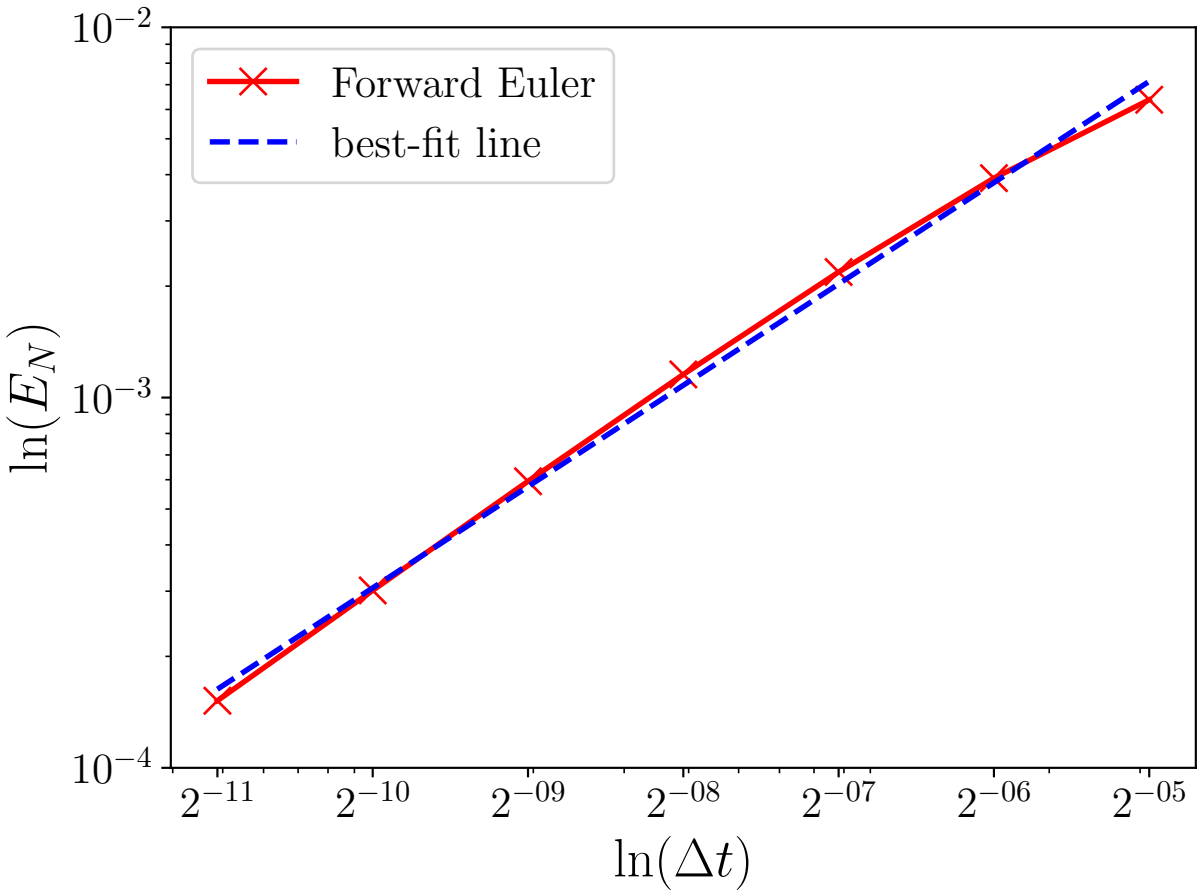

Figure 1: Order of accuracy of Forward-Euler Method

- The order of accuracy Forward-Euler method is 1.
- This is reflected in the slope of the best-fit line which is $0.9089449553882955 \approx 1$.

1.2 Heun's Method

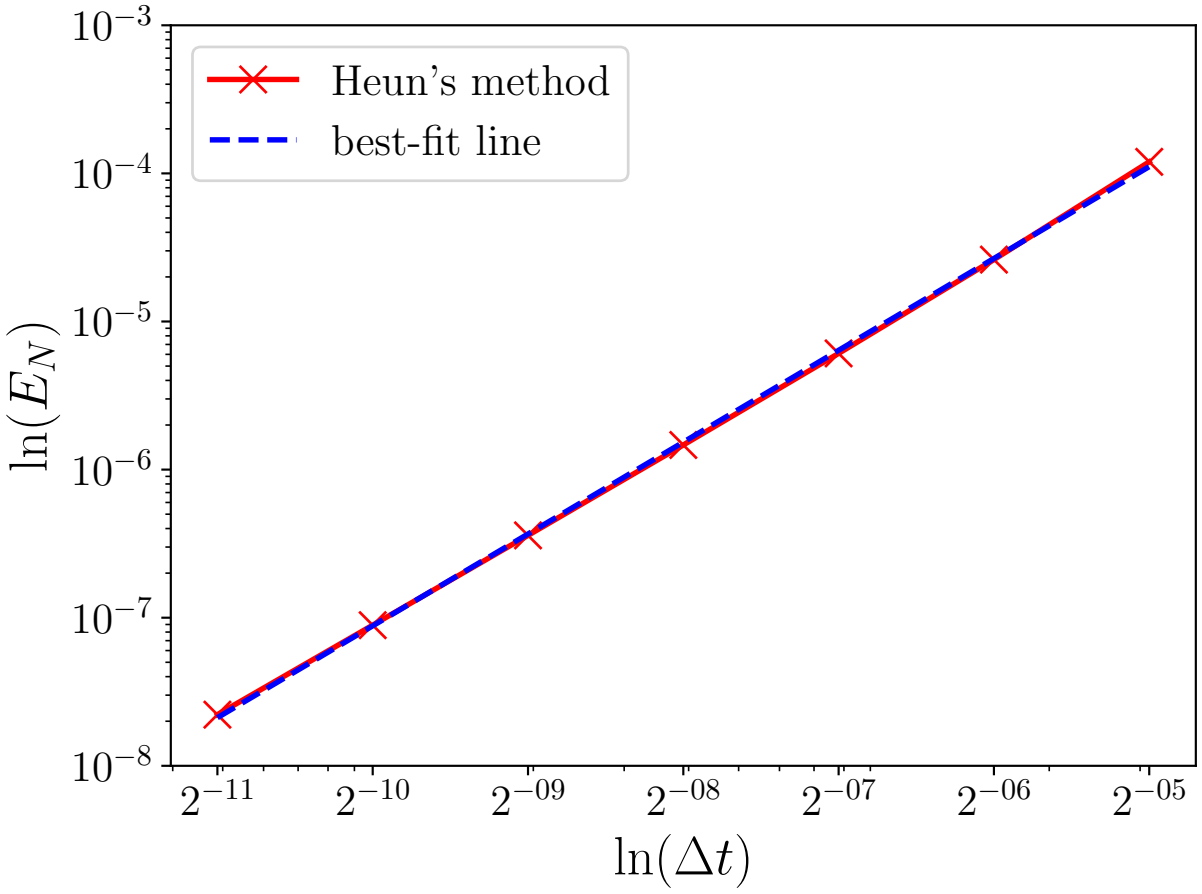

Figure 2: Order of accuracy of Heun's Method

- The order of accuracy Heun's method is 2.
- This is reflected in the slope of the best-fit line which is $2.0599552814134734 \approx 2$.

1.3 2^{nd} order Runge-Kutta (RK2) method

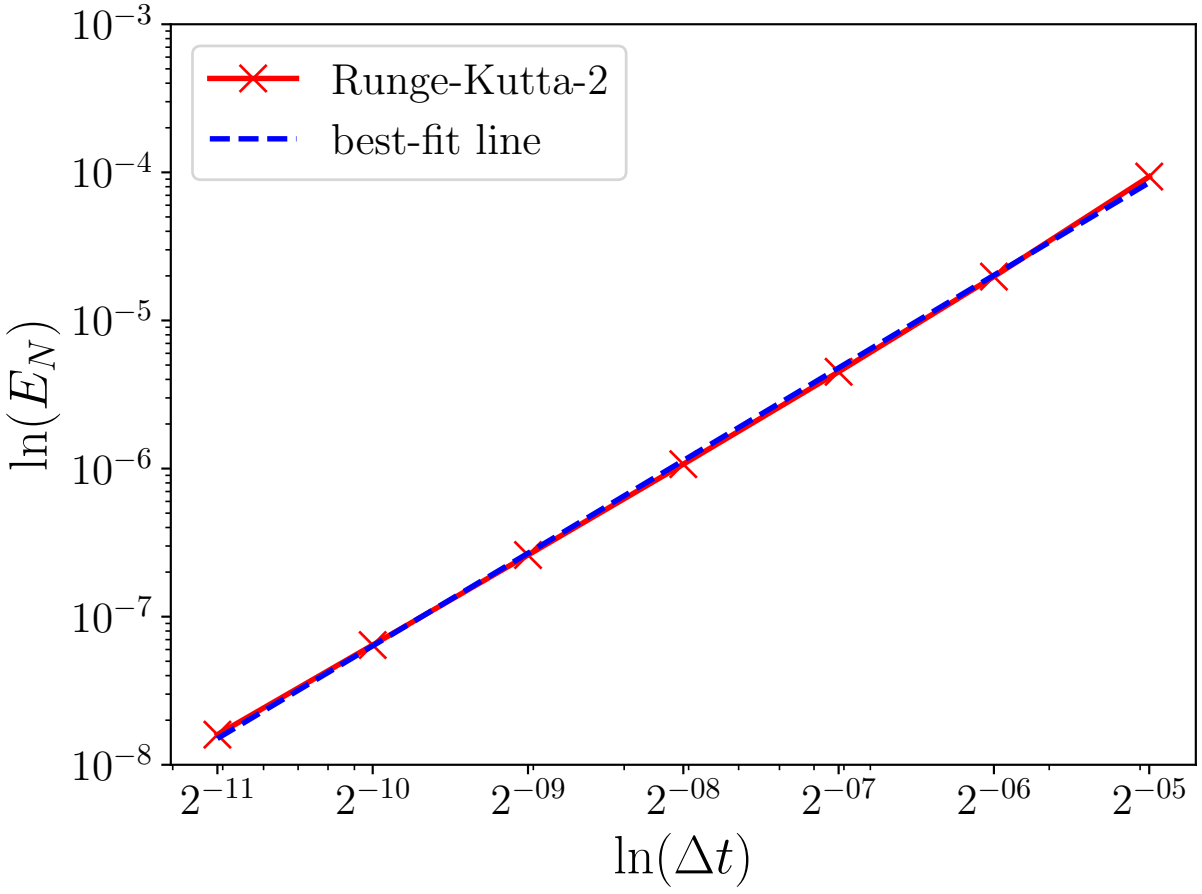

Figure 3: Order of accuracy of 2nd order Runge-Kutta Method

- The order of accuracy 2^{nd} order Runge-Kutta method, as the name suggests, is 2.
- This is reflected in the slope of the best-fit line which is $2.0781952218961823 \approx 2$.

2 Mathematical Explanation

- We observed in section 1.2 of AMATH 581 notes that for several methods, Global Error E_N is of the form $O((\Delta t)^n)$.
- For example:
- For Forward-Euler $E_N = O((\Delta t)^1)$
- For Heun's Method $E_N = O((\Delta t)^2)$
- For 2^{nd} order Runge-Kutta (RK2) $E_N = O((\Delta t)^2)$
- For a generalised method or order **n**, $E_N = O((\Delta t)^n)$
- It can be considered of the form

$$(E_N) = k((\Delta t)^n)$$

- Applying natural logarithm on both sides, we get,

$$ln(E_N) = ln(k((\Delta t)^n))$$

- Using the identity $ln(AB) = ln(A) + ln(B)$, we get,

$$ln(E_N) = ln(k) + ln((\Delta t)^n)$$

- Using the identify $ln(A^n) = n \times ln(A)$, we get,

$$ln(E_N) = ln(k) + n \times ln(\Delta t)$$

- Comparing this to the equation of a straight line $y = mx + c$, it is observed that the slope of the log-log plot of E_N and Δt results in a straight line with slope $m = n$, the order of accuracy of the method.