

## Module 4

**Name:** Nithin Dsouza

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**Topic:** Mathematical foundations-Linear algebra & probability

1. Two friends, friend 1 and friend 2 appear in an interview for two vacancies for the same post. The probability of friend 1 getting selected is  $(1/7)$  and the probability of friend 2 getting selected is  $(1/5)$ . What is the probability that only one of them is selected?

**Solution:**

Let A = the event that the friend 1 is selected

Let B = the event that the friend 2 is selected

We have,

$$P(A) = 1/7$$

$$P(B) = 1/5$$

The event that only one of them is selected (let it be 'C') can be interpreted as,

Statements:

1. The friend 1 is selected and the friend 2 is not. OR
2. The friend 2 is selected and the friend 1 is not.

Mathematically, statement 1 is equivalent to,  $P(A \text{ \& not B})$

Statement 2 is equivalent to  $P(B \text{ \& not A})$ .

$$\text{Then, } P(C) = P(A \text{ \& not B}) + P(B \text{ \& not A})$$

$$= P(A) \cdot P(\text{not B}) + P(B) \cdot P(\text{not A}) \dots\dots\dots (1)$$

Since the events are independent.

$$P(\text{not B}) = 1 - P(B) = 1 - 1/5 = 4/5$$

$$P(\text{not A}) = 1 - P(A) = 1 - 1/7 = 6/7$$

$\therefore$  Eqn 1 becomes:

$$P(C) = 1/7 \times 4/5 + 1/5 \times 6/7 = 0.284$$

2. Find the determinant of
- $$\begin{bmatrix} 1, 3, 2 \\ -3, -1, -3 \\ 2, 3, 1 \end{bmatrix}$$

**Solution:**

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\begin{vmatrix} 1 & 3 & 2 \\ -3 & -1 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 1 \begin{vmatrix} -1 & -3 \\ 3 & 1 \end{vmatrix} - 3 \begin{vmatrix} -3 & -3 \\ 2 & 1 \end{vmatrix} + 2 \begin{vmatrix} -3 & -1 \\ 2 & 3 \end{vmatrix}$$

$$= 1(-1 \cdot 9) - 3(-3 \cdot 6) + 2(-9 \cdot 2)$$

$$= 9 - 54 + 36$$

$$= -9$$

3. If a number is picked at random from a set  $\{1, 2, 3, \dots, 70\}$ , then the probability that the picked number is a cube?

**Given:** If a number is chosen at random from the set  $\{1, 2, 3, \dots, \text{and } 70\}$ .

**To find:** The probability that the chosen number is a perfect cube?

**Solution:**

**Total number of outcome** is set  $\{1, 2, 3, \dots, 70\} = 70$

**Numbers having a perfect cube are,**

$$1^3 = 1$$

$$2^3 = 8$$

$$3^3 = 27$$

$$4^3 = 64$$

**Favorable outcome** of chosen number is a perfect cube = 4

**So, Probability that the chosen number is a perfect cube is**

$$\text{Probability} = \frac{\text{Favorable Outcome}}{\text{Total outcome}}$$

$$\text{Probability} = \frac{4}{100}$$

$$\text{Probability} = \frac{1}{25}$$

Therefore, Probability that the chosen number is a perfect cube is  $\frac{1}{25}$

4. If two dice are thrown together, what is the probability of getting an odd number on one die and an even number on the other die?

**Solution:**

The number of exhaustive outcomes is 36.

Let E be the event of getting an even number on one die and an odd number on the other.

Let the event of getting either both even or both odd then =  $18/36 = \frac{1}{2}$

$$P(E) = 1 - 1/2 = \frac{1}{2}$$

5. Ajay took two tests. The probability of him passing both tests is 0.9. The probability of him passing the first test is 0.7. What is the probability of him passing the second test given that he has passed the first test?

**Solution:**

$$P(\text{second}|\text{first}) = \frac{P(\text{first and second})}{P(\text{first})}$$
$$= \frac{0.9}{0.7}$$