Introduction to Quantum computing (From a layperson to programmer in 30 steps)

This is one of the greatest books for beginners in quantum computing, in my opinion, especially for those who might feel intimidated by the abundance of complicated online resources. The book's Analogies section, which simplified complicated topics for laypeople, was its strongest point. The majority of the book's issues are resolved below with little to no prior knowledge required, and repeated reads of the book made nearly every topic easier.

Note:- This book is solved according to my understanding. please email me for any comments or feedback.

Book link

https://link.springer.com/book/10.1007/978-3-030-98339-0

Chapter 1

Basic intro

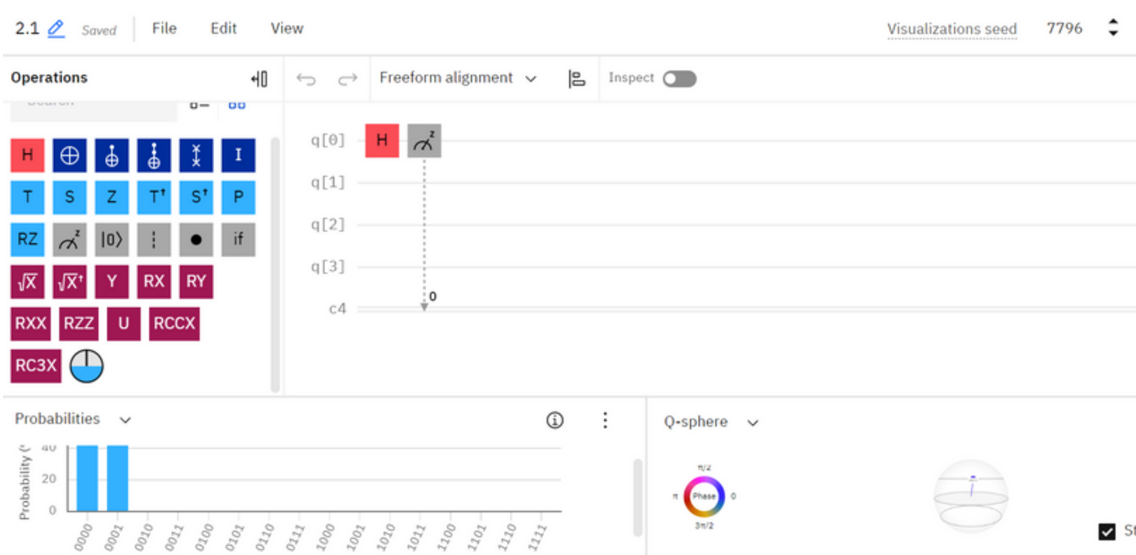
Code link

https://github.com/nithingovindugari/IntrotoQC

Chapter 2

2.1) Created IBM account

2.2)



2.3)

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Details

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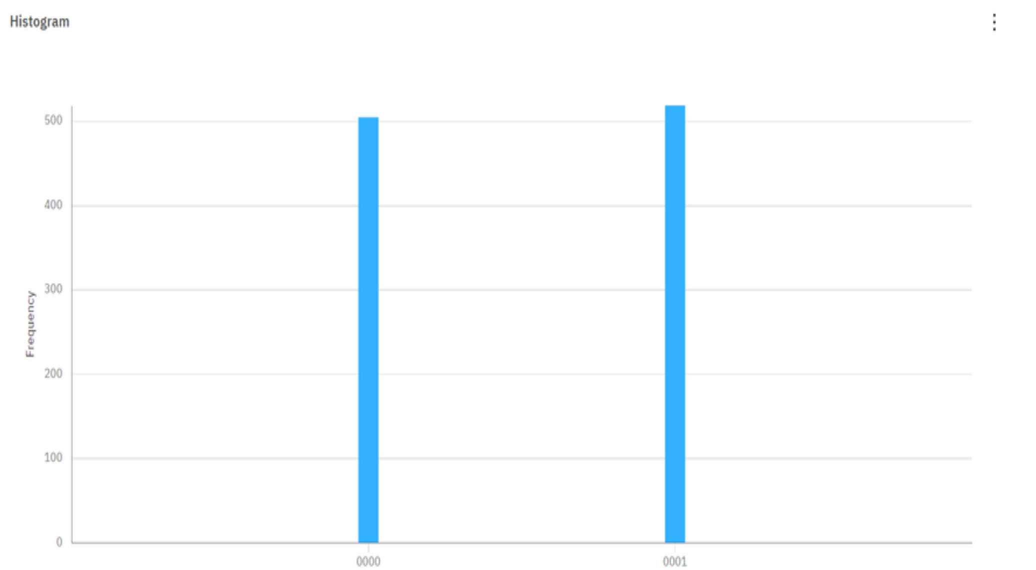


Chapter 3

3.1) Had a google collab account already

3.2 & 3.3 & 3.4)

```
import numpy as np
from sklearn.preprocessing import normalize

#problem3
v1 = np.array([[np.sqrt(3)],[1]])
v2 = np.array([[1],[1]])
inner_result = np.vdot(v1,v2)
print(inner_result)
print(v1.shape)


v3 = np.array([[1j],[1]])
v4 = np.array([[-1j],[1]])
inner_result1 = np.vdot(v3,v4)
print(inner_result1)

v5 = np.array([[1+2j],[1]])
v6 = np.array([[1-1j],[1]])
inner_result2 = np.vdot(v5,v6)
print(inner_result2)
```

Chapter 4

4.1)

from 4.18 we know

$$|v'\rangle = \frac{|v\rangle}{\sqrt{\sum_{i=0}^{n-1} |a_i|^2}}$$

we can derive numerators of above co-efficients

$$|\vec{v_1}\rangle = 2|\uparrow\rangle + 1|\downarrow\rangle$$

$$= \left(\frac{2}{\sqrt{2^2+1}}, \frac{1}{\sqrt{2^2+1}}\right) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

$$= (0.8944 \quad 0.4472)$$

4.3)

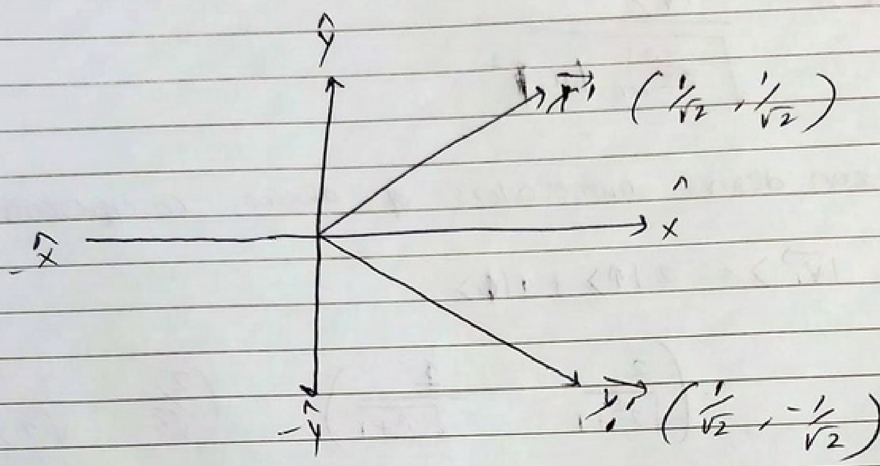
$$|\vec{v_1}\rangle = 2|\uparrow\rangle + (1+i)|\downarrow\rangle.$$

$$= \left(\frac{2}{\sqrt{2^2+(1+i)^2}}, \frac{1+i}{\sqrt{2^2+(1+i)^2}}\right) = \left(\frac{2}{\sqrt{4+2i}}, \frac{1+i}{\sqrt{4+2i}}\right)$$

$$= (0.816+0.j \quad 0.4082+0.408j)$$

4.5) Given $|\vec{x'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle + |\hat{y}\rangle)$

$|\vec{y'}\rangle = \frac{1}{\sqrt{2}}(|\hat{x}\rangle - |\hat{y}\rangle)$



$\langle \vec{x'} | \vec{x'} \rangle = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{2} + \frac{1}{2} = 1$ (Normalized)

$\langle \vec{y'} | \vec{y'} \rangle = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{2} + \frac{1}{2} = 1$ (Normalized)

$\langle \vec{x'} | \vec{y'} \rangle = \left(\frac{1}{\sqrt{2}} \quad \frac{1}{\sqrt{2}}\right) \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{2} - \frac{1}{2} = 0$ (orthogonal)

$\langle \vec{y'} | \vec{x'} \rangle = \left(\frac{1}{\sqrt{2}} \; -\frac{1}{\sqrt{2}}\right) \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = +\frac{1}{2} - \frac{1}{2} = 0$ (orthogonal)

It is satisfying both orthoganality and normalization

It satisfies orthonormality

```
#Problem 4
v1 = np.array([[2],[1]])
print(v1)
v2 = v1/np.linalg.norm(v1)
print(v2)

v3 = np.array([[2],[1+1j]])
print(v3)
v4 = v3/np.linalg.norm(v3)
print(v4)
```

```
[[2]
 [1]]
[[0.89442719]
 [0.4472136 ]]
[[2.+0.j]
 [1.+1.j]]
[[0.81649658+0.j        ]
 [0.40824829+0.40824829j]]
```

Chapter 5

5.1) we know from 36 page.

$$|0\rangle = \frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle \quad \& \quad |1\rangle = \frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle \quad \text{①}$$

we know $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ & $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$ ②

Given $|\vec{v_1}\rangle = \begin{pmatrix}1\\2\end{pmatrix}$ in $|+\rangle / |-\rangle$ basis

$$|\vec{v_1}\rangle = |+\rangle + 2|-\rangle$$

we get $|0\rangle = \frac{1}{\sqrt{2}}(|+\rangle + 2|-\rangle)$ $|1\rangle = \frac{1}{\sqrt{2}}(|+\rangle - 2|-\rangle)$ ③

converting back by substituting ③ in ②

$$|+\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(2|+\rangle)\right) = 1$$

$$|-\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(4|-\rangle)\right) = 2$$

$$= \begin{pmatrix}1\\2\end{pmatrix}$$

5.2)

Given $|\vec{v_1}\rangle = \begin{pmatrix}1\\2\end{pmatrix}$

Assumed it is in $|0\rangle, |1\rangle$ basis

$$|\vec{v_1}\rangle = 1|0\rangle + 2|1\rangle$$

Converting it to $|+\rangle$ $|-\rangle$ basis

$$= \left(\frac{1}{\sqrt{2}}|+\rangle + \frac{1}{\sqrt{2}}|-\rangle\right) + 2\left(\frac{1}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle\right)$$

$$= \frac{3}{\sqrt{2}}|+\rangle - \frac{1}{\sqrt{2}}|-\rangle$$

Normalized values are

~~5.9~~)

$$\left(\frac{3/\sqrt{2}}{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}, \frac{\cancel{+}1/\sqrt{2}}{\left(\frac{3}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2}\right)$$

$$= (0.948, \cancel{-}0.316)$$

Probability is positive as it can never be negative

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5.3) Probability to find $|+\rangle$ in $(0.948, 0.316)$ is 0.948

5.4) Probability to find $|0\rangle$ in $|0\rangle / |1\rangle$ basis

$$|\vec{v_1}\rangle = 1|0\rangle + 2|-\rangle$$

$$= \left(\frac{1}{\sqrt{1+4}}, \frac{2}{\sqrt{1+4}}\right) = \left(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$$

$$= (0.447, 0.894)$$

Probability of finding $|0\rangle$ is 0.447

```
#Problem 5.2
v1 = np.array([[3/np.sqrt(2)],[1/np.sqrt(2)]])
print(v1)
v2 = v1/np.linalg.norm(v1)
print(v2)

# Problem 5.4
v3 = np.array([[1] , [(2 )]])
print(v3)
v4 = v3/np.linalg.norm(v3)
print(v4)
```

```
[[2.12132034]
 [0.70710678]]
[[0.9486833 ]
 [0.31622777]]
[[1]
 [2]]
[[0.4472136 ]
 [0.89442719]]
```

Chapter 6

6.1 sol.

Given operator or matrix

$$\sigma_y |eigenvector\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$= \begin{pmatrix} -iy \\ ix \end{pmatrix}$$

we can assume eigen value as α

$$\begin{pmatrix} -iy \\ ix \end{pmatrix} = \alpha \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow (1)$$

$\alpha x = -iy$ & $\alpha y = ix$

we know from equ (1)

$\alpha x = -iy \qquad \alpha y = ix$

we can substitute and derive $\alpha^2 = 1$

and $\alpha = \pm 1$

when $\alpha = 1$ we know 1st vector is

x part of matrix $-i$ & y part is 1

and after normalization we can derive

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} -i \\ 1 \end{pmatrix}$$

when $\alpha = -1$ we can derive by equating

them to

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ 1 \end{pmatrix}$$

```
#Lecture_6-Problems
import numpy as np
sigmay = np.array([[0, -1j],[1j,0]])
print(sigmay)
eigenvalues,eigenvectors = np.linalg.eig(sigmay)
print(eigenvalues)
print(eigenvectors)
```

```
[[ 0.+0.j -0.-1.j]
 [ 0.+1.j  0.+0.j]]
[ 1.+0.j -1.+0.j]
[[-0.        -0.70710678j  0.70710678+0.j        ]
 [ 0.70710678+0.j          0.        -0.70710678j]]
```

Chapter 7

Problems

7.1) $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

To find. $\sigma_y^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 - i^2 & 0+0 \\ 0+0 & -i^2+0 \end{pmatrix}$ $(\because i^2 = -1)$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$\sigma_y^2 = I$

7.2) $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$\sigma_z^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$= \begin{pmatrix} 1*1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$\sigma_z^2 = I$

7.3) σ_y is Hermitian

we know $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$

Firstly conjugation

$$\sigma_y^* = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$$

Transpose

$$\sigma_y^{*T} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \sigma_y^{*T}$$

7.4) $\langle 0 | \sigma_y$

$$\begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{bmatrix} 1\times 0 + 0\times i & 1\times -i + 0\times 0 \end{bmatrix}$$

$$\begin{pmatrix} 0 & -i \end{pmatrix}$$

It is in bra form and it is equal to $\sigma_y |0\rangle$ only if we do transpose and complex conjugate to $(\langle 0|\sigma_y)^\dagger$ i.e, $(\langle 0|\sigma_y)^{T*} = \sigma_y|0\rangle$

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```
#Lecture_7-Problems
#7.1
sigmay=np.array([[0,-1j],[1j,0]])
print(sigmay)
print(np.matmul(sigmay, sigmay))

#7.2
sigmaz=np.array([[1,0],[0,-1]])
print(sigmaz)
print(np.matmul(sigmaz, sigmaz))

#7.3

sigmayh = np.conjugate(np.transpose(sigmay))
print(sigmayh)

#7.4
zerospin=np.array([[1],[0]])
print(zerospin.shape)
print(np.dot(sigmay,zerospin))
```

```
[[ 0.+0.j -0.-1.j]
 [ 0.+1.j  0.+0.j]]
[[1.+0.j 0.+0.j]
 [0.+0.j 1.+0.j]]
[[ 1  0]
 [ 0 -1]]
[[1 0]
 [0 1]]
[[ 0.-0.j  0.-1.j]
 [-0.+1.j  0.-0.j]]
(2, 1)
[[0.+0.j]
 [0.+1.j]]
```

Chapter 8

```python
#Lecture_8-Problems

import numpy as np
sigmax=np.array([[0,1],[1,0]])
print(sigmax)
eigenvalues, eigenvectors=np.linalg.eig(sigmax)
print(eigenvalues)
print(eigenvectors)

sigmaxp=np.array([1/2,1/2])
sigmaxn=np.array([1/2,-1/2])
eigenvalues, eigenvectors=np.linalg.eig(sigmax)
print(eigenvalues)
print(eigenvectors)
print(np.dot(sigmaxp,sigmaxp))
print(np.dot(sigmaxn,sigmaxn))
```

```
[[ 0.+0.j -0.-1.j]
 [ 0.+1.j  0.+0.j]]
[[1.+0.j 0.+0.j]
 [0.+0.j 1.+0.j]]
[[ 1  0]
 [ 0 -1]]
[[1 0]
 [0 1]]
[[ 0.-0.j  0.-1.j]
 [-0.+1.j  0.-0.j]]
(2, 1)
[[0.+0.j]
 [0.+1.j]]
```

8.2) Given σ_z

we know $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ it is ket notation

and even if find adjoint to get bra it is sar

so it is Hermitanian matrix

we know project operator is

$$P_{\sigma_z} = |\sigma z\rangle\langle\sigma z|$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$= \begin{pmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} =$$

$$I = P_{\sigma z}$$

8.3)

Given

eigen vectors of σ_x

$|+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ $|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

we know for Probability

$P_i = |i\rangle\langle i|$

probability of $|+\rangle$ ~~is $|-\rangle$~~

$= \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix} \cdot \frac{1}{\sqrt{2}}(1 \; 1)$

$= \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$

So it is 50%. & similarly for $|-\rangle$ is

$= \frac{1}{2}\begin{pmatrix}1 & -1\\-1 & 1\end{pmatrix}$

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```
#8.4
sigmaxp=np.array([1/np.sqrt(2),1/np.sqrt(2)])
sigmaxn=np.array([1/np.sqrt(2),-1/np.sqrt(2)])
eigenvalues, eigenvectors=np.linalg.eig(sigmax)
print(eigenvalues)
print(eigenvectors)
print(np.dot(sigmaxp,sigmaxp))
print(np.dot(sigmaxn,sigmaxn))
.
```

result

```
-
0.9999999999999998
0.9999999999999998
```

Chapter 9

9.1 sol

we know from eq 9.15

that $U^{+} U = I$

we know from page no 24 we can use

eq 4.1 $\hat{v}_i \cdot \hat{v}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

so when we consider a row in 9.15 like

we can say if it is in same row

example $b_{0,0}$ is 0^{th} row it is 1 etc

9.2) Given 9.2 used it in (i) (ii) (iii)

$$\begin{vmatrix} a_{0,0}-\lambda_i & a_{0,1} \\ a_{1,0} & a_{1,1}-\lambda_i \end{vmatrix}$$

i) $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$= \begin{vmatrix} 0-\lambda_i & 1 \\ 1 & 0-\lambda_i \end{vmatrix} = 0 \qquad +\lambda_i^2 - 1 = 0$$

$$\boxed{\lambda_i = \pm 1}$$

ii) $\sigma_y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$

$$= \begin{vmatrix} 0-\lambda_i & i \\ -i & 0-\lambda_i \end{vmatrix} = 0 \Rightarrow \lambda_i^2 - (i^2) = 0$$

$$\lambda^2 = -1$$

$$\boxed{\lambda = \pm 1}$$

iii) $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$= \begin{vmatrix} 1-\lambda_i & 0 \\ 0 & -1-\lambda_i \end{vmatrix} = 0 \Rightarrow -(1-\lambda^2) = 0$$

$$\lambda^2 = 1$$

$$\boxed{\lambda = \pm 1}$$

eigen values are same for all

```
#Lecture_9-Problems
#9.3
# M=np.array([[1,0],[0,-1]])
sigmax=np.array([[0,1],[1,0]])
sigmay=np.array([[0,-1j],[1j,0]])
sigmaz=np.array([[1,0],[0,-1]])



print(np.linalg.det(sigmax))
print(np.linalg.det(sigmay))
print(np.linalg.det(sigmaz))
```

```
-1.0
(-1+0j)
-1.0
```

9.4)

For example we can take

A as $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ i.e, σx

we know pauli matrices are unitary

we can prove it now

$$|v_i\rangle = \frac{1}{\sqrt{2}}|\alpha\rangle + \frac{1}{\sqrt{2}}|\beta\rangle$$

$$|v_j\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}$$

$$A|v_j\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

we can prove orthonormality by Adding squares of cofficients of transformed $A|v_i\rangle$ and outer product with other matrices will be zero

$$\left(-\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 = 1$$

Chapter 10

10.1)

Transformation matrix from basis σ_y to σ_x

we know from problem 6.1 that eigen vectors of σ_y are $\frac{1}{\sqrt{2}}\left(\begin{pmatrix}-i\\1\end{pmatrix}\begin{pmatrix}i\\1\end{pmatrix}\right)$

$|+\rangle$ for $\sigma_x = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ & $|-\rangle$ is $\frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$

$$u = (\langle new, y \rangle | old, x \rangle) = \begin{bmatrix} \frac{1}{2}(-i \; 1)^{+}\begin{pmatrix}1\\1\end{pmatrix} & \frac{1}{2}(-i, 1)^{+}\begin{pmatrix}1\\-1\end{pmatrix} \\ \frac{1}{2}(i \; 1)^{+}\begin{pmatrix}1\\1\end{pmatrix} & \frac{1}{2}(i \; 1)^{+}\begin{pmatrix}1\\-1\end{pmatrix} \end{bmatrix}$$

$$= \left[\frac{1}{2} \begin{bmatrix} i+1 & i-1 \\ -i+1 & -i-1 \end{bmatrix} \right]$$

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10.2) Given in Page 87

· complete basis is defined by outer product of each vectors sums to Identity matrix

~~Ider~~ outer product is Nothing but Ket x bra

$$\sum_{i=0}^{n-1} |i\rangle\langle i| = |-\rangle\langle -| + |+\rangle\langle +|$$

By deriving

$$= \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

10.3) For σ_z we know

$$|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad |+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

weknow from 10.19

$$\sigma_x = \sum_{i=0}^{2-1} \lambda_i |i\rangle\langle i|$$

$$= 1\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} * \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \end{pmatrix}\right) + -1\left(\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \end{pmatrix}\right)$$

$$= \left(\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right) - \left(\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right)$$

$$\sigma_x = \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

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```
#Lecture_10-Problems

#10.4

import numpy as np
sigmaxn=np.array([1/np.sqrt(2),-1])
sigmaxp=np.array([1/np.sqrt(2),1])
sigmayp=np.array([1/np.sqrt(2),+1j])
sigmayn=np.array([1/np.sqrt(2),-1j])

print(np.matmul(sigmaxn, sigmayn))
print(np.matmul(sigmaxn, sigmayp))
print(np.matmul(sigmaxp, sigmayn))
print(np.matmul(sigmaxp, sigmayp))
```

```
(0.4999999999999999+1j)
(0.4999999999999999-1j)
(0.4999999999999999-1j)
(0.4999999999999999+1j)
```

Chapter 11

11.1)

Given $|f\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $|g\rangle = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$

From 11.17 & 11.14

we know $\begin{pmatrix} \alpha_3 \\ \beta_3 \\ \gamma_3 \\ \delta_3 \end{pmatrix} = \begin{pmatrix} \alpha_1 \alpha_2 \\ \alpha_1 \beta_2 \\ \beta_1 \alpha_2 \\ \beta_1 \beta_2 \end{pmatrix}$

i) $|f\rangle_1 \otimes |g\rangle_2$

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$$

ii) $|g\rangle_1 \otimes |f\rangle_2$

$$\begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix} \otimes \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 0 \\ i/\sqrt{2} \end{bmatrix}$$

Both are not same

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11.2)

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} \frac{1}{\sqrt{2}} \\ i/\sqrt{2} \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1/\sqrt{2} \\ 0 \\ i/\sqrt{2} \end{pmatrix}$$

11.3) we need to prove distribution rule

11.20

$$(|f\rangle + |e\rangle) \otimes |g\rangle = |f\rangle \otimes |g\rangle + |e\rangle \otimes |g\rangle$$

$$\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) \otimes \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ i/\sqrt{2} \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ i/\sqrt{2} \end{pmatrix}$$

$$\begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} + \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{bmatrix}$$

L.H.S = R.H.S

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11.4)

```
#Lecture_11-Problems

#11.4
#proving 11.1
import numpy as np
g=np.array([[0],[1]])
h=np.array([[1/np.sqrt(2)],[1j/np.sqrt(2)]])
print(np.kron(g,h))
print(np.kron(h,g))

# proving 11.2
x = np.kron(h,g)
print(np.kron(g,x))

#proving 11.3
y=np.array([[1],[0]])
z = (g + y)
print(np.kron(z,h))
a = np.kron(z,h)
```

```
l = np.kron(g,h)
m = np.kron(y,h)
b = l+m
print(l+m)

if  a.all() == b.all() :
  print(" L.H.S == R.H.S")
```

```
[[0.        +0.j        ]
 [0.        +0.j        ]
 [0.70710678+0.j        ]
 [0.        +0.70710678j]]
[[0.        +0.j        ]
 [0.70710678+0.j        ]
 [0.        +0.j        ]
 [0.        +0.70710678j]]
[[0.        +0.j        ]
 [0.        +0.j        ]
 [0.        +0.j        ]
 [0.        +0.j        ]
 [0.        +0.j        ]
 [0.70710678+0.j        ]
 [0.        +0.j        ]
 [0.        +0.70710678j]]
[[0.70710678+0.j        ]
 [0.        +0.70710678j]
 [0.70710678+0.j        ]
 [0.        +0.70710678j]]
[[0.70710678+0.j        ]
 [0.        +0.70710678j]
 [0.70710678+0.j        ]
 [0.        +0.70710678j]]
 L.H.S == R.H.S
```

11.5)

Given $n_1 = n_2 = 1$ & $|f\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$|e\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $|g\rangle = \begin{pmatrix} i \\ i \end{pmatrix}$ and $|k\rangle = \begin{pmatrix} i \\ 1 \end{pmatrix}$

<u>equ</u> 11.21

$$\langle h_1 | h_2 \rangle = \left\langle \sum_{j=1}^{n_1} |f_j\rangle \otimes |g_j\rangle \middle| \sum_{l=1}^{n_2} |e_l\rangle \otimes |k_l\rangle \right\rangle$$

$$= \sum_{j=1}^{n_1} \sum_{l=1}^{n_2} \langle f_j | e_l \rangle \langle g_j | k_l \rangle$$

$\because n_1 = n_2 = 1$

L.H.S

$$= \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} i \\ i \end{pmatrix} \middle| \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} i \\ 1 \end{pmatrix} \right\rangle$$

($\because$ It is inner product)

$$= \begin{pmatrix} i & i & i & i \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ i \\ i \end{pmatrix} = i - 1$$

R.H.S

$$= \begin{pmatrix} 1 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} i & i \end{pmatrix} \begin{pmatrix} i \\ 1 \end{pmatrix}$$

$$= (0 + 1)\,(i - 1)$$

$$= i - 1$$

Chapter 12

12.1 Sol)

Given $|f\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $|g\rangle = \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$

Initially

$\sigma_x |f_0\rangle_1$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_x |g\rangle_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} \\ \frac{i}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} i/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$$

$\sigma_x |f\rangle_1 \otimes \sigma_x |g\rangle_2$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix} \begin{pmatrix} i/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} i/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{pmatrix}$$

Now to find $|f\rangle_1 \otimes |g\rangle_2$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \otimes \begin{pmatrix} 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ i/\sqrt{2} \end{pmatrix}$$

$$\sigma_x \otimes \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

$$\sigma_x \otimes \sigma_x \left(|f\rangle_1 \otimes |g\rangle_2 \right)$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

So it is equal to $\sigma_x |f\rangle_1 \otimes \sigma_x |g\rangle_2$

$$\sigma_x |f\rangle_1 \otimes \sigma_x |g\rangle_2 = \sigma_x \otimes \sigma_x \left(|f\rangle_1 \otimes |g\rangle_2 \right)$$

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Chapter 13

13.1

As quantum computing is known for exponential speed and for quantum parallelism world can store up to
21021 quantum states due to superposition.

13.2)

13.2)

Here $|\phi^+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\0\\1\end{pmatrix}$ is proved already

$|\phi^-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\0\\0\\-1\end{bmatrix}$

$= \frac{1}{\sqrt{2}}\left(1|00\rangle + 0|01\rangle + 0|10\rangle - 1|11\rangle\right)$

we can assume $|\phi^+\rangle = |m\rangle \otimes |n\rangle$

$|m\rangle = a_0|0\rangle + a_1|1\rangle$ & $|n\rangle = b_0|0\rangle + b_1|1\rangle$

Given 13.11 $\rightarrow$ $|\phi^+\rangle = a_0b_0|00\rangle + a_0b_1|01\rangle + a_1b_0|10\rangle + a_1b_1|11\rangle$

Similar to 13.12 $a_0b_0 = 1$, $a_0b_1 = 0$, $a_1b_0 = 0$
$a_1b_1 = -1$

Similarly $a_0b_0 = 0$ or $a_1b_1 = 0$ should be here

So $|\phi^+|$ is entangled as our assumption is false

$|\psi^+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}0\\1\\1\\0\end{bmatrix}$

$= \frac{1}{\sqrt{2}}\left(0|00\rangle + 1|01\rangle + 1|10\rangle + 0|11\rangle\right)$

From 13.2

$a_0 b_0 = 0$, $a_0 b_1 = 1$

$a_1 b_0 = 1$ $a_1 b_1 = 0$

we can assume $a_0 b_0 = 0$ either of them should be zero it denies our above calculation

$|\psi^-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-1\\0\end{pmatrix}$

$= \frac{1}{\sqrt{2}}\left(0|00\rangle + 1|01\rangle - 1|10\rangle + 0|11\rangle\right)$

From 13.2

$a_0 b_0 = 0$, $a_0 b_1 = 1$, $a_1 b_0 = 1$

$a_1 b_1 = 0$

we can assume $a_0 b_0 = 0$ and one of them should be zero it denies calculation

13.3 sol) Yes, I believe entanglement is concept irresp. of distance we can expect the result. It will be proffiting for him

13.4)

Here as mentioned basis states are orthonormal and if we have the same state in both two spaces then the inner product will be 1

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Chapter 14

14.1

14.2

Given $\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$

From 14.12 $\Rightarrow$ $\frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|0\rangle\langle 1| + \frac{1}{2}|1\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1|$

$$\rho = \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}(1\ 0) + \frac{1}{2}\begin{pmatrix}1\\0\end{pmatrix}(0\ 1) + \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}(1\ 0) + \frac{1}{2}\begin{pmatrix}0\\1\end{pmatrix}(0\ 1)$$

$$= \frac{1}{2}\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix} + \frac{1}{2}\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$$

$$\rho = \frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$$

As we know entaglement entropy is zero from previous chapter that we know our given state is not product of two subspaces and it should be zero

14.2)

Given $\rho = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow 14.15$

- We know $\rho_B = Tr_A(\rho)$

Figure 14.2 for tracing second qubit

$$= \begin{pmatrix} 0 & 0 & \boxed{0} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & \boxed{0} \\ 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Above same shape elements can be added

From above tracing B space

$$= \begin{bmatrix} 0 & \times & 0 & \times \\ \times & \frac{1}{2} & \times & 0 \\ 0 & \times & \frac{1}{2} & \times \\ \times & 0 & \times & 0 \end{bmatrix}$$

Chapter 15

15.1)

we know

from 15.6

$$U_{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

To prove it as unitary we know

$$U^{\dagger} \cdot U = I$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{should be } I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \times 0 + 1 + 1 & 0 \times 0 + 1 \times 0 \\ 1 \times 0 * 0 \times 1 & 1 \times 1 + 0 \times 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Hence proved

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15.2)

we know from 15.15

$$\text{CNot is } U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

From 9.18 we know

$$U^{+}U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1\times1+0\times0+0\times0+0\times0 & 0 & 0 & 0 \\ 0 & 0\times0+1\times1+0\times0\times0 & 0 & 0 \\ 0 & 0 & 0\times0+0\times0+0\times0\times1 & 0 \\ 0 & 0 & 0 & 0\times0+0\times0+1\times1+0\times0 \end{pmatrix}$$

$$= I = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Hence proved

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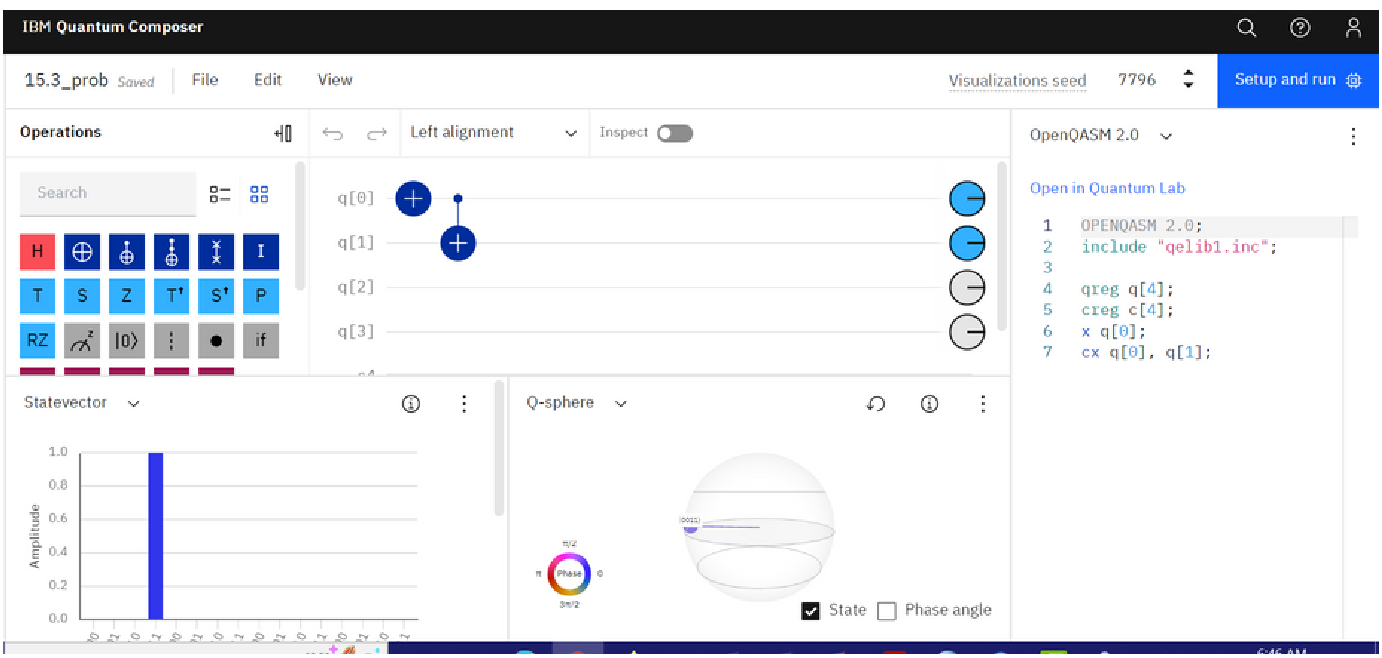
```python
#Chapter 15

#15.1
import numpy as np
sigmay=np.array([[0,1],[1,0]])   #Not gate and transpose is same for this matrix
print(sigmay)
print(np.matmul(sigmay, sigmay))

#15.2
sigmac=np.array([[1,0,0,0],[0,1,0,0],[0,0,0,1],[0,0,1,0]])   #CNot gate and transpose is same for this matrix
print(sigmac)
print(np.matmul(sigmac, sigmac))
```

```
[[0 1]
 [1 0]]
[[1 0]
 [0 1]]
[[1 0 0 0]
 [0 1 0 0]
 [0 0 0 1]
 [0 0 1 0]]
[[1 0 0 0]
 [0 1 0 0]
 [0 0 1 0]
 [0 0 0 1]]
```

15.3)



Chapter 16

16.1)

16.1) we know Z-gate is $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

and $|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ & $|+\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

when we apply Z gate to $|-\rangle$

$$Z|-\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1\times 1 + 0\times -1 \\ 0\times 1 - 1\times -1 \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= |+\rangle$$

16.2)

(16.2) we know control phase shift gate is

$$U_{cps,\phi} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{bmatrix}$$

In z-axis when angle is π

we can assume $e^{i\phi} = \cos\phi + i\sin\phi = \cos\pi + i\sin\pi = -1$

controlled z gate is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$

It is in $|0\rangle / |1\rangle$ basis

In order to prove it is CNOT in $|+\rangle\ |-\rangle$ we need to convert it $\Rightarrow$ we know $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

we know transformation matrix from $|0\rangle / |1\rangle$ to $|+\rangle\ |-\rangle$ is (Hadamard gate

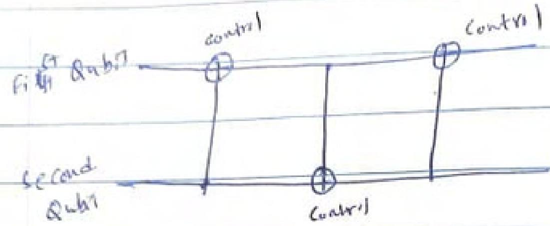
$$U = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

when we apply the above matrix to convert it to $|+\rangle\ |-\rangle$ is

$$CZ_{(|+\rangle/|-\rangle)} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

So Hence C_Z in $|+\rangle / |-\rangle$ is CNOT gate

16.3) Here I have analyzed 3 CNOT gates usage: building swap gate



Here 1^{st} Qubit converts (First Not gate)

1^{st} CNot gate $CNot_1(|00\rangle = |00\rangle, |01\rangle = |01\rangle, |10\rangle = |11\rangle,$

control is with 2^{nd} Qubit

2^{nd} CNot gate $CNot_2(|00\rangle = |00\rangle, |01\rangle = |11\rangle, |11\rangle = |01\rangle,$

control is with 1^{st} Qubit

3^{rd} CNot gate $CNot_3(|00\rangle = |00\rangle, |11\rangle = |10\rangle, |01\rangle = |01\rangle,$

we can see swap is done and it is proved

For CNot 2 using state representation we can derive

$$CNot_2 = \begin{array}{cc} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} & \begin{array}{c} |00\rangle \\ |11\rangle \\ |00\rangle \\ |10\rangle \end{array} \end{array}$$

Chapter 17

17.1)

17.1 Given

$H|\psi\rangle = \alpha|+\rangle + \beta|-\rangle$ From 17.4

Normally we can directly understand that co-efficient of $|0\rangle$ & $|1\rangle$ basis is from 17.4 $\frac{1}{\sqrt{2}}(\alpha+\beta)$ & $\frac{1}{\sqrt{2}}(\alpha-\beta)$ from understanding

we can even derive from 17.4

$= \alpha|+\rangle + \beta|-\rangle$

we know $|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$= \alpha\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right) + \beta\left(\frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\right)$

$= \frac{\alpha}{\sqrt{2}}|0\rangle + \frac{\alpha}{\sqrt{2}}|1\rangle + \frac{\beta}{\sqrt{2}}|0\rangle - \frac{\beta}{\sqrt{2}}|1\rangle$

$H|\psi\rangle = \frac{1}{\sqrt{2}}(\alpha+\beta)|0\rangle + \frac{1}{\sqrt{2}}(\alpha-\beta)|1\rangle$

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17.2)

17.2

Given $H^{\otimes n}|y\rangle = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} (-1)^{x \cdot y}|x\rangle$

Here n means n qubit Hadamard gate

$H^{\otimes 2}|2\rangle = \frac{1}{2^{2/2}} \sum_{x=0}^{2^2-1} (-1)^{x \cdot 2}$.

$= \frac{1}{2}\Big((-1)^{0\cdot 2}|0\rangle + (-1)^{1\cdot 2}|1\rangle + (-1)^{2\cdot 2}|2\rangle$

$+ (-1)^{3\cdot 2}|3\rangle\Big)$

From 17.3 we have

$0\cdot 2 = (0 \text{ AND } 1) \oplus (0 \text{ AND } 0) = 0 \oplus 0 = 0$

$1\cdot 2 = (0 \text{ AND } 1) \oplus (1 \text{ AND } 0) = 0 \oplus 0 = 0$

$2\cdot 2 = (1 \text{ AND } 1) \oplus (0 \text{ AND } 0) = 1 \oplus 0 = 1$

$3\cdot 2 = (1 \text{ AND } 1) \oplus (1 \text{ AND } 0) = 1 \oplus 0 = 1$

$= H^{\otimes 2}|2\rangle = \frac{1}{2}\left(|0\rangle + |1\rangle - |2\rangle - |3\rangle\right)$

17.3)

17.3)

From 17.2 Example using 17.19

$$H^{\otimes 2}|2\rangle = H^{\otimes 2}|10\rangle$$

$$= (H|1\rangle) \otimes (H|0\rangle)$$

$$= \frac{1}{2^{2/2}} (|0\rangle + (-1)^1|1\rangle) \otimes (|0\rangle + (-1)^0|1\rangle)$$

$$= \frac{1}{2} (|0\rangle - |1\rangle) \otimes (|0\rangle + |1\rangle)$$

$$= \frac{1}{2} (|00\rangle + |01\rangle - |10\rangle - |11\rangle)$$

Here we proved same solution we achieved in previous problems

17.4)

17.4

From 17.10 we know

$$H^{\otimes 2} = H \otimes H$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}$$

we need to find $H^{\otimes 2}|2\rangle$

we know $|2\rangle = |10\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

$$H^{\otimes 2}|10\rangle = \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1\times 0 + 1\times 0 + 1\times 1 + 1\times 0 \\ 1\times 0 - 1\times 0 + 1\times 1 + 1\times 0 \\ 1\times 0 + 1\times 0 - 1\times 1 - 1\times 0 \\ 1\times 0 - 1\times 0 - 1\times 1 + 1\times 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ -1 \\ -1 \end{pmatrix}$$

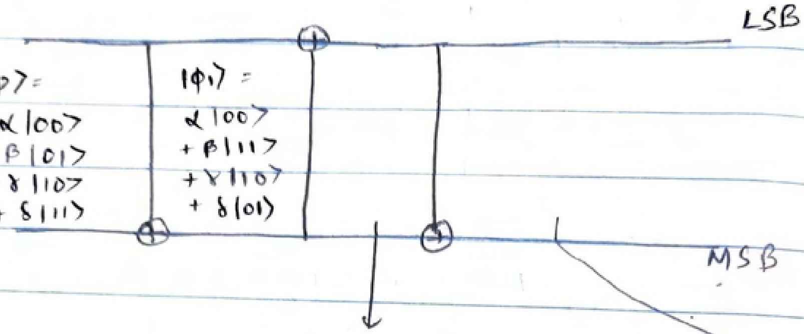
$$= \frac{1}{2}\left(|00\rangle + |01\rangle - |10\rangle - |11\rangle\right)$$

Hence proved

Chapter 18

18.1)

18.1)



$|\psi\rangle = \alpha|00\rangle + \beta|01\rangle + \gamma|10\rangle + \delta|11\rangle$

$|\phi_1\rangle = \alpha|00\rangle + \beta|11\rangle + \gamma|10\rangle + \delta|01\rangle$

$|\phi_2\rangle = \alpha|00\rangle + \beta|10\rangle + \gamma|11\rangle + \delta|01\rangle$

$|\phi\rangle = \alpha|00\rangle + \beta|10\rangle + \gamma|01\rangle + \delta|11\rangle$

So it doesnot have any effect even if we imple- CNOT gates starting from LSB or MSB

$U_{xor,right},\ U_{xor,center},\ U_{xor,left}$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = U_{swap}$$

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18.2)

18.2)

1 Given LSB as controlling bit from 15.13 we can derive

$U_{XOR}|00\rangle = |0\oplus0, 0\rangle = |0,0\rangle = |00\rangle$

$U_{XOR}|01\rangle = |0\oplus1, 1\rangle = |1,1\rangle = |11\rangle$

$U_{XOR}|10\rangle = |1\oplus0, 0\rangle = |1,0\rangle = |10\rangle$

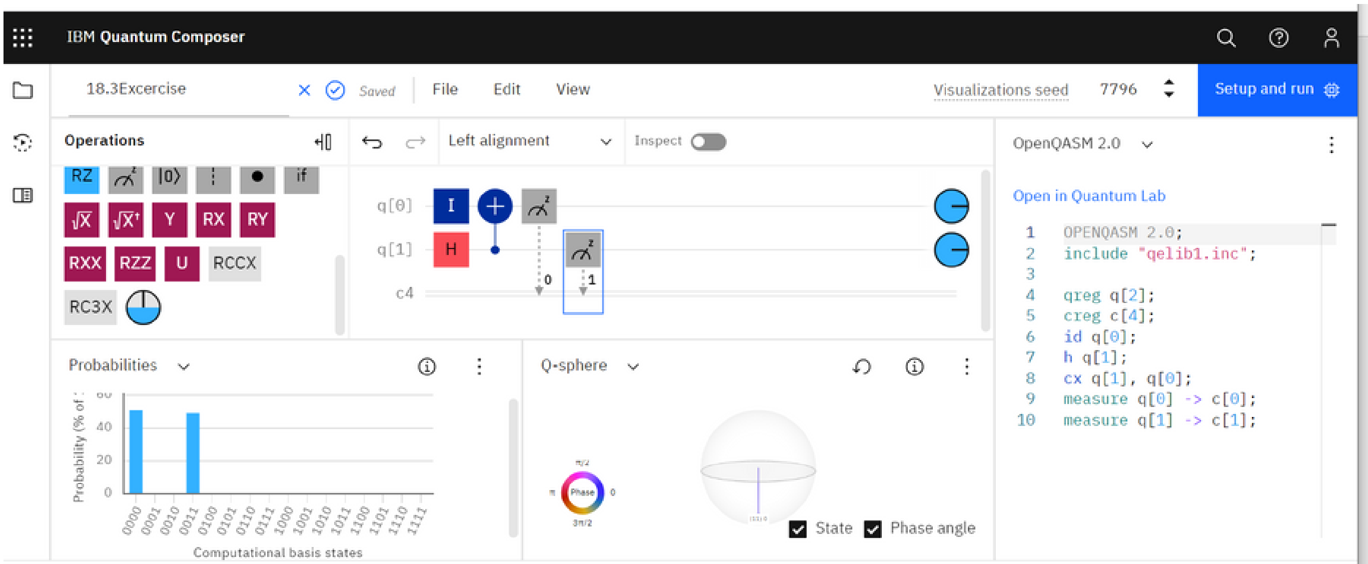
$U_{XOR}|11\rangle = |1\oplus1, 1\rangle = |0,1\rangle = |01\rangle$

we can derive matrix as

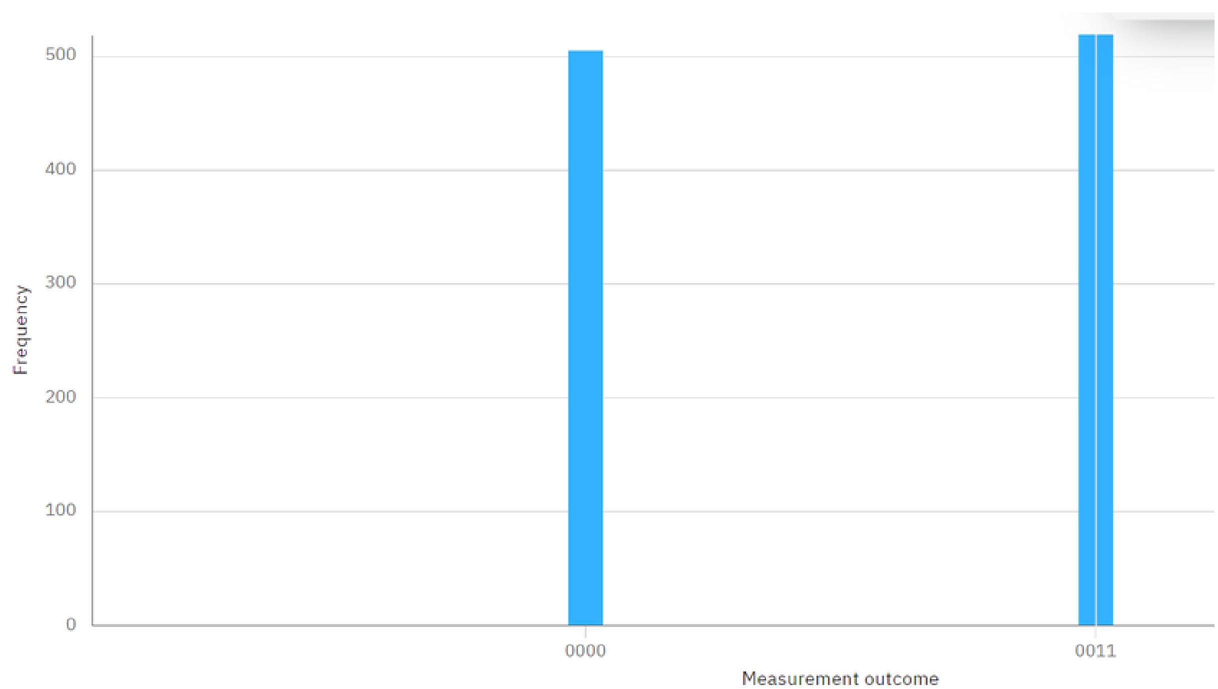
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

It matches our assumed value in previous problem

18.3)



Simulation output



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Hardware execution result

Details

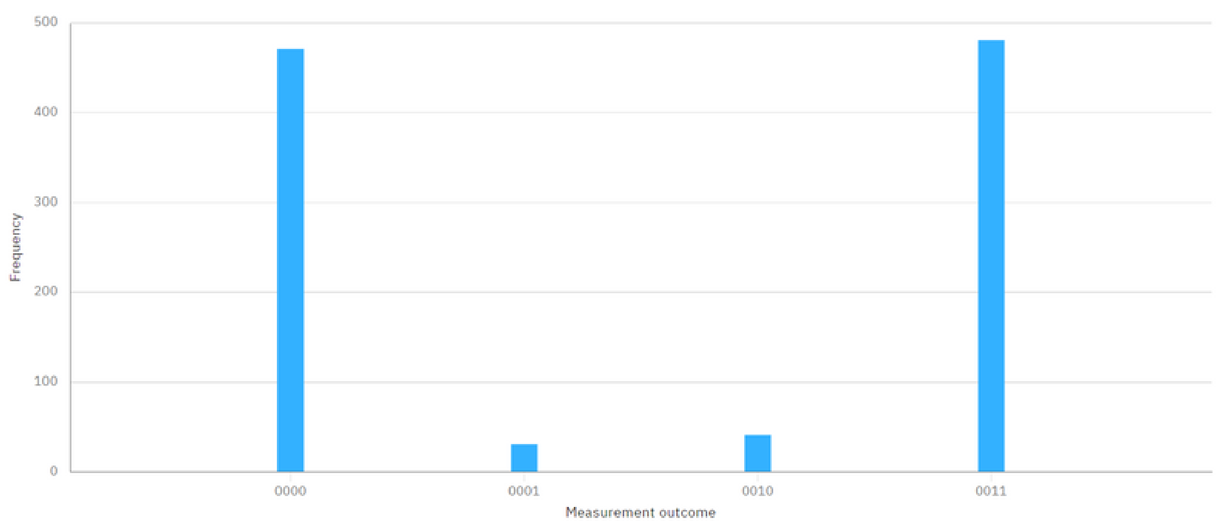
36m 30.5s
Total completion time

ibmq_manila
Backend

Sent from	Untitled circuit
Created on	Sep 07, 2022 11:50 AM
Sent to queue	Sep 07, 2022 11:50 AM
Provider	ibm-q/open/main
Run mode	fairshare
# of shots	1024
# of circuits	1

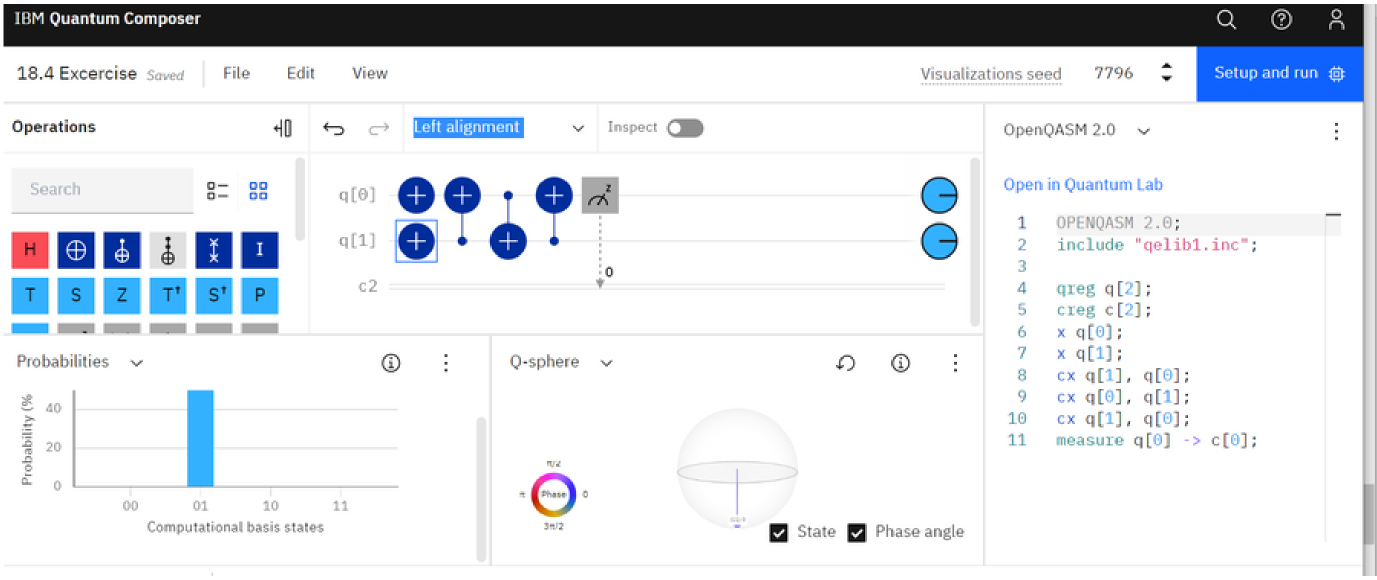
Status Timeline

- Created: Sep 07, 2022 11:50 AM
- Transpiling: 1.3s
- Validating: 1.1s
- In queue: 35m 51.6s
- Running: 4.9s
  time in system 4.9s
- Completed: Sep 07, 2022 12:26 PM



Introduction to Quantum computing (From a layperson to programmer in 30 steps)

18.4)



Simulation

Hardware execution result

Nithin Reddy Govindugari(nithinreddy1747@gmail.com)

Details

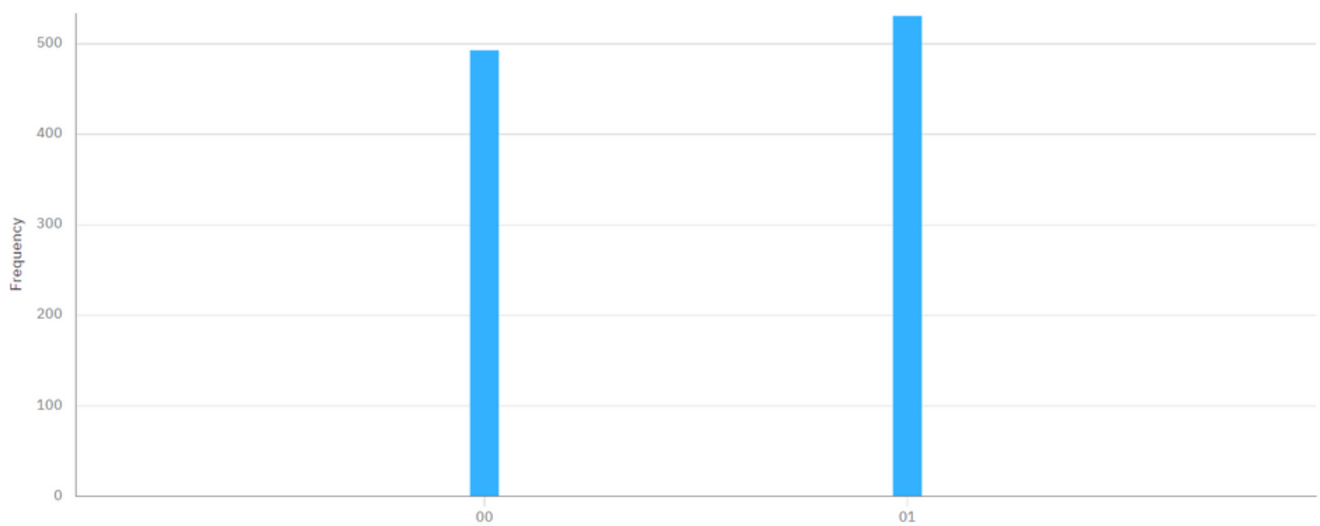
21h 2m 51.4s
Total completion time

ibm_oslo
Backend

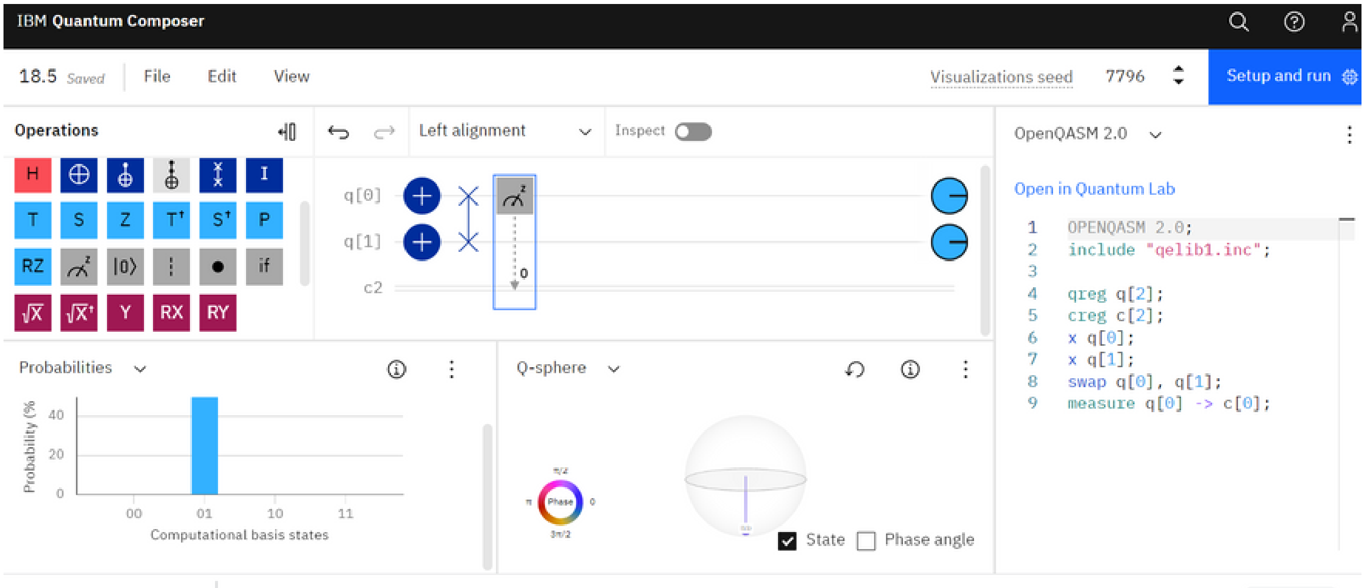
Sent from	18.4 Excercise
Created on	Sep 07, 2022 12:14 PM
Sent to queue	Sep 07, 2022 12:14 PM
Provider	ibm-q/open/main
Run mode	fairshare
# of shots	1024
# of circuits	1

Status Timeline

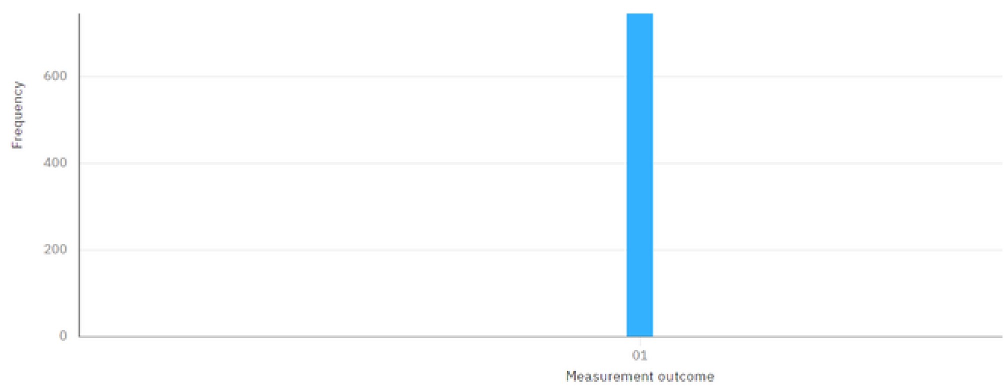
- Created: Sep 07, 2022 12:14 PM
- Transpiling: 1s
- Validating: 1.1s
- In queue: 21h 2m 41.1s
- Running: 4.4s
  time in system 4.4s
- Completed: Sep 08, 2022 9:16 AM



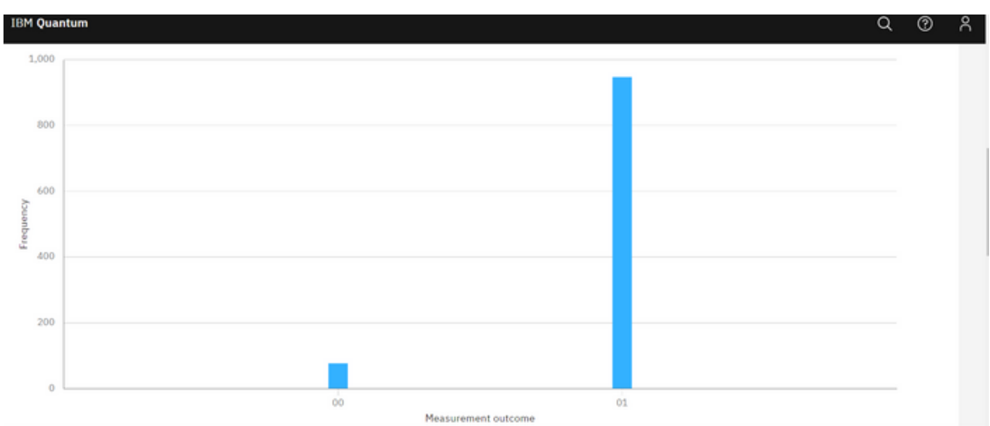
18.5)



Simulation results



Hardware execution result



Chapter 19
19.1)

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19.1)

Given

$$L\left(\tfrac{1}{\sqrt{2}}\left(|a\rangle_A + |b\rangle_A\right)|0\rangle_B\right) = \tfrac{1}{\sqrt{2}}\left(|a\rangle_A + |b\rangle_A\right) + \tfrac{1}{\sqrt{2}}\left(|a\rangle_B + |b\rangle_B\right)$$

$$L\left(|a\rangle_A |0\rangle_B + |b\rangle_A |0\rangle_B\right)$$

$$\Rightarrow |a\rangle_A |a\rangle_B + |b\rangle_A |b\rangle_B \rightarrow L.H.S \quad (1)$$

$\rightarrow$ R.H.S (Tensor product)

$$\Rightarrow |a\rangle_A |a\rangle_B + |a\rangle_A |b\rangle_B + |b\rangle_A |a\rangle_B + |b\rangle_A |b\rangle_B \quad (2)$$

So here L.H.S $\neq$ R.H.S because $|a\rangle_A |b\rangle_B$ & $|b\rangle_A |a\rangle_B$ probability is zero

Non cloning theorem survives on the concept of not able to copying linear combinations and linearity stands important

Ignored $1/\sqrt{2}$ for simplification

19.2)

19.2

a) Here we know when Hadamard gate is applied to $|1\rangle$ state it will be $|-\rangle$ state and before measuring the entangled or Bob qubit we are applying z gate we know z gate is Not gate in $|+\rangle / |-\rangle$ basis so it converts $|-\rangle$ to $|+\rangle$ before measurement

b) Here we have H_2 CNOT H_1

As we know CNOT is 2 qubit we can make Hadamard 2 qubit by applying Identity

$$= (H \otimes I)\ \text{CNOT}\ (H \otimes I)$$

$$H \otimes I \quad \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \\ -1 & 1 & 1 & 1 \end{pmatrix}$$

c) Here we know CNOT gate is used for entanglement and important part in Quantum Teleportation

In first step Hadamard gate is applied to Alice qubit and it creates superposition and later CNOT gate is applied for entanglement and Hadamard is applied to Alice qubit to bring back to original state. In LSB or Bob z gate is applied to switch the state

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D) Here as we have converted Hadamard gate to two qubit matrix as we are applying CNOT gate to it and entanglement avoids cancelling of two Hadamard gates

19.3)

19.3) Yes, It is possible to clone basis state as Non-cloning theorem only limits copying of linear combination of Alice to Bob but when we measure Alice state we can clone as many times as we want as when we ~~clone~~ measure the value remains same

You can't clone arbitrary state due to linearity and we proved it 1st Question

According to 19.7 we have to change basis and we can do teleportation but Alice won't be same after teleporting

Heinsbergs uncertainity principle states that you can't calculate position of a things exactly at a time in two basis. we can't have exact basis states of two different basis at a time. we need to have linear combination of one state and base state of one state

Chapter 20

20.1)

20.1)

$$= \frac{1}{\sqrt{2}}\left(\alpha|000\rangle + \beta|101\rangle + \alpha|011\rangle + \beta|110\rangle\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha|0\rangle\otimes|0\rangle\otimes|0\rangle + \beta|1\rangle\otimes|0\rangle\otimes|1\rangle + \alpha|0\rangle\otimes|1\rangle\otimes|1\rangle + \beta|1\rangle\otimes|1\rangle\otimes|0\rangle\right)$$

$$\frac{1}{\sqrt{2}}\left(\alpha\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) + \beta\left(\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right) + \alpha\left(\begin{pmatrix}1\\0\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right) + \beta\left(\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right)\right)$$

$$= \frac{1}{\sqrt{2}}\left(\alpha\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix}\otimes\begin{pmatrix}1\\0\end{pmatrix}\right) + \beta\left(\begin{bmatrix}0\\0\\1\\0\end{bmatrix}\otimes\begin{pmatrix}0\\1\end{pmatrix}\right)\right.$$

$$+\alpha\left(\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\otimes\begin{bmatrix}0\\1\end{bmatrix}\right)+\beta\left(\begin{bmatrix}0\\0\\0\\1\end{bmatrix}\otimes\begin{bmatrix}1\\0\end{bmatrix}\right)$$

$$\frac{1}{\sqrt{2}}\left[\alpha\begin{bmatrix}1\\0\\0\\0\\0\\0\\0\\0\end{bmatrix}+\beta\begin{bmatrix}0\\0\\0\\0\\0\\1\\0\\0\end{bmatrix}+\alpha\begin{bmatrix}0\\0\\0\\1\\0\\0\\0\\0\end{bmatrix}+\beta\begin{bmatrix}0\\0\\0\\0\\0\\0\\1\\0\end{bmatrix}\right]$$

$$=\frac{1}{\sqrt{2}}\begin{bmatrix}\alpha\\0\\0\\\alpha\\0\\\beta\\\beta\\0\end{bmatrix}$$

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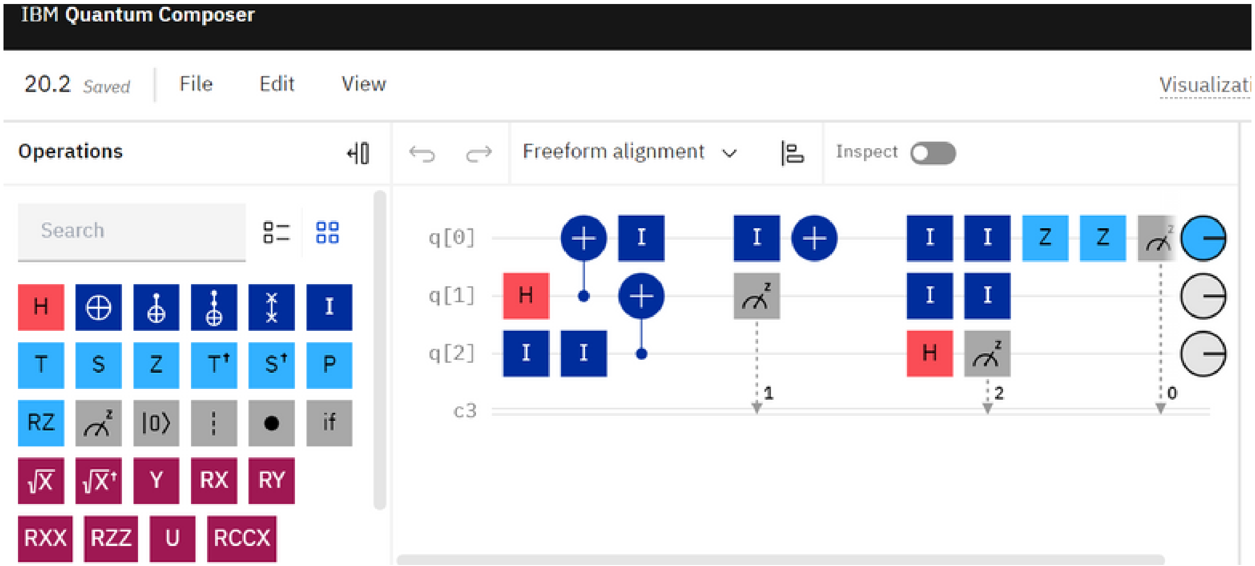
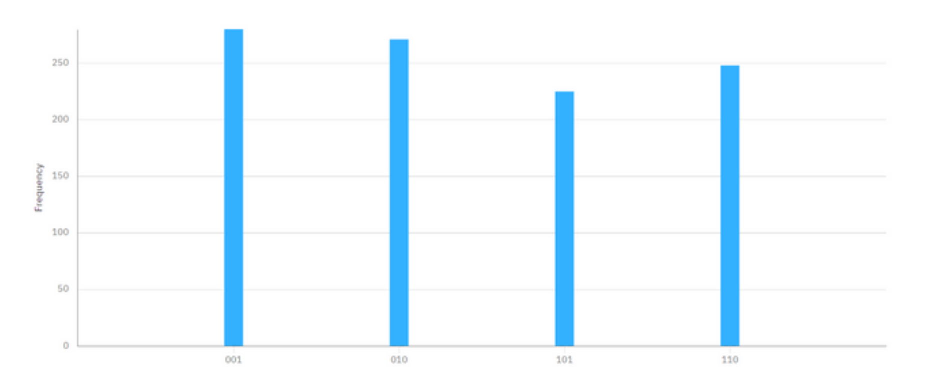
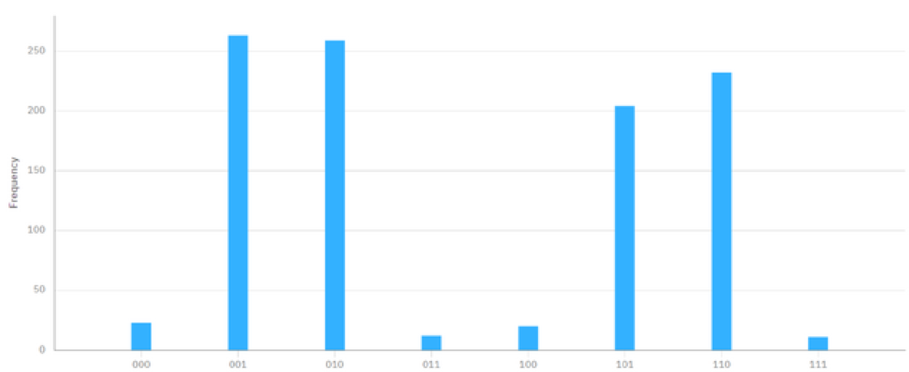
20.2)



Simulation result



IBM



20.3)

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20.3)

1) Due to Non cloning theorem it won't allow cloning of linear arbitrary states so we need to measure Alice

2) Entanglement swapping (20.5) allows us to transport / teleport

3) Default circuit represents Alice to teleport |0> to Bob. According to 20.4 so yes Quantum teleportation happens by default

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21.1)

21.1) ~~i) $f_B(x)$~~

$U_f|0\rangle|0\rangle = |0\rangle|0 \oplus f(0)\rangle$

$U_f|0\rangle|1\rangle = |0\rangle|1 \oplus f(0)\rangle$

$U_f|1\rangle|0\rangle = |1\rangle|0 \oplus f(1)\rangle$

$U_f|1\rangle|1\rangle = |1\rangle|1 \oplus f(1)\rangle$

i) $f_B(x) \Rightarrow f_B(0)=0$ & $f_B(1)=1$

$= |0\rangle|0 \oplus 0\rangle = |0\rangle|0\rangle$

$= |0\rangle|1 \oplus 0\rangle = |0\rangle|1\rangle$

$= |1\rangle|0 \oplus 1\rangle = |1\rangle|1\rangle$

$= |1\rangle|1 \oplus 1\rangle = |1\rangle|0\rangle$

ii) $f_C(x) \Rightarrow f_C(0)=1$ & $f_C(1)=0$

$= |0\rangle|0 \oplus 1\rangle = |0\rangle|1\rangle$

$= |0\rangle|1 \oplus 1\rangle = |0\rangle|0\rangle$

$= |1\rangle|0 \oplus 0\rangle = |1\rangle|0\rangle$

$= |1\rangle|1 \oplus 0\rangle = |1\rangle|1\rangle$

iii) $f_D(x) \Rightarrow f_D(0)=1$ & $f_D(1)=1$

$= |0\rangle|0 \oplus 1\rangle = |0\rangle|1\rangle$

$= |0\rangle|1 \oplus 1\rangle = |0\rangle|0\rangle$

$= |1\rangle|0 \oplus 1\rangle = |1\rangle|1\rangle$

$= |1\rangle|1 \oplus 1\rangle = |1\rangle|0\rangle$

21.2)

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21.2)

we know from 17.11 that

$$H^{\otimes 2}|00\rangle = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

$$= \frac{1}{2}\begin{pmatrix}1\\1\\1\\1\end{pmatrix}$$

$$L.H.S = \frac{1}{2}(H \cdot H)(1\ X)|0\rangle|0\rangle$$

$$= \frac{1}{2}\begin{pmatrix}1 & 1 & 1 & 1\\1 & -1 & 1 & -1\\1 & 1 & -1 & -1\\1 & -1 & -1 & 1\end{pmatrix}\begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\end{bmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

$$= \cancel{\tfrac{1}{2}}\ (H\ H)\ |0\rangle|1\rangle \rightarrow R.H.S$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

$$|+\rangle|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$= \frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ 1 \\ -1 \end{pmatrix}$$

Hence $(H\ H)(I\ X)\ |0\rangle|0\rangle = (H\ H)\ |0\rangle|1\rangle = |+\rangle|-\rangle$

21.3)

21.3) Given state $|01\rangle$

from 17.21

$$H^{\otimes n}|y\rangle = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} (-1)^{x \cdot y} |x\rangle$$

we kno $n=2$ & $|01\rangle_2 = 1_{10}$.

$$H^{\otimes 2}|1\rangle = \frac{1}{2^{2/2}} \Big((-1)^{0 \cdot 1}|0\rangle + (-1)^{1 \cdot 1}|1\rangle$$
$$+ (-1)^{2 \cdot 1}|2\rangle + (-1)^{3 \cdot 1}|3\rangle \Big)$$

$0 \cdot 1 = (0 \text{ AND } 0) \oplus (0 \text{ AND } 1) = 0 \oplus 0 = 0$

$1 \cdot 1 = (0 \text{ AND } 0) \oplus (1 \text{ AND } 1) = 0 \oplus 1 = 1$

$2 \cdot 1 = (1 \text{ AND } 0) \oplus (0 \text{ AND } 1) = 0 \oplus 0 = 0$

$3 \cdot 1 = (1 \text{ AND } 0) \oplus (1 \text{ AND } 1) = 0 \oplus 1 = 1$

$$H^{\otimes 2}|1\rangle = H^{\otimes 2}|01\rangle = \frac{1}{2}\big(|0\rangle - |1\rangle + |2\rangle - |3\rangle\big)$$

$$= \frac{1}{2}\big(|00\rangle - |01\rangle + |10\rangle - |11\rangle\big) \rightarrow ①$$

From : 21.6

$$|+\rangle|-\rangle = \frac{1}{2}\big(|00\rangle - |01\rangle + |10\rangle - |11\rangle\big) \rightarrow ②$$

Hence proved ① = ②

21.4)

21.4) From 21.5 let us try for balanced function $f(x)$ and gives equal number of 0 & 1

ex: $f_B(0) = 0$, $f_B(1) = 1$

U_f

$U_f |0\rangle|0\rangle = |0\rangle|0 \oplus f(0)\rangle = |0\rangle|0 \oplus 0\rangle = |0\rangle|0\rangle$

$U_f |0\rangle|1\rangle = |0\rangle|1 \oplus f(0)\rangle = |0\rangle|1 \oplus 0\rangle = |0\rangle|1\rangle$

$U_f |1\rangle|0\rangle = |1\rangle|0 \oplus f(1)\rangle = |1\rangle|0 \oplus 1\rangle = |1\rangle|1\rangle$

$U_f |1\rangle|1\rangle = |1\rangle|1 \oplus f(1)\rangle = |1\rangle|1 \oplus 1\rangle = |1\rangle|0\rangle$

Chapter 22

22.1) & 22.2)

22.1. Given $x = 5$, $f(5) = 3$, $n = 3$; $m = 3$

y assumed as $|0\rangle_m$

we know from 22.1

$$U_f |x\rangle_n |y\rangle_m = |x\rangle_n |y \oplus f(x)\rangle_m$$

$$U_f |5\rangle_3 |0\rangle_3 = |5\rangle_3 |0 \oplus f(5)\rangle_3$$

$$\cancel{= |100101\rangle}$$

$$= |101\rangle_3 |0 \oplus (011)\rangle \quad = |101\rangle_3 |000 \oplus$$

$$= |101\rangle |011\rangle$$

$$= |101011\rangle$$

22.2 If we are using same method as phase Quantum oracle

Two basis vectors $|a\rangle$ & $|b\rangle$ and inner product is $\langle a|b\rangle$

i) $|a\rangle$ transformed by XOR Quantum oracle

we can assume y as 0

$|a\rangle|0 \oplus f(a)\rangle$

$|a\rangle|f(a)\rangle$

ii) $|b\rangle \rightarrow |b\rangle|f(b)\rangle$

Innerproduct is (No matrix involved

$\langle a|b\rangle = \cancel{|a\rangle|f}$

$\langle f(a)|f(b)\rangle \langle a|b\rangle$

For example if $a = b$ then we know

inner product of same vectors is

$\langle f(a)|f(b)\rangle \langle a|b\rangle$

$\langle f(a)|f(a)\rangle \langle a|a\rangle$

$= 1 \cdot 1 = 1$

& If $a \neq b$ $\langle a|b\rangle = 0$ as we already know

Basic vector Inner product is Not changed

It is still unitary

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22.3)

22.3 $U_{swap} = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \langle j | U | i \rangle |j\rangle \langle i|$

$\boxed{N=4}$

For $i=0$, $j=0,1,2,3$

$0 = 00$
$1 = 01$
$2 = 10$
$3 = 11$

$: \langle 00| U_{swap} |00\rangle |00\rangle \langle 00| + \langle 01| U_{swap} |00\rangle |01\rangle \langle 00|$

$+ \langle 10| U_{swap} |00\rangle |10\rangle \langle 00|$

$+ \langle 11| U_{swap} |00\rangle |11\rangle \langle 00|$

$+$

For $i=1$, $j=0,1,2,3$

$\cancel{\langle 01| U_{swap} |10\rangle |00\rangle \langle 10| + \langle}$

$\langle 00| U_{swap} |01\rangle |00\rangle \langle 01| + \langle 01| U_{swap} |01\rangle |01\rangle \langle 01|$

$+ \langle 10| U_{swap} |01\rangle |10\rangle \langle 01| +$

$\langle 11| U_{swap} |01\rangle |11\rangle \langle 01|$

$+$

For $i=2$, $j=0,1,2,3$

$\langle 00| U_{swap} |10\rangle |00\rangle \langle 10| + \langle 01| U_{swap} |10\rangle |01\rangle \langle 10|$

$+ \langle 10| U_{swap} |10\rangle |10\rangle \langle 10|$

$+ \langle 11| U_{swap} |10\rangle |11\rangle \langle 10|$

$+$

$$+\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$U_{swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

from 16.3 → It is proved

22.4)

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for F_c $\quad f_c(0)=1$, $f_c(1)=0$

22.4) $\langle 0|\langle 1|(U_f|0\rangle|0\rangle) = \langle 0|\langle 1|\,|01\rangle = 1$

$\langle 0|\langle 0|(U_f|0\rangle|1\rangle) = \langle 0|\langle 0|\,|00\rangle = 1$

$\langle 1|\langle 0|(U_f|1\rangle|0\rangle) = \langle 1|\langle 0|\,|10\rangle = 1$

$\langle 1|\langle 1|(U_f|1\rangle|1\rangle) = \langle 1|\langle 1|\,|11\rangle = 1$

$$= |01\rangle|00\rangle + |00\rangle|01\rangle + |10\rangle|10\rangle + |11\rangle|11\rangle$$

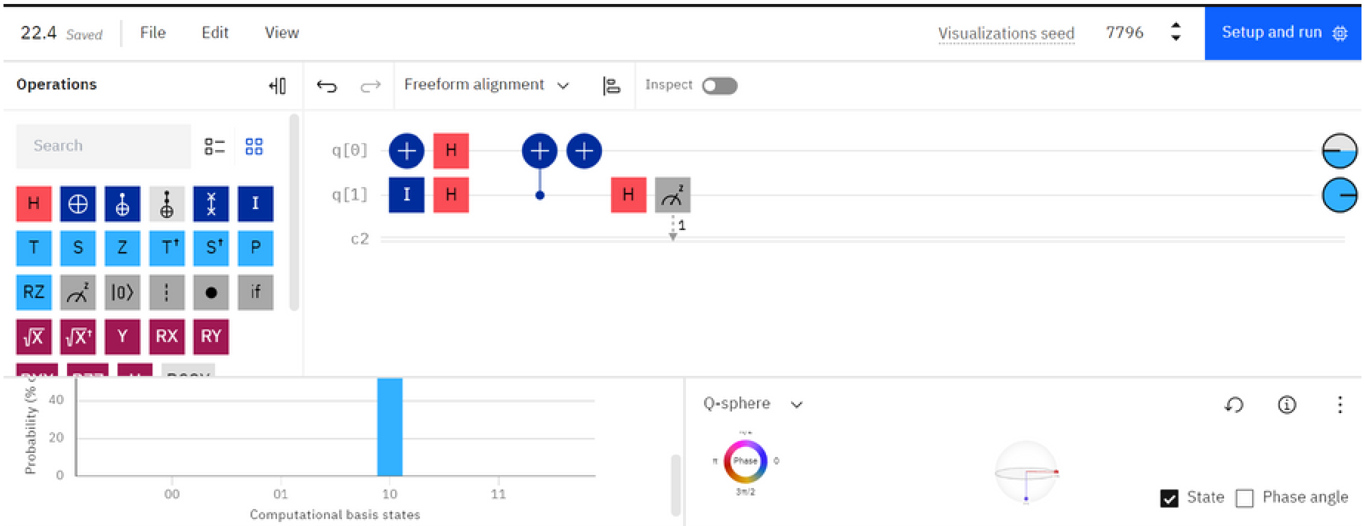
$$= \begin{pmatrix}0\\1\\0\\0\end{pmatrix}\begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}1\\0\\0\\0\end{bmatrix}\begin{bmatrix}0 & 1 & 0 & 0\end{bmatrix} + \begin{bmatrix}0\\0\\1\\0\end{bmatrix}\begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}$$

$$= \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\begin{bmatrix}0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}0 & 0 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}0 & 1 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & 0\end{bmatrix} + \begin{bmatrix}0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 0\end{bmatrix}$$

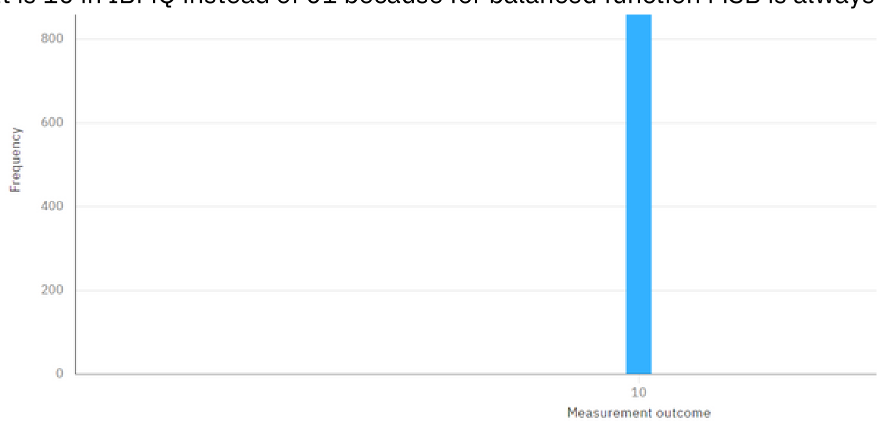
$$= \begin{bmatrix}0 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 0 & \\0 & 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}$$



Simulator outcome

It is 10 in IBMQ instead of 01 because for balanced function MSB is always 1 (page no 212)



for $f_D(0) = 1$ $f_d(1) = 1$

$\langle 0|\langle 1| \left(U_f|0\rangle|0\rangle\right) = \langle 01|01\rangle = 1$

$\langle 0|\langle 0| \left(U_f|0\rangle|1\rangle\right) = \langle 00|\cdot|00\rangle = 1$

$\langle 1|\langle 1| \left(U_f|1\rangle|0\rangle\right) = \langle 11|\cdot|11\rangle = 1$

$\langle 1|\langle 0| \left(U_f|1\rangle|1\rangle\right) = \langle 10|\cdot|10\rangle = 1$

$= |01\rangle|00\rangle + |00\rangle|01\rangle + |11\rangle|10\rangle$
$+ |10\rangle|11\rangle$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

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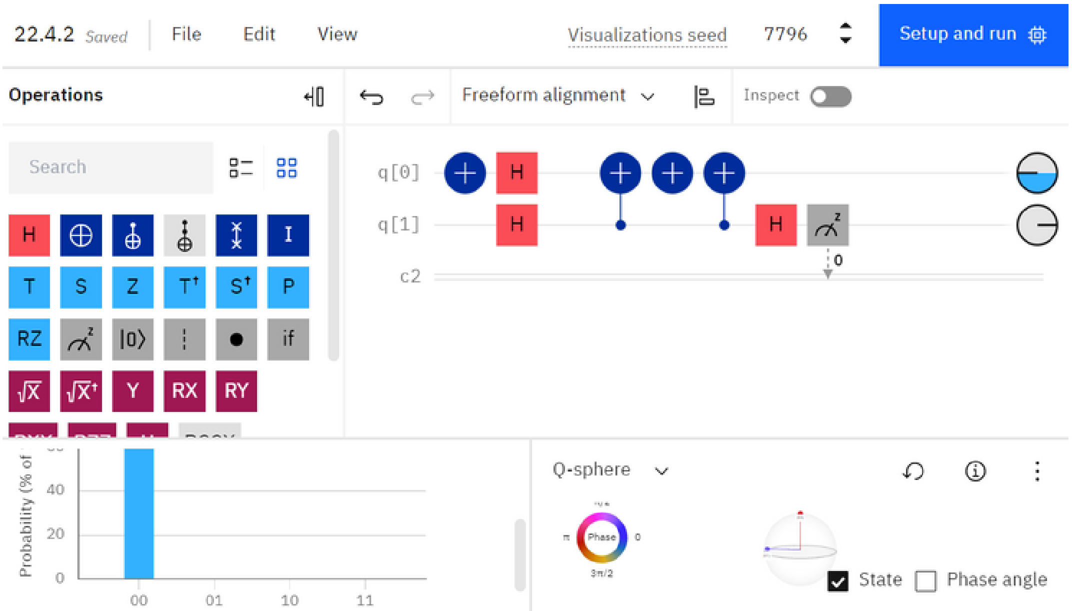
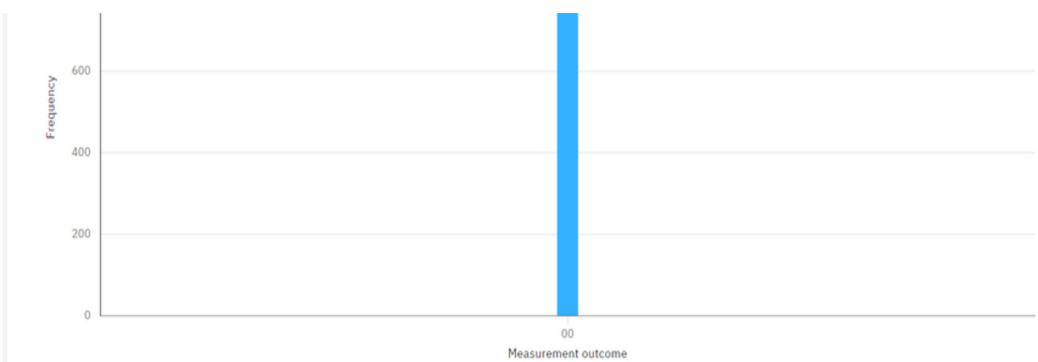
It is similar to above statement and as explained in page 200 it is always 0 for constant function here we can also remove both cnot gates as it wont make much difference. I have added it to describe the functionality



Simulator results



Chapter 23

23.1) Fro Fig 23.2

Let us assume angle between $|a\rangle$ & $|\phi\rangle$ as α

	angle between $|\phi\rangle$ & $|a\rangle$	between $|\phi\rangle$ & $|a\rangle_\perp$
$V\,|\phi\rangle$	$\alpha + 2\theta$	$-\theta$
$WV\,|\phi\rangle$	$\alpha - 2\theta$	$+3\theta$
$VW\,|\phi\rangle$	α	$+\theta$
$WVWV\,|\phi\rangle$	$\alpha - 4\theta$	$+3\theta$

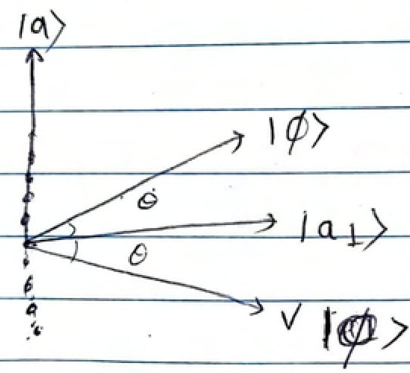
23.2)

23.2)

Given $V = I - 2|a\rangle\langle a|$

As we already know V is a quantum gate and it is linear combination of all basis vectors except our target entry $|a\rangle$

we know phase shift is added to target entry and it is multiplied by -1



~~From $V|\phi\rangle = (I - 2|a\rangle\langle a|)|\phi\rangle$~~

$= \cancel{\sum_{i=0, i\neq a}^{?} |\phi\rangle_i} -$

Here when operator V is applied to $|\phi\rangle$ then it reflects $|\phi\rangle$ towards negation (phase shift)

23.3)

23.3
Sol

Given $n=2$, $a=3$

$$|\phi\rangle = \frac{1}{2^{n/2}} \sum_{x=0}^{2^n-1} |x\rangle$$

we know $n=2$, $a=3$

$$= \frac{1}{2^{2/2}} \sum_{x=0}^{2^2-1} |x\rangle$$

$$= \frac{1}{2} \sum_{x=0}^{3} |x\rangle$$

$$= \frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)$$

$|a\rangle = |11\rangle$

$$|a_\perp\rangle = \frac{1}{\sqrt{2^n-1}} \sum_{x=0\,(x\neq a)}^{2^n-1} |x\rangle$$

$$\frac{1}{\sqrt{3}} \sum_{x=0\,(x\neq a)}^{3} |x\rangle$$

$$= \frac{1}{\sqrt{3}}\left(|00\rangle + |01\rangle + |10\rangle\right)$$

23.4 & 23.5)

23.4 Sol)

we know

$$V|\phi\rangle = V\left(\tfrac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\right)$$

$$= \tfrac{1}{2}\ |00\rangle + |01\rangle + |10\rangle - |11\rangle$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

23.5)

Algorithm B has linear speedup over Algorithm A

Chapter 24

24.1)

24.1)

$$U = \sum_{i=0}^{N-1}\sum_{j=0}^{N-1} \langle j|U|i\rangle |j\rangle\langle i| \longrightarrow 22.6$$

we know from 24.4 that

$$V|x\rangle_n = (-1)^{f(x)}|x\rangle_n \longrightarrow 23.9$$

$U_f|01\rangle = -|01\rangle$ and for every other base it will be same

$$\langle 0|\langle 0|(U_f|0\rangle|0\rangle) = \langle 00||00\rangle = 1$$

$$\langle 0|\langle 1|(U_f|0\rangle|1\rangle) = \langle 00|(-)01\rangle = -1 \quad \boxed{U_f(01) = -01}$$

$$\langle 1|\langle 1|(U_f|1\rangle|1\rangle) = \langle 11|11\rangle = 1$$

$$\langle 1|\langle 0|(U_f|1\rangle|1\rangle) = \langle 1|\langle 0|1\rangle|0\rangle = \langle 10|10\rangle = 1$$

$$U_f = \langle 0|\langle 0|(U_f|0\rangle|0\rangle)|00\rangle\langle 00| + \langle 0|\langle 1|(U_f|0\rangle|1\rangle)|01\rangle\langle 01| + \langle 1|\langle 0|(U_f|1\rangle|0\rangle)|10\rangle\langle 10| + \langle 1|\langle 1|(U_f|1\rangle|1\rangle)|11\rangle\langle 11|$$

$= |00\rangle\langle 00| - |01\rangle\langle 01| + |11\rangle\langle 11| + |10\rangle\langle 10|$

$\cancel{+ |10\rangle\langle 10|} + \cdots$

$$\begin{bmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} - \begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&1\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&1&0\\0&0&0&0\end{bmatrix}$$

$$= \begin{bmatrix}1&0&0&0\\0&-1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix} = U_f \quad \text{i.e, } 24.y$$

24.2)

we need to apply $H^{\otimes 2}(2|0\rangle_2\langle 0|_2 - I)H^{\otimes 2}(V|\phi\rangle)$

firstly $2|01\rangle_2\langle 01|_2 - I = 2\begin{bmatrix}0\\1\\0\\0\end{bmatrix}\begin{bmatrix}0&1&0&0\end{bmatrix} - \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$

$$= 2\begin{bmatrix}0&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} - \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$$2|01\rangle_2\langle 01|_2 - I = \begin{bmatrix}-1&0&0&0\\0&1&0&0\\0&0&-1&0\\0&0&0&-1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} \cancel{-1} \cancel{+1} \cancel{+1} \cancel{-1} \\ -1 -1 \cancel{+1} \cancel{+1} \\ \cancel{-1} \cancel{+1} \cancel{-1} \cancel{+1} \\ -1 -1 -1 -1 \end{bmatrix} = -\frac{1}{4}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$|11\rangle$ over rotated

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24.3)

24.3)

we know from 23.9 for $a=4$

$$U_f = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$V|\phi\rangle = U_f\left(\frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)\right)$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

for $a=3$

$$= H^{\otimes 2}(2|0\rangle_2 \langle 0|_2 - I) H^{\otimes 2}(V|\phi\rangle)$$

$$= \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+1-1 \\ 1-1+1+1 \\ 1+1-1+1 \\ 1-1-1-1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 \\ 2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$$

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$$= \frac{1}{4}\begin{bmatrix} 1-1-1+1 \\ 1+1-1-1 \\ 1-1+1-1 \\ 1+1+1+1 \end{bmatrix} = \frac{1}{4}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 4 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= |11\rangle$$

24.4)

from 24.15

$$C_Z = (I \otimes H)\, U_{XOR}\, (I \otimes H)$$

$$= \left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\right)\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\left(\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\right)$$

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$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & 0\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \\ 0\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & 1\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\end{bmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{pmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix} 1\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & 0\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} \\ 0\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix} & 1\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1 & 0 & 0\\ 1 & -1 & 0 & 0\\ 0 & 0 & 1 & 1\\ 0 & 0 & 1 & -1\end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & -2 \end{bmatrix}$$

$$= \frac{\cancel{2}\,1}{\cancel{2}} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = CZ$$

24.5)

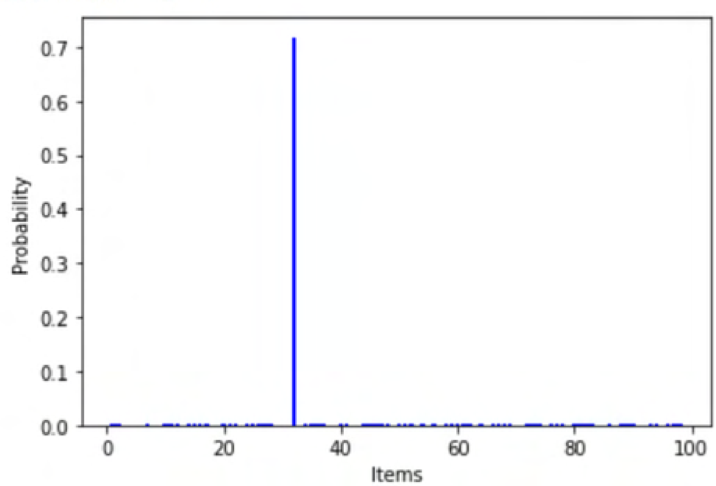
```
#24.5
#Keywords
#superposition is sup_pos
#amplitude  is amp

#Grovers Algorithm is implemented using standard procedure.
#Steps followed in books are implemented and tried to create classical grovers algorithm in different

plots = []

# Putting all qubits in superposition using hadamard gate
def  sup_pos(qubits):
    states = []
    total_states = int(math.pow(2,qubits))
    amp = 1/math.sqrt(total_states)
    for _ in range(0,total_states):
        states.append(amp)
    return states
```

```
# phase flip inverts the states amplitude
def grover_diffusion(states):
    average = sum(states)/len(states)
    for i in range(0,len(states)):
        states[i] = (average-states[i]) + average          #inversion about mean
    return states

#oracle to implement phase inversion
def oracle(states,datalist,key):
    for i in range(0,int(len(datalist))):
        if datalist[i] == key:
            states[i] *= -1                         #phase inversion
    return states
```

```python
#classical main grover search function
def grover_search(qubits,datalist,key):
    states = sup_pos(qubits)
    num_iterations  = math.ceil(math.sqrt(math.pow(2,qubits)))        #iterations
    probability_states = []
    for _ in range(0,num_iterations):
        states = oracle(states,datalist,key)
        states = grover_diffusion(states)
        probability_states = [states[i]*states[i] for i in range(0,len(states))]
        plots.append(probability_states)
    return probability_states


def grover(datalist,key):
    size_datalist = len(datalist)
    qubits_needed = math.ceil(math.log(size_datalist,2))
    paddings_required = int(math.pow(2,qubits_needed) - size_datalist)
    #required if the number of data items is not a power of 2.
    for _ in range(0,paddings_required):
         datalist.append(0)
    grover_search(qubits_needed,datalist,key)
```

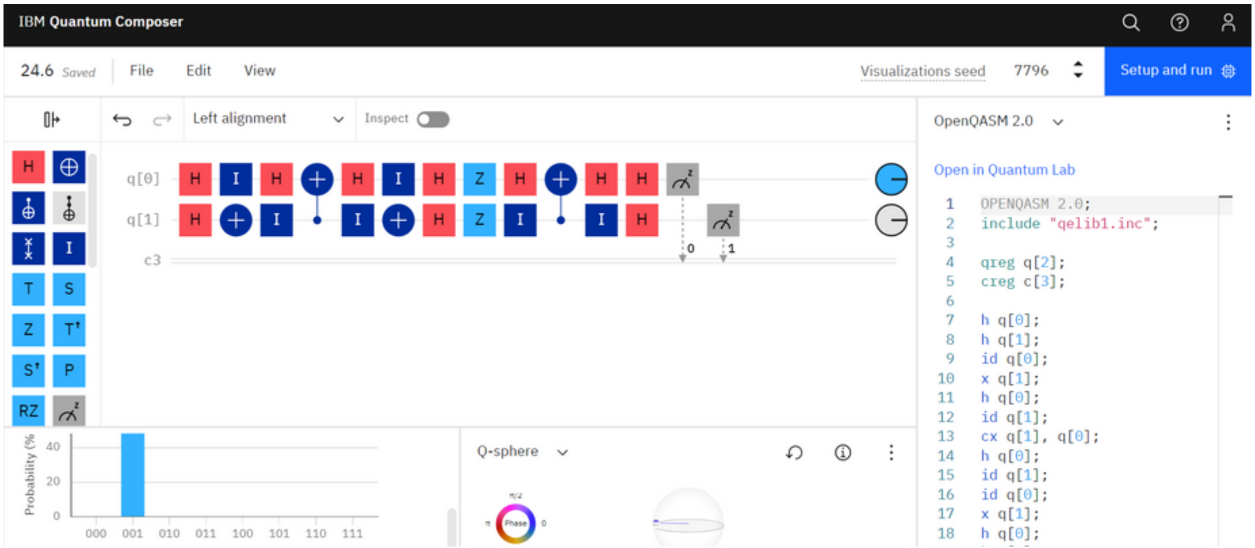
```python
datalist = random.sample(range(1, 100), 64)
grover(datalist,32)
print(datalist)
print("plots showing the change in probabilities of items after every grover's iteration :")
result ={}
iteration = 1
for plot in plots:
    for i in range(len(plot)):
        result[datalist[i]] = plot[i]
    print("-------------------------------------------------------------------------------------
    print("Iteration ",iteration," :")
    plt.bar(result.keys(), result.values(),color='b')
    plt.ylabel("Probability")
    plt.xlabel("Items")
    plt.show()
    iteration +=1
    print("-------------------------------------------------------------------------------------
```

Iteration 8 :

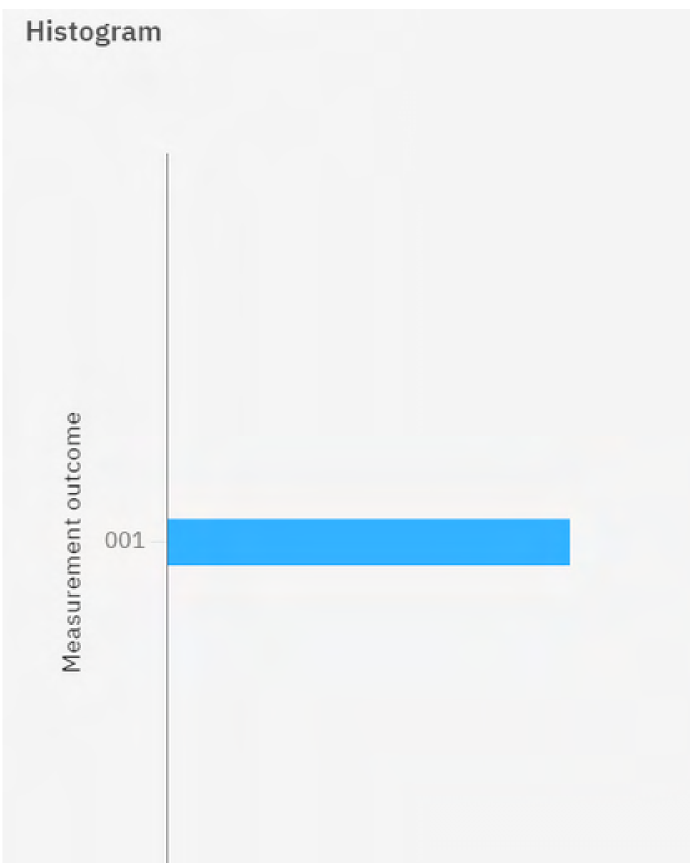


24.6)

Introduction to Quantum computing (From a layperson to programmer in 30 steps)



Simulation



Hardware circuit result

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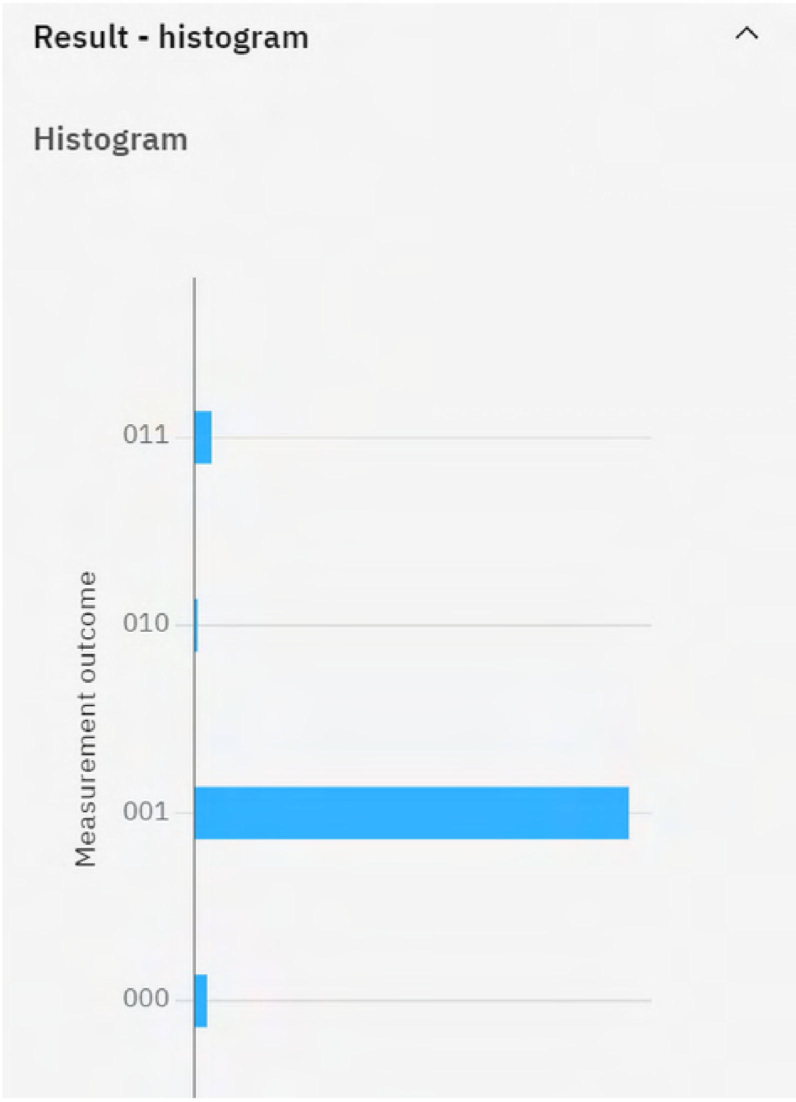
Result - histogram
Histogram
Measurement outcome
011
010
001
000

Chapter 25

25.1) Given that we have to prove

25.10 $\Rightarrow$ 25.9

$$10 \rightarrow \begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{pmatrix} = \frac{1}{\sqrt{N}} \begin{bmatrix} \omega^{-0.0} & \omega^{-0.1} & \cdots & \omega^{-0.(N-1)} \\ \omega^{-1.0} & \omega^{-1.1} & \cdots & \omega^{-1(N-1)} \\ \vdots & \vdots & & \vdots \\ \omega^{-(N-1)0} & \omega^{-(N-1)1} & \cdots & \omega^{-(N-1)\cdot(N-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

(1)

we know 25.9. is

$$y_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{-i2\pi kj/N} x_j$$

$$= \frac{1}{\sqrt{N}} \begin{bmatrix} e^{-i2\pi(0\cdot 0)/N} & e^{-2\pi(0.1)/N} & e^{-2\pi((0.N-1)} \\ e^{-i2\pi(1\cdot 0)/N} & e^{-2\pi(1\cdot 1)/N} & e^{-2\pi(1.(N-1))} \\ \vdots & \vdots & \vdots \\ e^{-i2\pi((N-1)\cdot 0)/N} & e^{-i2\pi((N-1)1)} & e^{-i2\pi(N-1)(N-1)} \end{bmatrix}$$

$$\begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix} \quad \text{—— (2)}$$

From 25.4 (First property we know)

$$e^{i2\pi m/N} = \omega^m$$

So it equals to (1) after substitution

$$= \frac{1}{\sqrt{N}} \begin{bmatrix} \omega^{-0.0} & \omega^{-0.1} & \cdots & \omega^{-0.(N-1)} \\ \omega^{-1.0} & \omega^{-1.1} & \cdots & \omega^{-1(N-1)} \\ \vdots & & & \vdots \\ \omega^{-(N-1).0} & \omega^{-(N-1)1} & & \omega^{-(N-1).(N-1)} \end{bmatrix} \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{N-1} \end{bmatrix}$$

25.2)

25.2 yes. U_{QFT} is Hermitian

From 25.20 we know

$$U_{QFT} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$ i.e, equals to Hadamard gate

$$U_{QFT}^{\dagger} = U_{QFT}$$ (Hermitian rule is satisfied)

Here U_{QFT} is symmetric (25.12) & satisfies Hermitian requirements. I believe it should be Hermitian because according to Fourier transforms domains changes i.e, bases in QFT it should remain same

25.3)

25.3) U_{QFT} is unitary considering row vectors

$$\langle x| = \frac{1}{\sqrt{N}}\left[\omega^{-x \cdot 0} \cdot \omega^{-x \cdot 1} \cdot \cdots \omega^{-x \cdot (N-1)}\right]^*$$

$$\langle y| = \frac{1}{\sqrt{N}}\left[\omega^{-y \cdot 0} \cdot \omega^{-y \cdot 1} \cdot \cdots \cdot \omega^{-y \cdot (N-1)}\right]^*$$

$$\langle x|y\rangle = \frac{1}{N}\sum_{i=0}^{N-1}\left(\omega^{-m \cdot L}\right)^* \omega^{-n \cdot L}$$

$$= \frac{1}{N}\sum_{i=0}^{N-1}\left(\left(e^{-i2\pi m L/N}\right)^*\right)^* e^{-i2\pi m \cdot L}$$

$$= \frac{1}{N}\sum_{i=0}^{N-1}\omega^{-m L} \cdot \omega^{-n \cdot -L}$$

$$\frac{1}{N}\sum_{i=0}^{N-1}\left(\omega^{-L(m-n)}\right)$$

if $m = n$ it is 1 else it is zero

U_{QFT} is unitary

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25.4)

25.3) U_{QFT} is unitary considering row vectors

$$\langle x| = \frac{1}{\sqrt{N}}\left[\omega^{-x\cdot 0}, \omega^{-x\cdot 1}, \ldots, \omega^{-x\cdot(N-1)}\right]^*$$

$$\langle y| = \frac{1}{\sqrt{N}}\left[\omega^{-y\cdot 0}, \omega^{-y\cdot 1}, \ldots, \omega^{-y\cdot(N-1)}\right]^*$$

$$\langle x|y\rangle = \frac{1}{N}\sum_{i=0}^{N-1}\left(\omega^{-m\cdot L}\right)^* \cdot \omega^{-n\cdot L}$$

$$= \frac{1}{N}\sum_{i=0}^{N-1}\left(\left(e^{-i2\pi m L/N}\right)^*\right)^{\dagger} e^{-i2\pi n\cdot L}$$

$$= \frac{1}{N}\sum_{i=0}^{N-1}\omega^{-mL}\cdot\omega^{-n\cdot -L}$$

$$\frac{1}{N}\sum_{i=0}^{N-1}\left(\omega^{-L(m-n)}\right)$$

if $m=n$ it is 1 else it is zero

U_{QFT} is unitary

25.5)

25.5)

Given $U_{IQFT} U_{QFT} = U_{QFT} U_{IQFT} = I$

we know

$$U_{QFT} = \frac{1}{\sqrt{N}} \begin{pmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \cdots & \omega^{-(N-1)} \\ 1 & \omega^{-2} & \omega^{-4} & \cdots & \omega^{-2(N-1)} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \omega^{-(N-1)} & \omega^{-2(N-1)} & \cdots & \omega^{-(N-1)(N-1)} \end{pmatrix}$$

$$U_{IQFT} = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{1} & \cdots & \cdots & \omega^{(N-1)} \\ 1 & \omega^{2} & \cdots & \cdots & \omega^{2(N-1)} \\ \vdots & \vdots & & & \vdots \\ 1 & \omega^{(N-1)} & \cdots & \cdots & \omega^{(N-1)(N-1)} \end{bmatrix}$$

$$U_{IQFT} \cdot U_{QFT} = \frac{1}{N} \begin{bmatrix} N & 1+\omega^1+\omega^2+\omega^3 & 1+\omega^2+\omega^4 & \cdots & 1+\omega^{(N-1)}+\omega^{2(N-1)} \\ \omega^1+\omega^{-2} & N & \cdots & \cdots & \vdots \\ & & & & \vdots \\ \cdots & \cdots & \cdots & \cdots & N \end{bmatrix}$$

From 25.5 we know $1+\omega^1+\omega^2 \ldots = 0$

$$= \frac{N}{N} \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \cdots & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & & \cdots & 0 \\ \vdots & & & & \vdots \\ 0 & \cdots & & \cdots & 1 \end{bmatrix}$$

$$U_{QFT} \cdot U_{IQFT} = \frac{1}{N}\begin{bmatrix} N & 1+\omega^{1}+\omega^{2} \ldots \omega^{(N-1)} & \cdots & 1+\omega^{(N-1)}+\omega^{\ldots} \\ 1+\omega^{1}+\omega^{2} & N & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \cdots & & & N \end{bmatrix}$$

From 25.5 & 25.6

$$= \frac{1}{N}\begin{bmatrix} N & 0 & \cdots & 0 \\ 0 & N & & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \cdots & \cdots & N \end{bmatrix}$$

25.6)

We know

$$U_{IQFT} = \frac{1}{\sqrt{N}}\begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega^{1} & \cdots & & & \omega^{(N-1)} \\ 1 & \omega^{2} & & \cdots & & \\ \vdots & \vdots & & & & \\ 1 & \omega^{(N-1)} & \cdots & & & \omega^{(N-1)(N-1)} \end{bmatrix}$$

For 2 qubit $\omega = i$

$$= \frac{1}{\sqrt{N}}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^{2} & i^{3} \\ 1 & i^{2} & i^{4} & i^{6} \\ 1 & i^{3} & i^{6} & i^{9} \end{bmatrix}$$

$$U_{IQFT} = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix}$$

$$U_{QFT} \cdot U_{IQT}$$

$$\frac{1}{2}\begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -i & -1 & i \\ 1 & -1 & 1 & -1 \\ 1 & i & -i & -i \end{pmatrix} \cdot \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & i & -1 & -1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 1\times1+1\times1+1\times1+1\times1 & 1\times1+i-1+1 & 1-1+1-1 & 1-i+i-1 \\ 1\times1-i-1+1 & 1+1+1+1 & 1+i-1-i & 1-1+i-1 \\ 1\times-1+1-1 & 1+1-i-i & 1+1+1+1 & 1+1-1-1 \\ 1\times i-1-i & 1-1+1-1 & 1-i-1+1 & 1+1+1+1 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
#25.7

from sympy.abc import w
N = int(input("Enter a value", ))
# w = 'w'

matrix=[]
row=[]
for i in range(N): #total row is 3
    row=[]
    for j in range(N): #total column is 3
        # row.append( a \N{SUPERSCRIPT (i*j)})
         row.append( (w)** (i*j) )

    matrix.append(row) #add fully defined column into the row
print( matrix )
```

```
Enter a value4
[[1, 1, 1, 1], [1, w, w**2, w**3], [1, w**2, w**4, w**6], [1, w**3, w**6, w**9]]
```

Chapter 26

26.1)

26.1) & 26.6)

According to method show in book
For 3 Qubit matrix $N = 8$

According to 25.22

$$U_{QFT} = \frac{1}{\sqrt{8}} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \omega^{-1} & \omega^{-2} & \omega^{-3} & \omega^{-4} & \omega^{-5} & \omega^{-6} & \omega^{-7} \\ 1 & \omega^{-2} & \omega^{-4} & \omega^{-6} & \omega^{-8} & \omega^{-10} & \omega^{-12} & \omega^{-14} \\ 1 & \omega^{-3} & \omega^{-6} & \omega^{-9} & \omega^{-12} & \omega^{-15} & \omega^{-18} & \omega^{-21} \\ 1 & \omega^{-4} & \omega^{-8} & \omega^{-12} & \omega^{-16} & \omega^{-20} & \omega^{-24} & [illegible] \\ 1 & \omega^{-5} & \omega^{-10} & \omega^{-15} & \omega^{-20} & \omega^{-25} & \omega^{-30} & [illegible] \\ 1 & \omega^{-6} & \omega^{-12} & \omega^{-18} & \omega^{-24} & \omega^{-30} & \omega^{-36} & \omega^{-42} \\ 1 & \omega^{-7} & \omega^{-19} & \omega^{-21} & \omega^{-25} & \omega^{-35} & \omega^{-40} & [illegible] \end{bmatrix}$$

In page 249 It is mentioned different sources has different Interpretation and am following method used in this text book

Given $\omega_{\theta} = e^{i2\pi/N} = e^{i.2\pi/8}$

If we convert it $\cos \pi/4 + i \sin \pi/4 = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}} = \frac{1+i}{\sqrt{2}}$

All even multiply $\omega^8 = \left(e^{i\frac{2\pi}{8}}\right)^8 = 1$, $\omega^{16} = 1$, $\omega^{24} = 1$, $\omega^{40} = e^{i\pi/2}$

$$= \frac{1}{\sqrt{8}} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{\sqrt{2}}{1+i} & \left(\frac{\sqrt{2}}{1+i}\right)^2 & \left(\frac{\sqrt{2}}{1+i}\right)^3 & \left(\frac{\sqrt{2}}{1+i}\right)^4 & \left(\frac{\sqrt{2}}{1+i}\right)^5 & \left(\frac{\sqrt{2}}{1+i}\right)^6 & \left(\frac{\sqrt{2}}{1+i}\right)^7 \\ 1 & \left(\frac{\sqrt{2}}{1+i}\right)^2 & \left(\frac{\sqrt{2}}{1+i}\right)^4 & \cancel{\left(\frac{\sqrt{2}}{1+i}\right)^6} = 1 & \left(\frac{\sqrt{2}}{1+i}\right)^8 & \left(\frac{\sqrt{2}}{1+i}\right)^{10} & \left(\frac{\sqrt{2}}{1+i}\right)^{12} & \left(\frac{\sqrt{2}}{1+i}\right)^{14} \\ 1 & \left(\frac{\sqrt{2}}{1+i}\right)^3 & \left(\frac{\sqrt{2}}{1+i}\right)^6 & \left(\frac{\sqrt{2}}{1+i}\right)^9 & \left(\frac{\sqrt{2}}{1+i}\right)^{12} & \left(\frac{\sqrt{2}}{1+i}\right)^{15} & \left(\frac{\sqrt{2}}{1+i}\right)^{18} & \left(\frac{\sqrt{2}}{1+i}\right)^{21} \\ 1 & \left(\frac{\sqrt{2}}{1+i}\right)^4 & \cancel{\left(\frac{\sqrt{2}}{1+i}\right)^8} 1 & \left(\frac{\sqrt{2}}{1+i}\right)^{12} & \cancel{\left(\frac{\sqrt{2}}{1+i}\right)^{16}} = 1 & \left(\frac{\sqrt{2}}{1+i}\right)^{20} & \left(\frac{\sqrt{2}}{1+i}\right)^{24} & \left(\frac{\sqrt{2}}{1+i}\right)^{28} \\ 1 & \left(\frac{\sqrt{2}}{1+i}\right)^5 & \left(\frac{\sqrt{2}}{1+i}\right)^{10} & \left(\frac{\sqrt{2}}{1+i}\right)^{15} & \left(\frac{\sqrt{2}}{1+i}\right)^{20} & \left(\frac{\sqrt{2}}{1+i}\right)^{25} & \left(\frac{\sqrt{2}}{1+i}\right)^{30} & \left(\frac{\sqrt{2}}{1+i}\right)^{35} \\ 1 & \left(\frac{\sqrt{2}}{1+i}\right)^6 & \left(\frac{\sqrt{2}}{1+i}\right)^{12} & \left(\frac{\sqrt{2}}{1+i}\right)^{18} & \cancel{\left(\frac{\sqrt{2}}{1+i}\right)^{24}} = 1 & \left(\frac{\sqrt{2}}{1+i}\right)^{30} & \left(\frac{\sqrt{2}}{1+i}\right)^{36} & \left(\frac{\sqrt{2}}{1+i}\right)^{42} \\ 1 & \left(\frac{\sqrt{2}}{1+i}\right)^7 & \left(\frac{\sqrt{2}}{1+i}\right)^{14} & \left(\frac{\sqrt{2}}{1+i}\right)^{21} & \left(\frac{\sqrt{2}}{1+i}\right)^{28} & \left(\frac{\sqrt{2}}{1+i}\right)^{35} & \left(\frac{\sqrt{2}}{1+i}\right)^{40} & \left(\frac{\sqrt{2}}{1+i}\right)^{49} \end{pmatrix}$$

Here given input $|000\rangle \rightarrow$
As we know when apply Hadamard gate to individually it ~~q~~ will be $\left(\frac{|1+\rangle + |-i\rangle}{\sqrt{2}}\right)$
($\therefore$ QFt = Hadamard for 1 qubit)
and we know their will be no phase shift if all bits is zero then we know their will equal probability in all states if all the given inputs are in starting state

26.2)

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26.2) Given that we have to prove

$$U_{cps,\phi}\, U_{cps,-\phi} = I$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\phi} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\phi} \end{bmatrix}$$

$$= \begin{bmatrix} 1\times1+0\times0+0\times0+0\times0 & 0\times[0+1+0+0] & 0[0+0+1+0] & 0[0+0+0+e^{-i\phi}] \\ 0\times1+0\times0+0\times0+0\times0 & 1[0+1+0+0] & 0[0+0+1+0] & 0[0+0+0+e^{-i\phi}] \\ 0\times1+0\times0+0\times0+0\times0 & 0[0+1+0+0] & 1[0+0+1+0] & 0[0+0+0+e^{-i\phi}] \\ 0\times1+0\times0+0\times0+0\times0 & 0[0+0+0+0] & 0[0+0+1+0] & e^{i\phi}\cdot e^{-i\phi} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{-i\phi+i\phi} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

26.3)

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26.3) Given that we have to prove

$$U_{swap} = U^{-1}_{swap} \Rightarrow U_{swap} \cdot U_{swap} = I$$

$$U_{swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1st method

$$U^{-1}_{swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{T}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

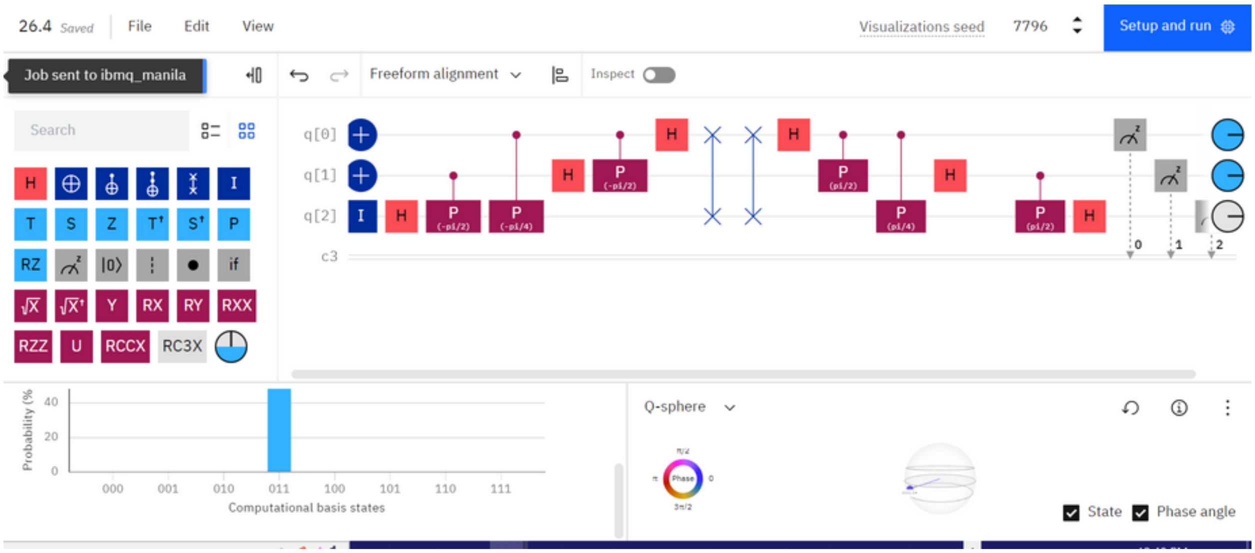
2nd method

$$U_{swap} U_{swap} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

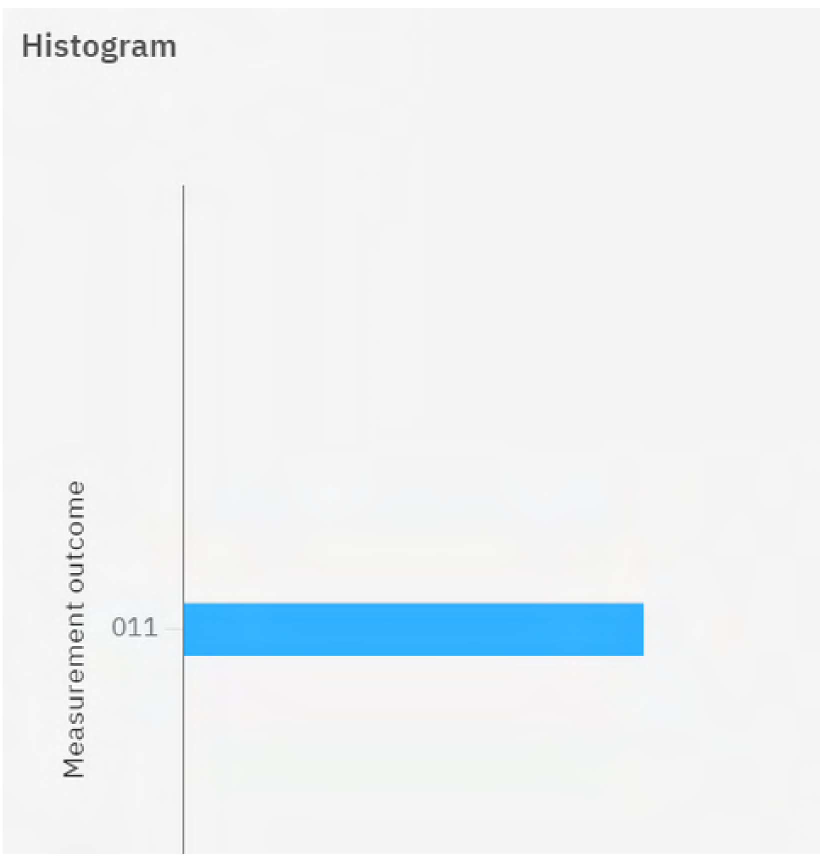
$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = I$$

26.4)

Introduction to Quantum computing (From a layperson to programmer in 30 steps)



Simulation result



Hardware circuit result

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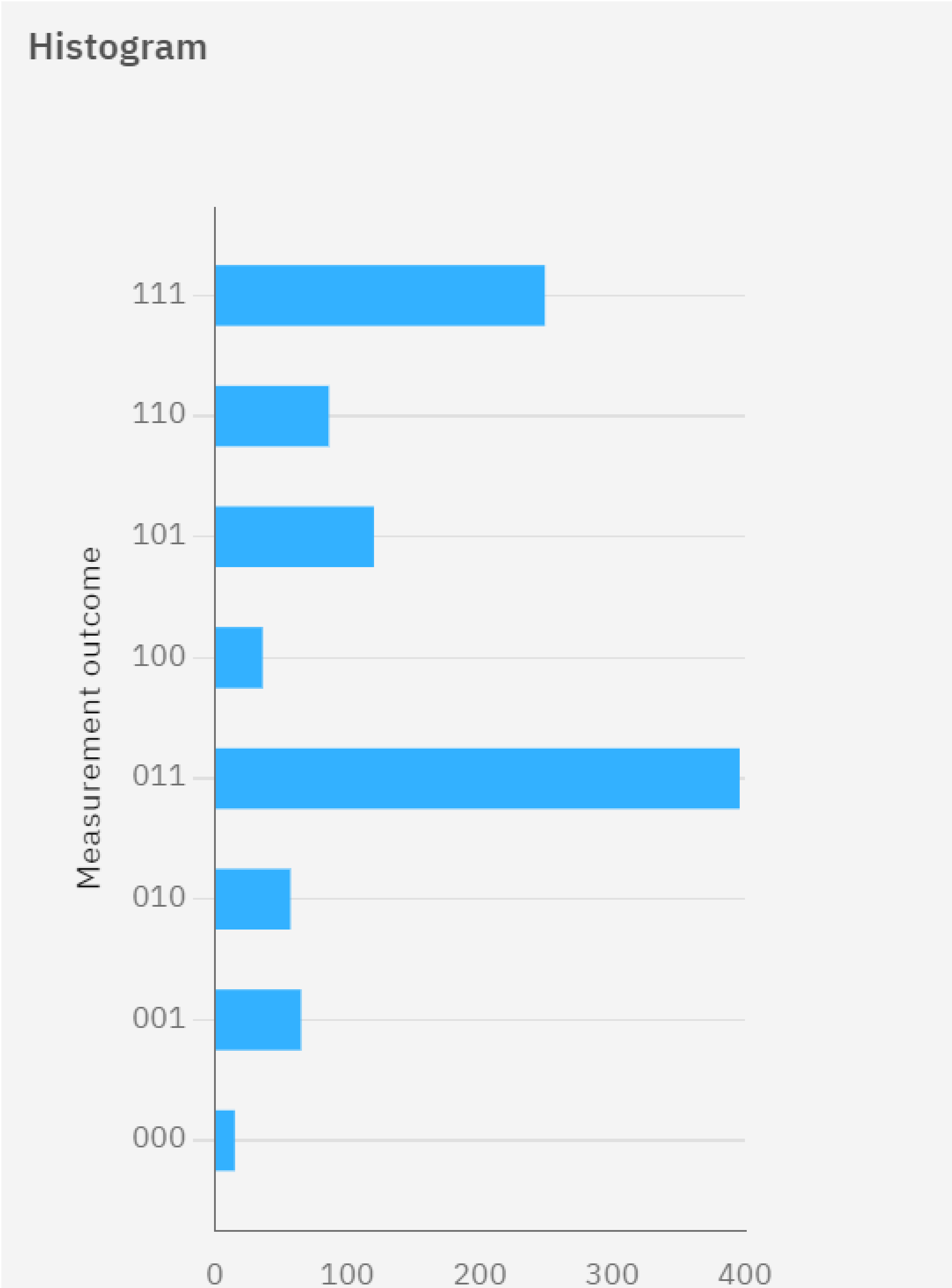
Histogram
Measurement outcome
111
110
101
100
011
010
001
000
0
100
200
300
400

26.5) according ~~to~~ to figure 26.7 we need $n-1$ $U_{c.p.s,\phi}$ gates with ϕ from $-\frac{2\pi}{2^2}$ to $\frac{-2\pi}{2^n}$

~~If~~ execution time is $y = 2^n$ ~~seconds~~ then i believe as we know QF has $O(n^2)$ speed

26.6) we have derived and elaborated matrix in 26.1

26.7) $|11001100\rangle \xrightarrow{swap} |10011001\rangle$

Introduction to Quantum computing (From a layperson to programmer in 30 steps)

Chapter 27

27.1)

27th Chapter

27.1) Given σ_z of $|\psi\rangle$

From example 27.2 we know

$$|\psi\rangle = \cos\theta/_2 \, e^{-i\phi/2}|0\rangle + \sin\theta/_2 \, e^{i\phi/2}|1\rangle$$

If we include phase angle as well

$$|\psi\rangle = e^{i\gamma}\left(\cos\theta/_2 \, e^{-i\phi/2}|0\rangle + \sin\theta/_2 \, e^{i\phi/2}|1\rangle\right)$$

↳ from $\boxed{27.1}$

$$\langle\psi|\sigma_z|\psi\rangle = e^{-i\gamma}\left(\cos\theta/_2 \, e^{i\phi/2} \quad \sin\theta/_2 \, e^{-i\phi/2}\right) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} e^{i\gamma} \begin{pmatrix} \cos\theta/_2 \, e^{-i\phi/2} \\ \sin\theta/_2 \, e^{i\phi/2} \end{pmatrix}$$

$$= e^{-i\gamma + i\gamma} \cdot \cos\theta \qquad \text{from} \rightarrow 27.6$$

Global phase ~~effect~~ has no effect proved below

$$= 1 \cdot \cos\theta = \cos\theta$$

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27.2)

27.2) we have three cases proved and we need to prove remaining three cases :

i) $z=-1, x=0, y=0$ i.e, $\theta=\pi, \phi=0$

It is also called South pole

$$|\psi_{\theta=\pi,\phi=0}\rangle = \cos\theta/2\, e^{-i\phi/2}|0\rangle + \sin\frac{\theta}{2} e^{i\phi/2}|1\rangle$$

$$= \cos\pi/2\, e^{-i0/2}|0\rangle + \sin\pi/2\, e^{i(0)/2}|1\rangle$$

$$= 0\cdot|1\rangle + 1\cdot1\cdot|1\rangle$$

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$\frac{\pi}{2} \to \frac{\pi}{4}$

$\frac{2}{8}$

$3\pi/4$

$= |1\rangle$

iii) consider $x = 1$, $y = z = 0$

$\theta = \pi/2 \quad \phi = = 0$

$$|\psi_{\theta=\pi/2, \phi=0}\rangle = \cos\frac{\theta}{2} e^{-i\phi/2}|0\rangle + \sin\theta/_2\, e^{i\phi/2}|1\rangle$$

$$= \cos\frac{\pi}{4} e^{-i0}|0\rangle + \sin\frac{\pi}{4} e^{i0}|1\rangle$$

$$= \frac{1}{\sqrt{2}}(1)|0\rangle + \cancel{\sin}\frac{1}{\sqrt{2}}(1)|1\rangle$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= |+\rangle$$

iv) consider $y = -1$, $x = z = 0$

$\theta = \pi/2, \ \phi = -\pi/2$

$$|\psi_{\theta=\pi/2, \phi=\pi/2}\rangle = \cos\frac{\pi}{4} e^{i\pi/4}|0\rangle + \sin\frac{\pi}{4} e^{-i\pi/4}|1\rangle$$

$$= \frac{1}{\sqrt{2}} e^{i\pi/4}(|0\rangle + e^{-i\pi/2}|1\rangle)$$

$$= e^{i\pi/4}\frac{1}{\sqrt{2}}(|0\rangle - i|1\rangle)$$

~~Here~~ we are ignoring global phase in all scenarios

27.3)

27.3)

$$|\psi\rangle = \cos\theta/_2\, e^{-i\phi/2}|0\rangle + \sin\theta/_2\, e^{i\phi/2}|1\rangle$$

$$= \begin{pmatrix} \cos\theta/_2\, e^{-i\phi/2} \\ \sin\theta/_2\, e^{i\phi/2} \end{pmatrix}$$

i) σ_x

$$\langle\psi|\sigma_x|\psi\rangle = \left(\cos\theta/_2\, e^{i\phi/2}\ \sin\theta/_2\, e^{-i\phi/2}\right) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \cos\theta/_2\, e^{-i\phi/2} \\ \sin\theta/_2\, e^{i\phi/2} \end{pmatrix}$$

$$= \left(\cos\theta/_2\, e^{i\phi/2}\ \sin\theta/_2\, e^{-i\phi/2}\right) \begin{pmatrix} \sin\theta/_2\, e^{i\phi/2} \\ \cos\theta/_2\, e^{-i\phi/2} \end{pmatrix}$$

$$= \cos\theta/_2\, e^{i\phi/2}\, \sin\theta/_2\, e^{i\phi/2} + \sin\theta/_2\, e^{-i\phi/2} \cos\theta/_2\, e^{-i\phi/2}$$

$e^{i\phi}$ ~~sin $2i\phi/2$ cos~~

$$= \cos\theta/_2 \sin\theta/_2 \cdot e^{i\phi} + \sin\theta/_2 \cos\theta/_2\, e^{-i\phi}$$

$$= \cos\theta/_2 \sin\theta/_2 \left(e^{i\phi} + e^{-i\phi} \right)$$

$$= \cos\theta/_2 \sin\theta/_2 \left(\cos\phi/_2 + i\sin\phi/_2 + \cos\phi/_2 - i\sin\phi/_2 \right)$$

$$= \cos\theta/_2 \sin\theta/_2 \left(\cos\phi \right)$$

$$= \sin\theta \cos\phi$$

ii) $\langle \psi | \sigma_y | \psi \rangle = \cos\theta/_2 e^{i\phi/2} \sin\theta/_2 e^{-i\phi/2} \cdot \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{bmatrix} \cos\theta/_2 e^{-i\phi/2} \\ \sin\theta/_2 e^{i\phi/2} \end{bmatrix}$

$$= \left(\cos\theta/_2 e^{i\phi/2} \quad \sin\theta/_2 e^{-i\phi/2} \right) \begin{bmatrix} -i\sin\theta/_2 e^{i\phi/2} \\ i\cos\theta/_2 e^{-i\phi/2} \end{bmatrix}$$

$$= \cos\theta/_2 e^{i\phi/2} \cdot -i\sin\theta/_2 e^{+i\phi/2} + \sin\theta/_2 e^{-i\phi/2} \cdot i\cos\theta/_2 e^{-i\phi/2}$$

$$i \sin\theta/_2 \cos\theta/_2 \, e^{-i\phi} - i \cos\theta/_2 \sin\theta/_2 \, e^{i\phi}$$

$$= i \sin\theta/_2 \cos\theta/_2 \left(e^{-i\phi} - e^{i\phi} \right)$$

$$= i \sin\theta/_2 \cos\theta/_2 \left(\cancel{\cos\phi} - i\sin\phi/_2 - \cancel{\cos\theta} - i\sin\phi \right)$$

$$= i \sin\theta/_2 \cos\theta/_2 \left(-i \sin\phi \right)$$

$$= -i^2 \sin\theta \sin\phi \; \Big($$

$$= \sin\theta \sin\phi$$

27.4

27.4)
sol

Given $|\psi\rangle = \left(\frac{3}{4} - i\frac{\sqrt{3}}{4}\right)|0\rangle + \left(\frac{\sqrt{3}}{4} + i\frac{1}{4}\right)|1\rangle$

$= \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}-i}{2}\right)|0\rangle + \frac{1}{2}\left(\frac{\sqrt{3}+i}{2}\right)|1\rangle \rightarrow 1$

We know From 27.2

$|\psi\rangle = \cos\theta/2 \, e^{-i\phi/2}|0\rangle + \sin\theta/2 \, e^{i\phi/2}|1\rangle \rightarrow 2$

we need to compare 2 with 1

$\cos\theta/2 = \sqrt{3}/2$

$\theta/2 = \pi/6 \therefore \theta = \pi/3$

$\sin\theta/2 = 1/2$

$\theta/2 = \pi/6 \therefore \theta = \pi/3$

$e^{-i\phi/2} = \cos\phi/2 - i\sin\phi/2$

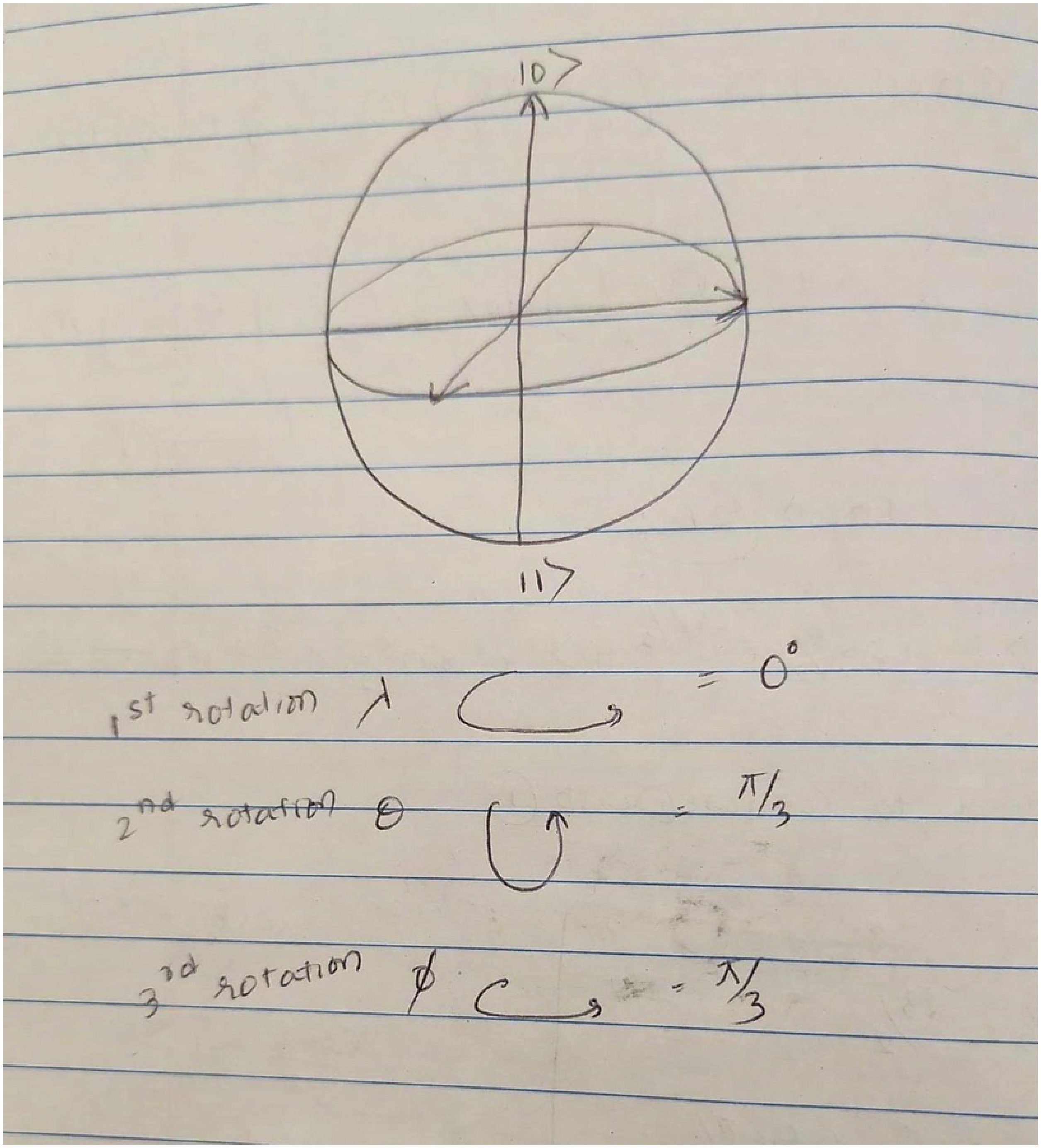
$= \frac{\sqrt{3}-i}{2}$

$\sqrt{3}/2 - i/2$

$\cos\phi/2 = \sqrt{3}/2$ $\quad$ $\sin\phi/2 = 1/2$

$\phi = \pi/3$ $\quad$ $\phi = \pi/3$

$|\psi\rangle = \cos\pi/6 \, e^{-i\pi/6}|0\rangle + \sin\pi/6 \, e^{i\phi/2}|1\rangle$


|0>
|1>
1st rotation λ = 0°
2nd rotation θ = π/3
3rd rotation ϕ = π/3

27.5)

27.5)

$$H(\psi) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$U_{\theta,\phi,\lambda} = U_{Hadamard}$$

$$\begin{pmatrix} \cos\theta/_2 & -e^{-i\lambda}\sin\theta/_2 \\ e^{i\phi}\sin\theta/_2 & e^{i(\lambda+\phi)}\cos\theta/_2 \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}$$

$\cos\theta/_2 = 1/\sqrt{2}$, $-e^{i\lambda}\sin\theta/_2 = 1/\sqrt{2}$

$e^{i\phi}\sin\theta/_2 = 1/\sqrt{2}$ $\quad e^{i(\lambda+\phi)}\cos\theta/_2 = 1/\sqrt{2}$

$\theta/_2 = 1/\sqrt{2} \Rightarrow \theta = \pi/2$ we can derive $\lambda = \pi$

and $-e^{i\lambda} = 1 \Rightarrow \phi = 0$

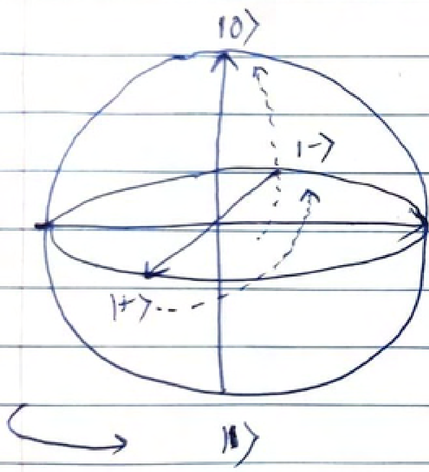
$$\Rightarrow U_{\theta,\phi,\lambda} = U_{\pi/2, 0, \pi}$$

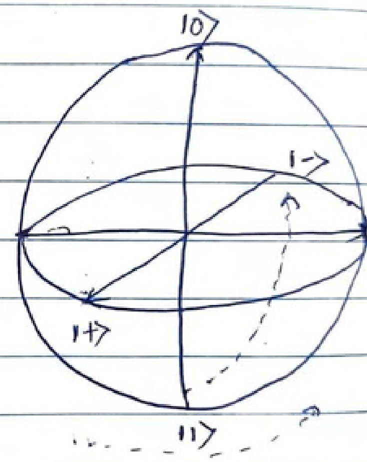
27.6)

27.6) Hadamard applied to $|+\rangle$ to $|1\rangle$

we know Hadamard's $U_{\theta,\phi,\lambda}$ is $U_{\pi/2,0,\pi}$

$H \rightarrow |+\rangle \longrightarrow |0\rangle$ $\quad$ $H \rightarrow |1\rangle \longrightarrow |-\rangle$

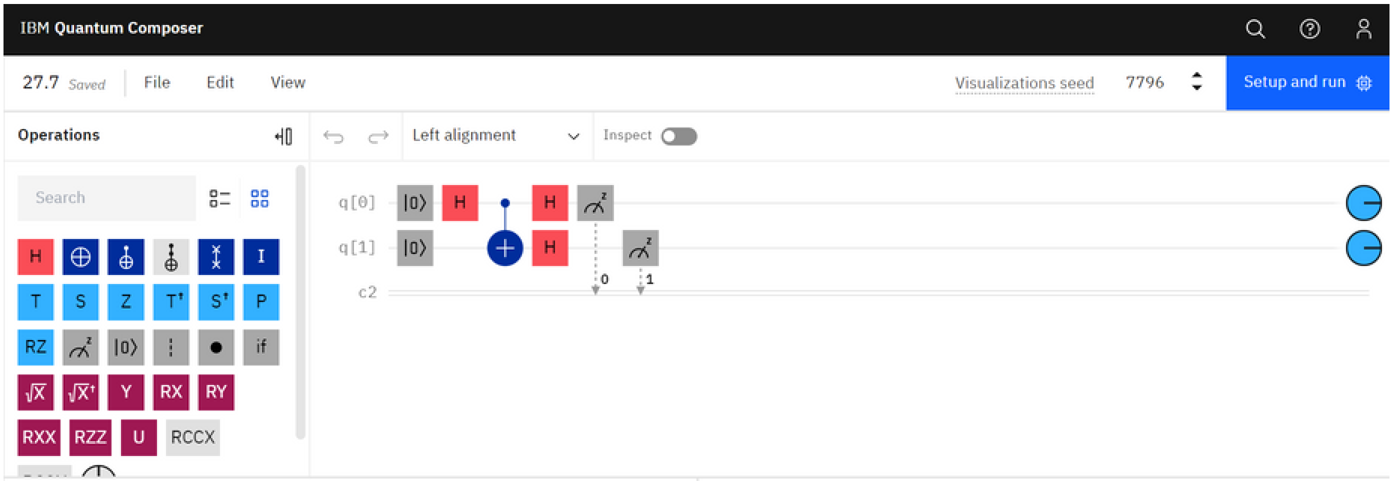




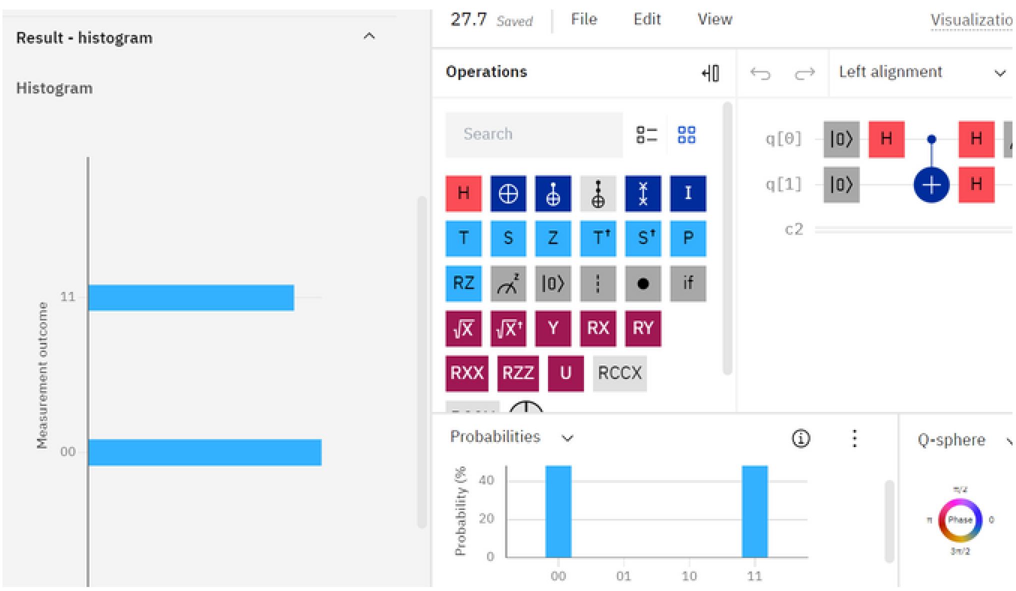
1st rotation $\lambda = \pi$
2nd rotation $\theta = \pi/2$
3rd rotation $\phi = 0$

1st rotation $\lambda = \pi$ (No use)
2nd rotation $\theta = \pi/2$
3rd rotation $\phi = 0$

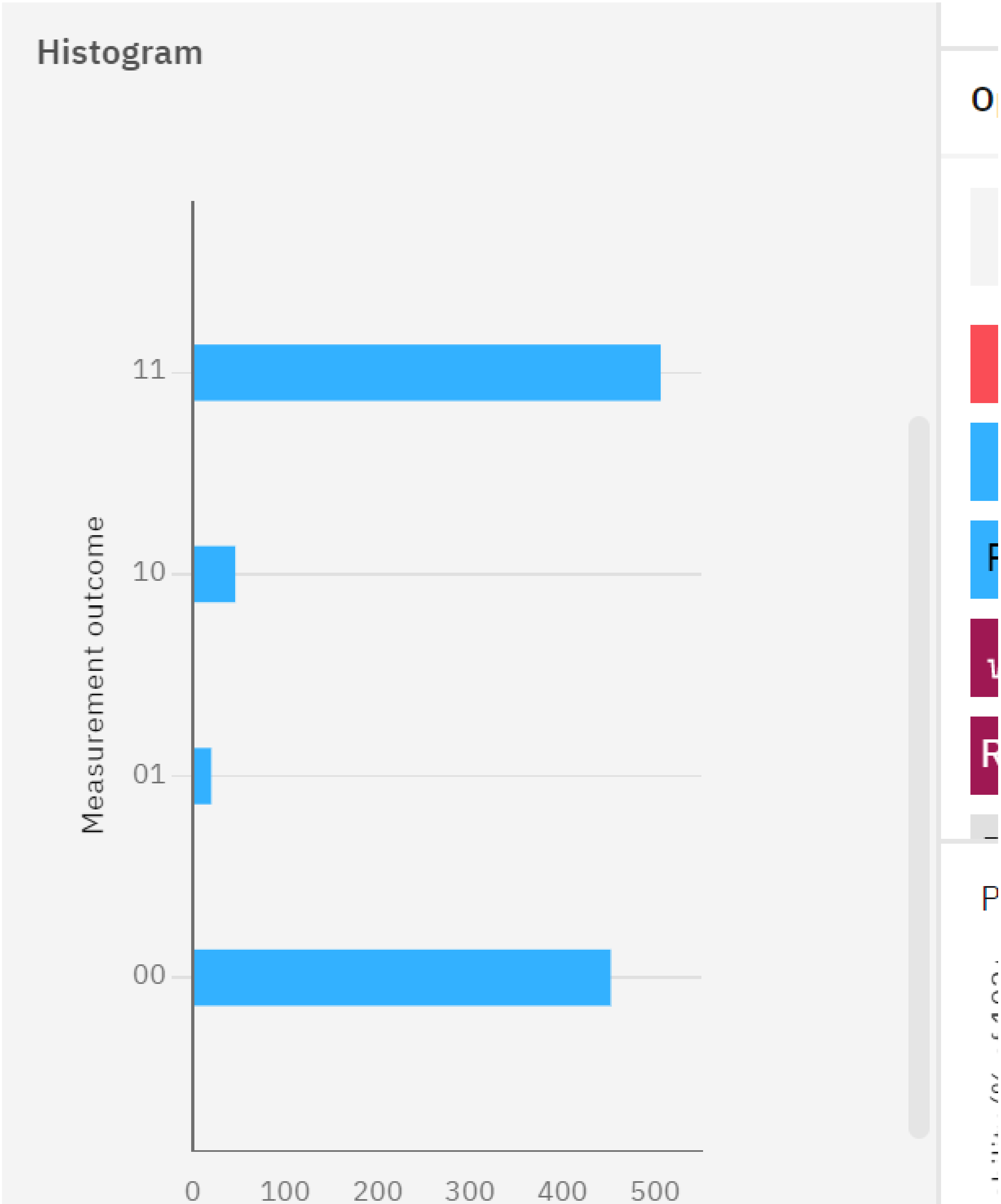
27.7



Result



Hardware execution result



Chapter 28

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28.1)

Given $\frac{1}{\sqrt{2}}\begin{pmatrix}-i & -i \\ -i & i\end{pmatrix}$

we can extract $-i$ from the above gate it becomes hadamard gate

$$-i \cdot \frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1 \\ 1 & -1\end{pmatrix}$$

$$-i\begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2}\end{pmatrix} = e^{i\gamma}\begin{pmatrix}\cos\theta/2 & -e^{-i\lambda}\sin\theta/2 \\ e^{i\phi}\sin\theta/2 & e^{i(\lambda+\phi)}\cos\theta/2\end{pmatrix}$$

$\cos\theta/2 = 1/\sqrt{2} \Rightarrow \theta/2 = 1/\sqrt{2}$ $\boxed{\theta = \pi/2}$

$-e^{-i\lambda}\sin\theta/2 \Rightarrow \boxed{\sin\theta/2 = 1/\sqrt{2}} \Rightarrow -e^{-i\lambda} \Rightarrow \lambda = \pi$

$e^{i\phi}\sin\theta/2 = e^{i\phi} \Rightarrow \phi = 0$

$\therefore e^{i\gamma} = -i \Rightarrow -(\cos\gamma + i\sin\gamma) = \gamma = \pi/2$

$\boxed{U_{\theta,\phi,\lambda,\gamma} = U_{\pi/2,0,\pi,\pi/2}}$

28.2)

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28.2

Given

$$C\text{-}U_{\theta,\phi,\lambda,\gamma} \quad \begin{array}{c|cccc} & 00 & 01 & 10 & 11 \\ \hline 00 & 1 & 0 & 0 & 0 \\ 01 & 0 & 1 & 0 & 0 \\ 10 & 0 & 0 & e^{i\gamma}\cos\theta/2 & -e^{i(\lambda+\gamma)}\sin\theta/2 \\ 11 & 0 & 0 & e^{i(\phi+\gamma)}\sin\theta/2 & e^{i(\lambda+\phi+\gamma)}\cos\theta/2 \end{array}$$

From 28.7 we can derive

we know

$$c_{2,3} = \langle 11| \langle 10| C\text{-}U_{\theta,\phi,\lambda,\gamma} |11\rangle = -e^{i(\lambda+\gamma)}\sin\theta/2$$

which is in $|10\rangle$ basis after inner product

$$c_{3,3} = \langle 11| \langle 11| C\text{-}U_{\theta,\phi,\lambda,\gamma} |11\rangle = e^{i(\lambda+\phi+\gamma)}\cos\theta/2$$

$$c_{2,2} = \langle 10| \langle 10| C\text{-}U_{\theta,\phi,\lambda,\gamma} |10\rangle = e^{i\gamma}\cos\theta/2$$

$$c_{3,2} = \langle 10| \langle 11| C\text{-}U_{\theta,\phi,\lambda,\gamma} |10\rangle = e^{i(\phi+\gamma)}\sin\theta/2$$

28.3)

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28.3) Given

$$C-U_{\theta,\phi,\lambda,\gamma} = |0\rangle\langle 0| \otimes I + |1\rangle\langle 1| \otimes U_{\theta,\phi,\lambda,\gamma}$$

$$|0\rangle\langle 0| \Rightarrow |0\rangle\langle 0| \Rightarrow \begin{bmatrix}1\\0\end{bmatrix}(1\ 0) = \begin{bmatrix}1&0\\0&0\end{bmatrix}$$

$$|1\rangle\langle 1| \Rightarrow |1\rangle\langle 1| \Rightarrow \begin{bmatrix}0\\1\end{bmatrix}(0\ 1) = \begin{bmatrix}0&0\\0&1\end{bmatrix}$$

$$= \begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}1&0\\0&1\end{bmatrix} + \begin{bmatrix}0&0\\0&1\end{bmatrix} \cdot e^{i\gamma}\begin{bmatrix}\cos\theta/2 & -e^{i\lambda}\sin\theta/2\\ e^{i(\phi+\theta)}\sin\theta/2 & e^{i(\lambda+\phi)}\cos\theta/2\end{bmatrix}$$

Anything multiplied with Identity is same

$$= \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix} + \begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&e^{i\gamma}\cos\theta/2&-e^{i(\lambda+\gamma)}\sin\theta/2\\0&0&e^{i(\phi+\gamma)}\sin\theta/2&e^{i(\lambda+\phi+\gamma)}\cos\theta/2\end{bmatrix}$$

$$= \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&e^{i\gamma}\cos\theta/2&-e^{i(\lambda+\gamma)}\sin\theta/2\\0&0&e^{i(\phi+\gamma)}\sin\theta/2&e^{i(\lambda+\phi+\gamma)}\cos\theta/2\end{bmatrix}$$

28.4)

28.4)

We know

$$U_{P_S,\pi}|e_1\rangle = +1\begin{pmatrix}1\\0\end{pmatrix} = +1|e_0\rangle$$

$$Z|e_0\rangle = e^{i2\pi.0}|e_0\rangle = |e_0\rangle$$

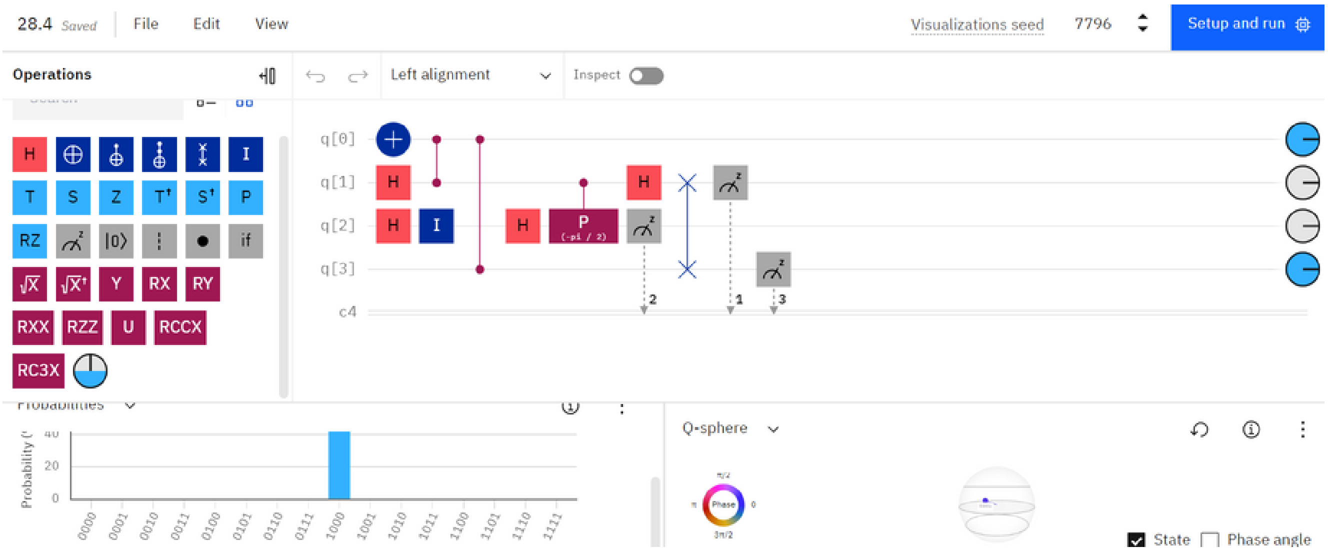
$$Z^2|e_0\rangle = \left(e^{i2\pi.0}\right)^2|e_0\rangle = |e_0\rangle$$

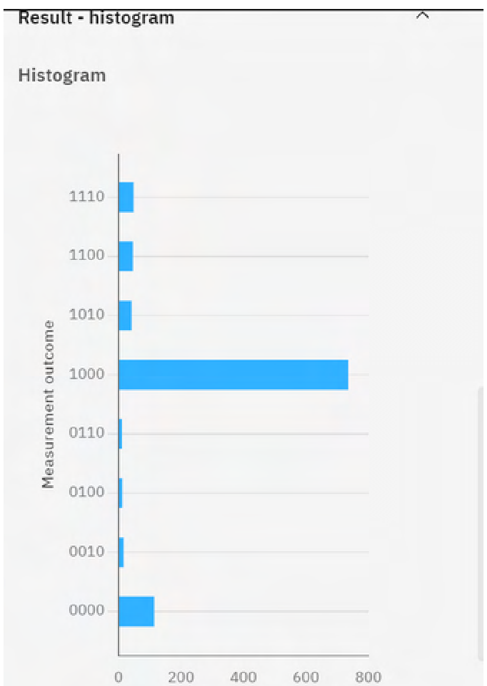
$Z = e^{i.2\pi.0}$ i.e, $e^0 = \cos 0 + i\sin 0 = 1$

If control Qubit is 0 OR e^0 it does Nothing

28.5)

Introduction to Quantum computing (From a layperson to programmer in 30 steps)





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28.6) Equation in $2\pi\psi = 2\pi K / 2^n$ is proved in 28.3.2

$$2\pi\psi = 2\pi K / 2^n$$

Here $|K\rangle = |10\rangle = |2\rangle$

$$2\pi\psi = 2\pi K / 2^n = 2\pi \cdot \cancel{2} / \cancel{4}_{2} = \pi$$

which has been proved

Chapter 29

29.1) By using method in (29.2)
177 mod 25

$177 = 25 \times 7 + 2$

i.e, 2

29.2 we cand find GCD by eucladean method

firstly we have 1242 & 242

i) Divide 1242 by 242 and we get
remainder 32

ii) Dividide 242 by 32 (untill last value 0)
242 % 32 → remainder is 18

iv) Divide 32 by 18
32 % 18 → remainder 14

iv) Divide 18 by 14

$18 \% 14 \rightarrow$ remainder is $4 > 0$

v) Divide 18 by 4

$18 \% 4 \rightarrow$ remainder is $2 > 0$

vi) Divide $4 \% 2 \rightarrow$ remander is 0

This will become GCD as remainder is zero (Value of GCD $\longrightarrow$ 2)

29.3)

```
# Shors simplified algorithm in python but cant handle big prime numbers
#29.3
# gcf is calculated
def get_gcf(a, b):
    while a != b:
        if a > b:
            a -= b
        elif b > a:
            b -= a
    return a

# I have used below integers as a in our text book we can add any number of numbers

# 12 is not suitable a so i have used incrementation to get better a

Randomint = [8, 10, 12]

for g in Randomint:
  N = 63
  done = False
  p = 0
  p_values = []
  while not done:
    p += 1
    r = g**p % N
    if g**p % N == 1:
        print(g, p)
        p_values.append(p)
    if len(p_values) >= 1:
        p = p_values[0]
        if p_values[0] % 2 != 0:
            g += 1
            p = 0
            p_values = []
        else:
            x = g**(p/2)+1
            y = g**(p/2)-1
            a = int(get_gcf(N, x))
            b = int(get_gcf(N, y))
            if a == 1 or b == 1:
                g += 1
                p = 0
                p_values = []
            else:
                print(g, p, a, b)
                done = True
```

```
    if p > 500:
        g += 1
        p = 0
        p_values = []

print(a, "x", b, "=", N)
```

```
8 2
8 2 9 7
9 x 7 = 63
10 6
10 6 7 9
7 x 9 = 63
13 6
13 6 7 9
7 x 9 = 63
```

29.4)

29.4)

Given $n=1$ and we have $n=1$ i.e, 2 for MSB & 1 for LSB

$|\varepsilon_0\rangle$ is initial setup

Basis state is

$$\varepsilon_0 = |0\rangle^{\otimes 2} \otimes |0\rangle = |00\rangle \otimes |0\rangle = |000\rangle$$

In First step

we apply two Hadamard gates in MSB

$$\varepsilon_1 = (H^{\otimes 2} \otimes I)(|0\rangle^{\otimes 2} |0\rangle)$$

we know $= \frac{1}{2^n} \sum_{x=0}^{2^{2n}-1} |x\rangle_{2n} |0\rangle_n$

Here n is 1 so when $n=1$

$$= \frac{1}{2} \sum_{x=0}^{3} |x\rangle_2 |0\rangle = \frac{1}{2}(|0\rangle + |1\rangle + |2\rangle + |3\rangle) \otimes |0\rangle$$

$$= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle) \otimes |0\rangle$$

Second step

ε_2 is basically applying oracle to the above state

$$|\varepsilon_2\rangle = U_f |\varepsilon_1\rangle$$

Here we are implementing quantum oracle as XOR oracle

Here we know $f(|00\rangle)$, $f(|01\rangle)$ and $f(|11\rangle)$ is zero (0)

and $f(|10\rangle)$ is 1

$$= \frac{1}{2^n} \sum_{x=0}^{2^{2n}-1} |x\rangle_{2n} |f(x)\rangle_n$$

$$= \frac{1}{2}(|00\rangle \otimes |f(|00\rangle)\rangle + |01\rangle \otimes |f(|01\rangle)\rangle + |10\rangle \otimes f(|10\rangle) + |11\rangle \otimes f(|11\rangle))$$

$$= \frac{1}{2}(|00\rangle \otimes |0\rangle + |01\rangle \otimes |0\rangle + |10\rangle \otimes |1\rangle + |11\rangle \otimes |0\rangle)$$

$$|\varepsilon_2\rangle = \frac{1}{2}(|000\rangle + |010\rangle + |101\rangle + |110\rangle)$$

Third step

QFT is applied to above state and it is the last step before measurement

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we know $U_{QFT}|x\rangle = \frac{1}{\sqrt{2^{2n}}} \sum_{k=0}^{2^{2n}-1} \omega^{-kx}|k\rangle$

$$|\varepsilon_3\rangle = (U_{QFT} \otimes I^{\otimes n})|\varepsilon_2\rangle$$

$$= \frac{1}{2^{2n}} \sum_{x=0}^{2^{2n}-1} \sum_{k=0}^{2^{2n}-1} \omega^{-kx}|k\rangle_{2n}|f(x)\rangle_n \quad (\because \omega = e^{i2\pi/2^{2n}})$$

$$= \frac{1}{2^2} \sum_{x=0}^{2^2-1} \sum_{k=0}^{2^2-1} e^{\frac{-i2\pi kx}{4}}|k\rangle|f(x)\rangle$$

$$= \frac{1}{4} \sum_{x=0}^{3} \sum_{k=0}^{3} e^{-i2\pi kx/4}|k\rangle|f(x)\rangle$$

$$= \frac{1}{4}\Big[(|000\rangle + |010\rangle + |110\rangle + |110\rangle)$$

$$+ [|000\rangle + i|010\rangle + |100\rangle + i|110\rangle) + [|001\rangle - |011\rangle + |110\rangle + |111\rangle]$$

$$+ [|000\rangle + i|010\rangle - |100\rangle - i|110\rangle]\Big)$$

It is not possible to have (in my opinion as we need to have atleast 5 qubits to execute shors Algorithm

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