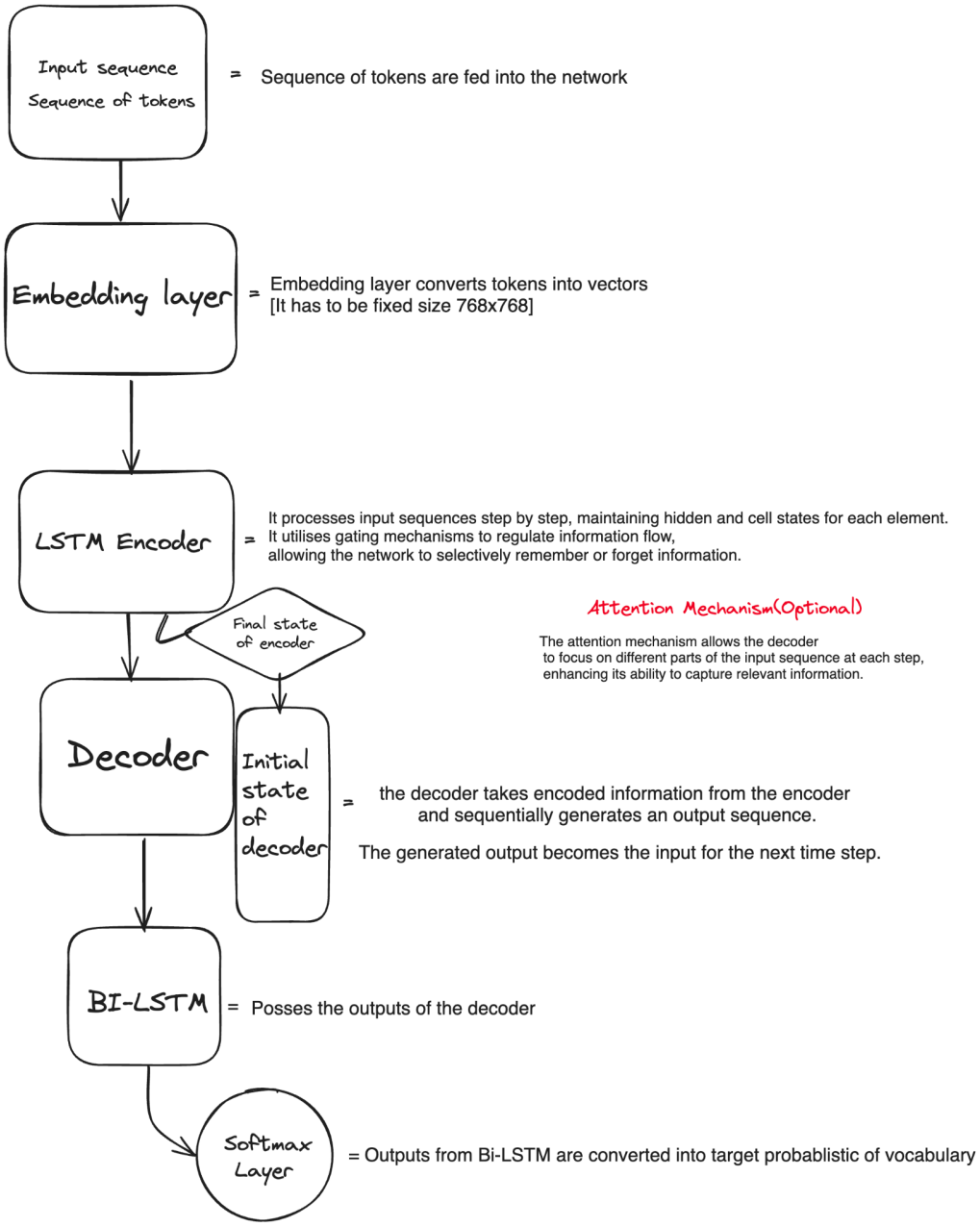
Research Assignment Solutions

P1. Develop a simple Sequence to Sequence model and connect the outputs to a BiLSTM (block diagram)



P2

1. Differentiate *Calculus* [1, 2] from *Calculus of Variations*
2. Can you explain how the Brachistochrone *problem* posed during the late 1600s led to the birth of *Calculus of Variations* ?

Solution: Mechanism, literature review

Sol: ***Calculus[1,2]***

- Calculus [1,2] is the branch of mathematics that deals with the study of limits, continuity, derivatives, integrals, and infinite series.
- Calculus [1] typically includes topics such as limits, derivatives of elementary functions, and basic integration techniques, while
- Calculus [2] extends these concepts to cover more advanced integration techniques, applications of integration, sequences, and series.

Calculus of Variations:

- The calculus of variations (or variational calculus) is a field of mathematical analysis that uses variations, which are small changes in functions and functionals, to find maxima and minima of functionals
- Calculus of variations deals with functionals which are functions of functions
- It majorly focuses on finding the function that minimizes or maximizes a certain functional.

Example:

Imagine if you wanna cross a river with least amount of time,

Calculus[1] : use math to figure out how fast the boat is moving, taking the river's current into account.

Write an equation that describes where the boat is, given at the moment.

Calculus[2]: set up a math problem to find the total time taken to cross the river including adding up smaller journeys.

Solve this math problem using more advanced techniques to find the fastest path.

Calculus of Variations:

In calculus of variations instead of finding one specific path, think about all possible paths.

Example, finding the most optimal path instead of one specific path

Come up with a rule or formula that tells you how to pick the best path. Use this rule to figure out the path that takes the least amount of time.

In simple two lines, calculus [1,2] lets you solve problems step by step, while calculus of variations lets you think about all possible solutions and find the best/optimized solution.

Brachistochrone problem:

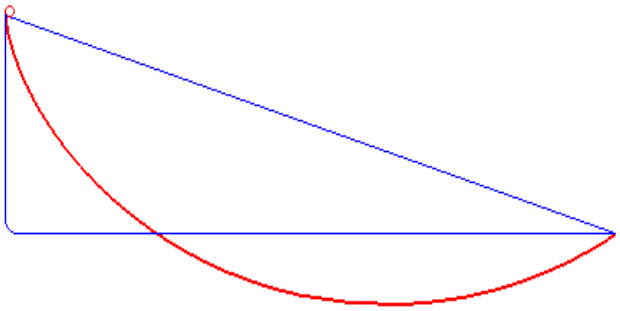
Brachistochrone problem involves finding the curve that a particle, acted upon only by gravity, would take between two points in the least amount of time.

The main objective is to determine the shape of the curve that minimizes the time to travel from one point to another.

Proposed by Johann Bernoulli in 1696

This led to the birth of calculus which involves optimizing the functionals,

The Euler-Lagrange equation, a fundamental equation in the Calculus of Variations.



$$\text{Time} = \int_{x_A}^{x_B} \sqrt{\frac{1+\left(\frac{dy}{dx}\right)^2}{2gy}}\, dx$$

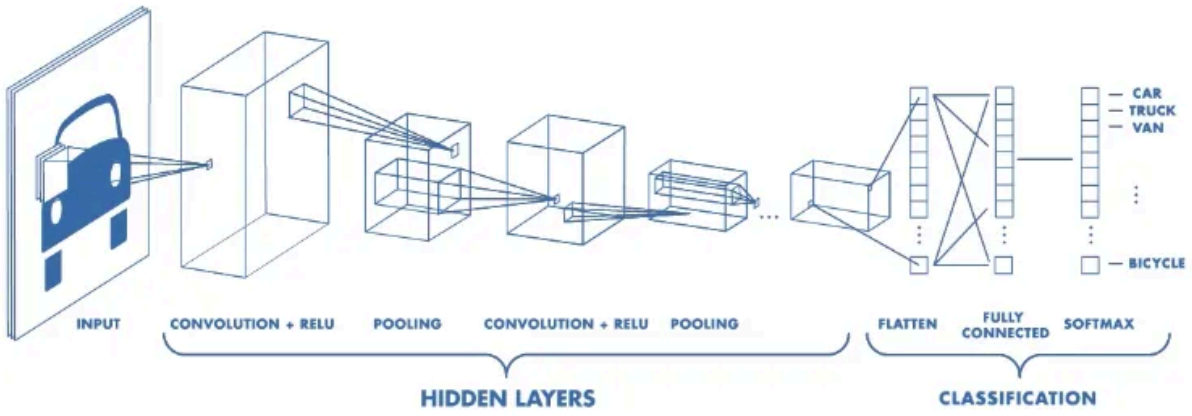
- Where xA, xB represents start and end points
- y(x) is the height of cycloid at each position x
- dy/dx is the slope of the cycloid indicating how steep it is
- g is the acceleration due to gravity

P3

Explain Convolutional Neural Networks:

Cnn is a type of neural network designed for processing and analyzing visual data. It is particularly effective for tasks related to image recognition, classification, and feature extraction. CNNs are characterized by their use of convolutional layers, which apply filters or kernels to input data, enabling the network to automatically and adaptively learn hierarchical representations of features from the data.

A CNN typically has three layers: a convolutional layer, a pooling layer, and a fully connected layer.



Mechanism:

Filters(core feature) applied across the image, extracting features like edges, textures, and color patterns. Imagine sliding a magnifying glass over a painting to examine details.

Pooling: Downsamples feature maps, reducing complexity and computational burden. It's like summarizing key points from each filtered image section.

Activation: Introduces non-linearity, enabling the network to learn more complex relationships. Imagine adding decision points based on extracted features, like "is this edge vertical or horizontal?"

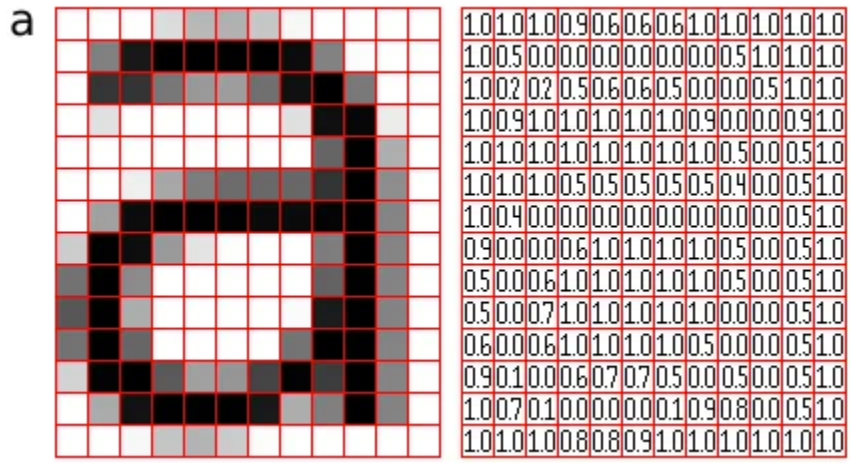
Layers: Stacked layers progressively extract higher-level features, similar to how your brain builds understanding layer by layer.

Learning: Through training (showing the CNN many images and their labels), the network adjusts its

filter weights and biases to become adept at identifying relevant features. Imagine your brain adapting based on new experiences.

Architecture's include AlexNet, Resnet, VGGNet, etc.

Ex: The below image is divided into Pixels and features are extracted from each pixel to identify patterns, textures etc.



Literature Survey:

- "**ImageNet Classification with Deep Convolutional Neural Networks**" (Krizhevsky et al., 2012): Introduced AlexNet, demonstrating the breakthrough potential of CNNs.
- "**Very Deep Convolutional Networks for Large-Scale Image Recognition**" (Simonyan & Zisserman, 2014): Showcased the power of depth in CNNs with VGGNet.
- "**Deep Residual Learning for Image Recognition**" (He et al., 2015): Presented ResNet, addressing vanishing gradients and achieving new state-of-the-art accuracy.

Involutional Neural Networks :

Involutional neural networks are a type of neural networks which are location specific and channel agnostic. A convolution kernel is spatial-agnostic and channel-specific. Because of this, it isn't able to adapt to different visual patterns with respect to different spatial locations. Along with location-related problems, the receptive field of convolution creates challenges with regard to capturing long-range spatial interactions.

In CNNs, features are processed spatially by applying filters across the image. In INNs, features are processed within each spatial location before combining them spatially.

Filter Design: CNNs use filters shared across channels, while INN filters can be different for each spatial location, offering more flexibility.

Feature Dependencies: CNNs might struggle with long-range dependencies, while INNs have the potential to capture them more effectively.