Digital System Design

Module 1 - NUMBER SYSTEM

17/08/2020

ITT203 DIGITAL SYSTEM DESIGN

Course Outcomes: After the completion of the course, the student will be able

No.	Course Outcome	Bloom's Taxonomy	Level
CO 1	To perform base conversion and arithmetic operations in various number systems.	Apply	K3
CO 2	To design digital circuits using simplified Boolean functions	Create	K4
CO 3	To develop simple design of combinational circuits	Apply	K3
CO 4	To develop simple design of sequential circuits	Apply	K3
CO 5	To interpret the generalization of synchronous and asynchronous sequential circuits	Understand	K2

Digital System

AIM

To understand the operation of each digital systems, it is necessary to have a basic knowledge of digital circuits and their logical function.

- Digital systems is their ability to represent and manipulate discrete elements of information.
- Binary
- Bit - 0 and 1
- Binary code eg. 0110

Understanding Numbers

A Decimal Number 5736 $= 5X10^3 + 7X10^2 + 3X10^1 + 6X10^0$
Consider a number $a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3}$
Then
$a_5a_4a_3a_2a_1a_0.a_{-1}a_{-2}a_{-3} = 10^5a_5 + 10^4a_4 + 10^3a_3 + 10^2a_2 + 10^1a_1 + 10^0a_0 + 10^{-1}a_{-1} + 10^{-2}a_{-2} + 10^{-3}a_{-3}$

Understanding Numbers

The decimal number system is said to be of *base*, or *radix*, $\mathbf{10}$ because it uses 10 digits and the coefficients are multiplied by powers of $\mathbf{10}$.

Base or Radix

- There are many different number systems

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- A number is expressed in a base-r system
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Base or Radix

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- A number is expressed in a base-r system
- Coefficients multiplied by powers of r
- $a_n.r^n + a_{n-1}.r^{n-1} + ... + a_2.r^2 + a_1.r + a_0 + a_{-1}.r^{-1} + a_{-2}.r^{-2} + ... + a_{-m}.r^{-m}$
- $(a_n a_{n-1} ... a_1 a_0 . a_{-1} a_{-2} ... a_m)_r$

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- Example:
  - Consider the number 301.4 to the base 5

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- $(a_n a_{n-1} ... a_1 a_0 . a_{-1} a_{-2} ... a_m)_r$
- Example:
  - Consider the number 301.4 to the base 5
  - $(301.4)_5$

Base or Radix

- There are many different number systems
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- $a_n.r^n + a_{n-1}.r^{n-1} + ... + a_2.r^2 + a_1.r + a_0 + a_{-1}.r^{-1} + a_{-2}.r^{-2} + ... + a_{-m}.r^{-m}$
- $(a_n a_{n-1} ... a_1 a_0 . a_{-1} a_{-2} ... a_m)_r$
- Example:
  - Consider the number 301.4 to the base 5
  - $(301.4)_5$
  - $(301.4)_5 = 5^2.3 + 5^1.0 + 5^0.1 + 5^{-1}.4 = (76.8)_{10}$

Number Systems

- Decimal Number System
  - 10 symbols
  - 0 1 2 3 4 5 6 7 8 9
- Binary Number System
  - 2 symbols
  - 0 1
- Octal Number System
  - 8 symbols
  - 0 1 2 3 4 5 6 7
- HexaDecimal Number System
  - 16 symbols
  - 0 1 2 3 4 5 6 7 8 9 A B C D E F

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Example

1. $(1101.1)_2 = 2^3.1 + 2^2.1 + 2^1.0 + 1 + 2^{-1}.1 = (13.5)_{10}$

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Example

1. $(1101.1)_2 = 2^3.1 + 2^2.1 + 2^1.0 + 1 + 2^{-1}.1 = (13.5)_{10}$
2. $(1101.1)_8 = 8^3.1 + 8^2.1 + 8^1.0 + 1 + 8^{-1}.1 = (577.125)_{10}$
3. $(1101.1)_{16} = 16^3.1+16^2.1+16^1.0+1+16^{-1}.1 = (4353.0625)_{10}$

Test Time.
You have 5 Minutes

NUMBER-BASE CONVERSIONS
Decimal to Binary:

Question: Convert $(55)_{10}$ to binary

Number	Quotient	Remainder	Coefficient
$55/2$	27	1	$a_0 = 1$
$27/2$	13	1	$a_1 = 1$
$13/2$	6	1	$a_2 = 1$
$6/2$	3	0	$a_3 = 0$
$3/2$	1	1	$a_4 = 1$
$1/2$	0	1	$a_5 = 1$

$$(55)_{10} = (110111)_2$$

NUMBER-BASE CONVERSIONS
Decimal to Binary:

Question: Convert $(0.6875)_{10}$ to binary

Number	Integer	decimal	Coefficient
0.6875×2	1	0.375	$a_{-1} = 1$
0.375×2	0	0.75	$a_{-2} = 0$
0.75×2	1	0.5	$a_{-3} = 1$
0.5×2	1	0	$a_{-4} = 1$

$(0.6875)_{10} = (0.1011)_2$

$(55.6875)_{10} = (110111.1011)_2$

NUMBER-BASE CONVERSIONS
Decimal to Octal:

Question: Convert $(155)_{10}$ to Octal

Number	Quotient	Remainder	Coefficient
$155/8$	19	3	$a_0 = 3$
$19/8$	2	3	$a_1 = 3$
$2/8$	0	2	$a_2 = 2$

$(155)_{10} = (233)_8$

NUMBER-BASE CONVERSIONS
Decimal to Hexadecimal:

Question: Convert $(2410)_{10}$ to hexadecimal

Number	Quotient	Remainder	Coefficient
$2410/16$	150	10	$a_0 = A$
$150/16$	9	6	$a_1 = 6$
$9/16$	0	9	$a_2 = 9$

$$(2410)_{10} = (96A)_{16}$$

NUMBER-BASE CONVERSIONS

Decimal to Hexadecimal:

Question: Convert $(0.6875)_{10}$ to Hexadecimal

Number	Integer	decimal	Coefficient
0.6875×16	11	0	$a_{-1} = B$

$$(0.6875)_{10} = (0.B)_{16}$$

NUMBER-BASE CONVERSIONS

Decimal to Hexadecimal:

Question:Convert $(0.7865)_{10}$ to Hexadecimal

Number	Integer	decimal	Coefficient
0.7865×16	12	0.584	$a_{-1} = C$
0.584×16	9	0.344	$a_{-2} = 9$
0.344×16	5	0.504	$a_{-3} = 5$
0.504×16	8	0.064	$a_{-4} = 8$
0.064×16	1	0.024	$a_{-5} = 1$

$$(0.7865)_{10} = (0.C9581)_{16}$$

$$(2410.7865)_{10} = (96A.C9581)_{16}$$

NUMBER-BASE CONVERSIONS
Binary to Hexadecimal:

Groups of four

Question: Convert $(1110101011.011011)_2$ to Hexadecimal

$(11, 1010, 1011.0110, 11)_2$

$$(0011 \quad 1010 \quad 1011 \quad .0110 \quad 1100)_2 =$$
$$(3 \quad A \quad B \quad .6 \quad C)_{16}$$

NUMBER-BASE CONVERSIONS
Binary to Octal:

Groups of three

Question: Convert $(1110101011.0110101)_2$ to octal

$(1, 110, 101, 011.011, 010, 1)_2$

(001	110	101	011	.011	010	$100)_2 =$
(1	6	5	3	.3	2	$4)_8$

Test Time.

Digital System Design

Module 1 - NUMBER SYSTEM

18.08.2020

Arithmetic Operations - Binary Addition

Augend	Addend	Carry	Sum
0	0	0	0
0	1	0	1
1	0	0	1
1	1	1	0

Binary Addition

An Example: $(1010)_2 + (1100)_2$

Carry	1	0	0	0	
augend		1	0	1	0
addend	$+$	1	1	0	0
Sum	1	0	1	1	0

Another example: $(101101)_2 + (110111)_2$

Carry	1	1	1	1	1	1	
augend		1	0	1	1	0	1
addend	$+$	1	1	0	1	1	1
Sum	1	1	0	0	1	0	0

Binary Subtraction

minuend	subtrahend	difference	borrow
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

Binary Subtraction

borrow				1			
borrow		0	0	10	10	0	0
minuend		1	1	0	0	1	0
subtrahend	-	1	0	0	1	0	0
difference		0	0	1	1	1	0

Binary Multiplication

Multiplicand	Multiplier	Product
0	0	0
0	1	0
1	0	0
1	1	1

Binary Multiplication

Multiplicand		1	0	1	0	1	1
Multiplier				×	1	0	1
		1	0	1	0	1	1
	0	0	0	0	0	0	
	1 0	1	0	1	1		
	1 1	0	1	0	1	1	1

	1	0	1	0	1	1		
101	1	1	0	1	0	1	1	1
	-1	0	1					
	0	0	1	1				
		0	0	0				
		0	1	1	0			
			-1	0	1			
			0	0	1	1		
				0	0	0		
				0	1	1	1	
					-1	0	1	
					0	1	0	1
						-1	0	1
						0	0	0

Hexadecimal Addition

Carry		1	1	0	
augend		4	A	2	5
addend	+	8	9	E	3
Sum		D	4	0	8

Octal Addition

Carry	1	0	1		
augend		6	2	5	0
addend	+	5	1	3	4
Sum	1	3	4	0	4

Test Time

Digital System Design

Module 1 - NUMBER SYSTEM

Dr. Deepthi Sasidharan, GEC Barton Hill, Thiruvananthapuram

August 19, 2020

COMPLEMENTS OF NUMBERS

- Complements are used in digital computers to simplify the subtraction operation and for logical manipulation.

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  - Radix complement ($r's$ complement)
  - Diminished radix complement ($(r-1)'s$ complement)

Diminished radix complement

N in base r having n digits

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$N = 6481$, $n = 4$, and $r = 10$ then $(r - 1) = 9$

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Example: Find the diminished radix complement of $(6481)_{10}$

$N = 6481$, $n = 4$, and $r = 10$ then $(r - 1) = 9$

Question: Find the $9's$ complement of $(6481)_{10}$

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$N = 6481$, $n = 4$, and $r = 10$ then $(r - 1) = 9$

Question: Find the $9's$ complement of $(6481)_{10}$

$$
\begin{aligned}
(r^n - 1) - N = (10^4 - 1) - 6481 &= (10000 - 1) - 6481 \\
&= 9999 - 6481 \\
&= (3518)_{10}
\end{aligned}
$$

9's Complement

Number	9's Complement
0	9
1	8
2	7
3	6
4	5
5	4
6	3
7	2
8	1
9	0

1's Complement

Example: Find the diminished radix complement of $(1011000)_2$

1's Complement

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$N = 1011000$, $n = 7$, and $r = 2$ then $(r - 1) = 1$

1's Complement

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Question: Find the $1's$ complement of $(1011000)_2$

1's Complement

Example: Find the diminished radix complement of $(1011000)_2$

$N = 1011000$, $n = 7$, and $r = 2$ then $(r - 1) = 1$

Question: Find the $1's$ complement of $(1011000)_2$

$$\begin{aligned}(r^n - 1) - N &= (2^7 - 1) - 1011000_2 \\ &= (10000000 - 1)_2 - (1011000)_2 \\ &= (1111111)_2 - (1011000)_2 \\ &= (0100111)_2\end{aligned}$$

1's Complement

Example: Find the diminished radix complement of $(1011000)_2$

$N = 1011000$, $n = 7$, and $r = 2$ then $(r-1) = 1$

Question: Find the $1's$ complement of $(1011000)_2$

$$
\begin{aligned}
(r^n - 1) - N &= (2^7 - 1) - 1011000_2 \\
&= (10000000 - 1)_2 - (1011000)_2 \\
&= (1111111)_2 - (1011000)_2 \\
&= (0100111)_2
\end{aligned}
$$

Another example:

$1's$ complement of $(0011010)_2 = (1100101)_2$

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Radix complement, is defined as $r^n - N$

Example: Find the radix complement of $(6481)_{10}$

$N = 6481$, $n = 4$, and $r = 10$

Question: Find the $10's$ complement of $(6481)_{10}$

Radix complement

N in base r having n digits

Radix complement, is defined as $r^n - N$

Example: Find the radix complement of $(6481)_{10}$

$N = 6481$, $n = 4$, and $r = 10$

Question: Find the $10's$ complement of $(6481)_{10}$

$r^n - N = 10^4 - 6481 = 10000 - 6481 = 3519$

Radix complement

N in base r having n digits

Radix complement, is defined as $r^n - N$

Example: Find the radix complement of $(6481)_{10}$

$N = 6481$, $n = 4$, and $r = 10$

Question: Find the $10's$ complement of $(6481)_{10}$

$r^n - N = 10^4 - 6481 = 10000 - 6481 = 3519$

$r^n - N = (r^n - 1) - N + 1$

2's Complement

Example: Find the radix complement of $(1011011)_2$

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Question: Find the $2's$ complement of $(1011011)_2$

2's Complement

Example: Find the radix complement of $(1011011)_2$

$N = 1011000$, $n = 7$, and $r = 2$

Question: Find the $2's$ complement of $(1011011)_2$

$$r^n - N = (2^7)_{10} - 1011000_2 = (10000000)_2 - (1011011)_2$$
$$= (0100101)_2$$

2's Complement

Example: Find the radix complement of $(1011011)_2$

$N = 1011000$, $n = 7$, and $r = 2$

Question: Find the $2's$ complement of $(1011011)_2$

$$r^n - N = (2^7)_{10} - 1011000_2 = (10000000)_2 - (1011011)_2$$
$$= (0100101)_2$$

$$r^n - N = (r^n - 1) - N + 1$$

2's Complement

Example: Find the radix complement of $(1011011)_2$

$N = 1011000$, $n = 7$, and $r = 2$

Question: Find the $2's$ complement of $(1011011)_2$

$$r^n - N = (2^7)_{10} - 1011000_2 = (10000000)_2 - (1011011)_2$$
$$= (0100101)_2$$

$$r^n - N = (r^n - 1) - N + 1$$

$1's$ complement of $(1011011)_2 = (0100100)_2$

$2's$ complement of
$(1011011)_2 = (0100100)_2 + (1)_2 = (0100101)_2$

Test Time

Digital System Design

Module 1 - NUMBER SYSTEM

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August 20, 2020

USING COMPLEMENT FOR SUBTRACTION

- The subtraction of two n-digit unsigned numbers $M - N$ in base r can be done as follows:

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  1. Add the minuend M to the $r's$ complement of the subtrahend N.

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  2. If $M \geq N$, the sum will produce an end carry r^n, which can be discarded; what is left is the result $M - N$.

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- The subtraction of two n-digit unsigned numbers $M - N$ in base r can be done as follows:
  1. Add the minuend M to the $r's$ complement of the subtrahend N.

     Mathematically, $M + (r^n - N) = M - N + r^n$.
  2. If $M \geq N$, the sum will produce an end carry r^n, which can be discarded; what is left is the result $M - N$.
  3. If $M < N$, the sum does not produce an end carry and is equal to $r^n - (N - M)$, which is the $r's$ complement of $(N - M)$. To obtain the answer in a familiar form, take the $r's$ complement of the sum and place a negative sign in front.

Example 1

Using 10's complement, subtract $52037 - 2300$

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Here $M = 52037$ and $N = 2300$

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Find the 10's complement of N

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$M = 52037$ and $N = 02300$

Find the 10's complement of N

$10's\ C\ of\ 02300 = 10^5 - 02300 = 97700$

Example 1

Using 10's complement, subtract $52037 - 2300$

Here $M = 52037$ and $N = 2300$

Both M and N should have same number of digits

$M = 52037$ and $N = 02300$

Find the 10's complement of N

$10's\ C\ of\ 02300 = 10^5 - 02300 = 97700$

$52037 - 2300 = 52037 + (10's\ C\ of\ 02300) = 52037 + 97700$

$= \boxed{1}49737$ We discard the carry as $M \geq N$

$52037 - 2300 = \underline{\underline{49737}}$

Example 2

Using 2's complement, subtract $1101101 - 10101$

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$M = 1101101$ and $N = 0010101$

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Using 2's complement, subtract $1101101 - 10101$

Here $M = 1101101$ and $N = 10101$

Both M and N should have same number of digits

$M = 1101101$ and $N = 0010101$

Find the 2's complement of N

Example 2

Using 2's complement, subtract $1101101 - 10101$

Here $M = 1101101$ and $N = 10101$

Both M and N should have same number of digits

$M = 1101101$ and $N = 0010101$

Find the 2's complement of N

$2's\ C\ of\ 0010101 = 1101010 + 1 = 1101011$

Example 2

Using 2's complement, subtract $1101101 - 10101$

Here $M = 1101101$ and $N = 10101$

Both M and N should have same number of digits

$M = 1101101$ and $N = 0010101$

Find the 2's complement of N

$2's\ C\ of\ 0010101 = 1101010 + 1 = 1101011$

$$\begin{aligned} 1101101 - 10101 &= 1101101 + (2's\ C\ of\ 0010101) \\ &= 1101101 + 1101011 \\ &= \boxed{1}1011000 \end{aligned}$$ We discard the carry as $M \geq N$

$1101101 - 10101 = \underline{\underline{1011000}}$

Example 3

Using 2's complement, subtract $11011 - 10101010$

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Here $M = 11011$ and $N = 10101010$

Both M and N should have same number of digits

$M = 00011011$ and $N = 10101010$

Example 3

Using 2's complement, subtract $11011 - 10101010$

Here $M = 11011$ and $N = 10101010$

Both M and N should have same number of digits

$M = 00011011$ and $N = 10101010$

Find the 2's complement of N

Example 3

Using 2's complement, subtract $11011 - 10101010$

Here $M = 11011$ and $N = 10101010$

Both M and N should have same number of digits

$M = 00011011$ and $N = 10101010$

Find the 2's complement of N

$2's\ C\ of\ 10101010 = 01010101 + 1 = 01010110$

Example 3

Using 2's complement, subtract $11011 - 10101010$

Here $M = 11011$ and $N = 10101010$

Both M and N should have same number of digits

$M = 00011011$ and $N = 10101010$

Find the 2's complement of N

$2's\ C\ of\ 10101010 = 01010101 + 1 = 01010110$

$$\begin{aligned} 11011 - 10101010 &= 11011 + (2's\ C\ of\ 10101010) \\ &= 11011 + 01010110 \\ &= 01110001 \text{ No carry since } M < N \end{aligned}$$

Example 3 contd..

$$2's\ C\ of\ 01110001 = 10001110 + 1 = 10001111$$

$$11011 - 10101010 = \underline{\underline{-10001111}}$$

Subtraction Using $(r-1)'s$ complement

- $(r-1)'s$ complement is one less than the $r's$ complement.
- The result of adding the minuend to the complement of the subtrahend produces a sum that is one less than the correct difference when an end carry occurs.
- Removing the end carry and adding 1 to the sum is referred to as an **end-around carry**.

Example 4

Using 1's complement, subtract $11011 - 10101$

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1's complement of $10101 = 01010$

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Using 1's complement, subtract $11011 - 10101$

1's complement of $10101 = 01010$

SUM $= 11011 + 01010 = \boxed{1}00101$

End-around carry $(+1) = 00101 + 1$

Answer $= \underline{\underline{00110}}$

SIGNED BINARY NUMBERS

Positive integers (including zero) can be represented as unsigned numbers

We need a notation for representation of negative values

Because of hardware limitations, computers must represent everything with binary digits

- Unsigned numbers
- Signed numbers

REPRESENTATION OF SIGNED BINARY NUMBERS

Consider we have 8 bits to represent a number

$+9$ can be represented as $(00001001)_2$

-9 can be represented in three ways:

signed-magnitude representation:	10001001
signed-1's-complement representation:	11110110
signed-2's-complement representation:	11110111

Signed Binary Numbers [1]

Table 1.3
Signed Binary Numbers

Decimal	Signed-2's Complement	Signed-1's Complement	Signed Magnitude
+7	0111	0111	0111
+6	0110	0110	0110
+5	0101	0101	0101
+4	0100	0100	0100
+3	0011	0011	0011
+2	0010	0010	0010
+1	0001	0001	0001
+0	0000	0000	0000
−0	—	1111	1000
−1	1111	1110	1001
−2	1110	1101	1010
−3	1101	1100	1011
−4	1100	1011	1100
−5	1011	1010	1101
−6	1010	1001	1110
−7	1001	1000	1111
−8	1000	—	—

[1] From: M. M. Mano, M. D. Ciletti," Digital Design With an Introduction to the Verilog HDL", 5th Edition

Test Time

Digital System Design

Module 1 - NUMBER SYSTEM

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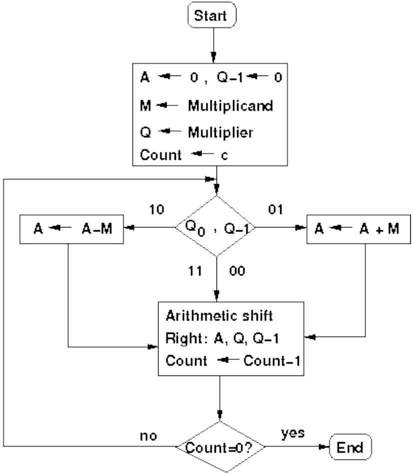
August 24, 2020

Booth Algorithm



Example 1

Question: $(12)_{10} \times (6)_{10}$

$$(12)_{10} = (01100)_2$$
$$(6)_{10} = (00110)_2$$
$$(-12)_{10} = (10100)_2$$

Initialization

$$M = 01100$$
$$-M = 10100$$
$$A = 00000$$
$$Q = 00110$$
$$Q_{-1} = 0$$

	A	Q	Q_{-1}	Operation
0	00000	00110	0	Initialization
1	00000	00011	0	Arithmetic Right Shift
2	10100	00011	0	$A \leftarrow A - M$
	11010	00001	1	Arithmetic Right Shift
3	11101	00000	1	Arithmetic Right Shift
4	01001	00000	1	$A \leftarrow A + M$
	00100	10000	0	Arithmetic Right Shift
5	00010	01000	0	Arithmetic Right Shift

$$(01100)_2 \times (00110)_2 = (00\ 0100\ 1000)_2$$

	A	Q	Q_{-1}	Operation
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$$(01100)_2 \times (00110)_2 = (00\ 0100\ 1000)_2$$

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$$(01100)_2 \times (00110)_2 = (00\ 0100\ 1000)_2$$

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$$(01100)_2 \times (00110)_2 = (00\ 0100\ 1000)_2$$

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3	11101	00000	1	Arithmetic Right Shift
4	01001	00000	1	$A \leftarrow A + M$
	00100	10000	0	Arithmetic Right Shift
5	00010	01000	0	Arithmetic Right Shift

$(01100)_2 \times (00110)_2 = (00\ 0100\ 1000)_2$

Example 2

Question: $(12)_{10} \times (-6)_{10}$

$$(12)_{10} = (01100)_2$$
$$(-6)_{10} = (11010)_2$$
$$(-12)_{10} = (10100)_2$$

Initialization

$$M = 01100$$
$$-M = 10100$$
$$A = 00000$$
$$Q = 11010$$
$$Q_{-1} = 0$$

	A	Q	Q_{-1}	Operation
0	00000	11010	0	Initialization
1	00000	01101	0	Arithmetic Right Shift
2	10100	01101	0	$A \leftarrow A - M$
	11010	00110	1	Arithmetic Right Shift
3	00110	00110	1	$A \leftarrow A + M$
	00011	00011	0	Arithmetic Right Shift
4	10111	00011	0	$A \leftarrow A - M$
	11011	10001	1	Arithmetic Right Shift
5	11101	11000	1	Arithmetic Right Shift

$$(01100)_2 \times (11010)_2 = (11\ 1011\ 1000)_2$$

Example 3

Question: $(-12)_{10} \times (+6)_{10}$

$$(-12)_{10} = (10100)_2$$
$$(12)_{10} = (01100)_2$$
$$(+6)_{10} = (00110)_2$$

Initialization

$$M = 10100$$
$$-M = 01100$$
$$A = 00000$$
$$Q = 00110$$
$$Q_{-1} = 0$$

	A	Q	Q_{-1}	Operation
0	00000	00110	0	Initialization
1	00000	00011	0	Arithmetic Right Shift
2	01100	00011	0	$A \leftarrow A - M$
	00110	00001	1	Arithmetic Right Shift
3	00011	00000	1	Arithmetic Right Shift
4	10111	00000	1	$A \leftarrow A + M$
	11011	10000	0	Arithmetic Right Shift
5	11101	11000	0	Arithmetic Right Shift

$$(10100)_2 \times (00110)_2 = (11\ 1011\ 1000)_2 = (-72)_{10}$$

Test Time

Question: Perform Multiplication using Booth Algorithm

$$(-12)_{10} \times (-6)_{10}$$

Digital System Design

Module 1 - NUMBER SYSTEM

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August 25, 2020

BINARY CODES

- An n-bit binary code is a group of n bits that assumes up to 2^n distinct combinations
- A set of eight elements requires a three-bit code and a set of 16 elements requires a four-bit code
- Although the minimum number of bits required to code 2n distinct quantities is n, there is no maximum number of bits that may be used for a binary code
- For example, the 10 decimal digits can be coded with 10 bits, and each decimal digit can be assigned a bit combination of nine 0's and a 1. In this particular binary code, the digit 6 is assigned the bit combination 0001000000

Binary-Coded Decimal Code

Decimal	BCD Code
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

The binary combinations 1010 through 1111 are not used and have no meaning in BCD.

Example 1:

$$(582)_{10} = (0101\ 1000\ 0010)_{BCD} = (1001000110)_2$$

Example 2:

$$(109)_{10} = (0001\ 0000\ 1001)_{BCD} = (1101101)_2$$

BCD numbers are decimal numbers and not binary numbers

BCD Addition

- Addition of two decimal digits in BCD
- Sum cannot be greater than $9 + 9 + 1 = 19$
- The binary sum will produce a result in the range from 0 to 19
- This range will be from 0000 to 10011, but in BCD, it is from 0000 to 1 1001
- When the binary sum is equal to or less than 1001 (without a carry), the corresponding BCD digit is correct
- When the binary sum is greater than or equal to 1010, the result is an invalid BCD digit
- The addition of $6 = (0110)_2$ to the binary sum converts it to the correct digit and also produces a carry as required

Examples

$$\begin{array}{r} 3 \\ +6 \\ \hline 9 \end{array} \qquad \begin{array}{cccc} 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 0 & 1 \end{array}$$

$$(0011)_{BCD} + (0110)_{BCD} = (1001)_{BCD}$$

$$\begin{array}{r} 9 \\ 6 \\ \hline 15 \end{array} \qquad \begin{array}{ccccc} & 1 & 0 & 0 & 1 \\ & 0 & 1 & 1 & 0 \\ \hline & 1 & 1 & 1 & 1 \\ + & 0 & 1 & 1 & 0 \\ \hline 1 & 0 & 1 & 0 & 1 \end{array}$$

$$(1001)_{BCD} + (0110)_{BCD} = (0001\ 0101)_{BCD}$$

Question: $(5942)_{10} + (4963)_{10}$

		1	1	1		
5942			0101	1001	0100	0010
4963		+	0100	1001	0110	0011
10905			1010	10011	1010	0101
			0110	0110	0110	
	0001		0000	1001	0000	0101

Other Binary Codes

Table 1.5
Four Different Binary Codes for the Decimal Digits

Decimal Digit	BCD 8421	2421	Excess-3	8, 4, −2, −1
0	0000	0000	0011	0000
1	0001	0001	0100	0111
2	0010	0010	0101	0110
3	0011	0011	0110	0101
4	0100	0100	0111	0100
5	0101	1011	1000	1011
6	0110	1100	1001	1010
7	0111	1101	1010	1001
8	1000	1110	1011	1000
9	1001	1111	1100	1111
	1010	0101	0000	0001
Unused	1011	0110	0001	0010
bit	1100	0111	0010	0011
combi-	1101	1000	1101	1100
nations	1110	1001	1110	1101
	1111	1010	1111	1110

Gray Code

Gray Code	Decimal Equivalent
0000	0
0001	1
0011	2
0010	3
0110	4
0111	5
0101	6
0100	7
1100	8
1101	9
1111	10
1110	11
1010	12
1011	13
1001	14
1000	15