BINARY HEAP

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Abstract. This report explains about bianry heap as a data structure.It explains the working of heap taking into consideration a min heap.Finally it deals with an analysis regarding height of the binary heap and explains different heap operation in that context.

1. Introduction

A bianry heap is a data structure created using binary tree that is a tree with atmost two children.A binary heap needs to satisfy two properties:

1: The tree is a complete bianry tree. A complete binary tree is one in which all levels are completely filled except possibly the bottom most one .

2: All nodes are either greater than or equal to each of its children called as Max-heap or less than or equal to each of its children called as min-heap.

2. Working of a Binary Heap

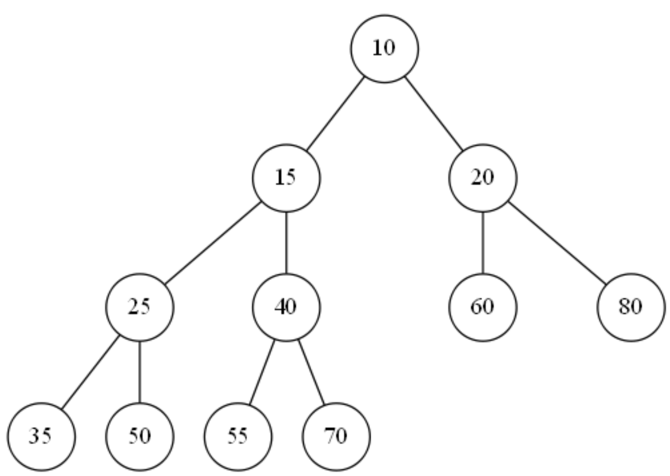

Figure 1. min heap

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. The above diagram shows a min heap satisfying the two essential properties of binary heap.
The essential operations on a binary heap are:

-INSERTION
-DELETION
-Extract Min (if min heap)
-Extract Max (if max heap)

Now we will analyse all these operation in a min heap.

INSERTION
. Suppose we want to insert element 5 into the min heap shown in figure 1.
Steps to be followed are:

- Insert new node 5 into the tree maintaining complete binary tree property.
- Make necessary swaps to maintain heap property.

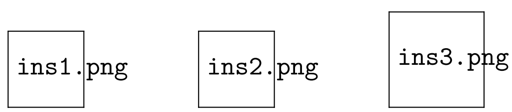

FIGURE 2. Steps in Inserting 5 into min heap

As depicted by above figure insertion of 5 causes 2 swaps and finally it becomes the new root.
For a min heap after inserting the new node it is compared with its parent.If parent is greater than newnode then swap and repeat the process for parent and grand parent and so on. The process stops once we reach the root or parent becomes smaller than its child.

DELETION
. Deletion in a heap always delete the root node.In case of a min-heap root is the smallest element.So deletion of root deletes the minimum that is Extract Minimum operation. In case of a max-heap root is the largest element. So deletion of root deletes the maximum element that is Extract Maximum operation.

Steps to be followed in delete operation are :

- replace the root node by the rightmost node in the bottommost level of the complete binary tree.
- compare the new root with its children and make necessary swapping to maintain heap property.

Suppose we execute delete operation on the min heap obtained in Figure 2. The steps involved in the deletion process are shown in Figure 3.

del1.png del2.png del3.png

FIGURE 3. Steps in Delete operation in min heap

3. Height of a Binary Heap

Height is O(logn) :

Proof. The height of a binary heap is the maximum number of levels from root to the leaves.
Suppose the heap contains only one node that is only the root,then height is 0.If it contains 2 or 3 nodes then height is 2.
Root is always at level 0. So for a heap with height **h** maximum number of nodes in the last level that is maximum number of leaves can be 2^h. If **n** is the total number of nodes and height of the heap is **h** then we have

$2^0+2^1+2^2+\text{- - - - - - -}+2^h$=n
2^{h+1}=n+1
h+1=log(n+1)
h=O(logn)

While inserting a new node, in the worst case the node may move upto root.So insert operation is of the order of height of the heap that is O(logn).
Similarly while deletion, the node replacing the root may come down to leaf level in the worst case. So again deletion is of the order of height of the heap that is O(logn).

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References

[1] Thomas H. Cormen ,Charles E. Leiserson ,Ronald L. Rivest,Clifford Stein, *Introduction to Algorithms, 3rd edition*,3rd edition, 2001
[2] web resource *wikipedia*

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