Module: 2 Topic: Boolean Algebra

Introduction

Boolean Algebra

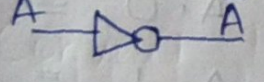
Boolean algebra is an algebra that may be defined with a set of elements, a set of operators and a number of unproved axioms or postulates.

→ A set of elements is any collection of objects having common property.

→ The set of operators AND (Boolean product) operation (·), the OR (Boolean Sum) operation (+) and the NOT (complement operation) (') are defined in the set.

Logic Gates

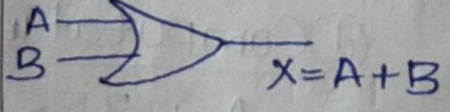
NOT



Truth table

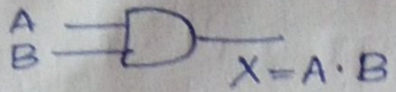
A	$\bar{A}$
0	1
1	0

OR



A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

AND



A	B	X
0	0	0
0	1	0
1	0	0
1	1	1

A truth table can be used to describe the operation of a gate circuit.
2nd way of indicating gate operation is with BOOLEAN EQUATION.

A Boolean Equation contain 3 elements input variable, output variable and boolean operators.

Laws of Boolean Algebra

ⓐ OR's LAWS

$A + 0 = A$

$A + 1 = 1$

$A + A = A$

$A + \bar{A} = 1$

ⓑ AND's LAWS

$A \cdot 0 = 0$

$A \cdot 1 = A$

$A \cdot A = A$

$A \cdot \bar{A} = 0$

ⓒ Complementation LAW

$\bar{0} = 1$

$\bar{1} = 0$

ⓓ Computating LAW

$A + B = B + A$

$AB = BA$

(e) Associative LAW

$(A+B)+C = A+(B+C)$

$(A\cdot B)\cdot C = A\cdot(B\cdot C)$

(f) Distributive LAW

$A\cdot(B+C) = AB + AC$

$A+(BC) = (A+B)\cdot(A+C)$

(g) Double Neogation

$\overline{\overline{A}} = A \quad \overline{\overline{0}} = 0 \quad \overline{\overline{1}} = 1$

(h) Identity LAW

$A\cdot 1 = A$

$A+1 = 1$

(i) NULL LAW

$A\cdot 0 = 0$

$A + 0 = A$

(j) Absorption LAW

(1) $A + AB = A$

proof $A(1+B) = \underline{\underline{A}}$

(2) $A(A+B) = A$

proof $AA + AB$

$= A + AB$

$= A(1+B)$

$= \underline{\underline{A}}$

Using the truth table show that

$A + AB = A$

A	B	AB	A+ AB
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1
(1)	(2)	(3)	(4)

Here column (1) and (4) are equal.

Hence proved.

Postulates of Boolean algebra

The postulates are the basic assumptions from which it is possible to deduce the rules, theorems and properties of the system.

1. Closure

A set S is closed w.r.t a binary operator if for every pair of elements of S, the binary operator specifies a rule for obtaining a unique element of S. Set of natural numbers $N = \{1, 2, 3, 4, \ldots\}$ is closed under binary operator (+) since any $a, b \in N$ we obtain a unique $c \in N$ by the operation $a+b=c$. Set of natural numbers is not closed w.r.t binary operator minus (−) because $2-3=-1$ where $-1 \notin N$.

2. Associative Law

A binary operator $*$ on a set S is said to associative whenever

$$(x*y)*z = x*(y*z) \text{ for all } x, y, z \in S.$$

3. Commutative Law

A binary operator $*$ on a set S is said to be commutative whenever

$$x*y = y*x \text{ for all } x, y \in S.$$

(4) Identity Element

A set S is said to have an identity element respect to a binary operation $*$ on S if there exists an element $e \in S$ with the property

$$e * x = x * e = x \quad \text{for every } x \in S$$

(5) Inverse

A set S having the identity element e with respect to a binary operator $*$ is said to have an inverse whenever, for every $x \in S$, there exists an element $y \in S$ such that

$$x * y = e$$

eg: $a + (-a) = 0$ { In the set of integers I with $e=0$, the inverse of an element a is $(-a)$ }

(6) Distributive Law

If $*$ and $\cdot$ are two binary operators on a set S $*$ is said to be distributive over $\cdot$, whenever:

$$x * (y \cdot z) = (x * y) \cdot (x * z)$$

POSTULATES OF BOOLEAN ALGEBRA

Boolean algebra is an algebric structure defined on a set of elements B together with two binary operators + and ·

1 a) Closure with respect to the operator +
b) Closure with respect to the operator ·

2 a) An identity element with respect to +, designated by 0:

$$x + 0 = 0 + x = x$$

b) An identity element w.r.t · designated by 1:

$$x \cdot 1 = 1 \cdot x = x$$

3 a) Commutative w.r.t +:

$$x + y = y + x$$

b) Commutative w.r.t ·:

$$x \cdot y = y \cdot x$$

4 a) · is distributive over +:

$$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$$

b) + is distributive over ·:

$$x + (y \cdot z) = (x + y) \cdot (x + z)$$

5. For every element $x \in B$, there exists an element $x' \in B$ (called the complement of x) such that:

a) $x + x' = 1$

b) $x \cdot x' = 0$

6. There exists at least two elements $x, y \in B$, such that $x \neq y$.

Basic Theorems and Properties of Boolean Algebra

1. Duality

The dual of a logical expression is obtained by simply interchanging OR and AND operators. Replace 1 by 0 and 0 by 1.

	Dual
eg: 1) $x + x = 1$	$x \cdot x = 0$
2) $x + 1 = 1$	$x \cdot x = 0$
3) $x + x = x$	$x \cdot x = x$
4) $x + xy = x$	$x \cdot (x + y) = x$

Find the complement by taking dual?

Rules 1. Find dual

2. Take complement of each literal.

Ques) $F_1 = \bar{x} y \bar{z} + \bar{x} \bar{y} z$

Step 1: $F_1 = (\bar{x} + y + \bar{z}) \cdot x + \bar{y} + z$

Step 2: $F_1' = (x + \bar{y} + z) \cdot (x + y + \bar{z})$

2. **Demorgan's Theorem**

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

$$\overline{AB} = \bar{A} + \bar{B}$$

I theorem

Any two element A and B, the complement of the sum is equal to the product of their complement.

II theorem

Any two element A and B, the complement of product is equal to the sum of their complement.

De-Morganize the following Expression

① $\overline{A + B + C + D}$

$= \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D}$

② $\overline{A + \overline{B + \overline{CD}}}$

$= \bar{A} \cdot \overline{\overline{B + \overline{CD}}}$

$= \bar{A} \cdot B + \overline{CD}$

$= \bar{A} \cdot B + \bar{C} + \bar{D}$

POSTULATES AND THEOREMS OF Boolean Algebra

Postulate 2 a) $x+0=x$ b) $x \cdot 1 = x$

Postulate 3 commutative a) $x+y=y+x$ b) $x \cdot y = y \cdot x$

Postulate 4 Distributive a) $x(y+z)=xy+xz$ b) $x+yz = (x+y)(x+z)$

Postulate 5 a) $x+x'=1$ b) $x \cdot x' = 0$

Theorem 1 a) $x+x=x$ b) $x \cdot x = x$

Theorem 2 a) $x+1=1$ b) $x \cdot 0 = 0$

Theorem 3 $(x')' = x$

Theorem 4 Associative a) $x+(y+z)=(x+y)+z$ b) $x \cdot (y \cdot z) = (x \cdot y) \cdot z$

Theorem 5 De morgan a) $(x+y)' = x' \cdot y'$ b) $(xy)' = x'+y'$

Theorem 6 Absorption a) $x+xy=x$ b) $x(x+y)=x$

Note ÷ $x+xy = x$

$x+x'y = x+y$

$x'+xy = x'+y$

Theorems and their proof

Theorem 1(a) : $x + x = x$

$x+x = (x+x)\cdot 1$ Postulate 2(b)

$= (x+x)\cdot(x+x')$ 5(a)

$= \cancel{xx + xx'} + xx + xx'$ 4(a)

$= xx + xx'$ Theorem 1(a)

$= x + 0$ Theorem 1(b)

Postulate 5(b)

$= x$ Postulate 2(a)

Theorem 1(b) : $x \cdot x = x$

$x\cdot x = xx + 0$ postulate 2(a) $x+0=x$

$= xx + xx'$ postulate 5(b) $x\cdot x' = 0$

$= x(x+x')$ Postulate 4(a) $x(y+z) = xy + xz$

$= x\cdot 1$ Postolate 5(a) $x+x' = 1$

$= x$ postulate 2(b) $x\cdot 1 = x$

Theorem 2(a) : $x + 1 = 1$

$x+1 = 1 \cdot (x+1)$ Postulate 2(b) $x \cdot 1 = x$

$= (x+x') \cdot (x+1)$ Postulate 5(a) $x + x' = 1$

$= xx + x + xx' + x'$ Postulate 4(a) $x(y+z) = xy + xz$

$= x + xx' + x'$ Theorem 1 a) $x + x = x$ b) $x \cdot x = x$

$= x + 0 + x'$ Postulate 5(b) $x \cdot x' = 0$

$= 1$ Postulate 5(a) $x + x' = 1$

Theorem 2(b) : $x \cdot 0 = 0$ by duality.

Theorem 6(a) : $x + xy = x$

$x + xy = x \cdot 1 + xy$ Postulate 2(b) $x \cdot 1 = x$

$= x(1+y)$ Postulate 4(a) $x(y+z) = xy + xz$

$= x \cdot 1$ by theorem 2(a) $x + 1 = 1$

$= x$ Postulate 2(b) $x \cdot 1 = x$

Theorem 6(b) : $x(x+y) = x$ by duality.

Operator Precedence

The operator precedence for evaluating Boolean expression is
1) Parentheses
2) NOT
3) AND
4) OR

Boolean Functions

A binary variable can take the value 0 or 1. A boolean function is an expression formed with binary variables, the two binary operators OR and AND, the unary operator NOT, parentheses and equal sign

$$F_1 = xyz'$$

The function F_1 is equal to 1, if $x=1$ and $y=1$ and $z=1$; otherwise $F_1 = 0$.

Literal

A literal is a primed or unprimed variable. When a boolean function is implemented with logic gates, each literal in the function designates an input to a gate and each term is implemented with a gate.

The minimization of the number of literals and the number of terms results in a circuit with less equipment.

Complement of a Function

The complement of a function F is F' and is obtained from an interchange of 0's for 1's and 1's for 0's in the value of F. The complement of a function may be derived algebraically through De Morgan's theorem.

$$(A+B+C)' = (A+X)' \quad \text{let } B+C=X$$
$$= A'X' \quad \text{by DeMorgan's theorem}$$
$$= A'\cdot(B+C)' \quad \text{substitute } B+C=X$$
$$= A'\cdot(B'\cdot C') \quad \text{by De Morgan theorem}$$
$$= \underline{\underline{A'\cdot B'\cdot C'}} \quad \text{by associative theorem.}$$

Find the complement of the function F_1 by taking their duals and complementing each literal.

$F_1 = x'yz' + x'y'z$

The dual of $F_1 = (x'+y+z')(x'+y'+z)$

Complement each literal:

$(x+y'+z)(x+y+z')$

UQ * Q) Find the complement of the given Boolean function using De Morgan's theorem.

$$F(x, y, z) = x(y'+z)$$

$$F'(x, y, z) = [x(y'+z)]'$$

$$= x' + (y'+z)'$$

$$= x' + y'' \cdot z'$$

$$= x' + y \cdot z'$$

$$= \underline{\underline{x' + yz'}}$$

U.Q * Prove the given Boolean identity using laws of Boolean algebra

$$x + x'y = x + y$$

L.H.S $x + x'y = x \cdot 1 + x'y$; $x \cdot 1 = x$

$= x \cdot (1+y) + x'y$; $1 + y = 1$

$= x + xy + x'y$

$= x + y(x + x')$

$= x + y \cdot 1$; $x + x' = 1$

$= x + y$

$=$ R.H.S Hence proved

More Questions

1) Simplify the Boolean function $(x+y)(x+y')$ to a minimum no: of literals. Ans: x.

2) Find Complement of $F = A\bar{B} + B\bar{C} + \bar{A}C$
Ans: $(\bar{A}+B)(\bar{B}+C)(A+\bar{C})$

3) Prove $AB + \bar{A}C = (A+C)(\bar{A}+B)$

4) Using Boolean Postulates simplify the expressions.
(i) $xy + x'z + yz$ Ans: $xy + x'z$
(ii) $x'y'z + x'yz + xy'$ Ans: $x'z + xy'$

5) Simplify
(i) $(x+y)(x+z)$ Ans: $x+yz$
(ii) $xy + xz + yz' = xz + yz'$. Prove.

6) Reduce the expression $A + B[AC + (B+C')D]$
Ans: $A + BD$

7) Reduce the expression $(\overline{A+\overline{BC}})(A\bar{B} + ABC)$
Ans: 0

8) Reduce the expression $(B+BC)(B+\bar{B}C)(B+D)$
Ans: B

9) S.T $AB + A\bar{B}C + B\bar{C} = AC + B\bar{C}$

10) S.T $A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C = B + C$

11) Reduce $x(x'+y)$ Ans: xy

12) Simplify

(i) $A\bar{B}D + A\bar{B}\bar{D}$ Ans: $A\bar{B}$

(ii) $AB + A(B+C) + B(B+C)$ Ans: $B + AC$

13) P.T $\bar{A}\bar{B}C + \bar{A}BC + A\bar{B} = \bar{A}C + A\bar{B}$

14) P.T $ABC + \bar{A}B + AB\bar{C} = B$.

15) P.T $C(A\bar{B} + AB) + BC = C(A+B)$

16) P.T $(A + BC' + C)C' = ABC' + AB'C' + A'BC'$

17) P.T $\bar{A}\bar{B}\bar{C} + \bar{A}BC + ABC + A\bar{B}\bar{C} + A\bar{B}C$

$= \bar{B}\bar{C} + BC + A\bar{B}C$

18) P.T $(B+\bar{C})(\bar{B}+C) + \overline{\bar{A}+B+\bar{C}} = C(A+B) + \bar{B}\bar{C}$

19) P.T $(x+y)[\bar{x}(\bar{y}+\bar{z})] + \bar{x}\bar{y} + \bar{x}\bar{z} = 1$

20) P.T $\overline{(\overline{A\bar{B}C + \bar{A}\bar{B}}) + AC} = \bar{A} \cdot \bar{B}$

21) Use Boolean Algebra to Show that

$\bar{A}B\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} + ABC = A + B\bar{C}$

Q) Simplify the Boolean function $(x+y)(x+y')$ to a minimum no. of literals,

$(x+y)(x+y') = xx + xy' + yx + yy'$

$= x + xy' + xy + 0$ → $\begin{cases} x \cdot x = x \\ y \cdot y' = 0 \\ y \cdot x = x \cdot y \end{cases}$

$= x(1+y') + xy + 0$

$= x \cdot 1 + xy$ → $1 + y' = 1$

$= x(1+y)$ → $1 + y = 1$

$= \underline{\underline{x}}$

U.Q) Find complement function

$F = A\bar{B} + B\bar{C} + \bar{A}C$

$F' = (A\bar{B} + B\bar{C} + \bar{A}C)'$

$= \overline{A\bar{B}} \cdot \overline{B\bar{C}} \cdot \overline{\bar{A}C}$

$= \underline{\underline{(\bar{A}+B)(\bar{B}+C)(A+\bar{C})}}$

U.Q) Prove $AB + \bar{A}C = (A+C)(\bar{A}+B)$

R.H.S $= (A+C)(\bar{A}+B)$

$= A\bar{A} + AB + \bar{A}C + BC$ $\quad A \cdot \bar{A} = 0$

$= 0 + AB + \bar{A}C + BC$ $\quad A + \bar{A} = 1$

$= AB + \bar{A}C + BC(A + \bar{A})$

$= AB + \bar{A}C + ABC + \bar{A}BC$

$= AB + ABC + \bar{A}C + \bar{A}BC$

$= AB(1 + C) + \bar{A}C(1 + B)$

$= \underline{\underline{AB + \bar{A}C}}$ $\quad 1 + C = 1$
$\quad 1 + B = 1$

(Q.Q) Using Boolean Postulates simplify the following expressions:

(i) $xy + x'z + yz$ (ii) $x'y'z + x'yz + xy'$

(i) $xy + x'z + yz = xy(z+z') + x'z(y+y') + yz(x+x')$

$= \underset{(1)}{xyz} + xyz' + \underset{(1)}{x'yz} + x'y'z + \underset{(2)}{xyz} + \underset{(2)}{x'yz}$

$= xyz + xyz' + x'yz + x'y'z$

$= xy(z+z') + x'z(y+y')$ $\quad y + y' = 1$
$\quad z + z' = 1$

$= xy \cdot 1 + x'z \cdot 1$

$= \underline{\underline{xy + x'z}}$

(ii) $x'y'z + x'yz + xy'$

$x'y'z + x'yz + xy'$

$= x'z(y' + y) + xy'$

$= x'z \cdot (1) + xy'$ $\quad \because y + y' = 1$

$= \underline{\underline{x'z + xy'}}$

Texbook
Morris mano

Simplify (i) $(x+y)(x+z) = x+yz$

$$
\begin{aligned}
(x+y)(x+z) &= xx + xz + xy + yz \\
&= x + xz + xy + yz \quad \because x \cdot x = x \\
&= x(1+z) + xy + yz \\
&= x \cdot 1 + xy + yz \quad \because 1+z=1 \\
&= x + xy + yz \\
&= x(1+y) + yz \\
&= \underline{\underline{x + yz}} \quad \because 1+y=1
\end{aligned}
$$

(ii) $xy + xz + yz' = xz + yz'$

$$
\begin{aligned}
xy+xz+yz' &= xy(z+z') + xz(y+y') + yz'(x+x') \\
&= \underset{(1)}{xyz} + \underset{(1)}{xyz'} + \underset{(2)}{xyz} + xy'z + \underset{(2)}{xyz'} + x'yz' \\
&= xyz + xyz' + xy'z + x'yz' \\
&= xyz + xy'z + xyz' + x'yz' \\
&= xz(y+y') + yz'(x+x') \\
&= xz \cdot 1 + yz' \cdot 1 \\
&= \underline{\underline{xz + yz'}}
\end{aligned}
$$

(rearranging due to reach the final answer)

T.B
(anand kumar)

① Reduce the expression $A + B[AC + (B+C')D]$

$$
\begin{aligned}
A + B[AC + (B+C')D] &= A + B[AC + BD + C'D] \\
&= A + ABC + BD + BC'D = A(1+BC) + BD(1+C') \\
&= \underline{\underline{A + BD}}
\end{aligned}
$$

Texbook
Morris mano

Simplify (i) $(x+y)(x+z) = x + yz$

$$(x+y)(x+z) = xx + xz + xy + yz$$
$$= x + xz + xy + yz \quad \because x \cdot x = x$$
$$= x(1+z) + xy + yz$$
$$= x \cdot 1 + xy + yz \quad \because 1+z=1$$
$$= x + xy + yz$$
$$= x(1+y) + yz$$
$$\because 1+y=1$$
$$= \underline{\underline{x + yz}}$$

(ii) $xy + xz + yz' = xz + yz'$

$$xy + xz + yz' = xy(z+z') + xz(y+y') + yz'(x+x')$$
$$= \underset{(1)}{xyz} + \underset{(1)}{xyz'} + \underset{(2)}{xyz} + xy'z + \underset{(2)}{xyz'} + x'yz'$$
$$= xyz + xyz' + xy'z + x'yz'$$
$$= xyz + xy'z + xyz' + x'yz'$$

(rearranging) due to reach the final answer

$$= xz(y+y') + yz'(x+x')$$
$$= xz \cdot 1 + yz' \cdot 1$$
$$= \underline{\underline{xz + yz'}}$$

T.B
(anandkumar)

① Reduce the expression $A + B[AC + (B+C')D]$

$$A + B[AC + (B+C')D] = A + B[AC + BD + C'D]$$
$$= A + ABC + BD + BC'D = A(1+BC) + BD(1+C')$$
$$= \underline{\underline{A + BD}}$$

(2) Reduce the expression $\overline{(A+\overline{BC})}(A\overline{B}+ABC)$

$\overline{(A+\overline{BC})}\ (A\overline{B}+ABC)$

$=(\overline{A}\cdot\overline{\overline{BC}})(A\overline{B}+ABC) = (\overline{A}\cdot BC)(A\overline{B}+ABC)$

$= \overline{A}BCA\overline{B} + \overline{A}BCABC$ $\quad B\cdot\overline{B}=0$

$= 0+0 = 0$ $\quad A\cdot\overline{A}=0$

(3) Reduce the expression $(B+BC)(B+\overline{B}C)(B+D)$

$(B+BC)(B+\overline{B}C)(B+D)$

$=(BB+B\overline{B}C+BC+B\overline{B}C)(B+D)$

$=(B+0+BC+0)(B+D)$

$=(B+BC)(B+D) = B+BD+BC+BCD$

$= B(1+D)+BC(1+D)$ $\quad \because 1+D=1$

$= B+BC = B(1+C) = \underline{\underline{B}}$ $\quad \because 1+C=1$

(4) Show that $AB+A\overline{B}C+B\overline{C} = AC+B\overline{C}$

$AB+A\overline{B}C+B\overline{C} = A(B+\overline{B}C)+B\overline{C}$

$= A(B+C)+B\overline{C}$ $\quad B+\overline{B}C = B+C$

$= AB+AC+B\overline{C}$ $\quad 1+B=1$, $\quad 1+A=1$

$= AB(C+C')+AC+B\overline{C}$

$= ABC+AB\overline{C}+AC+B\overline{C} = AC(1+B)+B\overline{C}(1+A)$

$= AC+B\overline{C}$

(5) Show that $A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C = B + C$

$A\bar{B}C + B + B\bar{D} + AB\bar{D} + \bar{A}C$

$= A\bar{B}C + \bar{A}C + B(1 + \bar{D} + A\bar{D})$

$= A\bar{B}C + \bar{A}C + B$

$= C(\bar{A} + A\bar{B}) + B$ $\quad = C(\bar{A} + \bar{B}) + B$

$= \bar{A}C + \bar{B}C + B$ $\quad = \bar{A}C + B + C \quad \because B + \bar{B}C = B + C$

$= C(\bar{A} + 1) + B$ $\quad = \underline{\underline{B + C}}$

(6) Reduce $x(x' + y)$.

$x(x' + y) = xx' + xy$

$= 0 + xy$

$= \underline{\underline{xy}}$

(7) Simplify (i) $A\bar{B}D + A\bar{B}\bar{D}$?

$A\bar{B}D + A\bar{B}\bar{D} = A\bar{B}(D + \bar{D}) = A\bar{B}(1) = A\bar{B}$

(ii) $AB + A(B + C) + B(B + C)$?

$AB + A(B+C) + B(B+C) = AB + AB + AC + B + BC$

$= AB + AC + B + BC$

$= AB + AC + B(1 + C)$

$= AB + AC + B$

$= B(1 + A) + AC \quad \because 1 + A = 1$

$= \underline{\underline{B + AC}}$

(8) Prove that $\bar{A}\bar{B}C + \bar{A}BC + A\bar{B} = \bar{A}C + A\bar{B}$

$\bar{A}\bar{B}C + \bar{A}BC + A\bar{B}$

$= \bar{A}C(\bar{B}+B) + A\bar{B}$ $\quad \because B+\bar{B}=1$

$= \underline{\bar{A}C + A\bar{B}}$

(9) P.T $ABC + \bar{A}B + AB\bar{C} = B$

$ABC + \bar{A}B + AB\bar{C}$

$= AB(C+\bar{C}) + \bar{A}B$ $\quad C+\bar{C}=1$

$= AB + \bar{A}B$

$= B(A+\bar{A}) = B \cdot 1 = \underline{B}$

(10) $C((A\bar{B} + AB)) + BC = C(A+B)$

$C((CA\bar{B} + AB)) + BC = A\bar{B}C + ABC + BC$

$= AC(B+\bar{B}) + BC$

$= AC + BC$ $\quad \because B+\bar{B}=1$

$= \underline{C(A+B)}$

(11) Prove the following

$(A+BC'+C)C' = ABC' + AB'C' + A'BC'$

$L.H.S = AC' + BC' + 0 = AC'(B+B') + BC'(A+$

$= \underset{(1)}{ABC'} + AB'C' + \underset{(2)}{ABC'} + A'BC'$

$= \underline{ABC' + AB'C' + A'BC'}$

R.H.S simplify

$\Rightarrow ABC' + A'BC' + AB'C'$

$= BC'(A + A') + AB'C'$

$= BC' + AB'C'$

$= C'(B + B'A)$ $\quad \because x + x'y = x + y$

$= \underline{\underline{C'(A+B)}}$

(12) Prove that

$A'B'C' + A'BC + ABC + AB'C' + AB'C$
$= B'C' + BC + AB'C$

rearrange as

$A'B'C' + AB'C' + A'BC + ABC + AB'C$

$= B'C'(A' + A) + BC(A' + A) + AB'C$

$= \underline{\underline{B'C' + BC + AB'C}}$

(13) Prove that

$(B+\bar{C})(\bar{B}+C) + \overline{\bar{A} + B + \bar{C}} = C(A+B) + \bar{B}\bar{C}$

L.H.S

$= (B+\bar{C})(\bar{B}+C) + \bar{\bar{A}} \cdot \bar{B} \cdot \bar{\bar{C}}$

$= B\bar{B} + BC + \bar{B}\bar{C} + C\bar{C} + A\bar{B}C$ $\quad \because B\bar{B} = 0$, $C\bar{C} = 0$

$= 0 + BC + \bar{B}\bar{C} + 0 + A\bar{B}C$

$= BC + A\bar{B}C + \bar{B}\bar{C}$ $\quad \because$ rearranging

$= C(B + \bar{B}A) + \bar{B}\bar{C}$ $\quad \because B + \bar{B}A = B + A$

$= C(B + A) + \bar{B}\bar{C}$

$= \underline{\underline{C(A+B) + \bar{B}\bar{C}}}$

(14) $(x+y)\,[\overline{\bar{x}\cdot(\bar{y}+\bar{z})}] + \bar{x}\bar{y} + \bar{x}\bar{z} = 1$

$(x+y)\,(x + \overline{\bar{y}+\bar{z}}) + \bar{x}\bar{y} + \bar{x}\bar{z}$

$= (x+y)(x + y\cdot z) + x'y' + x'z'$

$= (x+y)(x + yz) + x'y' + x'z'$

$= x + xyz + xy + yz + x'y' + x'z'$

$\because \begin{cases} xx = x \\ yy = y \\ x + xy = x \end{cases}$

$= \underbrace{x + xy} + yz(1+x) + x'y' + x'z'$

$= x + yz + x'y' + x'z'$

$= \underbrace{x + x'y'} + yz + x'z'$ $\quad \begin{cases} x + x'y' = x + y' \\ y' + yz = y' + z \end{cases}$

$= x + \underbrace{y' + yz} + x'z'$

$= x + y' + z + x'z'$

$= \underbrace{x + x'z'} + y' + z$

$x + x'z' = x + z'$

$= x + z' + y' + z$

$= x + y' + 1$ $\quad 1 + \underbrace{x} = 1$

$= \underline{\underline{1}}$

(15) $\overline{\overline{(ABC + \bar{A}\bar{B})} + AC} = \bar{A}\cdot\bar{B}$

L.H.S $= \overline{\overline{ABC + \bar{A}\bar{B}}}\cdot\overline{AC} = (ABC + \bar{A}\bar{B})(\bar{A} + \bar{C})$

$= A\bar{A}BC + ABC\bar{C} + \bar{A}\bar{B} + \bar{A}\bar{B}\bar{C}$ $\quad A\cdot\bar{A} = 0$, $C\cdot C = 0$

$= A'B' + A'B'C' = A'B'(1+C') = \underline{\underline{A'\cdot B'}}$ $\quad 1 + C' = 1$

(16) Q. Use Boolean Algebra to show that

$A'BC' + AB'C' + AB'C + ABC' + ABC = A + BC'$

Rearrange L.H.S

$A'\underline{BC'} + A\underline{BC'} + AB'C' + AB'C + ABC$

$= BC'(A' + A) + AB'(C' + C) + ABC$ $\quad A' + A = 1$, $C + C' = 1$

$= BC' + AB' + ABC$

$= BC' + A(B' + BC)$ $\quad x' + xy = x' + y$

$= BC' + A(B' + C)$

$= BC' + AB' + AC$

$= BC' + AB' + AC(B + B')$

$= BC' + AB' + ABC + AB'C$

$= BC' + AB' + AB'C + ABC$ rearrangement

$= BC' + AB'(1 + C) + ABC$

$= BC' + AB' + ABC$

$= B(C' + CA) + AB'$ $\quad \because x' + xy = x' + y$

$= B(C' + A) + AB'$

$= BC' + AB + AB'$

$= BC' + A(B + B')$ $\quad \because B + B' = 1$

$= \underline{\underline{BC' + A}}$

Topic: CANONICAL AND STANDARD FORMS

(i) Minterms and Maxterms

A binary variable may appear either in its normal form (x) or in its complement form (x'). Consider two binary variable x and y combined with an AND operation.

There are 4 possible combinations $x'y'$, $x'y$, xy' and xy. Each of these four AND terms is called a "minterm or a standard product." n variables can be combined to form 2^n minterms. Each minterm is obtained from an AND term of the n variables.

In a similar fashion, n variables forming an OR term with each variable provide 2^n possible combinations. These are called maxterms or standard sums.

Table: minterms & maxterms for three binary variables

x	y	z	Minterms Term	Minterms Designation	Maxterms Term	Maxterms Designation
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x'+y'+z$	M_6
1	1	1	xyz	m_7	$x'+y'+z'$	M_7

(ii) Canonical Form (std form of boolean Expression)

Boolean Functions expressed as a sum of minterms or product of maxterms are said to be in canonical form.

Sum of Product (SOP) eg: $AB + BC$

Product of Sum (POS) eg: $(A+B)\cdot(B+C)$

(iii) Sum of Minterms (Sum of Products - SOP)

Two or more AND functions ORed together

eg: $AB + CD$, $AB + BCD$, $AB\bar{C} + \bar{D}E\bar{F} + A\bar{F}G$

Note:

⇒ SOP form can also contain a term with a single variable.

eg: $A + BCD + E'FG$

⇒ SOP expression, a single overbar (complement) cannot extend over more than one variable.

$\bar{A}\,\bar{B}\,\bar{C}$ ✓ $\overline{ABC}$ ✗

⇒ It is an useful form of Boolean expression because of the straight forward manner in which it can be implemented with logic gates.

⇒ Canonical SOP, contains all the term with all the variables either in complemented or uncomplemented form.

eg: $A\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}BC$

1 - A, B, C
0 - A', B', C'

Representation of SOP

$$F = \underset{0\,0\,1}{A'B'C} + \underset{1\,0\,0}{AB'C'} + \underset{1\,0\,1}{AB'C} + \underset{1\,1\,0}{ABC'} + \underset{1\,1\,1}{ABC}$$

$$= m_1 + m_4 + m_5 + m_6 + m_7$$

minterms

or the short notation

$$F(A, B, C) = \Sigma\,(1, 4, 5, 6, 7)$$

$F(A,B,C)$ ↓ variables Σ ↓ ORing of terms

Qn) Conversion of a general expression to SOP form.

$(A+B)\;(C+\bar{B})$ $\because B\bar{B} = 0$

$$= AC + A\bar{B} + BC + B\bar{B} = \underline{\underline{AC + A\bar{B} + BC}}$$

Qn) ~~Convert~~ Express the given boolean function $F = A + B'C$ in a sum of minterms or std SOP form or Canonical SOP form.

Soln:

Function has 3 variables A, B, C: $A + B'C$ (A: 1st term; B'C: 2nd term)

First term: 'A'

missing two variables

$$A = A(B+B') = AB + AB'$$

missing one more variable 'C'.

$$AB(C+C') + AB'(C+C') = ABC + ABC' + AB'C + AB'C'$$

Second term: $B'C$

missing one variable

$B'C = B'C(A+A') = AB'C + A'B'C$

Combining first and second terms $\quad x+x=x$

$F = A + B'C$

$= ABC + ABC' + \underset{(1)}{AB'C} + AB'C' + \underset{(2)}{AB'C} + A'B'C$

Thus $F = \underset{111}{ABC} + \underset{110}{ABC'} + \underset{101}{AB'C} + \underset{100}{AB'C'} + \underset{001}{A'B'C}$

$\therefore F = A'B'C + AB'C' + AB'C + ABC' + ABC$

$= m_1 + m_4 + m_5 + m_6 + m_7$

(iv) Product of Maxterms (Products of Sum) - POS

Two or more OR functions ANDed together

eg: $(A+B)(B+C+D)$

$(A+B+\bar{C})(\bar{D}+\bar{E}+F)(F+G+H)$

Note ÷

⇒ POS expression can also contain a single variable term eg: $A(B+C+D)(E'+F'+G)$

⇒ In Canonical POS, each sum term contains all the variables either in complement or uncomplemented form.

eg: $(A+B)(\bar{A}+B)(A+\bar{B})$

POS 1 → A', B', C'
0 → A, B, C

Representation of POS

$$F = (x+y+z)(x+y'+z)(x'+y+z)(x'+y+z')$$
$$\quad\ 0\ 0\ 0 \quad\ 0\ 1\ 0 \quad\ 1\ 0\ 0 \quad\ 1\ 0\ 1$$

$$= M_0\, M_2\, M_4\, M_5$$ maxterms

or short notation

$$F(x, y, z) = \Pi(0, 2, 4, 5)$$

$\Downarrow$ variables $\Downarrow$ ANDing of the terms

(iv) Conversion Between CANONICAL FORMS

The complement of a function expressed as the sum of minterms equals the sum of minterms missing from the original function.

Consider the function

$$F(A, B, C) = \Sigma(1, 4, 5, 6, 7)$$

Complement is

$$F'(A, B, C) = \Sigma(0, 2, 3)$$
$$= m_0 + m_2 + m_3$$

Take the complement of F' by De Morgan's theorem,

$$F = (m_0 + m_2 + m_3)'$$
$$= m_0' \cdot m_2' \cdot m_3' = M_0\, M_2\, M_3 = \Pi(0, 2, 3)$$

$F(x,y,z) = \pi(0,2,4,5)$ is expressed in the product of maxterm form. Its conversion to sum of minterms is

$$F(x,y,z) = \Sigma(1,3,6,7)$$

Standard forms

Standard form is used to express Boolean functions. There are 2 types of std forms

(i) The Sum of Products

(ii) The Product of Sums

Sum of products is a Boolean expression containing AND terms.

eg: $F_1 = y' + xy + x'yz'$ → 3 product terms, → 3 literals

Product of sums is a Boolean expression containing OR terms.

eg: $F_2 = x(y'+z)(x'+y+z'+w)$ → 3 sum terms, → 4 literals

Qn) Expand of a boolean expansion to Std POS form or canonical POS form $A(\bar{B}+\bar{A})B$.

$$A(\bar{B}+\bar{A})B = (A+B\bar{B})(\bar{B}+\bar{A})(B+A\bar{A})$$

$$= \underset{(1)}{(A+B)}\underset{(1)}{(A+\bar{B})}\underset{(2)}{(\bar{B}+\bar{A})}\underset{(2)}{(B+A)}(B+\bar{A})$$

$$= \underset{0}{(A+B)}\underset{0}{(A+\bar{B})}(\bar{A}+B)$$

$$= \pi(0,1,2) = M_0 \cdot M_1 \cdot M_2$$

University Questions

① Express the given function in sum of minterms form. $F(x,y,z) = 1$ (1½ mark)

Ans: $\Sigma(0,1,2,3,4,5,6,7)$

② a) What is the difference b/w canonical form and standard form? Which form is preferable while implementing a boolean function with gates? (2 marks)

b) Simplify the given boolean function $F(w,x,y,z) = \Sigma(2,3,12,14,15)$ (i) Sum of Products and (ii) POS (use kmap) (7 marks)

③ Express the following Boolean function in canonical form

$$F(x,y,z) = x' + yz + xz' + xy'z' + xyz'$$

Ans: M_5 (3 marks)

④ Express the following function as sum of minterms and product of maxterms:

a) $F(A,B,C) = \bar{B} + A\bar{C} + A\bar{B}\bar{C}$ Ans: $M_2 \cdot M_3 \cdot M_7$ $m_0 + m_1 + m_4 + m_5 + m_6$

b) $F(A,B,C) = C(A+\bar{B})(\bar{A}+\bar{B}+\bar{C})$ Ans: $m_1 + m_5$ $M_0 \cdot M_2 \cdot M_3 M_4 M_6 M_7$

⑤ Express the following functions:

(i) $F_1 = AB + B'C$ in sum of minterms form

Ans: $m_0 + m_5 + m_6 + m_7$

(ii) $F_2 = A + B'C$ in product of maxterms form

Ans: $M_0 M_2 M_3$

6) Express the following functions as product of max-terms

a) $F(x,y,z) = y' + xz' + xy'z'$ Ans: $M_2 M_3 M_7$

Same

b) $F(A,B,C) = C(A+\bar{B})(\bar{A}+\bar{B}+\bar{C})$ Ans: $M_0 M_2 M_3 M_4 M_6 M_7$

*) c) $F(A,B,C,D) = D(\bar{A}+B) + \bar{B}D$ Ans: $\pi(0,2,4,6,8,10,12,14)$

d) $F(A,B,C) = (\bar{A}+B)(\bar{B}+C)$ express in SOP Form

Ans: $\Sigma(0,1,3,7)$

e) $F(x,y,z) = (xy+z)(y+xz)$ express in SOP form

Ans: $\Sigma(3,5,6,7)$

f) $F(A,B,C,D) = (A+\bar{B}+C)(A+\bar{B})(A+\bar{C}+\bar{D})$
$(\bar{A}+B+C+\bar{D})(B+\bar{C}+\bar{D})$

Anss. $\Sigma(0,1,2,8,10,12,13,14,15)$

U.Q

(i) $F(x,y,z) = xy + y + z$ into canonical PoS.

$\pi(0,4)$ 2.5 marks

(ii) $F(x,y,z) = (x+y+z)(x'+y+z)$
$(x+y'+z)(x+z)$ into std PoS.

Schem F $(x+z)(x+y)$ 2.5 marks

$F = (x+y+z)(x'+y+z)(x+y'+z)$

(6) Express the following functions as product of max-terms:

a) $F(X,Y,Z) = Y' + XZ' + XY'Z'$

b) $F(A,B,C) = C(A+B')(A'+B'+C')$

Answers

(1) $F(x,y,z) = 1$

$F(x,y,z) = 1$ mean, there is no maxterms

& $F(x,y,z) = 0$ mean, there is no minterms

$\therefore F(x,y,z) = \Sigma(0,1,2,3,4,5,6,7)$

$F(x,y,z) = x'y'z' + x'y'z + x'yz' + x'yz$
$\quad + xy'z' + xy'z + xyz' + xyz$

(3) $F(x,y,z) = x' + yz + xz' + xy'z' + xyz'$

first term

$x'(y+y') = x'y + x'y'$

$x'y + x'y'(z+z') = \underset{011}{x'yz} + \underset{010}{x'yz'} + \underset{001}{x'y'z} + \underset{000}{x'y'z'}$

Second term

$yz(x+x') = \underset{111}{xyz} + \underset{011}{x'yz}$

Third term

$xz'(y+y') = \underset{110}{xyz'} + \underset{100}{xy'z'}$

Fourth term

$\underset{100}{xy'z'}$

fifth term

$\underset{110}{xyz'}$

$F = x'y'z' + x'y'z + x'yz' + x'yz$
$\quad + xy'z' + xyz' + xyz$

$F(x,y,z) = \Sigma(0,1,2,3,4,6,\cancel{7})$

$$F(x,y,z) = m_0 + m_1 + m_2 + m_3 + m_4 + m_6 + m_7$$

$$F(x,y,z) = \pi(5)$$

$$F(x,y,z) = M_5 = \cancel{(A+B+C)} = \overset{1}{x'} + \overset{0}{y} + \overset{1}{z'}$$

④ a) $F(A,B,C) = \bar{B} + A\bar{C} + A\bar{B}\bar{C}$

first term $\bar{B}(A+\bar{A}) = A\bar{B} + \bar{A}\bar{B}$

$A\bar{B} + \bar{A}\bar{B}(C+\bar{C}) = \underset{101}{A\bar{B}C} + \underset{100}{A\bar{B}\bar{C}} + \underset{001}{\bar{A}\bar{B}C} + \underset{000}{\bar{A}\bar{B}\bar{C}}$

Second term $A\bar{C}(B+\bar{B}) = \underset{110}{AB\bar{C}} + \underset{100}{A\bar{B}\bar{C}}$

Third term $\underset{100}{A\bar{B}\bar{C}}$

SOP $\left\{ \begin{aligned} F(A,B,C) &= \Sigma(0,1,4,5,6) \\ &= m_0 + m_1 + m_4 + m_5 + m_6 \\ &= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + A\bar{B}\bar{C} + A\bar{B}C + AB\bar{C} \end{aligned} \right.$

POS $\left\{ \begin{aligned} F(A,B,C) &= \Pi(2,3,7) \\ &= M_2 \cdot M_3 \cdot M_7 \\ &= \underset{010}{(A+\bar{B}+C)}\ \underset{011}{(A+\bar{B}+\bar{C})}\ \underset{111}{(\bar{A}+\bar{B}+\bar{C})} \end{aligned} \right.$

4 b) $F(A,B,C) = C(A+\bar{B})(\bar{A}+\bar{B}+\bar{C})$

First term $\left\{ \begin{aligned} & C + (A\bar{A}) = (C+A)(C+\bar{A}) \\ & (C+A)(C+\bar{A}) + (B\bar{B}) = (C+A+B)(C+A+\bar{B}) \\ & \qquad (C+\bar{A}+B)(C+\bar{A}+\bar{B}) \end{aligned} \right.$

$= \underset{000}{(A+B+C)}\ \underset{010}{(A+\bar{B}+C)}\ \underset{100}{(\bar{A}+B+C)}\ \underset{110}{(\bar{A}+\bar{B}+C)}$

Second term $(A+\bar{B})+C\bar{C} = (A+\bar{B}+C)\ (A+\bar{B}+\bar{C})$

$\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad 0\ 1\ 0 \quad\quad 0\ 1\ 1$

Third term $(\bar{A}+\bar{B}+\bar{C})$

$\quad\quad\quad\quad 1\ 1\ 1$

$$F(A,B,C) = \pi(0,2,3,4,6,7)$$

$$= M_0 \cdot M_2 \cdot M_3 \cdot M_4 + M_6 \cdot M_7$$

SOP $F(A,B,C) = \Sigma(1,5)$

$$= m_1 + m_5$$

$$= \bar{A}\bar{B}C + A\bar{B}C$$

5 (i) $F_1 = AB + B'C$ in sum of minterms form

$F_1(A,BC) = AB + B'C$

First term

$AB(C+C') = ABC + ABC'$

$\quad\quad\quad\quad\quad 1\,1\,1 \quad 1\,1\,0$

Second term

$B'C(A+A' = AB'C + A'B'C$

$\quad\quad\quad\quad\quad 1\,0\,1 \quad 0\,0\,1$

$$F_1(A,B,C) = \Sigma(1,5,6,7)$$

$$= m_0 + m_5 + m_6 + m_7$$

$$= A'B'C + AB'C + ABC' + ABC$$

5(ii) $F_2(A, B, C) = A + B'C$ in product of max terms forms

SOP $F = A + B'C$

First term $A(B+B') = AB + AB'$

$AB + AB'(C+C') = \underset{111}{ABC} + \underset{110}{ABC'} + \underset{101}{AB'C} + \underset{100}{AB'C'}$

Second term

$B'C(A+A') = \underset{101}{AB'C} + \underset{001}{A'B'C}$

$F_2(A,B,C) = \Sigma(1, 4, 5, 6, 7)$

$= m_1 + m_4 + m_5 + m_6 + m_7$

POS $F_2(A,B,C) = \Pi(0, 2, 3)$

$A \to 0$

$A' \to 1$

$= M_0 \cdot M_2 \cdot M_3$

$= (A + B + C) \cdot (A + \bar{B} + C) \cdot (A + \bar{B} + \bar{C})$

(6) a) $F(X, Y, Z) = Y' + XZ' + \underset{100}{XY'Z'}$

SOP $F = Y' + XZ' + XY'Z'$

First term

$Y'(X + X') = XY' + X'Y'$

$(XY' + X'Y')(Z + Z') = \underset{101}{XY'Z} + \underset{100}{XY'Z'} + \underset{001}{X'Y'Z} + \underset{000}{X'Y'Z'}$

Second term $XZ'(Y+Y') = \underset{110}{XYZ'} + \underset{100}{XY'Z'}$

Third term $\underset{100}{XY'Z'}$

SOP $F(x,y,z) = \Sigma(0,1,4,5,6)$

$= m_0 + m_1 + m_4 + m_5 + m_6$

POS $F(x,y,z) = \Pi(2,3,7)$

$x \to 0$

$x' \to 1$

$= M_2 \cdot M_3 \cdot M_7$

$= (x+y'+z) \cdot (x+y'+z') \; (x'+y'+z')$

6 (b) $F(A,B,C) = C\,(A+B')\,(A'+B'+C')$

First term

$C + AA' = (A+C)\,(A'+C)$

$(A+C)\,(A'+C) + BB' = (A+C+BB')\,(A'+C+BB')$

$= \underset{0\;0\;0}{(A+B+C)}\;\underset{0\;1\;0}{(A+B'+C)}\;\underset{1\;0\;0}{(A'+B+C)}\;\underset{1\;1\;0}{(A'+B'+C)}$

Second term

$(A+B') = A + B' + CC'$

$= \underset{0\;1\;0}{(A+B'+C)} \cdot \underset{0\;1\;1}{(A+B'+C')}$

Third term

$\underset{1\;1\;1}{(A'+B'+C')}$

$F(A,B,C) = \Pi(0,2,3,4,6,7)$

$= M_0 \cdot M_2 \cdot M_3 \cdot M_4 \cdot M_6 \cdot M_7$

(7) $F(A, B, C, D) = D(A' + B) + B'D$

SOP $F = DA' + DB + B'D$

First term

$DA'(B+B') = A'BD + A'B'D$

$A'BD + A'B'D(C+C') = A'BCD + A'BC'D + A'B'CD + A'B'C'D$

0111 (7)   0101 (5)   0011 (3)   0001 (1)

Second term

$DB(A+A') = ABD + A'BD$

$ABD + A'BD(C+C') = ABCD + ABC'D + A'BCD + A'BC'D$

1111 (15)   1101 (13)   0111 (7)   0101 (5)

Third term

$B'D(A+A') = AB'D + A'B'D$

$AB'D + A'B'D(C+C') = AB'CD + AB'C'D + A'B'CD + A'B'C'D$

1011 (11)   1001 (9)   0011 (3)   0001 (1)

$F(A, B, C, D) = \Sigma m(1, 3, 5, 7, 9, 11, 13, 15)$

$= m_0 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$

POS $F(A, B, C, D) = \Pi(0, 2, 4, 6, 8, 10, 12, 14)$

$= M_0 \cdot M_2 \cdot M_4 + M_6 + M_8 + M_{10} + M_{12} + M_{14}$

(8) $F(A, B, C) = (A' + B)(B' + C)$

POS First term $A' + B + CC' = (A' + B + C)(A' + B + C')$ 100   101

Second term $B' + C + AA' = (A + B' + C)(A' + B' + C)$ 010   110

$F(A,B,C) = \prod M(2,4,5,6)$

$= M_2 \cdot M_4 \cdot M_5 \cdot M_6$

SOP $F(A,B,C) = \sum m(0,1,3,7)$

$= m_0 + m_1 + m_3 + m_7$

$= A'B'C' + A'B'C + A'BC + ABC$

8) $F(x,y,z) = (xy+z)(y+xz)$

POS $F = (x+z)(y+z)(x+y)(y+z)$

$= (x+y)(y+z)(x+z)$

First term

$(x+y+zz') = (x+y+z)(x+y+z')$

$\quad\quad\quad 0\ 0\ 0 \quad\quad 0\ 0\ 1$

Second term

$(y+z+xx') = (x+y+z)(x'+y+z)$

$\quad\quad\quad 0\ 0\ 0 \quad\quad 1\ 0\ 0$

Third term

$(x+z+yy') = (x+y+z)(x+y'+z)$

$\quad\quad\quad 0\ 0\ 0 \quad\quad 0\ 1\ 0$

$F(x,y,z) = \prod M(0,1,2,4)$

$= M_0 \cdot M_1 \cdot M_2 \cdot M_4$

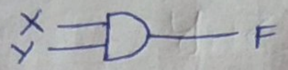
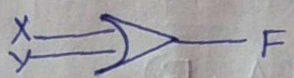
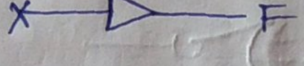
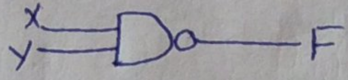
SOP $F(x,y,z) = \sum m(3,5,6,7)$

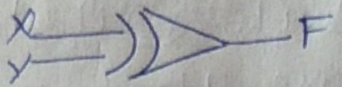
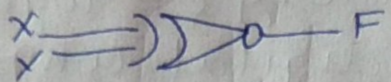
$= m_3 + m_5 + m_6 + m_7$

$= x'yz + xy'z + x'yz + xyz$

(V) LOGIC GATES

The manipulation of binary information is done by logic circuits called gates. Gates are blocks of hardware that produce signals of binary 1 or 0 when input logic requirements are satisfied. A variety of logic gates are commonly used in digital computer systems.

Name	Graphic symbol	Algebraic function	Truth table
AND		$F = X \cdot Y$	X Y | F 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1
OR		$F = X + Y$	X Y | F 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1
NOT		$F = X'$	X | F 0 | 1 1 | 0
Buffer		$F = X$	X | F 0 | 0 1 | 1
NAND		$F = (XY)'$	X Y | F 0 0 | 1 0 1 | 1 1 0 | 1 1 1 | 0
NOR		$F = (X+Y)'$	X Y | F 0 0 | 1 0 1 | 0 1 0 | 0 1 1 | 0

Name	Graphic symbol	Algebraic function	Truth table
XOR Exclusive OR		$F = xy' + x'y$ $= x \oplus y$	see below
Exclusive NOR or equivalence		$F = xy + x'y'$ $= x \odot y$	see below

XOR truth table:

x	y	F
0	0	0
0	1	1
1	0	1
1	1	0

Exclusive NOR truth table:

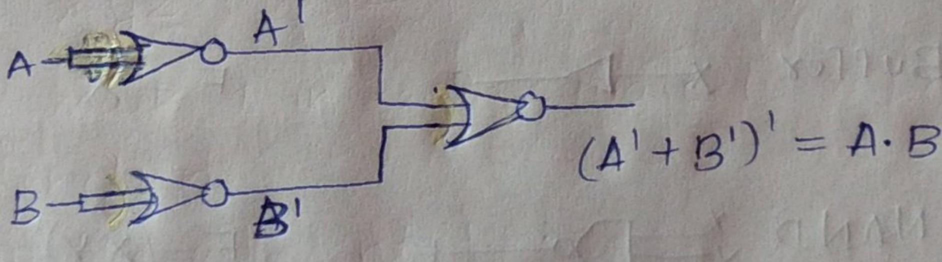
x	y	F
0	0	1
0	1	0
1	0	0
1	1	1

Qn Construct AND gate using NOR gate

Soln:

$$A \cdot B = \overline{\overline{(A \cdot B)}}$$
$$= (A' + B')'$$

$\Rightarrow$ Using De-Morgan's theorem



(VI) SIMPLIFICATION/MINIMIZATION OF BOOLEAN FUNCTIONS

(i) The Map Method/Veitch Diagram/Karnaugh Map

The complexity of the digital logic gates that implements a boolean function is directly related to the complexity of the algebraic expression from which the function is implemented. Boolean functions may be simplified by algebraic means but it lacks specific rules to predict each succeeding step. The map method provides a simple straight forward procedure for minimizing boolean functions. The map method first proposed by Veitch & slightly modified by Karnaugh, and is ~~known~~ known as the "Veitch diagram" or the "karnaugh map".

The map is a diagram made up of squares. Each square represents one minterm. Map presents a visual diagram of all possible ways a function may be expressed in a standard form.

(ii) Two and Three Variable Map

Two variable Map

m_0	m_1
m_2	m_3

$x \backslash y$	0	1
0	$x'y'$	$x'y$
1	xy'	xy

There are 4 minterms for two variables, hence map consist of 4 squares, one for each minterm

eg: $F = xy$

Since xy is equal to m_3, a 1 is placed inside the square that belongs to m_3.

x \ y	0	1
0		
1		1

Three Variable map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6

x \ yz	00	01	11	10
0	$x'y'z'$	$x'y'z$	$x'yz$	$x'yz'$
1	$xy'z'$	$xy'z$	xyz	xyz'

There are 8 minterms for three binary variables hence map consist of eight squares.

Simplify the boolean function:

$$F = x'yz + x'yz' + xy'z' + xy'z$$

$$\quad 011 \qquad 010 \qquad 100 \qquad 101$$

① First mark 1 in each square to represent the function.

② Group the adjacent 1's.

③ Take the area of the group and represent as the sum of two terms.

Combined as group of 2, 4, 8

x \ yz	$y'z'$ 00	$y'z$ 01	yz 11	yz' 10
x' 0			1	1
x 1	1	1		

$F = xy'z' + xyz + x'yz + x'yz'$

$= xy'(z+z') + x'y(z+z')$ $\because z+z'=1$

$= \underline{xy' + x'y}$

(1) Simplify the boolean function:

$F = x'yz + xy'z' + xyz + xyz'$

x \ yz	$y'z'$ 00	$y'z$ 01	yz 11	yz' 10
x' 0			1	
x 1	1		1	1

$F = \underline{xz' + yz}$

(2) $F = A'C + A'B + AB'C + BC$

A \ BC	$B'C'$ 00	$B'C$ 01	BC 11	BC' 10
A' 0	0	1	1	1
A 1	0	1	1	0

$A'C(B+B') = ABC + A'B'C$

$A'B(C+C') = A'BC + A'BC'$

$BC(A+A') = ABC + A'BC$

$F = \underline{C + A'B}$

(3) $F(x,y,z) = \Sigma(0,2,4,5,6)$

x \ yz	$y'z'$ 00	$y'z$ 01	yz 11	yz' 10
x' 0	1 (0)	(1)	(3)	1 (2)
x 1	1 (4)	1 (5)	(7)	1 (6)

$F = z' + xy'$

Four Variable Map

m_0	m_1	m_3	m_2
m_4	m_5	m_7	m_6
m_{12}	m_{13}	m_{15}	m_{14}
m_8	m_9	m_{11}	m_{10}

Qn 1) Simplify the boolean function

$$F = (w,x,y,z) = \Sigma(0,1,2,4,5,6,8,9,12,13,14)$$

wx \ yz	$y'z'$ 00	$y'z$ 01	yz 11	yz' 10
$w'x'$ 00	1	1		1
$w'x$ 01	1	1		1
wx 11	1	1		1
wx' 10	1	1		

$$\underline{\underline{F = y' + w'z' + xz'}}$$

2) Simplify the boolean function

$$F = A'B'C' + B'CD' + A'BCD' + AB'C' \ ?$$

$$F = \underset{0001}{\overset{(1)}{A'B'C'D}} + \underset{0000}{\overset{(0)}{A'B'C'D'}} + \underset{1010}{\overset{(10)}{AB'CD'}} + \underset{0010}{\overset{(2)}{A'B'CD'}}$$

$$+ \underset{0110}{\overset{(6)}{A'BCD'}} + \underset{1001}{\overset{(9)}{AB'C'D}} + \underset{1000}{\overset{(8)}{AB'C'D'}}$$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	1_{0}	1_{1}	$_{3}$	1_{2}
A'B 01	$_{4}$	$_{5}$	$_{7}$	1_{6}
AB 11	$_{12}$	$_{13}$	$_{15}$	$_{14}$
AB' 10	1_{8}	1_{9}	$_{11}$	1_{10}

$$F = B'C' + B'D' + A'CD'$$

(Q3) $F = xyz + xy'z + xyz' + x'y'z$

$\quad 111 \qquad 101 \qquad 110 \qquad 001$

x \ yz	y'z' 00	y'z 01	yz 11	yz' 10
x' 0	$_{0}$	1_{1}	$_{3}$	$_{2}$
x 1	$_{4}$	1_{5}	1_{7}	1_{6}

$$F = y'z + xy$$

(Q4) $F = ab'c + a'bc + a'b'c + a'b'c' + ab'c'$

$\quad 101 \qquad 011 \qquad 001 \qquad 000 \qquad 100$

a \ bc	b'c' 00	b'c 01	bc 11	bc' 10
a' 0	1_{0}	1_{1}	1_{3}	$_{2}$
a 1	1_{4}	1_{5}	$_{7}$	$_{6}$

$$F = b' + a'c$$

Q5 $F = \sum(2, 5, 6, 9, 12, 13)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	0	1	3	1 2
A'B 01	4	1 5	7	1 6
AB 11	1 12	1 13	15	14
AB' 10	8	1 9	11	10

$F = ABC' + BC'D + AC'D + A'CD'$

Q6 $F = \sum(0, 1, 2, 3, 8, 9, 10, 11)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	1 0	1 1	1 3	1 2
A'B 01	4	5	7	6
AB 11	12	13	15	14
AB' 10	1 8	1 9	1 11	1 10

$F = \overline{B}$

Q7 $F = \sum(4, 5, 6, 7, 12, 13, 14, 15)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	0	1	3	2
A'B 01	1 4	1 5	1 7	1 6
AB 11	1 12	1 13	1 15	1 14
AB' 10	8	9	11	10

$F = B$

More Questions

1) Simplify using K-Map

a) $F = \Sigma(2,5,6,9,12,13)$ Ans: $AB\bar{C} + B\bar{C}D + A\bar{C}D + \bar{A}C\bar{D}$

b) $F = \Sigma(0,1,2,3,8,9,10,11)$ Ans: $F = \bar{B}$

c) $F = \Sigma(4,5,6,7,12,13,14,15)$ Ans: $F = B$

d) $F = \Sigma(2,6,8,9,10,11,14)$ Ans: $A\bar{B} + C\bar{D}$

e) $F = \Sigma(0,1,4,5,8,9,10,11,14,15)$ Ans: $\bar{A}\bar{C} + A\bar{B} + AC$

f) $F = \Sigma(0,2,5,7,8,10,13,15)$ Ans: $F = B$.

g) $F = \Sigma(1,3,4,6,9,11,12,14)$ Ans: $F = B'D + BD'$

h) Simplify $F(W,x,y,z) = \Sigma(2,3,12,13,14,15)$

Q8) $F = \Sigma(2, 6, 8, 9, 10, 11, 14)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	(0)	(1)	(3)	1 (2)
A'B 01	(4)	(5)	(7)	1 (6)
AB 11	(12)	(13)	(15)	1 (14)
AB' 10	1 (8)	1 (9)	1 (11)	1 (10)

$F = AB' + CD'$

Q9) $F = \Sigma(0, 1, 4, 5, 8, 9, 10, 11, 14, 15)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	1 (0)	1 (1)	(3)	(2)
A'B 01	1 (4)	1 (5)	(7)	(6)
AB 11	(12)	(13)	1 (15)	1 (14)
AB' 10	1 (8)	1 (9)	1 (11)	1 (10)

$F = A'C' + AB' + AC$

Q10) $F = \Sigma(0, 2, 5, 7, 8, 10, 13, 15)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	1 (0)	(1)	(3)	1 (2)
A'B 01	(4)	1 (5)	1 (7)	(6)
AB 11	(12)	1 (13)	1 (15)	(14)
AB' 10	1 (8)	(9)	(11)	1 (10)

$F = B$

Q11) $F = \Sigma(1, 3, 4, 6, 9, 11, 12, 14)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	0	1 1	1 3	2
A'B 01	1 4	5	7	1 6
AB 11	1 12	13	15	1 14
AB' 10	8	1 9	1 11	10

$$F = B'D + BD'$$

U.Q Q12) Simplify the given Boolean function

$F(w, x, y, z) = \Sigma(2, 3, 12, 13, 14, 15)$

(i) Sum of Products (use K Map)

wx \ yz	y'z' 00	y'z 01	yz 11	yz' 10
w'x' 00	0	1	1 3	1 2
w'x 01	4	5	7	6
wx 11	1 12	1 13	1 15	1 14
wx' 10	8	9	11	10

$$F = wx + w'x'y$$

U.Q Q13) Reduce the following expression using K-map and implement the real minimal expression in universal logic

1) $F(A, B, C, D) = \Sigma m(0, 1, 2, 3, 5, 7, 8, 9, 10, 12, 13)$

Q). $F(A,B,C,D) = \Sigma m(0,1,2,3,5,7,8,9,10,12,13)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	1 (0)	1 (1)	1 (3)	1 (2)
A'B 01	(4)	1 (5)	1 (7)	(6)
AB 11	1 (12)	1 (13)	(15)	(14)
AB' 10	1 (8)	1 (9)	(11)	1 (10)

$$F = AC' + A'D + B'D'$$

VII PRODUCT OF SUMS SIMPLIFICATION

Minimization of POS is similar to SOP except plotting of 0's instead of 1's.

Note ÷ In POS, $A \rightarrow 0$ & $A' \rightarrow 1$.

1) $F = \pi(0,1,3,4,7)$

A \ BC	BC 00	BC' 01	B'C' 11	B'C 10
A 0	0 (0)	0 (1)	0 (3)	(2)
A' 1	0 (4)	(5)	0 (7)	(6)

$$F = (B+C)(A+B)(B'+C')$$

2) $F = \pi(0,4,6,7,8,12,13,14,15)$

AB \ CD	CD 00	CD' 01	C'D' 11	C'D 10
AB 00	1 (0)	(1)	(3)	(2)
AB' 01	1 (4)	(5)	1 (7)	1 (6)
A'B' 11	1 (12)	1 (13)	1 (15)	1 (14)
A'B 10	1 (8)	(9)	(11)	(10)

$F = (C+D)$
$(B'+C')$
$(A'+B')$

U.Q 3) Simplify the given Boolean function

$F(w, x, y, z) = \Sigma(2, 3, 12, 13, 14, 15)$

Product of Sums (Use K map)

wx \ yz	yz 00	yz' 01	y'z' 11	y'z 10
wx 00	0	1	0 3	0 2
wx' 01	4	5	7	6
w'x' 11	0 12	0 13	0 15	0 14
w'x 10	8	9	11	10

$(w' + x') \cdot (w + x + y')$

U.Q 4) $F(A, B, C, D) = \Pi M(2, 8, 9, 10, 11, 12, 14)$

AB \ CD	CD 00	CD' 01	C'D 11	C'D 10
AB 00	0	1	3	0 2
AB' 01	4	5	7	6
A'B' 11	0 12	13	15	0 14
A'B 10	0 8	0 9	0 11	0 10

$(A' + B) \cdot (A' + D) \cdot (B + C' + D)$

(5) $F(x,y,z) = \pi(0,2,5,7)$

x \ yz	yz 00	yz' 01	y'z' 11	y'z 10
x 0	0 (0)	(1)	(3)	0 (2)
x' 1	(4)	0 (5)	0 (7)	(6)

$F = (x' + z')(x + z)$

(6) Simplify $\pi(0,2,3,6)$ by K-Map

A \ BC	BC 00	BC' 01	B'C' 11	B'C 10
A 0	0 (0)	(1)	0 (3)	0 (2)
A' 1	(4)	(5)	(7)	0 (6)

$F = (A + C)(A + B')(B' + C)$

(7) Simplify $f = \pi(0,1,2,3,4,5,6,7,8,9,10,14)$

AB \ CD	CD 00	CD' 01	C'D' 11	C'D 10
AB 00	0 (0)	0 (1)	0 (3)	0 (2)
AB' 01	0 (4)	0 (5)	0 (7)	0 (6)
A'B' 11	(12)	(13)	(15)	0 (14)
A'B 10	0 (8)	0 (9)	(11)	0 (10)

$F = A(C' + D)(B + C)$

(8) $F = \pi(0,3,5,7)$

A \ BC	BC 00	BC' 01	B'C' 11	B'C 10
A 0	0 (0)	(1)	0 (3)	(2)
A' 1	(4)	0 (5)	0 (7)	(6)

$F = (A + B + C)(A' + C')(B' + C')$

VIII

DON'T CARE CONDITIONS / CANNOT HAPPEN STATES

Some logic circuits can be designed so that there are certain input conditions for which there are no specified o/p levels, or we "don't care". In such cases, the o/p level is not defined. It can be either HIGH or LOW. These output levels are indicated by 'X' or 'd' or 'φ' in the truth tables and are called "don't care outputs" and such combinations are called don't care conditions.

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	X
1	1	0	X
1	1	1	X

When choosing adjacent squares to simplify the function in the map, the X's may be assumed to be either 0 or 1. Whichever gives the simplest expression. An X need not be used at all if it does not contribute to covering a larger area.

A \ BC	B'C' 00	B'C 01	BC 11	BC' 10
A' 0		1		
A 1	1	X	X	X

$F = A + B'C$

① $F(A,B,C) = \sum m(0,1,5) + \sum d(4,7)$

A \ BC	B'C' 00	B'C 01	BC 11	BC' 10
A' 0	1	1		
A 1	X	1	X	

$F = B'$

② $F(A,B,C,D) = \sum m(0,1,3,7,15) + \sum d(2,11,12)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00	1	1	1	X
A'B 01			1	
AB 11	X		1	
AB' 10			X	

$F = AB' + CD$

③ $F(A,B,C,D) = \sum m(4,6,7,13,14) + \sum d(5,10,12,15)$

AB \ CD	C'D' 00	C'D 01	CD 11	CD' 10
A'B' 00				
A'B 01	1	X	1	1
AB 11	X	1	X	1
AB' 10				X

$F = B$

④ $F(A,B,C,D) = \Pi M(0,2,6,8,12) \cdot \Pi d(3,4,7,10,14)$

AB \ CD	00	01	11	10
AB 00	O		X	O
AB' 01	X		X	O
A'B 11	O			X
A'B 10	O			X

$F = D$

(5) $F(w,x,y,z) = \Sigma m(1,3,7,11,15) + d(0,2,5)$

wx \ yz	y'z' 00	y'z 01	yz 11	yz' 10
w'x' 00	X (0)	1 (1)	1 (3)	X (2)
w'x 01	(4)	X (5)	1 (7)	(6)
wx 11	(12)	(13)	1 (15)	(14)
wx' 10	(8)	(9)	1 (11)	(10)

$$F = w'z + yz$$

UQ April 2018

Using k-map, simplify the boolean function F in sum of products form, using the don't care condition d:

$$F(w,x,y,z) = w'(x'y + x'y' + xyz) + x'z'(y+w)$$

$$d(w,x,y,z) = w'x(y'z + yz') + wyz$$

SOP $3 \to 1$, $3 \to 0$

$$F = w'x'y + w'x'y' + w'xyz + x'z'y + x'z'w$$

First term

$$w'x'y(z+z') = \underset{0\,0\,1\,1\ (3)}{w'x'yz} + \underset{0\,0\,1\,0\ (2)}{w'x'yz'}$$

Second term

$$w'x'y'(z+z') = \underset{0\,0\,0\,1\ (1)}{w'x'y'z} + \underset{0\,0\,0\,0\ (0)}{w'x'y'z'}$$

Third term

$$\underset{0\,1\,1\,1}{w'xyz}$$

Fourth term

$$(w+w')x'z'y = \underset{1\,0\,1\,0\ (10)}{wx'yz'} + \underset{0\,0\,1\,0\ (2)}{w'x'yz'}$$

Fifth term

$$wx'z'(y+y') = wx'yz' + wx'y'z'$$

1010 (10) $\quad 1000$ (8)

$$d(w,x,y,z) = w'xy'z + w'xyz' + wyz$$

$0101 \quad 0110$

Third term $wyz(x+x') = wxyz + wx'yz$

$1111 \quad 1011$

$$F(w,x,y,z) = \sum m(0,1,2,3,\overset{7}{\wedge}8,10) + d(5,\cancel{6},11,15)$$

$wx \backslash yz$	$y'z'$ 00	$y'z$ 01	yz 11	yz' 10
$w'x'$ 00	1 (0)	1 (1)	1 (3)	1 (2)
$w'x$ 01	(4)	X (5)	1 (7)	$\cancel{X}$ (6)
wx 11	(12)	(13)	X (15)	(14)
wx' 10	1 (8)	(9)	X (11)	1 (10)

$F = \cancel{w'x' + x'z'}$

$F = w'z + x'z'$

SOP

POS → $F = (w+z')(x+z)$

Q) $F(x,y,z) = \Pi(0,2,5,7)$ Ans: $(x'+z')(x+z)$

Q) $\Pi(0,2,3,6)$ Ans: $F=(A+C)(A+\bar{B})+\bar{B}+$

Q) $F=\Pi(0,1,2,3,4,5,6,7,8,9,10,14)$ Ans: $F=A(\bar{C}+D)(B+$

Q) $F=\Pi(0,3,5,7)$ Ans: $F=(A+B+C)(\bar{A}+C)(\bar{B}+\bar{C})$

Q) $F(w,x,y,z) = \Sigma m(1,3,7,11,15) + d(0,2,5)$

Ans: $F = w'z + yz$

Q) $F(w,x,y,z) = w'(x'y + x'y' + xyz) + x'z'(y+w)$

$d(w,x,y,z) = w'x(y'z + yz') + wyz$

Ans: $F = (w' \cdot z)(x' \cdot z')$

or

$(w'x' + x'z)$

Q) $F = A'B'D' + A'CD + A'BC$

$d = A'BC'D + ACD + AB'D'$

Ans: SOP

$F = B'D' + A'C$

② $F = A'B'D' + A'CD + A'BC$

$d = A'BC'D + ACD + AB'D'$

$F = A'B'D' + A'CD + A'BC$

First item

$$A'B'D'(C+C') = \underset{0010}{\overset{(2)}{A'B'CD'}} + \underset{0000}{\overset{(0)}{A'B'C'D'}}$$

Second term

$$A'CD(B+B') = \underset{0111}{\overset{(7)}{A'BCD}} + \underset{0011}{\overset{(3)}{A'B'CD}}$$

Third term

$$A'BC(D+D') = \underset{0111}{\overset{(7)}{A'BCD}} + \underset{0110}{\overset{(6)}{A'BCD'}}$$

$d = A'BC'D + ACD + AB'D'$

First term

$$\underset{0101}{\overset{(5)}{A'BC'D}}$$

Second term

$$ACD(B+B') = \underset{1111}{\overset{(15)}{ABCD}} + \underset{1011}{\overset{(11)}{AB'CD}}$$

Third term

$$AB'D'(C+C') = \underset{1010}{\overset{(10)}{AB'CD'}} + \underset{1000}{\overset{(8)}{AB'C'D'}}$$

$F(A,B,C,D) = \Sigma m(0, 2, 3, 6, 7) + \Sigma d(5, 8, 10, 11, 15)$

AB \ CD	$C'D'$ 00	$C'D$ 01	CD 11	CD' 10
$A'B'$ 00	1_{0}	$_{1}$	1_{3}	1_{2}
$A'B$ 01	$_{4}$	X_{5}	1_{7}	1_{6}
AB 11	$_{12}$	$_{13}$	X_{15}	$_{14}$
AB' 10	X_{8}	$_{9}$	X_{11}	X_{10}

SOP

$F = B'D' + A'C$

Quine Mc- Cluskey Method (Tabulation Method)

Minimization by K-map is very efficient, if no. of variables in the function is less than 4. If the no. of variable higher, K-map become tedious & inconvient, it is basically trial & error method used for grouping.

Tabulation Method is developed by Quine, which was modified by Mc-Clusky so it is known as Quine - McCluskey minimization techniques.

It consists of two parts

① To find all the terms that are to be included in the simplified fonction. These terms are called prime implicants.

② To choose among the P.M those that give an expression with least no. of literals.

The sum of P.M gives a

In SOP simplified expression for the boolean function

Use a K-map to find the simplest possible logic expression involving the variables S, M, T, and H for turning on the sprinkler system.

Solution

The given circumstances 1, 2, 3, 4, and 5 are expressed in terms of the defined variables S, M, T, and H as $\overline{M}\,\overline{S}$, $T\overline{M}S$, THS, $\overline{T}\,\overline{M}S$, and $T\overline{H}$, respectively.

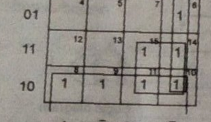
The Boolean expression is

$$\overline{S}\,\overline{M} + S\overline{M}T + STH + S\overline{M}\,\overline{T} + T\overline{H}$$
$$= 0\,0\,X\,X + 1\,0\,1\,X + 1\,X\,1\,1 + 1\,0\,0\,X + X\,X\,1\,0$$

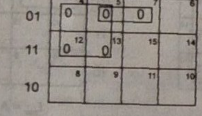
The expressions in terms of minterms and maxterms are

$$\Sigma\, m(0, 1, 2, 3, 6, 8, 9, 10, 11, 14, 15)$$
$$\Pi\, M(4, 5, 7, 12, 13)$$

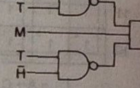
The K-maps in the SOP and POS forms, their minimization, the minimal expressions obtained from each, and the logic diagram in the SOP form are all shown below. Both the SOP and POS forms give the same minimum.



$f_{min} = \overline{M} + ST + T\overline{H}$



$f_{min} = (\overline{M} + T)(S + \overline{M} + \overline{H})$



Logic diagram

6.15 QUINE-McCLUSKY METHOD

The minimization of Boolean expressions using K-maps is usually limited to a maximum of six variables. The Quine-McClusky method, also known as the tabular method, is a more systematic method of minimizing expressions of even larger number of variables. This is suitable for hand computation as well as computation by machines.

The fundamental idea on which this tabulation procedure is based is that, repeated application of the combining theorem $PA + P\overline{A} = P$ (where P is a set of literals) on all adjacent pairs of terms, yields the set of all prime implicants, from which a minimal sum may be selected.

Consider the minimization of the expression

$$\Sigma\, m(0, 1, 4, 5) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}C + A\overline{B}\,\overline{C} + A\overline{B}C$$

The first two terms, and the third and fourth terms can be combined to yield

$$\overline{A}\,\overline{B}(C + \overline{C}) + A\overline{B}(C + \overline{C}) = \overline{A}\,\overline{B} + A\overline{B}$$

This expression can further be reduced to

$$\overline{B}(\overline{A} + A) = \overline{B}$$

In the first step, we combined two pairs of adjacent terms, each of 3 literals per term, into two terms each of two literals. In the second step, these two terms are combined again and reduced to one term of a single variable.

The same result can be obtained by combining m_0 and m_4 and m_1 and m_5 in the first step and the resulting terms in the second step. Minterms $m_0(\overline{A}\,\overline{B}\,\overline{C})$ and $m_1(\overline{A}\,\overline{B}C)$ are adjacent to each other because they differ in only literal C. Similarly, minterms $m_4(A\overline{B}\,\overline{C})$ and $m_5(A\overline{B}C)$ are adjacent to each other because they differ in only one literal C. Minterms $m_0(\overline{A}\,\overline{B}\,\overline{C})$ and $m_5(A\overline{B}C)$ or $m_1(\overline{A}\,\overline{B}C)$ and $m_4(A\overline{B}\,\overline{C})$ cannot be combined, being not adjacent to each other since they differ in more than one variable. If we consider the binary representation of minterms, $m_0(0\ 0\ 0)$ and $m_1(0\ 0\ 1)$, i.e. 0 0 0 and 0 0 1, they differ in only one position. When combined they result in 0 0 –, i.e. variable C is absorbed. Similarly, $m_4(1\ 0\ 0)$ and $m_5(1\ 0\ 1)$, i.e. 1 0 0 and 1 0 1 differ in only one position. So, when combined, they result in 10 –. Now 0 0 – and 1 0 – can be combined because they differ in only one position. The result is a – 0 –.

For the binary representation of two minterms to be different in just one position, it is necessary (but not sufficient) that the number of 1s in those two minterms differ exactly by one. Consequently, to facilitate the combination process, the minterms are arranged in groups according to the number of 1s in their binary representation.

The procedure for the minimization of a Boolean expression by the tabular method may, therefore, be described as below.

1. Arrange all minterms in groups of the same number of 1s in their binary representation. Start with the least number of 1s group and continue with groups of increasing number of 1s. The number of 1s in a term is called the ***index*** of the term.
2. Compare each term of the lowest index group with every term in the succeeding group. Whenever possible, combine the two terms being compared by means of the combining theorem. Repeat this by comparing each term in a group of index i with every term in the group of index $i + 1$, until all possible applications of the combining theorem have been exhausted.

   Two terms from adjacent groups are combinable, if their binary representations differ by just a single digit in the same position; the combined terms consist of the original fixed representation with the differing one replaced by a dash (–). Place a check mark (✓) next to every term, which has been combined with at least one term (each term may be combined with several terms, but only a single check is required).
3. Compare the terms generated in step 2 in the same fashion; combine two terms which differ by only a single 1 and whose dashes are in the same position to

generate a new term. Continue the process until no further combinations are possible. The remaining unchecked terms constitute the set of prime implicants of the expression. They are called ***prime implicants*** because they are not covered by any other term with fewer literals.

Quine Mc-cluskey Method

U.Q) 1) Simplify the boolean Function $F(w,x,y,z) = \sum m(0,5,7,8,9,10,11,14,15)$ using Quine McCluskey Method.

U.Q2) Simplify the following Boolean function by means of tabulation method.

$$F(w,x,y,z) = \sum(1,4,6,7,8,9,10,11,15)$$

U.Q3) Simplify the boolean function $F(A,B,C,D) = \sum m(1,3,4,5,10,12,13,15)$ using Quine - McCluskey Method.

U.Q 4) Given $F(A,B,C,D) = \sum(1,4,6,7,8,9,10,11,15)$. Simplify using Quine-McCluskey method and determine the prime implicants, essential prime implicants and the minimized Boolean expression.

solve in T.B eg 3-14

5) Simplify the following expression using Quine- McCluskey method.

$$F = \sum m(0,1,2,8,9,15,17,21,24,25,27,31)$$

U.Q 6) Use tabulation method to identify the simplified Boolean expression for the function, $F(w,x,y,z) = \pi(1,3,4,6,9,11,12,14)$.

1. Simplify the following Boolean function by using the tabulation method.

$$F = \sum (0, 1, 2, 3, 8, 10, 11, 14, 15)$$

0 → 0000
1 → 0001
2 → 0010
3 → 0011
8 → 1000
10 → 1010
11 → 1011
14 → 1110
15 → 1111

Index → no. of 1's

Column 1			(1 bit change) Column 2		1 bit change of same dash Column 3 (ABCD)	
Index 0	0	0000	0, 1	000_	(0, 1, 2, 3)	00_ _
Index 1	1	0001	0, 2	00_0	(0, 2, 1, 3)	00_ _
	2	0010	0, 8	_000	(0, 2, 8, 10)	_0_0
	8	1000	1, 3	00_1	(0, 8, 2, 10)	_0_0
Index 2	3	0011	2, 3	001_	(2, 3, 10, 11)	_01_
	10	1010	2, 10	_010	(2, 10, 3, 11)	_01_
Index 3	11	1011	8, 10	10_0	(10, 11, 14, 15)	1_1_
	14	1110	3, 11	_011	(10, 14, 11, 15)	1_1_
Index 4	15	1111	10, 11	101_		
			10, 14	1_10		
			11, 15	1_11		
			14, 15	111_		

$A'B' + B'D' + B'C + AC$

	0	1	2	3	8	10	11	14	15
✓ (0, 1, 2, 3)	X	X	X	X					
✓ (0, 2, 8, 10)	X		X		X	X			
(2, 3, 10, 11)			X	X		X	X		
✓ (10, 11, 14, 15)						X	X	X	X
	✓✓	✓	✓✓	✓✓	✓	✓✓	✓✓	✓	✓

Essential P.I → 1, 8, 14, 15

minimized form $F = A'B' + B'D' + AC$

(2) Simplify the following expression using Quine-Mccluskey method.

$F = \Sigma m(0, 1, 2, 8, 9, 15, 17, 21, 24, 25, 27, 31)$

0	00000
1	00001
2	00010
8	01000
9	01001
15	01111
17	10001
21	10101
24	11000
25	11001
27	11011
31	11111

2 | 24 0
2 | 12 0
2 | 6 0
2 | 3 1
1

2 | 15 1
2 | 7 1
2 | 3 1
1

2 | 17 1
2 | 8 0
2 | 4 0
2 | 2 0
1

2 | 31 1
2 | 15 1
2 | 7 1
2 | 3 1
1

2 | 21 1
2 | 10 0
2 | 5 1
2 | 2 0
1

Lecture Note

Column1

Index 0
0 00000

Index 1
1 00001
2 00010
8 01000

Index 2
9 01001
17 10001
24 11000

Index 3
21 10101
25 11001

Index 4
15 01111
27 11011

Index 5
31 11111

Column 2
(1 bit change)

(0, 1) 0000_
(0, 2) 000_0
(0, 8) 0_000

(1, 9) 0_001
(1, 17) _0001

(8, 9) 0100_
(8, 24) _1000

(9, 25) _1001

(17, 21) 10_01
(17, 25) 1_001

(24, 25) 1100_

(25, 27) 110_1

(15, 31) _1111

(27, 31) 11_11

Column 3
1 bit change & same dash

(0, 1, 8, 9) 0_00_
(0, 8, 1, 9) 0_00_
(1, 9, 17, 25) __001
(1, 17, 9, 25) __001
(8, 9, 24, 25) _100_
(8, 24, 9, 25) _100_

missing → 2, 15, 21, 27, 31

(0, 2) 000_0
(15, 31) _1111
(17, 21) 10_01
(25, 27) 110_1
(27, 31) 11_11

	0	1	2	8	9	15	17	21	24	25	27	31
(0,1,8,9)	X	X		X	X							
(1,9,17,25)		X			X		X			X		
(8,9,24,25)				X	X				X	X		
(0,2)	X		X									
(15,31)						X						X
(17,21)							X	X				
(25,27)										X	X	
(27,31)											X	X

Essential P.I $\rightarrow$ 2, 15, 21, 24

$$F = A'C'D' + C'D'E + BC'D' + A'B'C'E' + BCDE + AB'D'E + ABC'E + ABDE$$

minimized

$$F = BC'D' + A'B'C'E' + BCDE + AB'D'E + A'C'D' + ABC'E$$

U.Q 1) Simplify the boolean function
F(w, x, y, z) = $\sum m(0,5,7,8,9,10,11,14,15)$
using Quine McClusky Method.

0	0000
5	0101
7	0111
8	1000
9	1001
10	1010
11	1011
14	1110
15	1111

Column 1			Column 2		Column 3 (wxyz)	
Index 0	0	0000	(0,8)	_000	(8,9,10,11)	10_ _
Index 1	8	1000	(8,9)	100_	(10,11,14,15)	1_1_
Index 2	5	0101	(5,7)	01_1	(10,14,11,15)	1_1_
	9	1001	(9,11)	10_1		
	10	1010	(10,11)	101_		
Index 3	7	0111	(10,14)	1_10		
	11	1011	(7,15)	_111		
	14	1110	11,15	1_11		
Index 4	15	1111	14,15	111_		

missing
(0,8) _000
(5,7) 01_1
(7,15) _111

F = wx' + wy + x'y'z' + w'xz + xyz

	0	5	7	8	9	10	11	14	15
✓ (8, 9, 10, 11)				X	X	X	X		
✓ (10, 11, 14, 15)						X	X	X	X
✓ (0, 8)	X			X					
✓ (5, 7)		X	X						
✓ (7, 15)			X						X
	✓	✓	✓	✓	✓	✓	✓	✓	✓

Essential P.I = 0, 5, 9, 14

$F_{minimized} = wx' + wy + x'y'z' + w'xz$

Verification Q2

AB \ CDE	000	001	011	010	110	111	101	100
00	1 (0)	1 (1)	3	1 (2)	6	7	5	4
01	1 (8)	1 (9)	11	10	14	1 (15)	13	12
11	1 (24)	1 (25)	1 (27)	26	30	1 (31)	29	28
10	16	1 (17)	19	18	22	23	1 (21)	20

$A'B'C'E' + A'C'D' + BC'D' + BCDE$
$+ ABC'E + AB'D'E$

16	10000
17	10001
18	10010
19	10011
20	10100
21	10101
22	10110
23	10111
24	11000
25	11001
26	11010
27	11011
28	11100
29	11101
30	11110
31	11111

2 | 24 0
2 | 12 0
2 | 6 0
2 | 3 1
1

Q) Find the minimal expression for

$\Pi M(2,3,8,12,13) \cdot d(10,14)$

2	0010
3	0011
8	1000
12	1100
13	1101
10	1010
14	1110

Column 1			Column 2	Column 3 (ABCD)
Index 1	✓2	0010	✓(2,3) 001_	(8,10,12,14) 1_ _0
	✓8	1000	✓(2,10) _010	(8,12,10,14) 1_ _0
Index 2	✓3	0011	✓(8,10) 10_0	
	✓12	1010	✓(8,12) 1_00	
	12	1100	✓(10,14) 1_10	
Index 3	13	1101	(12,13) 110_	
	14	1110	✓(12,14) 11_0	

missing

2, 3, 13,

(2,3) 001_

(2,10) _010

(12,13) 110_

$$F = (A' + D)(A + B + C')(B + C' + D)(A' + B' + C)$$

"Prime Implicant table"

	2	3	8	12	13	10	14
✓ (8, 10, 12, 14)			✓	✓		✓	✓
✓ (2, 3)	✓	✓					
(2, 10)	✓					✓	
✓ (12, 13)				✓	✓		
	✓✓	✓	✓	✓✓	✓	✓✓	✓

Essential P.I → 3, 8, 13, 14

Minimized $F = (A'+D)(A+B+C')(A'+B'+C)$

Verification

AB \ CD	00	01	11	10
00			0	0
01				
11	0	0		X
10	0			X

$F = (A'+D)(A'+B'+C)$
$(A+B+C')$