MATHEMATICS – (4 th semester)

(For Information Technology)

CODE	COURSE NAME	CATEGORY	L	T	P	CREDIT
MAT 208	PROBABILITY,STATISTICS AND ADVANCED GRAPH THEORY	BASIC SCIENCE COURSE	3	1	0	4

Preamble: This course introduces students to the modern theory of probability and statistics, covering important models of random variables and techniques of parameter estimation and hypothesis testing. This course introduce fundamental concepts in Graph Theory, including properties and characterisation of Graph/Trees and Graph theoretic algorithms, which are widely used in Mathematical modelling and has got applications across **Information Technology**

Prerequisite: A basic course in one-variable and multi-variable calculus, knowledge of elementary set theory, matrices

Course Outcomes: After the completion of the course the student will be able to

CO 1	Understand the concept, properties and important models of discrete random variables and, using them, analyse suitable random phenomena.
CO 2	Understand the concept, properties and important models of continuous random variables and, using them, analyse suitable random phenomena.
CO 3	Perform statistical inferences concerning characteristics of a population based on attributes of samples drawn from the population
CO 4	Understand the basic concept in Graph theory, Understand planar graphs and it's properties. Demonstrate the knowledge of fundamental concepts of matrix representation of graphs, Apply fundamental theorems on Eularian graphs and Hamiltonian graphs.
CO 5	Understand the basic concept in Trees,coloring of graphs. Apply coloring of graphs, Apply algorithm to find the minimum spanning tree

Mapping of course outcomes with program outcomes

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2	2	2	2					2		1
CO 2	3	2	2	2	2					2		1
CO 3	3	2	2	2	2					2		1
CO 4	3	2	2	2	2					2		1
CO 5	3	2	2	2	2					2		1

Assessment Pattern

Bloom's Category	Continuous Assessment Tests(%)		End Semester Examination(%)
	1	2	
Remember	10	10	10
Understand	30	30	30
Apply	30	30	30
Analyse	20	20	20
Evaluate	10	10	10
Create			

End Semester Examination Pattern: There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question. Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks.

Course Level Assessment Questions

Course Outcome 1 (CO1):

1. Let X denote the number that shows up when an unfair die is tossed. Faces 1 to 5 of the die are equally likely, while face 6 is twice as likely as any other. Find the probability distribution, mean and variance of X.

2. An equipment consists of 5 components each of which may fail independently with probability 0.15. If the equipment is able to function properly when at least 3 of the components are operational, what is the probability that it functions properly?

3. X is a binomial random variable $B(n,p)$with $n = 100$and $p = 0.1$. How would you approximate it by a Poisson random variable?

4. Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X,Y)

Course Outcome 2 (CO2)

1. What can you say about $P(X = a)$ ↑for any real number $a$when $X$is a (i) discrete random variable? (ii) continuous random variable?

2. A string, 1 meter long, is cut into two pieces at a random point between its ends. What is the probability that the length of one piece is at least twice the length of the other?

3. A random variable has a normal distribution with standard deviation 10. If the probability that it will take on a value less than 82.5 is 0.82, what is the probability that it will take on a value more than 58.3?

4. X and Y are independent random variables with X following an exponential distribution with parameter μ and Y following and exponential distribution with parameterλ. Find $P(X+Y \leqslant 1)$

Course Outcome 3(CO3):

1. In a random sample of 500 people selected from the population of a city 60 were found to be left-handed. Find a 95% confidence interval for the proportion of left-handed people in the city population.

2. What are the types of errors involved in statistical hypothesis testing? Explain the level of risks associated with each type of error.

3. A soft drink maker claims that a majority of adults prefer its leading beverage over that of its main competitor's. To test this claim 500 randomly selected people were given the two beverages in random order to taste. Among them, 270 preferred the soft drink maker's brand, 211 preferred the competitor's brand, and 19 could not make up their minds. Determine whether there is sufficient evidence, at the 5% level of significance, to support the soft drink maker's claim against the default that the population is evenly split in its preference.

4. A nutritionist is interested in whether two proposed diets, *diet A* and *diet B* work equally well in providing weight-loss for customers. In order to assess a difference between the two diets, she puts 50 customers on diet A and 60 other customers on diet B for two weeks. Those on the former had weight losses with an average of 11 pounds and a standard deviation of 3 pounds, while those on the latter lost an average of 8 pounds with a standard deviation of 2 pounds. Do the diets differ in terms of their weight loss?

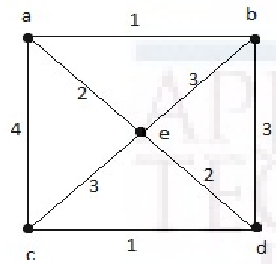
Course Outcome 4(CO4):

1. How many edges are there in a graph with ten vertices each of degree six?
2. Prove that a simple graph with n vertices must be connected, if it has more than $\frac{(n-1)(n-2)}{2}$ edges
3. Prove that a connected graph G is an Euler graph if all vertices of G are of even degree.
4. Use Kuratowski'stheorem to determine whether $K_{4,4}$ is planar.

Course Outcome 5 (CO5):

1. Prove that a tree with n vertices has $n-1$ edges.
2. Find the chromatic number of $K_{m,n}$

3. Using graph model, how can the final exam at a university be scheduled so that no student has two exams at the same time?
4. Explain Prim's algorithm and use it to find the minimum spanning tree for the graph given below



Syllabus

Module 1 (Discrete probability distributions) **9 hours**

(Text-1: *Relevant topics* from sections-3.1-3.4, 3.6, 5.1)

Discrete random variables and their probability distributions, Expectation, mean and variance, Binomial distribution, Poisson distribution, Poisson approximation to the binomial distribution, Discrete bivariate distributions, marginal distributions, Independent random variables, Expectation -multiple random variables.

Module 2 (Continuous probability distributions) **9 hours**

(Text-1: *Relevant topics* from sections-4.1-4.4, 3.6, 5.1)

Continuous random variables and their probability distributions, Expectation, mean and variance, Uniform, exponential and normal distributions, Continuous bivariate distributions, marginal distributions, Independent random variables, Expectation-multiple random variables, i.i.d random variables and Central limit theorem (**without proof**).

Module 3 (Statistical inference) **9 hours**

(Text-1:*Relevant topics from sections*-5.4, 3.6, 5.1, 7.2, 8.1, 8.3, 9.1-9.2,9.4)

Population and samples, Sampling distribution of the mean and proportion (for large samples only), Confidence interval for single mean and single proportions (for large samples only). Test of hypotheses: Large sample test for single mean and single proportion, equality of means and equality of proportions of two populations, small sample t-tests for single mean of normal population, equality of means (**only pooled t-test, for independent samples from two normal populations with equal variance**)

Module 4 (Advanced Graph theory -I) **9 hours**

(Text-2: *Relevant topics of sections* -10.1, 10.2, 10.3, 10.4, 10.5, 10.7)

Introduction- Basic definitions, Directed graphs, pseudo graph, multigraph, Graph models, Graph terminology-vertex degree, simple graph, Complete graphs, cycles, bipartite graph, new graphs from old-union, complement, Representing graph-Adjacency matrix, Incidence Matrix , Isomorphism, Connectivity, path , cut vertices , cut edges ,connectedness in directed and undirected graphs, Counting paths between vertices-Euler paths and circuits , Fleury's algorithm**(proof of algorithm omitted)** , Hamiltonian paths and circuits. Ore's theorem, Planar graph, -Euler's formula on planar graphs, Kuratowski's theorem **(Proof of theorem omitted)**

Module 5 (Advanced Graph theory -II) (9 hours)

(Text-2: *Relevant topics* of *sections* –(10.8,11.1, 11.4, 11.5)

Graph colouring, dual graph, chromatic number, chromatic number of completegraphK_n, chromatic number of complete bipartite graph $K_{m,n}$, chromatic number of cycle C_n , Four color theorem, applications of graph colouring-scheduling and assignments

Trees-rooted trees, Properties of trees-level, height, balanced rooted tree, Spanning tree- basic theorems on spanning tree (**DFS, BFS algorithms and it's applicationsomitted**), Minimum spanning tree, Prim's algorithm and Kruskal's algorithm(**proofs of algorithms omitted)**

(9 hours)

Text Books

1. (Text-1) Jay L. Devore, *Probability and Statistics for Engineering and the Sciences*, 8th edition, Cengage, 2012
2. (Text-2) Kenneth H Rosen, *Discrete Mathematics and it's applications*, Tata Mc Graw Hill, 8 th Edition,

Reference Books

1. Hossein Pishro-Nik, *Introduction to Probability, Statistics and Random Processes*, Kappa Research, 2014 (Also available online at www.probabilitycourse.com)
2. Sheldon M. Ross, *Introduction to probability and statistics for engineers and scientists*, 4th edition, Elsevier, 2009.
3. T.Veera Rajan, *Probability, Statistics and Random processes,* Tata McGraw-Hill, 2008
4. Ralph P Grimaldi, *Discrete and Combinatorial Mathematics, An applied Introduction*, 4th edition, Pearson
5. C L Liu, *Elements of Discrete Mathematics*, Tata McGraw Hill, 4th edition,2017
6. NarasinghDeo, *Graph theory*, PHI, 1979
7. John Clark , Derek Allan Holton, *A first look at Graph Theory.*

Assignments

Assignments should include specific problems highlighting the applications of the methods introduced in this course in physical sciences and engineering.

Course Contents and Lecture Schedule

No	Topic	No. of Lectures
1	**Discrete Probability distributions**	**9 hours**
1.1	Discrete random variables and probability distributions, expected value, mean and variance (discrete)	3
1.2	Binomial distribution-mean, variance, Poisson distribution-mean, variance, Poisson approximation to binomial	3
1.3	Discrete bivariate distributions, marginal distributions, Independence of random variables (discrete), Expected values	3
2	**Continuous Probability distributions**	**9 hours**
2.1	Continuous random variables and probability distributions, expected value, mean and variance (continuous)	2
2.2	Uniform, exponential and normal distributions, mean and variance of these distributions	4
2.3	Continuous bivariate distributions, marginal distributions, Independent random variables, Expected values, Central limit theorem.	3
3	**Statistical inference**	**9 hours**
3.1	Population and samples, Sampling distribution of single mean and single proportion(large samples)	1
3.2	Confidence interval for single mean and single proportions (large samples)	2
3.3	Hypothesis testing basics, large sample test for single mean, single proportion	2
3.4	Large sample test for equality of means and equality of proportions of two populations	2
3.5	t-distribution and small sample t-test for single mean and pooled t-test for equality of means	2
4	**Advanced Graph Theory -I**	**9 hours**
4.1	**Introduction**- Basic definition – Application of graphs Incidence	1

	and Degree – Isolated vertex, pendent vertex and Null graph	
4.2	Theorems connecting vertex degree and edges, bipartite graphs.	1
4.3	Adjacency matrix, incidence matrix, Isomorphism	1
4.4	Path, cut set, cut edges, Connectedness of directed and undirected graphs ,path isomorphism	2
4.5	Euler paths and circuits , Fleury's algorithm**(proof of algorithm omitted)** , Hamiltonian paths and circuits. Ore's theorem(proof omitted)	3
4.6	Planar graph, - Euler's theorem on planar graph , applications of Kuratowski's theorem	1
5	**Advanced Graph Theory -II**	**9 hours**
5.1	Graph colouring, dual graph	1
5.2	Chromatic number, chromatic number of K_n, $K_{m,n}$, C_n ,	2
5.3	Four colour theorem, applications of graph colouring-scheduling and assignments,	2
5.4	Trees-spanning trees-definition and example, minimum spanning tree,	2
5.5	Prim's algorithm and Kruskal's algorithm(**proofs of algorithms omitted)**	2

MODEL QUESTION PAPER (2019 Scheme)

Reg. No: .... Total Pages: 4

Name : ....

APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY

FOURTH SEMESTER B.TECH DEGREE EXAMINATION (Month & year)

Course Code: MAT208

Course Name: PROBABILITY, STATISTICS AND ADVANCED GRAPH THEORY

(For Information Technology)

Max Marks:100Duration : 3 Hours

PART A (Answer all questions. Each question carries 3 marks)

1. Suppose X is a Poisson random variable find $P(X = 1) = P(X = 2)$ .Find the mean and variance. (3)
2. The diameter of a circular metallic discs produced by a machine is a random variable with mean 6cm and variance 2cm. Find the mean area of the discs. (3)
3. If the cumulative distribution of a continuous random variable is given by

$$F(x) = \begin{cases} 0 & x \leq 1 \\ 0.5 & 1 < x < 3, \\ 1 & x \geq 3 \end{cases}$$

find$P(X \leq 2)$(3)

4. The random variable X is exponentially distributed with mean 3. Find $P(X > t + 3|X > t)$ where t is any positive real number. (3)
5. The 95% confidence interval for the mean mass (in grams) of tablets produced by a machine is [0.56 0.57], as calculated from a random sample of 50 tablets. What do you understand from this statement? (3)
6. The mean volume of liquid in bottles of lemonade should be at least 2 litres. A sample of bottles is taken in order to test whether the mean volume has fallen below 2 litres. Give a null and alternate hypothesis for this test and specify whether the test would be one-tailed or two-tailed. (3)
7. Draw the graph represented by the following adjacency matrix

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 0 & 0 \\ 0 & 2 & 2 \end{bmatrix}$$ (3)

8. Give an example of a graph which has a circuit that is (i) Eularian but not Hamiltonian(ii)Hamiltonian but not Eularian (iii) neither Eularian nor Hamiltonian (3)
9. Find the value of $\chi_2(K_3)$ (3)

10. How many non isomorphic spanning tree does K_3have ?. Justify your answer (3)

PART B (Answer one question from each module)

MODULE 1

11. (a) Verify that $p(x) = \left(\frac{8}{7}\right)\left(\frac{1}{2}\right)^x, x = 1, 2, 3$is a probability distribution. Find (i) $P(X \leq 2)$ (ii) $E[X]$ and (iii) $var(X)$. (7)

(b) Find the mean and variance of a binomial random variable (7)

OR

12. (a) Accidents occur at an intersection at a Poisson rate of 2 per day. What is the probability that there would be no accidents on a given day? What is the probability that in January there are at least 3 days (not necessarily consecutive) without any accidents? (7)

(b) Two fair dice are rolled. Let X denote the number on the first die and Y = 0 or 1, according as the first die shows an even number or odd number. Find (i) the joint probability distribution of X and Y, (ii) the marginal distributions. (iii) Are X and Y independent? (7)

MODULE 2

13. (a) The IQ of an individual randomly selected from a population is a normal distribution with mean 100 and standard deviation 15. Find the probability that an individual has IQ (i) above 140 (ii) between 120 and 130. (7)

(b) A continuous random variable X is uniformly distributed with mean 1 and variance 4/3. Find P(X < 0) (7)

OR

14. (a) Determine the value of c so that $f(x,y) = cxy$ for $0 < x < 3$, $0 < y < 3$ and $f(x,y) = 0$ otherwise satisfies the properties of a joint density function of random variables X and Y. Also find $P(X + Y \leq 1)$. Are X and Y independent? Justify your answer (7)

(b) The lifetime of a certain type of electric bulb may be considered as an exponential random variable with mean 50 hours. Using central limit theorem, find the approximate probability that 100 of these electric bulbs will provide a total of more than 6000 hours of burning time. (7)

MODULE 3

15. (a) The mean blood pressure of 100 randomly selected persons from a target population is 127.3units. Find a 95% confidence interval for the mean blood pressure of the population. (7)

(b) The CEO of a large electric utility claims that 80 percent of his 1,000,000 customers are very satisfied with the service they receive. To test this claim, the local newspaper surveyed 100 customers, using simple random sampling. Among the sampled customers, 73 percent say they are very satisfied. Based on these findings, do you think that the CEO is making a false claim of high satisfaction levels among his customers? Use a 0.05 level of significance. (7)

OR

16. (a) A magazine reported the results of a telephone poll of 800 adult citizens of a country. The question posed was: "Should the tax on cigarettes be raised to pay for health care reform?" The results of the survey were: Out of the 800 persons surveyed, 605 were non-smokers out of which 351 answered "yes" and the rest "no". Out of the remaining 195, who were smokers, 41 answered "yes" and the remaining "no". Is there sufficient evidence, at the 0.05 significance level, to conclude that the two populations smokers and non-smokers differ significantly with respect to their opinions? (7)

(b) Two types of cars are compared for acceleration rate. 40 test runs are recorded for each car and the results for the mean elapsed time recorded below:

	Sample mean	Sample Standard Deviation
Car A	7.4	1.5
Car B	7.1	1.8

Determine if there is a difference in the mean elapsed times of the two car models at 95% confidence level. (7)

MODULE 4

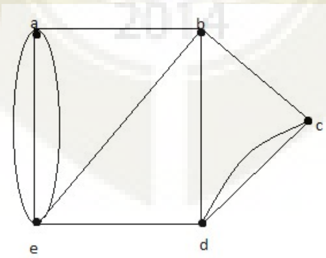
17. (a) Prove that an undirected graph has an even number of odd degree vertices (7)

(b)Show that a bipartite graph with an odd number of vertices does not have a Hamilton circuit (7)

OR

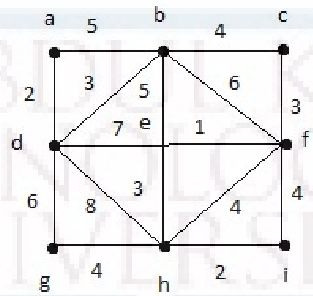
18. (a) Show that an edge in a simple graph is a cut edge if and only if this edge is not part of any simple circuit in the graph. (7)

(b) Use Fleury's algorithm to find an Euler circuit in the following graph (7)



MODULE 5

19. (a) Prove that a simple graph is a tree if and only if it is connected, but the deletion of any of it's edges produces a graph that is not connected (7)(b) Find the minimal spanning tree for the following graph by Prim's algorithm (7)



OR

20. (a)Show that a connected bipartite graph has a chromatic number of 2. (7)
(b) Prove that a full m-ary tree with l leaves has $n = \frac{ml-1}{m-1}$ vertices and $i = \frac{l-1}{m-1}$ internal vertices (7)
