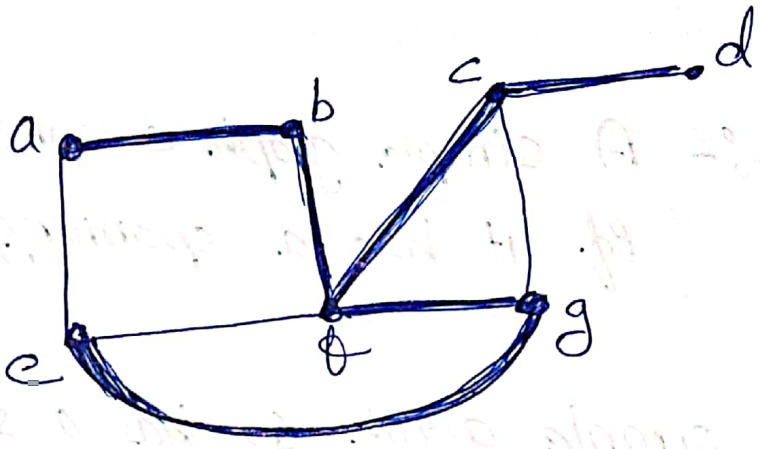
Spanning Tree :- Let G be a simple graph.

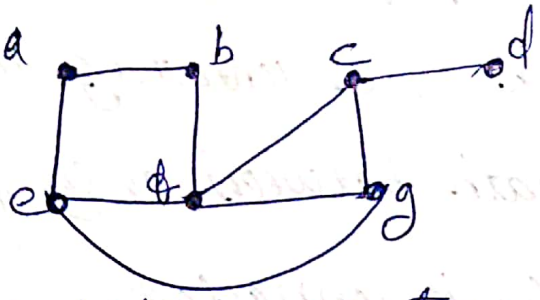
A spanning tree of G is a subgraph of G is a tree containing every vertex of G

Example :

1)

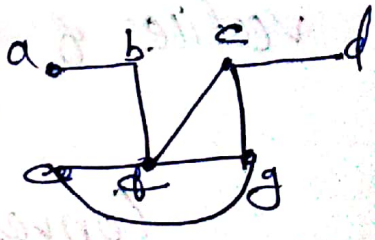


2)

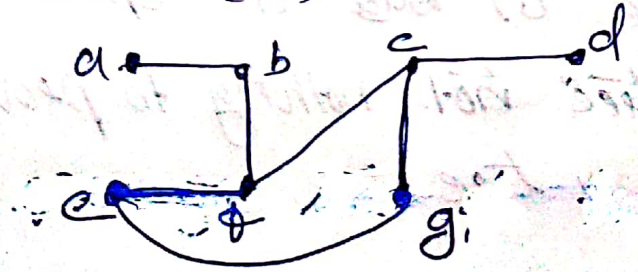


Steps to find spanning tree

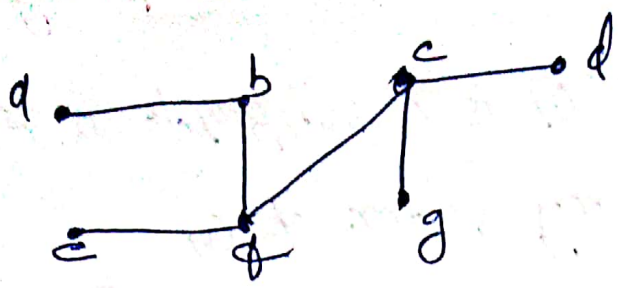
i) Remove edge (a,e) to remove a cycle without making graph disconnected



ii) Remove edge (f,e)



(iii) Remove edge $\{e, f\}$



is another spanning tree of the given graph

Theorem :- A simple graph is connected iff and only iff it has a spanning tree

Suppose simple graph G has a spanning tree T.
ie it has a tree with all vertices of G.
$\exists$ a path between every pair of vertices in T
since T is a subgraph of G containing all the vertices of G, $\exists$ a path between every pair of vertices of G. Hence G is connected.

Conversely suppose that G is connected. i.e., G is not To prove G has a spanning tree
If G itself is a tree nothing to prove
If G is not a tree

it has a circuit

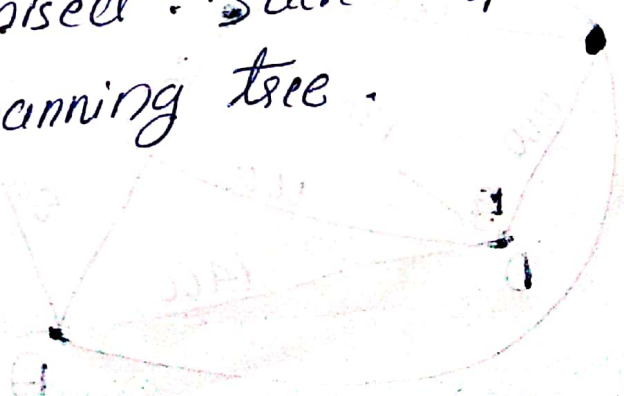
Remove any edge from the circuit

The new graph is still connected since no edge of a circuit makes a graph disconnected

If the new graph is a tree nothing to prove if not it has a cycle. Remove an edge from the cycle to obtain a new graph. If it is a tree nothing to prove otherwise repeat the process until we find a spanning tree.

Minimum Spanning tree

In a connected weighted graph, ie a graph with a particular weight for each edge - we can find spanning tree so that sum of the weights of the tree is minimised. Such a spanning tree is called minimal spanning tree.



Algorithms

1) Prim's Algorithm :

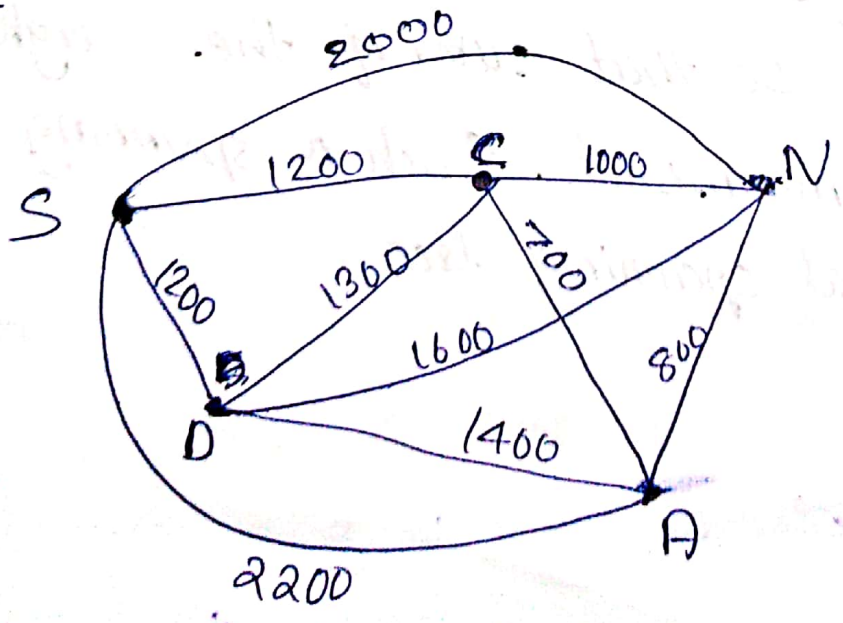
Steps : 1) First choose any edge with the smallest weight

2) Now consides ~~next~~ vertices already in the tree and find an edge incident with these vertices of minimum weight, add that edge to spanning tree

3) Continue this without forming a cycle

4) Stop when $n-1$ edges are added to the spanning tree

Example

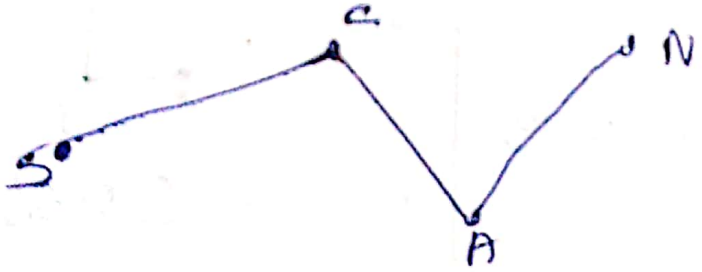


Total 5 vertices

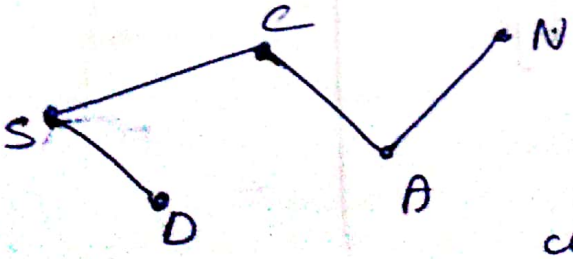
1) minimum weight 700, choose edge {C, A}

2) A is incident with edge of minimum weight 800 choose {A, N}

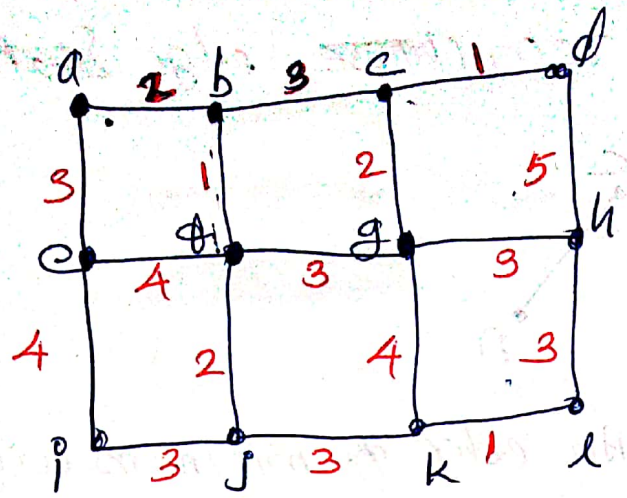
3) N is incident with edge of weight 1000 but choosing that will make a circuit so choose {S, C} of weight 1200 (next minimum)



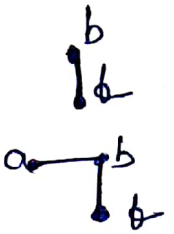
4) S is incident with edge {S, D} of minimum weight 900 Above

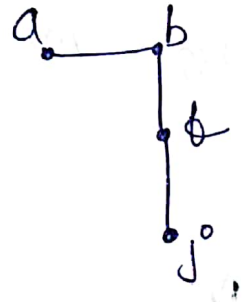


minimum spanning tree with 5 - 1 = 4 edges

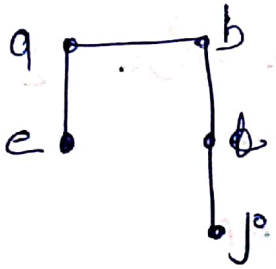


Choice		edge		weight
1	⟶	{b, f}	⟶	1
2	⟶	{a, b}	⟶	2
3	⟶	{f, j}	⟶	2

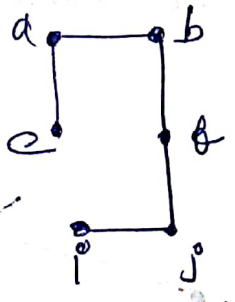




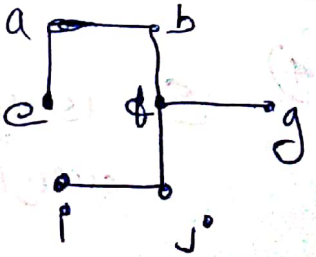
4 ⟶ {a, e} ⟶ 3



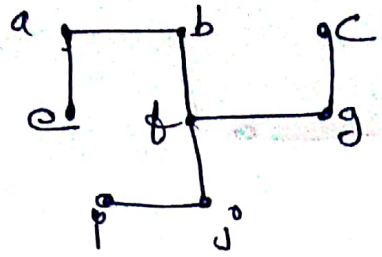
5 ⟶ {i, j} ⟶ 3



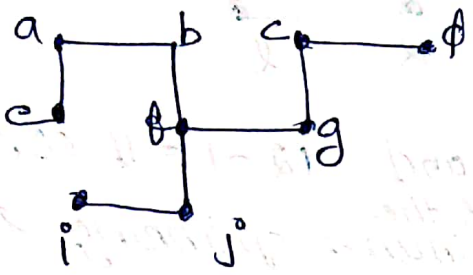
6 ⟶ {f, g} ⟶ 3



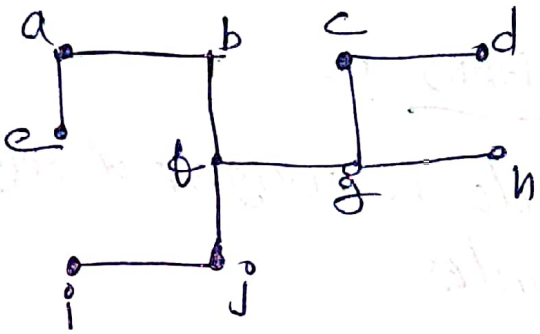
7 ⟶ {c, f, g} ⟶ 2



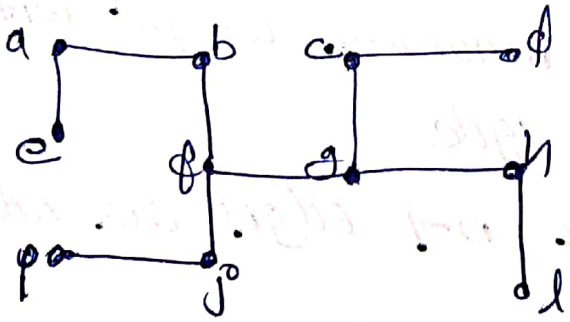
8 ⟶ {c, d} ⟶ 1



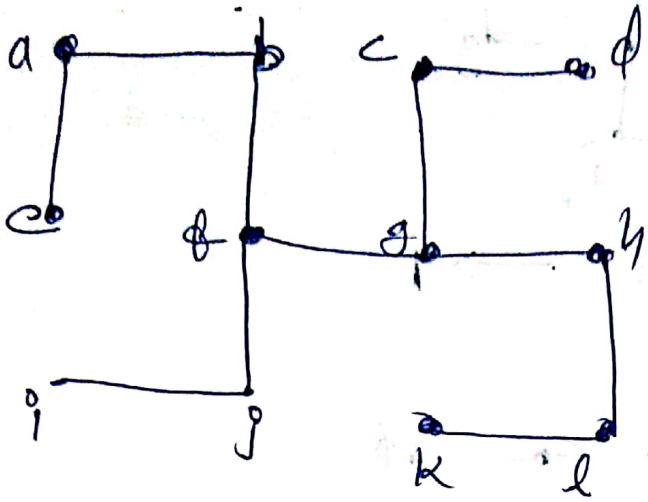
9 ⟶ {g, h} ⟶ 3



10 ⟶ {h, l} ⟶ 3



11 ⟶ {k, l} ⟶ 1



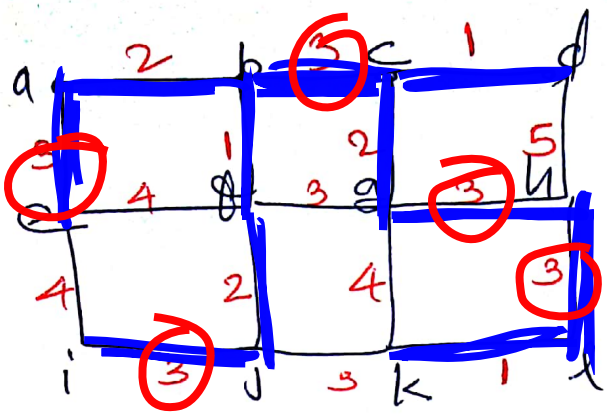
Total 12 vertices and 12 − 1 = 11 edges added
we obtained one of the minimum spanning trees
Total weight

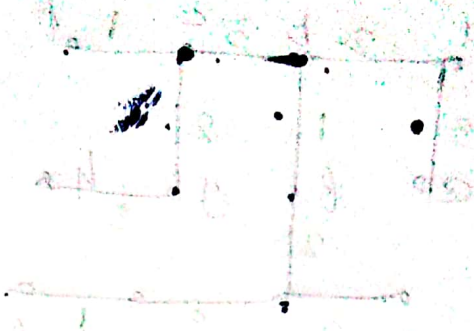
$$1+2+2+3+3+3+2+1+3+3+1 = 24$$

2) Kruskal's Algorithm :-

1) Arrange write down weights of edges in increasing order
2) select an edge with minimum weight
3) Add another edge to the graph with the next minimum weight without forming a cycle
4) stop when n−1 edges are added

Example

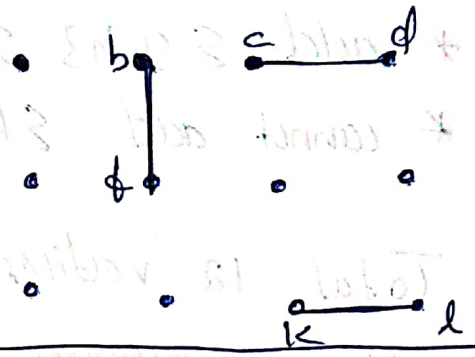




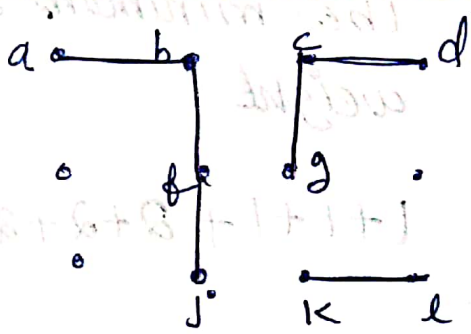
edges and weights

$\{c, d\} \longrightarrow 1$
$\{k, l\} \longrightarrow 1$
$\{b, f\} \longrightarrow 1$
$\{a, b\} \longrightarrow 2$
$\{c, g\} \longrightarrow 2$
$\{f, j\} \longrightarrow 2$
$\{b, c\} \longrightarrow 3$
$\{a, e\} \longrightarrow 3$
$\{f, g\} \longrightarrow 3$
$\{g, h\} \longrightarrow 3$
$\{i, j\} \longrightarrow 3$
$\{j, k\} \longrightarrow 3$
$\{h, l\} \longrightarrow 3$
$\{e, f\} \longrightarrow 4$
$\{e, i\} \longrightarrow 4$
$\{g, k\} \longrightarrow 4$
$\{d, h\} \longrightarrow 5$

1) choose $\{c, d\}$, $\{k, l\}$ $\{b, f\}$ of weights 1

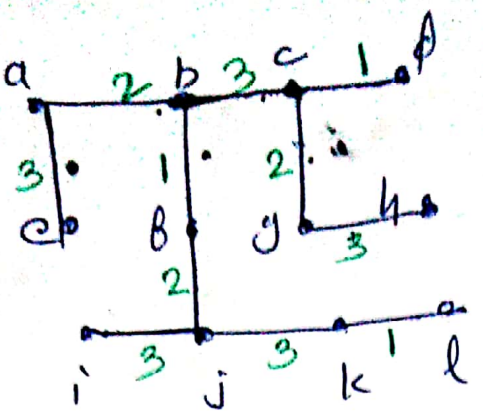


2)



add $\{a, b\}$, $\{c, g\}$, $\{f, j\}$ of weights 2

3)



* add $\{b, c\}$, $\{a, e\}$ of length weight 3
* cannot add $\{b, g\}$ it will form circuit
* add $\{g, h\}$, $\{i, j\}$ $\{j, k\}$ of ~~len~~ weight 3
* cannot add $\{h, l\}$ it will form circuit

Total 12 vertices and 12 - 1 edges covered
The minimum spanning tree obtained of weight

$1+1+1+2+2+2+3+3+3+3+3 = 24$