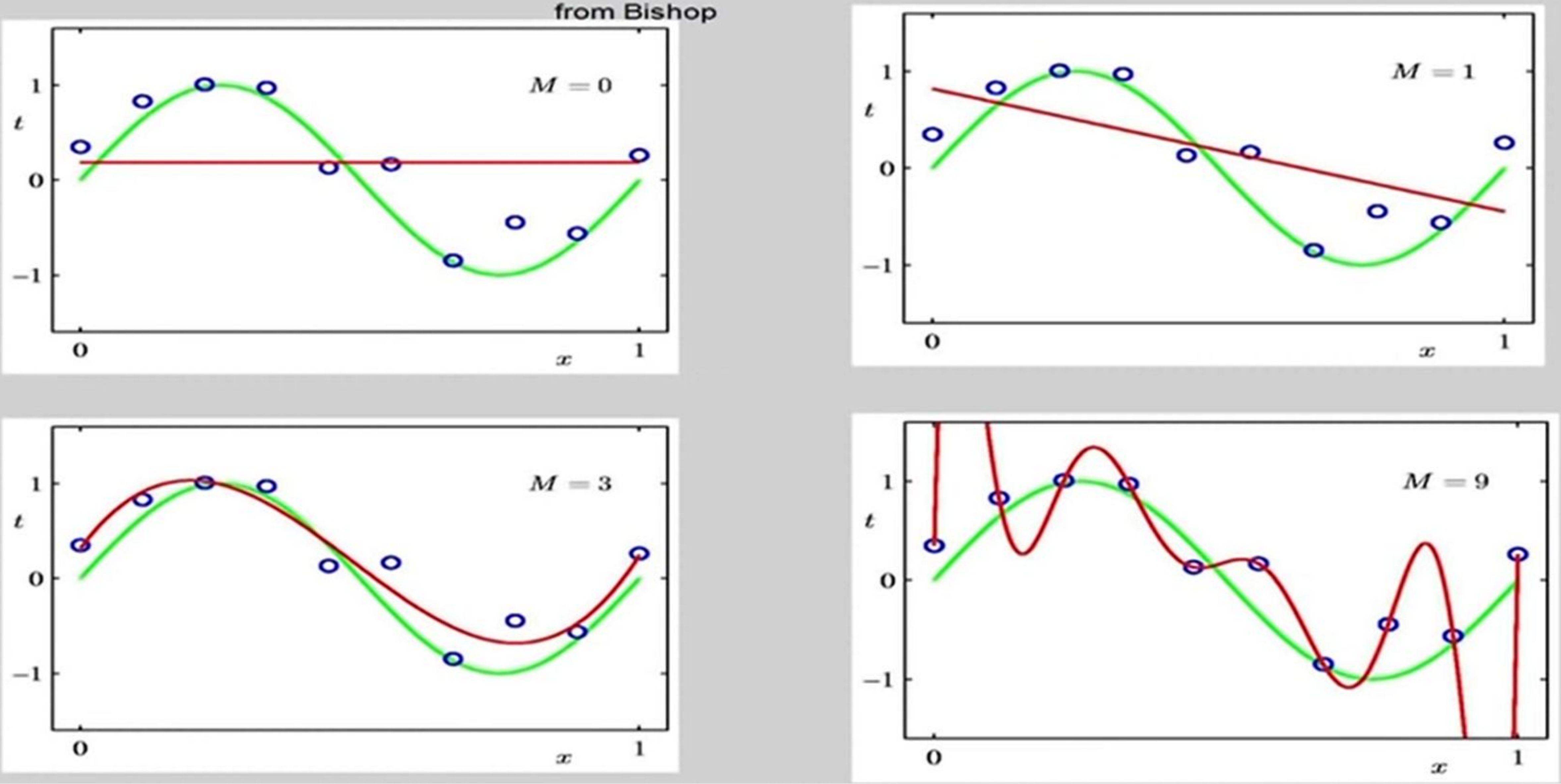
Module 2: Statistical Machine Learning

Introduction, **Regression**, and Classification, Decision Trees, Random Forests

Reference: James, G., Witten, D., Hastie, T., Tibshirani, R. (2017). An Introduction to Statistical Learning: with Applications in R., Springer.)

Some fits to the data: which is best?

from Bishop



Extensions of the Linear Model

Assumptions - between the predictors and response relationship are:

1. Additive
2. Linear

Additive assumption means that the effect of changes in a predictor Xj on the response Y is independent of the values of the other predictors

Removing the Additive Assumption

Synergy effect or interaction effect

Synergy Effect or Interaction Effect is a phenomenon that arises in the multiple linear regression, when increase in the value of one Independent variable increases the impact of another Independent variable on the dependent variable

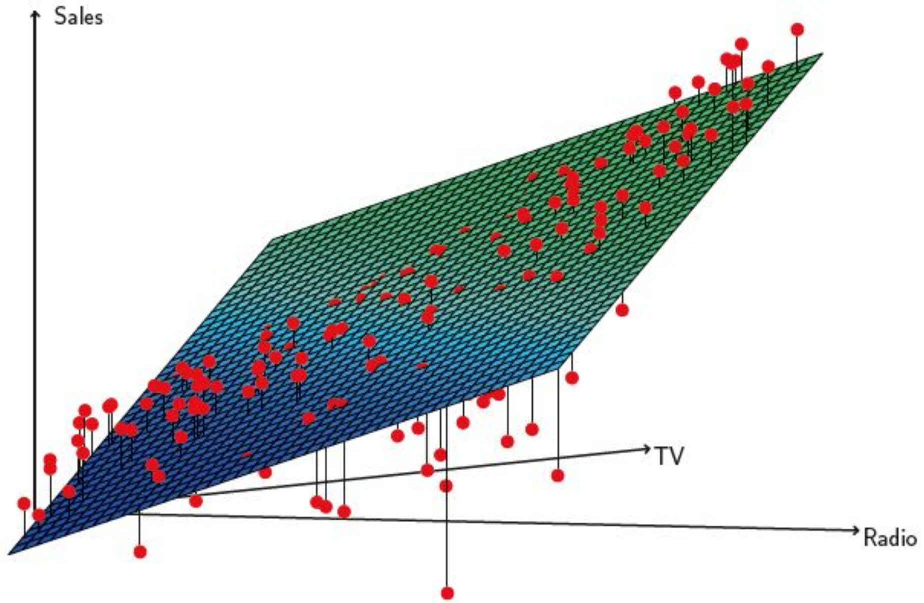


FIGURE 3.5. *For the* `Advertising` *data, a linear regression fit to* `sales` *using* `TV` *and* `radio` *as predictors. From the pattern of the residuals, we can see that there is a pronounced non-linear relationship in the data. The positive residuals (those visible above the surface), tend to lie along the 45-degree line, where TV and Radio budgets are split evenly. The negative residuals (most not visible), tend to lie away from this line, where budgets are more lopsided.*

Consider the standard linear regression model with two variables,

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon.$$

According to this model, if we increase X_1 by one unit, then Y will increase by an average of β_1 units. Notice that the presence of X_2 does not alter this statement—that is, regardless of the value of X_2, a one-unit increase in X_1 will lead to a β_1-unit increase in Y. One way of extending this model to allow for interaction effects is to include a third predictor, called an *interaction term*, which is constructed by computing the product of X_1 and X_2. This results in the model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_1 X_2 + \epsilon. \tag{3.31}$$

How does inclusion of this interaction term relax the additive assumption? Notice that (3.31) can be rewritten as

$$\begin{aligned} Y &= \beta_0 + (\beta_1 + \beta_3 X_2)X_1 + \beta_2 X_2 + \epsilon \\ &= \beta_0 + \tilde{\beta}_1 X_1 + \beta_2 X_2 + \epsilon \end{aligned} \tag{3.32}$$

where $\tilde{\beta}_1 = \beta_1 + \beta_3 X_2$. Since $\tilde{\beta}_1$ changes with X_2, the effect of X_1 on Y is no longer constant: adjusting X_2 will change the impact of X_1 on Y.

The hierarchical principle states that if we include an interaction in a model, we should also include the main effects, even if the p-values associated with their coefficients are not significant.

Non-linear Relationships

Polynomial Regression

$$\texttt{mpg} = \beta_0 + \beta_1 \times \texttt{horsepower} + \beta_2 \times \texttt{horsepower}^2 + \epsilon$$

But it is still a linear model! That is, this equation is simply a multiple linear regression model with

X1 = horsepower and **X2 = horsepower².**

So we can use standard linear regression software to estimate β0, β1, and β2 in order to produce a non-linear fit.

If including **horsepower²** led to such a big improvement in the model, why not include **horsepower³**, **horsepower⁴**, or even **horsepower⁵**?

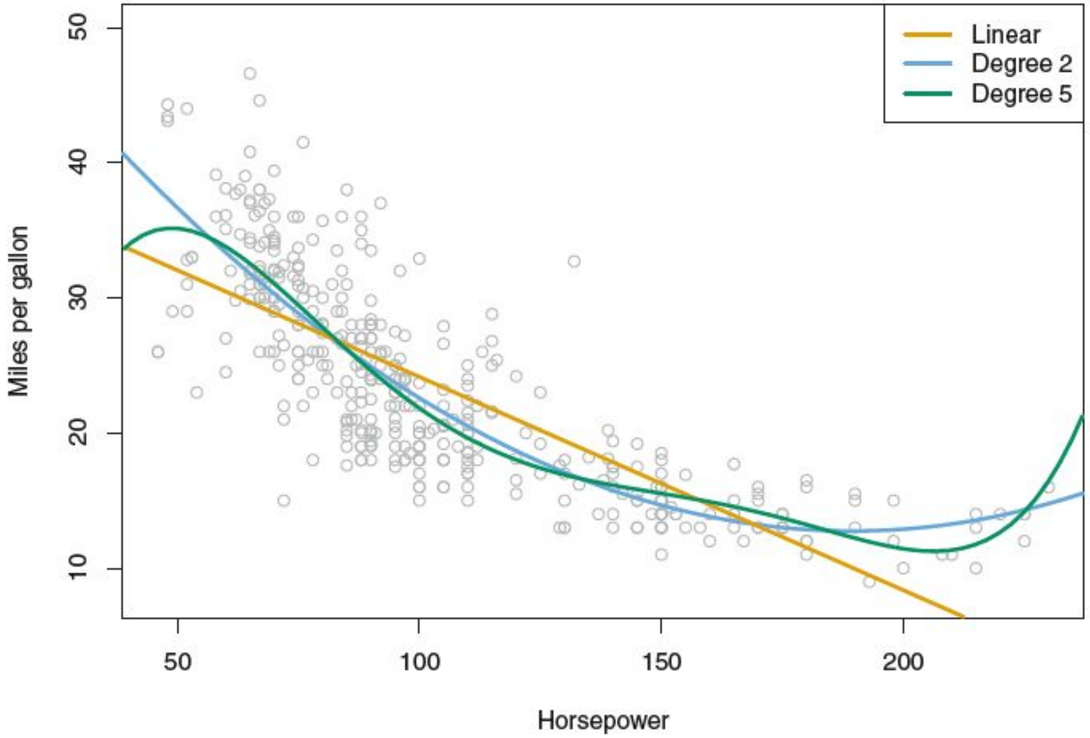


FIGURE 3.8. *The* `Auto` *data set. For a number of cars,* `mpg` *and* `horsepower` *are shown. The linear regression fit is shown in orange. The linear regression fit for a model that includes* $\texttt{horsepower}^2$ *is shown as a blue curve. The linear regression fit for a model that includes all polynomials of* `horsepower` *up to fifth-degree is shown in green.*