Module 3

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1. Find the maximum and minimum value of the function

x3 - 3x2 - 9x + 12

Solution:

$f(x) = x^3 - 3x^2 - 9x + 12$ ________________ (1)
On diff. both sides we get,
$f'(x) = 3x^2 - 6x - 9$
for Maxima and minima,
put $f'(x) = 0$,
$3x^2 - 6x - 9 = 0$
$x^2 - 2x - 3 = 0$
$x^2 - 3x + x - 3 = 0$
$x (x-3) + 1 (x-3) = 0$
$(x-3) (x+1) = 0$
$X = 3$ or $x = -1$
Now diff eqn (1) wrt x we get,
$f'' (x) = 6x - 9$
case 1:
when $x = 3$,
$f'' (3) = 6 * 3 - 9 = 9$
$f'' (3) > 0$
f(x) is minimum at x = 3
Minimum value = f (3)
$(3)^3 - 3(3)^2 - 9 * 3 + 12$
$27 - 27 - 27 + 12$
$= -15$
Minimum value of f(x) is -15
Case 2:
When $x = -1$,
$f'' (-1) = 6 * (-1) - 9 = - 6 - 9 = -15$
$f'' (-1) < 0$
f(x) is maximum at x = -1
Maximum value = f (-1)
$(-1)^3 - 3(-1)^2 - 9 * (-1) + 12$
$(-1) - 3 + 9 + 12$
$= 17$
Maximum value of f(x) is 17

2. Calculate the slope and the equation of a line which passes through the points (-1, -1) (3,8)

   Solution:

   Slop m of a line passing through two points (x1, y1) and (x2, y2) is given by,

   $m = (y_2 – y_1) / (x_2 – x_1)$

   then the slop of the line passing through the points (-1, -1) and (3, 8) is,

   m = 8 - (- 1) / 3 – (-1)

   m = 9 / 4

   m = 2.25

3. Solve for w'(z) when

   $$w(z) = \frac{4z-5}{2-z}$$

   Solution:

   Quotient Rule:

   $dy / dx = [v * du/dx – u * dv/dx] / v^2$

   w(z) = (4z – 5) / (2 – z)

   $w' (z) = [(4z – 5) (-1) – (2-z) * 4] / (2-z)^2$

   $w' (z) = [(5 – 4z) – (8 – 4z)] / (2-z)^2$

   $w' (z) = [5 – 4z – 8 + 4z] / (2-z)^2$

   $w' (z) = -3 / (2-z)^2$

4. Consider Y(x)= $2x^3 + 6x^2 + 3x$. Identify the critical values and verify if it gives maxima or minima.

   Solution:

   Y(x) = $2x^3 + 6x^2 + 3x$ _________________ (1)

   $Y'(x) = 6x^2 + 12x + 3$ _____________________ (2)

   Y" (x) = 12x + 12 ________________________ (3)

   Critical points,

   Putting Eqn (2) = 0,

   $Y'(x) = 6x^2 + 12x + 3 = 0$

   $Y'(x) = 2x^2 + 4x + 1 = 0$

   x = -1(-1/ (1/√2)) or x = -1(+1/ (1/√2))

   Maxima or Minima,

   At x = -1(-1/ (1/√2)),

   From eqn (3),

   Y" (-1(-1/ (1/√2))) = 12 (-1 / √2) < 0

   Y(x) has maximum value at x = -1(-1/ (1/√2))

   At x = -1(+1/ (1/√2)),

   From eqn (3),

   Y" (-1(+1/ (1/√2))) = 12 (+1 / √2) > 0

   Y(x) has minimum value at x = -1(+1/ (1/√2))

5. Determine the critical points and obtain relative minima or maxima or saddle points of function f defined by

$$y = 2x_1^2 + 2x_1x_2 + 2x_2^2 + 6x_1$$

$Y = 2x_1^2 + 2x_1x_2 + 2x_2^2 + 6x_1$
$\partial y / \partial x_1 = 4x_1 + 2x_2 + 6$
$\partial y / \partial x_2 = 2x_1 + 4x_2$
Critical points,

$\partial y / \partial x_1 = 0$ and $\partial y / \partial x_2 = 0$,

$\partial y / \partial x_2 = 2x_1 + 4x_2 = 0$

$x_1 + 2x_2 = 0$

$x_2 = - (1/2)\ x_1$

Substituting x_2 in $\partial y / \partial x_1$

$\partial y / \partial x_1 = 4x_1 + 2x_2 + 6 = 0$

$= 4x_1 + 2\ (- (1/2)\ x_1) + 6$

$= 4x_1 – x1 +6$

$x_1 = -2$

Substituting x_1 in x_2

$x_2 = - (1/2)\ x_1$

$x_2 = - (1/2)\ (-2)$

$x_2 = 1$

Critical points are, $x_1 = -2$ and $x_2 = 1$

Saddle Points,

$f_1 = \partial y / \partial x_1 = 4x_1 + 2x_2 + 6$
$f_2 = \partial y / \partial x_2 = 2x_1 + 4x_2$
Second order direct partials,

$\partial^2 y/\partial x_1^2 = f_{11} = 4$

$\partial^2 y/\partial x_2^2 = f_{22} = 4$

Second order Cross partials,

$\partial^2 y/\partial x_1 \partial x_2 = f_{12} = 2$

$\partial^2 y/\partial x_2 \partial x_1 = f_{21} = 2$

By using hessian determinant,

$$|\mathrm{H}| = \begin{vmatrix} f_{11} f_{12} \\ f_{21} f_{22} \end{vmatrix}$$

$$|\mathrm{H}| = \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix}$$

|H| = 16-4 = 12 is Saddle point.