Module: 1 Topic: <u>Number Systems</u>

<u>Lecture No: 1</u>

Number Systems

① Decimal Number system

* $0 \rightarrow 9$
* Position weighted system
* radix or base $\rightarrow 10$

eg: $(243.56)_{10} = 2\times10^2 + 4\times10^1 + 3\times10^0 + 5\times10^{-1} + 6\times10^{-2}$

② Binary Number System

* 0, 1
* Position weighted system
* Radix or base $\rightarrow 2$

eg: $(1\,0\,1\,0\,1)_2$

$\downarrow\ \downarrow\ \downarrow\ \downarrow\ \downarrow$

$2^4\ 2^3\ 2^2\ 2^1\ 2^0$

16 8 4 2 1

$1\times2^0 + 0\times2^1 + 1\times2^2 + 0\times2^3 + 1\times2^4$

$= 1+0+4+0+16$

$= (21)_{10}$

$(11011.101)_2$

$\downarrow\downarrow\downarrow\downarrow\downarrow\ \ \downarrow\downarrow\downarrow$

$2^4 2^3 2^2 2^1 2^0\ \ 2^{-1} 2^{-2} 2^{-3}$

$1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0 + 1\times2^{-1} + 0\times2^{-2} + 1\times2^{-3}$

$= 16 + 8 + 0 + 2 + 1 + 0.5 + 0 + 0.125$

$= (27.685)_{10}$

Note

$2^{-1} = 0.5$
$2^{-2} = 0.25$
$2^{-3} = 0.125$
$2^{-4} = 0.0625$
$2^{-5} = 0.03125$

③ Octal Number System

* 0 → 7
* Position Weighted System
* Radix or base → 8

$8^{-1} = 0.125$
$8^{-2} = 0.01562$
$8^{-3} = 0.00195$
$8^{-4} = 0.00024$

eg: $(360)_8$ $\quad 3\times8^2 + 6\times8^1 + 0\times8^0$

$\uparrow\uparrow\uparrow$

$8^2\ 8^1\ 8^0$

$= 192 + 48 + 0$

$= (240)_{10}$

④ Hexa decimal Number System

* 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
* 0 → 9 & A → F

10 → A
11 → B
12 → C
13 → D
14 → E
15 → F

* Postion Weighted System
* Radix or base → 16 or H

eg: $(7D0)_H$ $\quad 7\times16^2 + D\times16^1 + 0\times16^0$

$\downarrow\downarrow\downarrow$

$16^2\ 16^1\ 16^0$

$= 7\times256 + 13\times16 + 0$

$= 1792 + 208 + 0$

$= (2000)_{10}$

Decimal (base 10)	Binary (base 2)	Octal (base 8)	Hexadecimal (base 16)
00	0000	00	0
01	0001	01	1
02	0010	02	2
03	0011	03	3
04	0100	04	4
05	0101	05	5
06	0110	06	6
07	0111	07	7
08	1000	10	8
09	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

D B O H

10 2 8 16

DB ÷2	DO ÷8	DH ÷16
BD $\times 2^n$	BO 3 bit group	BH 4 bit group
OD $\times 8^n$	OB 3 bit eqn	OH O→B→H
HD $\times 16^n$	HB 4 bit eqn	HO H→B→O

Decimal to Binary Conversion

Step-1 **Repeated Division by 2 Method**

① Begin by dividing the decimal no. by 2.

② Divide each resulting quotient by 2 until there is 0 (zero) whole no quotient.

③ Reading the remainders from bottom to top constitute the required binary number.

$$(10)_{10} \longrightarrow (\ \ )_2$$

$$
\begin{array}{r|l l}
2 & 10 & 0 \\
2 & 5 & 1 \\
2 & 2 & 0 \\
 & 1 &
\end{array}
\qquad (1010)_2
$$

Decimal fraction can be converted to binary by repeated multiplication by 2.

Step1: Begin by multipling the given decimal fraction by 2. Then multipling each resulting fractional part of the product by 2.

Step2: Repeat step1 until the fractional product is 0 or until the desired no. of decimal place is reach.

Step3: Reading top to bottom constitute fractional binary.

eg: $(.75)_{10} \rightarrow (\ \ )_2$

$.75 \times 2 = 1.50$ 1

$.50 \times 2 = 1.00$ 1

$(0.75)_{10} \rightarrow (0.11)_2$

eg: $(35.625)_{10} \rightarrow (\ \ )_2$

2	35	1
2	17	1
2	8	0
2	4	0
2	2	0
	1	

$(35)_{10} \rightarrow (100011)_2$

$.625 \times 2 = 1.25$ 1

$.25 \times 2 = 0.50$ 0

$.50 \times 2 = 1.00$ 1

$(0.625)_{10} \rightarrow (0.101)_2$

combined

$(35.625)_{10} \rightarrow (100011.101)_2$

U.Q Convert the following decimal numbers to binary a) 12.0625 (b) 673.23

(a) $(12.0625)_{10} \longrightarrow (1100.0001)_2$

2	12	0
2	6	0
2	3	1
	1	

$0.0625 \times 2 = 0.125$ 0

$0.125 \times 2 = 0.25$ 0

$0.25 \times 2 = 0.5$ 0

$0.5 \times 2 = 1.0$ 1

(b) $(673.23)_{10} \longrightarrow ($

$$
\begin{array}{r|l}
2 & 673 \quad 1 \\
2 & 336 \quad 0 \\
2 & 168 \quad 0 \\
2 & 84 \quad 0 \\
2 & 42 \quad 0 \\
2 & 21 \quad 1 \\
2 & 10 \quad 0 \\
2 & 5 \quad 1 \\
2 & 2 \quad 0 \\
 & 1
\end{array}
$$

$0.23 \times 2 = 0.46 \quad 0$

$0.46 \times 2 = 0.92 \quad 0$

$0.92 \times 2 = 1.84 \quad 1$

$0.84 \times 2 = 1.68 \quad 1$

$0.68 \times 2 = 1.36 \quad 1$

$0.36 \times 2 = 0.72 \quad 0$

$0.72 \times 2 = 1.44 \quad 1$

$0.44 \times 2 = 0.88 \quad 0$

$0.88 \times 2 = 1.76 \quad 1$

$(673.23)_{10} \rightarrow (1010100001.001110101.....)_2$

U.Q) Perform the following conversions:
(Show the steps of conversion)

(i) $(463.25)_{10}$ to binary

$$
\begin{array}{r|l}
2 & 463 \quad 1 \\
2 & 231 \quad 1 \\
2 & 115 \quad 1 \\
2 & 57 \quad 1 \\
2 & 28 \quad 0 \\
2 & 14 \quad 0 \\
2 & 7 \quad 1 \\
2 & 3 \quad 1 \\
 & 1
\end{array}
$$

$0.25 \times 2 = 0.5 \quad 0$

$0.5 \times 2 = 1.0 \quad 1$

$(463.25)_{10} \rightarrow (111001111.01)_2$

Questions

a) $(12)_{10} \rightarrow (1100)_2$

(b) $(25)_{10} \rightarrow (11001)_2$

(c) $(58)_{10} \rightarrow (111010)_2$

(d) $(82)_{10} \rightarrow (1010010)_2$

(e) $(19)_{10} \rightarrow (10011)_2$

(f) $(45)_{10} \rightarrow (101101)_2$

(g) $(39)_{10} \rightarrow (100111)_2$

(h) $(23)_{10} \rightarrow (10111)_2$

(i) $(57)_{10} \rightarrow (111001)_2$

(j) $(45.5)_{10} \rightarrow (101101.1)_2$

(k) Convert the following decimal numbers to binary : a) 12.0625 b) 10^4 c) 673.23 d) 1998

a) $(1100.0001)_2$ b) $(10011100010000)_2$ c) $(1010100001.00111)_2$ d) $1998 \rightarrow (11111001110)_2$

(l) Convert the following binary numbers to decimal

10.10001, 101110.0101, 1110101.110, 1101101.111

$(2.53125)_{10}$ $(46.3125)_{10}$ $(117.75)_{10}$ $(109.875)_{10}$

(m) $(225.225)_{10} \rightarrow (\quad)_2$ $(11100001.001110011)_2$

(n) $(1101011.110)_2 \rightarrow (\quad)_{10}$ $(215.75)_{10}$

(o) $(1001001.011)_2 \rightarrow (\quad)_{10}$ $(73.375)_{10}$

U.Q) $(1010.011)_2 \rightarrow (10.375)_{10}$

$(630.4)_8 \rightarrow (408.5)_{10}$

Lecture No: 2

Topic: Decimal to Octal Conversion

Repeated division by 8 Method

$$\begin{array}{r|l l} 8 & 359 & 7 \\ 8 & 44 & 4 \\ & 5 & \end{array}$$

$(359)_{10} \rightarrow (547)_8$

Repeated Multiplication by 8 Method

$0.3125 \times 8 = 2.5$ 2

$.5 \times 8 = 4.0$ 4

$(0.3125)_{10} \longrightarrow (.24)_8$

eg: ① $(425.125)_{10} \longrightarrow (\quad)_8$

$$\begin{array}{r|l l} 8 & 425 & 1 \\ 8 & 53 & 5 \\ & 6 & \end{array}$$

$.125 \times 8 = 1.0$ 1

$(425.125)_{10} \longrightarrow (651.1)_8$

② $(225.225)_{10} \longrightarrow (\quad)_8$

$$\begin{array}{r|l l} 8 & 225 & 1 \\ 8 & 28 & 4 \\ & 3 & \end{array}$$

$.225 \times 8 = 1.8$ 1

$.8 \times 8 = 6.4$ 6

$.4 \times 8 = 3.2$ 3

$.2 \times 8 = 1.6$ 1

$.6 \times 8 = 4.8$ 4

$.8 \times 8 = 6.4$ 6

.....

$(341.163146)_8$

U.Q ① $(0.513)_{10}$ to Octal $(.406517\ldots)_8$

U.Q ② $(36.25)_{10}$ to octal

U.Q ③ $(378.93)_{10}$ to octal

U.Q ④ $(455)_{10}$ to octal

Topic: Decimal to hexadecimal

Repeated division by 16 Method

ⓐ $(650)_{10} \rightarrow (\quad)_{16}$

$$\begin{array}{r|l l}
16 & 650 & A \\
16 & 40 & 8 \\
 & 2 &
\end{array}$$

$(650)_{10} \rightarrow (28A)_H$

ⓑ $(2591)_{10}$

$$\begin{array}{r|l l}
16 & 2591 & F \\
16 & 161 & 1 \\
 & 10 &
\end{array}$$

$(2591)_{10} \rightarrow (A1F)_H$

U.Q ① $(250.55)_{10}$ to hexadecimal

U.Q ② $(455)_{10}$ to hexadecimal

T.B ③ $(225.225)_{10}$ to hexadecimal $(E1.399)_{16}$

T.B ④ $(215.75)_{10}$ to hexadecimal $(D7.C)_{16}$

T.B ⑤ $(403.9843)_{10}$ to " $(193.FC)_{16}$

T.B ⑥ $(10949.8125)_{10}$ to " $(2AC5.D)_{16}$

Topic: Binary to octal conversion (3 bit gp)

→ Each group of 3 bits indicate an octal digit

→ Integer part (Binary)

* Scan the B.N from right to left
* Translate each gp of 3 bits into the corresponding octal digit.
* Add leading zeros if necessary

→ For the fractional part

* Scan the B.N from left to right.
* Translate each gp of 3 bits into corresponding octal digit.
* Add trailing zeros if necessary to make a group of 3.

U.Q eg: $(110101.1011)_2$ to octal

$$\underbrace{110}_{6}\,\underbrace{101}_{5}\,.\,\underbrace{101}_{5}\,\underbrace{100}_{4} \qquad (65.54)_8$$

eg: $(10110.01)_2$ to octal

$$\underbrace{010}_{2}\,\underbrace{110}_{6}\,.\,\underbrace{010}_{2} \qquad (26.2)_8$$

T.B $(11010111.110)_2 \rightarrow (\quad)_8$

$\underbrace{011}_{3}\,\underbrace{010}_{2}\,\underbrace{111}_{7}\,.\,\underbrace{110}_{6}$

$(327.6)_8$

④ $(101111001)_2 \rightarrow (\quad)_H$

$\underbrace{101}_{5}\,\underbrace{111}_{7}\,\underbrace{001}_{1}$

$(571)_8$

Topic: Octal to Binary

To convert an octal no. to a binary number, replace each octal digit with 3 bits binary number.

0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

eg: ① $(13)_8 \rightarrow (\quad)_2$

1 3

$(001\,011)_2$

② $(723)_8 \rightarrow (\quad)_2$

7 2 3

$(111\,010\,011)_2$

③ $(1.54)_8 \rightarrow (\quad)_2$

1.5 4

$(001.101\,100)_2$

T.B $(623.77)_8 \longrightarrow (\quad)_2$

$$\begin{array}{ccccc} 6 & 2 & 3 & . & 7\ 7 \\ \downarrow & \downarrow & \downarrow & & \downarrow\ \downarrow \\ 110 & 010 & 011 & . & 111\ 111 \end{array}$$

$(110010011.111111)_2$

Topic: Binary to hexadecimal

Integer part of Binary	Fractional part of B.N
* Scan the binary number from right to left	* left to right
* Translate each gp of 4 bits into the corresponding hexadecimal digit	* → "
* Add leading zeros	* ⇒ Add trailing zeros

eg: $(10110.01)_2 \longrightarrow (\quad)_H$

$\underbrace{0001}_{1}\underbrace{0110}_{6}.\underbrace{0100}_{4}$ $= (16.4)_H$

T.B ① $(11010111.110)_2 \longrightarrow (\quad)_H$

$\underbrace{1101}_{D}\underbrace{0111}_{7}.\underbrace{1100}_{C}$ $= (D7.C)_H$

② $(0.1000010)_2 \rightarrow (\quad)_H$

$0.\underbrace{1000}_{8}\ \underbrace{0100}_{4}$ $= (.84)_H$

③ $(101.0101111)_2 \rightarrow (\quad)_H$

$\underbrace{0101}_{5}.\underbrace{0101}_{5}\ \underbrace{1110}_{E}$ $= (5.5E)_H$

Topic: <u>Hexadecimal to Binary</u>

Translate every hexadecimal digit into its 4-bit binary equivalent

eg: ① $(3A5)_{16} \Rightarrow (0011\,1010\,0101)_2$

T.B ② $(2AC5.D)_{16} \rightarrow (0010101011000101.1101)_2$

Binary	hexadecimal
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

U.Q) $(AF9.0C)_{16}$ to binary

$(AF9.0C)_{16} = (101011111001.00001100)_2$

Topic: Octal to Hexadecimal conversion

First step: Octal to binary

Second step: Binary to hexadecimal

T.B (1) $(623.77)_8 \longrightarrow (\quad)_{16}$

Octal to binary

$(623.77)_8$

↓

110010011.111111

Binary to hexadecimal

$\underbrace{0001}_{1}\underbrace{1001}_{9}\underbrace{0011}_{3} . \underbrace{1111}_{F}\underbrace{1100}_{C}$

Ans: $(193.FC)_{16}$

(2) $(327.6)_8 \longrightarrow (\quad)_{16}$

$(327.6)_8$

↓ Binary

$(\underbrace{1101}_{D}\underbrace{0111}_{7}.\underbrace{1100}_{C})_2$

Ans: $(D7.C)_{16}$

Topic: Hexadecimal to Octal Conversion

First step: Hexadecimal to binary

Second step: binary to Octal

T.B (1) $(2AC5.D)_{16} \longrightarrow (\quad)_8$

$$(2AC5.D)_{16}$$

$$\downarrow \text{binary}$$

$$(0010101011000101.110100)_2$$

$$\downarrow \text{octal}$$

$$(2\ 5\ 3\ 0\ 5.6\ 4)_8$$

Ans: $(25305.64)_8$

(2) $(E1.399)_{16} \longrightarrow (\quad)_8$

$$(E1.399)_{16}$$

$$\downarrow \text{binary}$$

$$(011100001.001110011001)_2$$

$$\downarrow \text{octal}$$

$$(3\ 4\ 1.1\ 6\ 3\ 1)_8$$

Ans: $(341.1631)_8$

Topic: Representation of negative numbers

When an integer binary no. is +ve, the sign is indicated by '0' and the magnitude by a positive binary number.

eg: consider the signed number 12 stored in an 8 bit register.

+12 : 00001100

'0' Indicate +ve number.

Although there is only one way to represent +12, there are 3 different ways to represent −12, with eight bits

① Signed magnitude form.

In this negative number consists of the magnitude and a negative sign.

∴ S.M.F of −12 is obtained from +12 by complementing only the sign bit.

sign bit (+ve)

+12 : 00001100

S.M.F −12 : 10001100

−ve (sign bit)

② 1's complement form

The signed 1's complement form of −12 is obtained by complementing all the bits of +12, including the sign bit. {0 → 1, 1 → 0}

+12 → 00001100

1's → 11110011

③ 2's complement form

Obtained by taking the 2's complement of the +ve no. including it's sign bit.

+12 → 00001100

1's → 11110011

2's → 11110011 +

1

11110100

Q) Express the decimal no −39 as an 8-bit number in the sign magnitude, 1's complement and 2's complement forms.

sign bit

+39 → 00100111

sign magnitude −39 → 10100111

1's (−39) → 11011000

2	39	1
2	19	1
2	9	1
2	4	0
2	2	0
	1	

2's complement →

+39 → 00100111

1's → 11011000

2's → 11011000 + (1's)

$$\begin{array}{r} 11011000 \\ +\quad 1 \\ \hline 11011001 \end{array}$$

U.Q) Obtain the 1's and 2's complements for the following binary numbers

(a) 1010101 (b) 0000001

(a) 1010101

1's → 0101010

2's → 0101010 +

$$\begin{array}{r} 0101010 \\ +\quad 1 \\ \hline 0101011 \end{array}$$

(b) 0000001

1's → 1111110

2's → 1111110 +

$$\begin{array}{r} 1111110 \\ +\quad 1 \\ \hline 1111111 \end{array}$$

U.Q) Represent +51 and −51 in 1's complement and 2's complement form.

Decimal	Signed magnitude	Signed 1's complement	Signed 2's complement
+51	00110011	00110011	00110011
−51	10110011	11001100	11001101

$$\begin{array}{r|rl} 2 & 51 & 1 \\ 2 & 25 & 1 \\ 2 & 12 & 0 \\ 2 & 6 & 0 \\ 2 & 3 & 1 \\ & 1 & \end{array}$$

U.Q) Represent the following decimal numbers in signed 2's complement 8-bit nos
(i) +43 (ii) −19

Decimal	Signed magnitude	1's	2's
+43	00101011	00101011	00101011
−43	10101011	11010100	11010101
+19	00010011	00010011	00010011
−19	10010011	11101100	11101101

2 | 43 1
2 | 21 1
2 | 10 0
2 | 5 1
2 | 2 0
1

2 | 19 1
2 | 9 1
2 | 4 0
2 | 2 0
1

T.B) Obtain the 1's and 2's complement of the following binary numbers:
(i) 1010101 (ii) 0111000, (iii) 0000001
(iv) 10000 (v) 00000

(i) 1010101
1's 0101010
2's 0101010 + 1
= 0101011

(ii) 0111000
1's 1000111
2's 1000111 + 1
= 1001000

(iii) 0000001
1's 1111110
2's 1111110 + 1
= 1111111

(iv) 10000

1's 01111

2's 01111 + 1

= 10000

(v) 00000

1's 11111

2's 11111 + 1

= 100000

Homework Question

U.Q ① $(0.513)_{10}$ to octal

$0.513 \times 8 = 4.104$ 4

$0.104 \times 8 = 0.832$ 0

$0.832 \times 8 = 6.656$ 6

$0.656 \times 8 = 5.248$ 5

$0.248 \times 8 = 1.984$ 1

$0.984 \times 8 = 7.872$ 7 . . . .

Ans: $(0.406517)_8$

U.Q ② $(36.25)_{10}$ to octal

$8 \underline{|36}$ 4

4

$.25 \times 8 = 2.00$ 2

Ans: $(44.2)_8$

U.Q ③ $(378.93)_{10}$ to octal

$8 \underline{|378}$ 2

$8 \underline{|47}$ 7

5

$.93 \times 8 = 7.44$ 7

$.44 \times 8 = 3.52$ 3

$.52 \times 8 = 4.16$ 4

$.16 \times 8 = 1.28$ 1

$.28 \times 8 = 2.24$ 2

$.24 \times 8 = 1.92$ 1

$(572.734121)_8$

④ $(455)_{10}$ to octal

Ans: $(707)_8$

$$\begin{array}{r|l l} 8 & 455 & 7 \\ 8 & 56 & 0 \\ & 7 & \end{array}$$

Homework Question

U.Q ① $(250.55)_{10}$ to $(\ )_{16}$

$$\begin{array}{r|l l} 16 & 250 & A \\ & 15(F) & \end{array}$$

$.55 \times 16 = 8.8 \quad 8$

$.8 \times 16 = 12.8 \quad 12 \quad C$

$.8 \times 16 = 12.8 \quad C \ldots$

Ans: $(FA.8CC\ldots)_{16}$

U.Q ② $(445)_{10}$ to $(\ )_{16}$

$$\begin{array}{r|l l} 16 & 445 & 13(D) \\ 16 & 27 & 11(B) \\ & 1 & \end{array}$$

Ans: $(1BD)_{16}$

U.Q ③ $(225.225)_{10}$ to $(\ )_{16}$

$$\begin{array}{r|l l} 16 & 225 & 1 \\ & 14(E) & \end{array}$$

$.225 \times 16 = 3.6 \quad 3$

$.6 \times 16 = 9.6 \quad 9$

$.6 \times 16 = 9.6 \quad 9 \ldots$

Ans: $(E1.399\ldots)_{16}$

U.Q ④ $(215.75)_{10}$ to $(\ )_{16}$

$$\begin{array}{r|l l} 16 & 215 & 7 \\ & 13(D) & \end{array}$$

$.75 \times 16 = 12(C)$

Ans: $(D7.C)_{16}$

Topic: Representation of BCD numbers

BCD → Binary Coded Decimal

Binary coded decimal (BCD) is a way to express each of the decimal digits with a binary code. These are only ten code groups inthe BCD system. →

The 8421 code is a type of BCD code. BCD means that each decimal digit 0 through 9 is represented by a binary code of 4 bits. The designation 8421 indicates the binary weight of the four bits ($2^3, 2^2, 2^1, 2^0$).

Decimal Digit	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Invalid Codes

With 4 bits, 16 nos. (0000 to 1111) can be represented, but in 8421 code, only ten of these are used. The six code combination that are not used:-
1010, 1011, 1100, 1101, 1110, 1111 are invalid in the 8421 BCD code

To express any decimal number in BCD simply replace each decimal digit with the appropriate 4-bit code.

eg: ① $(3)_{10} \longrightarrow 0011$

② $(9.2)_{10}$ → 1001 . 0010

③ $(34.8)_{10}$ → 00110100.1000

④ $(2469)_{10} \rightarrow 0010010001101001$

⑤ $(1472)_{10} \rightarrow 0001010001110010$

Convert BCD to Decimal

① $\underbrace{0011}_{3}\underbrace{0001}_{1}$ Ans: $(31)_{10}$

② 0001100001100000.0111 Ans: $(1860.7)_{10}$

Topic : Addition of Binary numbers

A (Augend)	B (Addend)	S (Sum)	C (Carry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

(a) $(1010)_2 + (0101)_2$

$$\begin{array}{r} 1010 + \\ 0101 \\ \hline 1111 \end{array}$$

(b) Add $(10)_{10} + (11)_{10}$ in binary

$$\begin{array}{r|l} 2 & 10 \quad 0 \\ 2 & 5 \quad 1 \\ 2 & 2 \quad 0 \\ & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 11 \quad 1 \\ 2 & 5 \quad 1 \\ 2 & 2 \quad 0 \\ & 1 \end{array}$$

$$\begin{array}{r} \overset{1}{1}010 + \\ 1011 \\ \hline 10101 \end{array}$$

(c) Add 1011.011 and 110.1

$$\begin{array}{r} \overset{1}{1}\overset{1}{0}11 \cdot 011 \\ 110 \cdot 100 \\ \hline 10001 \cdot 111 \end{array}$$

Topic: Subtraction of Binary Nos.

(a) Direct Subtraction

A (Minuend)	B (Subtrahend)	D (Difference)	B (Borrow)
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

(a) Subtract 10101 from 11011

```
 01
11011 -        27 -
10101          21
-----          --
00110           6
```

(b)
```
0222
10000 -
 1010
-----
00110
```

(c)
```
  2
0 10
10110 -
 1010
-----
 1100
```

(d)
```
  1 2    2
0 2 10 0 10
1001.10 -
0111.01
-------
0010.01
```

(e)
```
 2
0 10
1010.11 -
0110.01
-------
0100.10
```

(b) Complement Subtraction

1. 1's complement subtraction method

case(i) Subtraction of smaller number from larger number.

procedure

1. Determine 1's complement of the smaller number (subtrahend)
2. Add 1's complement to the larger number
3. Carry generated after the addition is called "End Around carry". Remove EAC and add it to the result.

① Subtract 101011 from 111101 using 1's

111101 − 101011

1's of 101011 = 010100

```
   111101 +
   010100
  --------
[1] 010001 +
        1   Add EAC
  --------
   010010   = (18)10
```

② 11100 − 01000

1's of 01000 = 10111

$(28)_{10} - (8)_{10}$
$= (20)_{10}$

```
   11100 +
   10111
  -------
[1] 10011 +
        1
  -------
   10100
```

③ 25.5 − 12.25

11001.1 − 01100.01

$$\begin{array}{r} 11001.1\ + \\ 10011.10 \\ \hline \boxed{1}\ 01101.00\ + \\ 1 \\ \hline 01101.01 \end{array}$$

$$\begin{array}{l|ll} 2 & 25 & 1 \\ 2 & 12 & 0 \\ 2 & 6 & 0 \\ 2 & 3 & 1 \\ & 1 & \end{array}$$

.25 × 2 = 0.5 0

.5 × 2 = 1.0 1

1's complement of

01100.01

10011.10

④ 10.625 − 8.75

1010.101 − 1000.110

$$\begin{array}{r} 1010.101\ + \\ 0111.001 \\ \hline \boxed{1}\ 0001.110\ + \\ 1 \\ \hline 0001.111 \end{array}$$ $(1.875)_{10}$

.625 × 2 = 1.25 1

.25 × 2 = 0.50 0

.50 × 2 = 1.00 1

.75 × 2 = 1.50 1

.50 × 2 = 1.00 1

1's comp of 1000.110

0111.001

H.W 30 − 25

(case ii) Subtraction of larger number from smaller number.

① Determine the 1's complement of the larger number (subtrahend)

② Add 1's complement to the smaller number (minuend)

③ After addition no carry will be generated by answer in 1's complement form (-ve no.)

To get the answer in true form, take the 1's complement of it and assign -ve sign to the answer.

a) 43 - 57

$(101011)_2 - (111001)_2$

1's of 111001

000110

$$\begin{array}{r} 101011 + \\ 000110 \\ \hline 110001 \end{array}$$

The result has no carry, so the answer is in 1's complement form

1's of 110001 → 001110

→ -001110

→ $(-14)_{10}$

$$\begin{array}{r|l} 2 & 43 \;\; 1 \\ 2 & 21 \;\; 1 \\ 2 & 10 \;\; 0 \\ 2 & 5 \;\; 1 \\ 2 & 2 \;\; 0 \\ & 1 \end{array}$$

$$\begin{array}{r|l} 2 & 57 \;\; 1 \\ 2 & 28 \;\; 0 \\ 2 & 14 \;\; 0 \\ 2 & 7 \;\; 1 \\ 2 & 3 \;\; 1 \\ & 1 \end{array}$$

(b) 8.75 - 10.625

1000.110 - 1010.101

1's of 1010.101

0101.010

$$\begin{array}{r} 1000.110 + \\ 0101.010 \\ \hline 1110.000 \end{array}$$

1's of 1110.000

1's (0001.111) -$(1.875)_{10}$

Ans: -0001.111

Ans: -$(1.875)_{10}$

.75×2 = 1.50 1

.5 ×2 = 1.00 1

$$\begin{array}{r|l} 2 & 8 \;\; 0 \\ 2 & 4 \;\; 0 \\ 2 & 2 \;\; 0 \\ & 1 \end{array}$$

.625×2 = 1.25 1

.25×2 = 0.5 0

.5 ×2 = 1.0 1

H.W ① $5/8 - 7/8$ Ans: $-(\cdot 010)_2$
$-(\cdot 25)_{10}$

② $16.875 - 11.125$ Ans: 0101.110
$(5.75)_{10}$

Topic 2's complement method

Case (i) Subtraction of a smaller number from larger number

1. Determine the 2's complement of the small number (subtrahend)
2. Add 2's complement to the minuend
3. Discard the carry generated.

ⓐ $(111001)_2 - (101011)_2$ $57 - 43 = 14$

2's of 101011 is 010100 + 1 = 010101

111001 +
010101

$\boxed{1}$ 001110

Discard carry

ⓑ $(100.5)_{10} - (50.75)_{10}$

1100100.10 − 0110010.11

$.75 \times 2 = 1.50$ 1
$.5 \times 2 = 1.00$ 1

2	100	0
2	50	0
2	25	1
2	12	0
2	6	0
2	3	1
	1	

2's of 0110010.11 is 1001101.00 +
1
1001101.01

1100100.10 +
1001101.01

$\boxed{1}$ 0110001.11 $(49.75)_{10}$

Discard carry

ⓒ $(1111)_2 - (1010)_2$

2's of 1010 is 0101 +
1
0110

1111 +
0110

$\boxed{1}$ 0101

H.W ① $(112)_{10} - (65)_{10}$ using 2's form.

Ans: $(0101111)_2$
$(47)_{10}$

② $(22)_{10} - (7)_{10}$ using 2's form.

Ans: $(1111)_2$, $(15)_{10}$

(case ii) Subtraction of a larger number from smaller number.

1. Determine the 2's complement of the subtrahend.
2. Add the 2's complement to minuend.
3. Answer is in 2's complement form. Take the 2's complement and assign negative sign to the answer.
   No carry will be generated.

(a) 7 − 22

$(00111)_2 - (10110)_2$

$$\begin{array}{r|l} 2 & 22 \quad 0 \\ 2 & 11 \quad 1 \\ 2 & 5 \quad 1 \\ 2 & 2 \quad 0 \\ & 1 \end{array}$$

2's of 10110 → 01001 +

$$\begin{array}{r} 01001 \\ +\ \ 1 \\ \hline 01010 \end{array}$$

$$\begin{array}{r} 01010 \\ +\ 00111 \\ \hline 10001 \end{array}$$

2's of 10001 is 01110 +

$$\begin{array}{r} 01110 \\ +\ \ 1 \\ \hline -(01111)_2 \end{array}$$

$= -(15)_{10}$

(b) 16.5 − 24.75

$(10000.10)_2 - (11000.11)_2$

.75 × 2 = 1.50   1

.5 × 2 = 1.00   1

$$\begin{array}{r|l} 2 & 16 \quad 0 \\ 2 & 8 \quad 0 \\ 2 & 4 \quad 0 \\ 2 & 2 \quad 0 \\ & 1 \end{array} \qquad \begin{array}{r|l} 2 & 24 \quad 0 \\ 2 & 12 \quad 0 \\ 2 & 6 \quad 0 \\ 2 & 3 \quad 1 \\ & 1 \end{array}$$

2's of 11000.11 is 00111.00 +
1

00111.01

10000.10 +
00111.01

10111.11

2's of 10111.11 → 01000.00 +
1

01000.01

Ans: $-(01000.01)_2$
$-(8.25)_{10}$

Topic: Multiplication of Binary Numbers

Multiplicant A	Multiplier B	Product P
0	0	0
0	1	0
1	0	0
1	1	1

a) 7×5
0111×0101

111 ×
101

111
000
111

100011 $= (35)_{10}$

(b) 4.75 x 3.625

100.110 x 011.101

```
         100.110 x
          11.101
      -----------
          100110
        0000000
        100110
       100110
      100110
  -----------------
  100001.001110
```

```
2|4  0
2|2  0
   1
```

```
  10
   1
  ---
  11
```

(c) 27 x 21

27 → 11011
21 → 10101

```
        11011 x
        10101
      --------
        11011
       00000
      11011
     00000
    11011
  -----------
  1000110111
```

```
2|27  1
2|13  1
2|6   0
2|3   1
   1
```

```
2|21  1
2|10  0
2|5   1
2|2   0
   1
```

Topic: Division of Binary Numbers

$0 \div 1 = 0$

$1 \div 1 = 1$

a) $50 \div 5$

```
        1010
101 ) 110010
      101
      00101
        101
         00
          0
          0
```

```
2 | 50   0
2 | 25   1
2 | 12   0
2 | 6    0
2 | 3    1
    1

2 | 5    1
2 | 2    0
    1
```

(b) Divide 101101 by 110

```
        111.1
110 ) 101101
      110
      1010
       110
      01001
        110
      00110
        110
        000
```

c) Divide $(110101 \cdot 11)_2$ by $(101)_2$

```
         1010.11
     ___________
101 | 110101.11
      101
      -----
      00110
        101
        -----
        00111
          101
          -----
          0101
           101
           -----
           000
```

d) $25 \div 4$

$11001 \div 100$

```
         110.01
     ___________
100 | 11001
      100
      -----
      0100
       100
      -----
         0100
          100
         -----
          000
```

Topic r's complement & (r − 1)'s complement

r's complement	(r − 1)'s complement
2's	1's
8's	7's
10's	9's
16's	15's
$r^n - N$	$r^n - r^{-m} - N$

Where r − base

n − no. of digits in the integer part of the given the number

N − Given number.

m − no. of digits in the fractional part of the given number

complement

① 10's complement of $(52520)_{10}$

$r^n - N$

here r − 10

n − 5

N − 52520

$10^5 - 52520$

$$\begin{array}{r} 100000 - \\ 52520 \\ \hline \underline{47480} \end{array}$$

② 10's complement of $(0.3267)_{10}$

$r^n - N$

here $n \rightarrow 0$

$r \rightarrow 10$

$N = 0.3267$

$10^0 - 0.3267$

$1 - 0.3267 = \underline{0.6733}$

③ 10's complement of $(25.639)_{10}$

$r^n - N$ here $r - 10$

$n - 2$

$10^2 - 25.639$ $N - 25.639$

$$\begin{array}{r} 100.000 \\ 25.639 \\ \hline 74.361 \end{array}$$

④ 16's complement of $(4B7)_{16}$

$r^n - N$ here $r - 16$

$n - 3$

$N = 4B7$

$(16^3)_{10} - (4B7)_{16}$ $16^3 = (4096)_{10}$

$(1000)_{16} - (4B7)_{16}$

$16 \lfloor 4096 \; 0$

hexadecimal subtraction

$16 \lfloor 256 \; 0$

carry - 16

$16 \lfloor 16 \; 0$

1

$$\begin{array}{r} 1000 - \\ 4B7 \\ \hline B49 \end{array}$$

$16 \lfloor 16 \; 0$

1

0 - 9

A - F

A - 10

B - 11

C - 12

D - 13

E - 14

F - 15

Ans: $(B49)_{16}$

⑤ 2's complement of $(101100)_2$

$r^n - N$ $n - 6$

$r - 2$

$N = 101100$

$(2^6)_{10} - (101100)_2$

$(64)_{10} - 101100$

$2 \lfloor 64 \; 0$

$2 \lfloor 32 \; 0$

$2 \lfloor 16 \; 0$

$2 \lfloor 8 \; 0$

$2 \lfloor 4 \; 0$

$2 \lfloor 2 \; 0$

1

$$\begin{array}{r} 1000000 - \\ 101100 \\ \hline 0010100 \end{array}$$

Another method

2's of $(101100)_2$

1's 010011 +

2's 1

010100

⑥ 8's of $(232)_8$

$r^n - N$

r - 8

n - 3

N = 232

$(8^3)_{10} - (232)_8$

$(512)_{10} - (232)_8$

$(1000)_8 - (232)_8$

$$\begin{array}{r|l} 8 & 512 \quad 0 \\ 8 & 64 \quad 0 \\ 8 & 8 \quad 0 \\ & 1 \end{array}$$

Octal subtraction

0 - 7

carry → 8

$$\begin{array}{r} 1000 - \\ 232 \\ \hline 546 \end{array}$$

$$\begin{array}{r|l} 8 & 8 \quad 0 \\ & 1 \end{array}$$

Ans: $(546)_8$

(r-1)'s complement

① 9's complement of $(52520)_{10}$

$r^n - r^{-m} - N$

here $n = 5$
$r = 10$
$m = 0$

$= 10^5 - 10^0 - 52520$

$= 10^5 - 1 - 52520$

$= 99999 - 52520$

$= \underline{\underline{47479}}$

$$\begin{array}{r} 100000 \\ -\quad 1 \\ \hline 99999 \end{array}$$

$$\begin{array}{r} 99999 \\ -\ 52520 \\ \hline 47479 \end{array}$$

② 9's complement of $(0.3267)_{10}$

$r^n - r^{-m} - N$

here $n \to 0$
$m \Rightarrow 4$
$N \Rightarrow (0.3267)_{10}$

$= 10^0 - 10^{-4} - (0.3267)_{10}$

$= 1 - 0.0001 - (0.3267)_{10}$

$= (0.9999)_{10} - (0.3267)_{10}$

$= \underline{0.6732}$

(3) 9's complement of $(25.639)_{10}$

$$r^n - r^{-m} - N$$

here $r \to 10$
$n \to 2$
$m \to 3$
$N \to (25.639)_{10}$

$$10^2 - 10^{-3} - (25.639)_{10}$$

$$= 100 - 0.001 - (25.639)_{10}$$

$$= (99.999)_{10} - (25.639)_{10}$$

$$= \underline{\underline{(74.360)_{10}}}$$

$$\begin{array}{r} 99.999 - \\ 25.639 \\ \hline 74.360 \\ \hline \end{array}$$

(4) 7's complement of $(76)_8$

$$r^n - r^{-m} - N$$

here $n = 2$
$m = 0$
$N = (76)_8$
$r = 8$

$$= 8^2 - 8^0 - (76)_8$$

$$= (64)_{10} - 1 - (76)_8$$

$$= (63)_{10} - (76)_8$$

$$= (77)_8 - (76)_8$$

$$= \underline{(01)_8}$$

$$\begin{array}{l|l} 8 & 63 \quad 7 \\ \hline 8 & 7 \quad 0 \\ \hline & \end{array}$$

$$\begin{array}{r} 77 - \\ 76 \\ \hline 01 \end{array}$$

(5) 15's complement of $(9F2C)_{16}$

$r^n - r^{-m} - N$

$r \to 16$
$n \to 4$
$m \to 0$
$N = (9F2C)_{16}$

$= 16^4 - 16^0 - (9F2C)_{16}$

$= (65536)_{10} - 1_{10} - (9F2C)_{16}$

$= (65535)_{10} - (9F2C)_{16}$

$$\begin{array}{r|l} 16 & 65535 \quad F \\ 16 & 4095 \quad F \\ 16 & 255 \quad F \\ & 15 \end{array}$$

$= (FFFF)_{16} - (9F2C)_{16}$

$$\begin{array}{r} FFFF - \\ 9F2C \\ \hline 60D3 \\ \hline \end{array}$$

A - 10
B - 11
C - 12
D - 13
E - 14
F - 15

Ans: 60D3

Topic : Subtraction with r's & (r-1)'s Complements

r's complement

The subtraction of two +ve numbers (M - N), both of base r, may be done as follows:

1. Add the minuend M to the r's complement of the subtrahend N
2. Inspect the result obtained in step 1 for an end carry.
   (a) If an end carry occurs, discard it
   (b) If an end carry does not occur, take the r's complement of the number obtained in step 1 and place a negative sign in front.

(r - 1)'s Complement

→ same as above

1. Add the minuend M to the (r - 1)'s complement of the subtrahend N
2. Inspect the result obtained in step 1 for an end around carry

$= (-15.6875)_{10}$

(a) If an end carry occurs, add 1 to the least significant digit

(b) If an end carry doesnot occur, take the (r − 1)'s complement of the number obtained in step 1 and place a negative sign in front.

U.Q ① 9's of $(13579)_{10}$

$$\begin{array}{r} 99999- \\ 13579 \\ \hline 86420 \end{array}$$

10's of $(13579)_{10}$

$$\begin{array}{r} 100000- \\ 13579 \\ \hline 86421 \end{array}$$

U.Q ② $(11010)_2 - (1101)_2$ using (i) 2's (ii) 1's

$(11010)_2 - (01101)_2$

(i) using 2's

2's of 01101 : 01101

$$\begin{array}{r} 10010+ \\ 1 \\ \hline 10011 \end{array}$$

$$\begin{array}{r} 11010+ \\ 10011 \\ \hline \boxed{1}\ 01101 \end{array}$$

Ans: $(01101)_2$

discard carry

(ii) using 1's

1's of 01101 : 01101
10010

$$\begin{array}{r} 11010+ \\ 10010 \\ \hline \boxed{1}\ 01100+ \\ \hline 01101 \end{array}$$

Ans: $(01101)_2$

University Question

(1) Find 9's and 10's complement of $(13579)_{10}$

(2) Subtract $(1101)_2$ from $(11010)_2$ using (i) 2's (ii) 1's

(3) Perform the subtraction of following binary numbers using 2's complement representation

(i) 11010 − 10000

(ii) 100 − 110000

(4) Obtain the 1's and 2's complement for the following binary numbers:

(a) 1010101 (b) 0000001

(5) Perform subtraction of the following using r's complement and (r − 1)'s complement methods:

(i) $(7235)_{10} - (346)_{10}$ (ii) $(1000100)_2 - (1110100)_2$

(6) Find the 9's and 10's complement of $(24579.12)_{10}$

(7) Subtract $(9F2C)_{16}$ from $(A96B)_{16}$ using 15's and 16's complement method.

$= (-15.6875)_{10}$

(8) ~~Sub~~ Perform $(417)_8 - (232)_8$ using 8's complement addition.

(9) Do the following operations:

(i) Compute 1's complement of the binary number 1101.01

(ii) Compute 8's complement of the octal number 672.23

(iii) Add base-16 numbers 1FE and EF1 (hexadecimal Addition)

Ans: (i) 1101.01

1's 0010.10

(ii) 8's of $(672.23)_8$

$$\begin{array}{r} 1000.00 - \\ 672.23 \\ \hline 105.55 \end{array}$$

Ans: $(105.55)_8$

Octal (0 - 7)

dec	octal
8	10
9	11
10	12
11	13
12	14
13	15

(iii) $(1FE)_{16} + (EF1)_{16}$

$$\begin{array}{r} 1FE + \\ EF1 \\ \hline 10EF \end{array}$$

Ans: $(10EF)_{16}$

16 | 30 E, 1

$(30)_{10} \rightarrow (1E)_{16}$

16 | 16 0, 1

0	9
A	10
B	11
C	12
D	13
E	14
F	15

① Using 10's complement, subtract $(72532-3250)_{10}$

72532 − 03250

10's complement of 3250 :

$$\begin{array}{r} 100000- \\ 03250 \\ \hline 96750 \\ \hline \end{array}$$

$$\begin{array}{r} 72532+ \\ 96750 \\ \hline \boxed{1}69282 \\ \hline \end{array}$$

discard carry

Ans: $(69282)_{10}$

② Using 10's complement, subtract $(3250-72532)_{10}$

$(03250 - 72532)_{10}$

10's complement of 72532 :

$$\begin{array}{r} 100000- \\ 72532 \\ \hline 27468 \\ \hline \end{array}$$

$$\begin{array}{r} 03250+ \\ 27468 \\ \hline 30718 \\ \hline \end{array}$$

10's complement of 30718 $\left\}\begin{array}{r} 100000- \\ 30718 \\ \hline 69282 \\ \hline \end{array}\right.$

Ans: $-(69282)_{10}$

③ Using 9's complement

ⓐ $(72532 - 3250)_{10}$

ⓑ $(3250 - 72532)_{10}$

ⓐ 72532 − 03250

9's complement of 03250 :

$$\begin{array}{r} 99999- \\ 03250 \\ \hline 96749 \end{array}$$

$$\begin{array}{r} 72532+ \\ 96749 \\ \hline \boxed{1}\,69281+ \\ 1 \\ \hline 69282 \end{array}$$

Ans: $(69282)_{10}$

ⓑ 03250 − 72532

9's complement of 72532 :

$$\begin{array}{r} 99999- \\ 72532 \\ \hline 27467 \end{array}$$

$$\begin{array}{r} 03250+ \\ 27467 \\ \hline 30717 \end{array}$$

→ 9's complement of 30717

$$\begin{array}{r} 99999- \\ 30717 \\ \hline 69282 \end{array}$$

Ans: $-(69282)_{10}$

V.Q

(3) Subtract (i) $(11010 - 10000)_2$
(ii) $(100 - 110000)_2$ using 2's.

(i) 11010 − 10000

2's complement of 10000 :

```
 10000
 01111+
     1
 -----
 10000
```

```
  11010+
  10000
 ------
[1] 01010
```

discard carry

(ii) 100 − 110000

$(000100)_2 - (110000)_2$

2's complement of $(110000)_2$:

```
 110000
 001111+
      1
 ------
 010000
```

```
 000100+
 010000
 ------
 010100
```

→ 2's complement :

```
 010100
 101011+
      1
 ------
 101100
```

Ans: $-(101100)_2$

V.Q

(4) 1's and 2's complement

(a) 1010101

1's 0101010

2's
```
 0101010+
       1
 -------
 0101011
```

(b) 0000001

1's 1111110

2's
```
 1111110+
       1
 -------
 1111111
```

U.Q (7) $(A96B)_{16} - (9F2C)_{16}$

Using 15's and 16's complement

① using 15's

0 – 9	0 – 9
A – F	
10 – A	
11 – B	
12 – C	
13 – D	
14 – E	
15 – F	
Now	
16 – 10	
17 – 11	
18 – 12	
19 – 13	

15's complement of 9F2C : FFFF − 9F2C = 6 0 D 3

$$\begin{array}{r} A96B + \\ 60D3 \\ \hline \boxed{1}\,0A3E + \\ 1 \\ \hline 0A3F \end{array}$$

$$\begin{array}{r} 13 + \\ 6 \\ \hline 19 \end{array}$$

$16 \,|\, 19 \; 3$, carry 1

② Using 16's

16's complement of 9F2C : 10000 − 9F2C = 060D4

$$\begin{array}{r} A96B + \\ 60D4 \\ \hline \boxed{1}\,0A3F \end{array}$$

discard carry

U.Q (8) Perform $(417)_8 - (232)_8$ using 7's and 8's complement.

octal (0 - 7)

dec	octal
8	10
9	11
10	12
11	13
12	14
13	15

$(417)_8 - (232)_8$

using 7's

7's complement of 232 :

$$\begin{array}{r} 777 - \\ 232 \\ \hline (545)_8 \end{array}$$

$8 \lfloor 8 \; 0$, 1

$$\begin{array}{r} 417 + \\ 545 \\ \hline \boxed{1}\ 164 + \\ 1 \\ \hline 165 \end{array}$$

Ans: $(165)_8$

using 8's

8's complement of 232

$$\begin{array}{r} 1000 - \\ 232 \\ \hline (546)_8 \end{array}$$

$$\begin{array}{r} 417 + \\ 546 \\ \hline \boxed{1}\ 165 \end{array}$$

Ans: $(165)_8$

using direct subtraction

$(417)_8 - (232)_8$

$$\begin{array}{r} 417 - \\ 232 \\ \hline 165 \end{array}$$

Ans: $(165)_8$

U.Q

5 (i) using 9's and 10's

$(7235)_{10} - (346)_{10}$

using 9's

9's complement of 0346 : 9999 − 0346 = 9653

$$\begin{array}{r} 7235 + \\ 9653 \\ \hline \boxed{1}\ 6888 + \\ 1 \\ \hline 6889 \end{array}$$

Ans: $(6889)_{10}$

using 10's

10's complement of 0346 $10000 - 0346 = 9654$

$$\begin{array}{r} 7235 + \\ 9654 \\ \hline \boxed{1}\ 6889 \end{array}$$

discard carry

Ans: $(6889)_{10}$

(ii) using 1's and 2's

$(1000100)_2 - (1110100)_2$

using 1's

1's of 1110100 : 0001011

$$\begin{array}{r} 1000100 + \\ 0001011 \\ \hline 1001111 \end{array}$$

1's form

1's of 1001111 is 0110000

Ans: $-(0110000)_2$

using 2's

2's of $(1110100)_2$: 1110100

0001011 +

1

$(0001100)_2$

1000100 +

0001100

1010000 2's form

so: 2's of 1010000

0101111 +

1

0110000

Ans: $-(0110000)_2$

V.Q (6) 9's and 10's of $(24579.12)_{10}$

(1) 9's

99999.99 −

24579.12

75420.87

(2) 10's

99999.90 −

24579.12

75420.88

<u>Text book</u> ① Obtain the 9's and 10's complement of the following decimal numbers

(i) 13579 (ii) 09900 (iii) 90090

(iv) 10000 (v) 00000

(i) $(13579)_{10}$

$$9\text{'s}: \begin{array}{r} 99999- \\ 13579 \\ \hline 86420 \\ \hline \end{array} \qquad 10\text{'s}: \begin{array}{r} 099991 \\ \cancel{1}00000- \\ 13579 \\ \hline 86421 \\ \hline \end{array}$$

(ii) $(09900)_{10}$

$$9\text{'s}: \begin{array}{r} 99999- \\ 09900 \\ \hline 90099 \\ \hline \end{array} \qquad 10\text{'s}: \begin{array}{r} 0991 \\ \cancel{1}00000- \\ 09900 \\ \hline 90100 \\ \hline \end{array}$$

(iii) $(90090)_{10}$

$$9\text{'s}: \begin{array}{r} 99999- \\ 90090 \\ \hline 09909 \\ \hline \end{array} \qquad 10\text{'s}: \begin{array}{r} 9991 \\ 100000- \\ 90090 \\ \hline 09910 \\ \hline \end{array}$$

(iv) 10000

$$9\text{'s}: \begin{array}{r} 99999- \\ 10000 \\ \hline 89999 \\ \hline \end{array} \qquad 10\text{'s}: \begin{array}{r} 10 \\ 0 \\ \cancel{1}00000- \\ 10000 \\ \hline 90000 \\ \hline \end{array}$$

(v) 00000

$$9\text{'s}: \begin{array}{r} 99999- \\ 00000 \\ \hline 99999 \\ \hline \end{array} \qquad 10\text{'s}: \begin{array}{r} 100000- \\ 00000 \\ \hline 100000 \\ \hline \end{array}$$

T.B (2) Perform the subtraction with the following decimal numbers using
(i) 10's complement (2) 9's complement

a) 5250 − 321 b) 753 − 864 c) 3570 − 2100

d) 20 − 1000

(a) 5250 − 321

(i) 10's:

$$\begin{array}{r} 10000 - \\ 0321 \\ \hline \underline{9679} \end{array}$$

$$\begin{array}{r} 5250 + \\ 9679 \\ \hline \boxed{1}\ 4929 \end{array}$$

discard carry

Ans: $(4929)_{10}$

(ii) 9's

$$\begin{array}{r} 9999 - \\ 0321 \\ \hline \underline{9678} \end{array}$$

$$\begin{array}{r} 5250 + \\ 9678 \\ \hline \boxed{1}\ 4928 + \\ 1 \\ \hline \underline{4929} \end{array}$$

Ans: $(4929)_{10}$

(b) 753 − 864

(i) 10's:

$$\begin{array}{r} 1000 - \\ 864 \\ \hline \underline{136} \end{array}$$

$$\begin{array}{r} 753 + \\ 136 \\ \hline \underline{889} \end{array}$$

10's:

$$\begin{array}{r} 1000 - \\ 889 \\ \hline \underline{111} \end{array}$$

Ans: $-(111)_{10}$

(ii) 9's:

$$\begin{array}{r} 999 - \\ 864 \\ \hline \underline{135} \end{array}$$

$$\begin{array}{r} 753 + \\ 135 \\ \hline \underline{888} \end{array}$$

9's:

$$\begin{array}{r} 999 - \\ 888 \\ \hline \underline{111} \end{array}$$

Ans: $-(111)_{10}$

(c) $(3570)_{10} - (2100)_{10}$

9's:
$$\begin{array}{r} 9999- \\ 2100 \\ \hline 7899 \end{array}$$

$$\begin{array}{r} 3570+ \\ 7899 \\ \hline \boxed{1}\ 1469+ \\ 1 \\ \hline 1470 \end{array}$$

10's:
$$\begin{array}{r} 10000 \\ 2100 \\ \hline 7900 \end{array}$$

$$\begin{array}{r} 3570+ \\ 7900 \\ \hline \boxed{1}\ 1470 \end{array}$$

discard carry

Ans: $(470)_{10}$

(d) $(20)_{10} - (1000)_{10}$

9's:
$$\begin{array}{r} 9999- \\ 0020 \\ \hline 9979 \end{array}$$

$$\begin{array}{r} 0020+ \\ 8999 \\ \hline 9019 \end{array}$$

no carry

9's:
$$\begin{array}{r} 9999- \\ 9019 \\ \hline 0980 \end{array}$$

Ans: $-(980)_{10}$

10's:
$$\begin{array}{r} 10000- \\ 1000 \\ \hline 9000 \end{array}$$

$$\begin{array}{r} 0020+ \\ 9000 \\ \hline 9020 \end{array}$$

No carry

Hence 10's:
$$\begin{array}{r} 10000- \\ 9020 \\ \hline 0980 \end{array}$$

Ans: $-(980)_{10}$

Topic : Addition of BCD Numbers

Dec	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

(i) BCD Addition

Procedure

1. Add the two BCD numbers, using the rules for binary addition.
2. If a 4 bit sum is equal to or less than 9 It is a valid BCD number.
3. If a 4 bit sum is greater than 9 or if a carry out of the 4-bit group is generated, It is an invalid result.

   Add 6(0110) to the 4- bit sum inorder to skip the six invalid states and return the code to 8421.

   If a carry results where 6 is added, simply add the carry to the next 4-bit group.

eg: a) $(2)_{10} + (6)_{10}$

$$\begin{array}{r} 0010 + \\ 0110 \\ \hline 1000 \\ \hline \end{array} = (8)_{10}$$

b) $(3)_{10} + (7)_{10}$

$$\begin{array}{r} 0011 \\ +\ 0111 \\ \hline 1010 \end{array} \quad 10 > 9 \text{ add } 6$$

$$\begin{array}{r} 1010 \\ +\ 0110 \\ \hline \underbrace{0001}_{(1}\ \underbrace{0000}_{0)_{10}} \end{array}$$

0001 0000 → BCD

$(1\ 0)_{10}$ → Decimal

c) $(57)_{10} + (26)_{10}$

$$\begin{array}{r} 0101\ 0111 \\ +\ 0010\ 0110 \\ \hline 0111\ 1101 \\ +\ 0110 \\ \hline \underbrace{1000}_{(8}\ \underbrace{0011}_{3)_{10}} \end{array} \quad 13 > 9 \text{ add } 6$$

1000 0011 → BCD

$(8\ 3)_{10}$ → Decimal

d) $(83)_{10} + (34)_{10}$

$$\begin{array}{r} 1000\ 0011 \\ +\ 0011\ 0100 \\ \hline 1\ 1011\ 0111 \\ 0110\ \\ \hline \underbrace{0001}_{(1}\ \underbrace{0001}_{1}\ \underbrace{0111}_{7)} \end{array}$$

0001 0001 0111 → BCD

$(1\ 1\ 7)$ → Decimal

e) $(67)_{10} + (53)_{10}$

$$\begin{array}{r} 0110\ 0111 \\ +\ 0101\ 0011 \\ \hline 1011\ 1010 \\ 0110\ 0110 \\ \hline \underbrace{0001}_{(1}\ \underbrace{0010}_{2}\ \underbrace{0000}_{0)_{10}} \end{array}$$

0001 0010 0000 → BCD

$(1\ 2\ 0)_{10}$ → Decimal

Topic: Subtraction of BCD

$M - N$
minuend ↗ ↖ subtrahend

using 9's complement

1. Take 9's complement for subtrahend.
2. Add it to the Minuend using BCD addition.
3. If the result is invalid BCD, then correct by adding 6.
4. Shift the carry to next bits.
5. If end around carry generated then add it to the result. Otherwise take 9's complement and put -ve sign in the decimal no.

Using 10's complement

1. Take 10's complement of subtrahend
2. Add it to the Minuend by using BCD addition.
3. If error BCD digits are ~~c~~corrected by adding 6.
4. Follow up the carry to next bits
5. If end around carry (EAC) is there, discard the carry. If not, say result is -ve and is in 10's complement form. Take 10's complement again to get actual result.

$= (-15.6875)_{10}$

Direct BCD subtraction

eg: $(206.7)_{10} - (147.8)_{10}$

```
0010 0000 0110 . 0111 -
0001 0100 0111 . 1000
---------------------
0000 1011 1110 . 1111 -
     0110 0110   0110
---------------------
0000 0101 1000 . 1001  -> BCD
(0    5    8   .  9)10 -> decimal
```

U.Q Convert the decimal numbers 596 and 386 into BCD and do the addition and subtraction operations in BCD arithmetic.

decimal	BCD Code (8421)
$(596)_{10}$ →	0101 1001 0110
$(386)_{10}$ →	0011 1000 0110

BCD addition

```
 0101 1001 0110 +
 0011 1000 0110
----------------
 1001 0001 1100
      0110 0110
----------------
 1001 1000 0010  -> BCD
  9    8    2
```

BCD Subtraction

$(596)_{10} - (386)_{10}$

```
 01
0101 1001 0110 -
0011 1000 0110
----------------
0010 0001 0000   → BCD
(2    1    0)10  → Decimal
```

U.Q

BCD addition of 01100111 and 01010011

```
 0110 0111 +
 0101 0011
-----------
 1011 1010
  11   10        11 & 10 > 9

 1011 1010 +
 0110 0110
-------------------
0001 0010 0000   → BCD
(1    2    0)    → Decimal
```

U.Q

Subtract 366 from 170 in BCD using 10's complement addition.

$(170)_{10} - (366)_{10}$

```
 1000 -          (170)10 + (634)10
  366
 ----            0001 0111 0000 +
  634            0110 0011 0100
                 ----------------
                 0111 1010 0100
                      0110
                 ----------------
                 1000 0000 0100
                 (8    0    4)10
```

$= (-15.6875)_{10}$

10's of $(804)_{10}$:

$$\begin{array}{r} 1000 \\ -\ 804 \\ \hline (196)_{10} \end{array}$$

UQ Subtract the BCD number 1671 from BCD number 837 using 10's complement addition.

837 − 1671

10's:

$$\begin{array}{r} 10000 \\ -\ 1671 \\ \hline 8329 \end{array}$$

ie $(0837)_{10} + (8329)_{10}$

$$\begin{array}{r} 0000\,1000\,0011\,0111 \\ +\ 1000\,0011\,0010\,1001 \\ \hline \underbrace{1000}\,\underbrace{1011}\,\underbrace{0110}\,\underbrace{0000} \\ +\ \quad\quad 0110 \quad\quad\quad 0110 \\ \hline \underbrace{1001}_{9}\,\underbrace{0001}_{1}\,\underbrace{0110}_{6}\,\underbrace{0110}_{6} \end{array}$$

$(9\ 1\ 6\ 6)_{10}$

10's:

$$\begin{array}{r} 10000 \\ -\ 9166 \\ \hline (0834)_{10} \end{array}$$

Ans: $-(834)_{10}$

T.B Perform the following decimal subtraction in BCD by the 9's complement

(a) 85 − 24 (b) 305.5 − 168.8 (c) 679.6 − 885.9

(a) 85 − 24

9's:
$$\begin{array}{r} 99 - \\ 24 \\ \hline \underline{\underline{75}} \end{array}$$

$(85)_{10} + (75)_{10}$

$$\begin{array}{r} 1000\ 0101\ + \\ 0111\ 0101 \\ \hline 1111\ 1010 \\ 0110\ 0110 \\ \hline \boxed{1}\ 0110\ 0000\ + \\ 1 \\ \hline 0110\ 0001 \end{array}$$

$\underbrace{0110}_{(6}\ \underbrace{0001}_{1)_{10}}$ → BCD

→ decimal

(b) 305.5 − 168.8

9's:
$$\begin{array}{r} 999.9 - \\ 168.8 \\ \hline \underline{\underline{831.1}} \end{array}$$

$(305.5)_{10} + (831.1)$

$$\begin{array}{r} 0011\ 0000\ 0101\ .\ 0101\ + \\ 1000\ 0011\ 0001\ .\ 0001 \\ \hline 1011\ 0011\ 0110\ .\ 0110 \\ 0110 \qquad\qquad\qquad\qquad \\ \hline \boxed{1}\ 0001\ 0011\ 0110\ .\ 0110\ + \\ 1 \\ \hline 0001\ 0011\ 0110\ .\ 0111 \end{array}$$

$\underbrace{0001}_{(1}\ \underbrace{0011}_{3}\ \underbrace{0110}_{6}\ .\ \underbrace{0111}_{7)_{10}}$ → BCD

→ decimal

$= (-15.6875)_{10}$

(c) 679.6 − 885.9

9's: 999.9 −
885.9
114.0

$(679.6)_{10} + (114.0)$

```
  0110 0111 1001 . 0110 +
  0001 0001 0100 . 0000
  ---------------------
  0111 1000 1101 . 0110
            0110
  ---------------------
  0111 1001 0011 . 0110
    7    9    3  .  6
```

```
  999.9
  793.6
  -----
  206.3
```

Ans: $-(206.3)_{10}$

Q.) Perform the following subtractions in 8421 code using the 10's complement method.

(a) 342.7 − 108.9 (b) 206.4 − 507.6

(a) 342.7 − 108.9

```
  1000.0 −
   108.9
  ------
   891.1
```

$(342.7)_{10} + (891.1)_{10}$

```
  0011 0100 0010 . 0111
  1000 1001 0001 . 0001
  ---------------------
  1011 1101 0011 . 1000
  0110 0110
  ---------------------
[1] 0010 0011 0011 . 1000
```

Ans: 233.8

(b) 206.4 − 507.6

$$\begin{array}{r} \overset{999}{\cancel{1}000}.\overset{1}{0} - \\ 507.6 \\ \hline 492.4 \end{array}$$

$(206.4)_{10} + (492.4)$

$$\begin{array}{r} 0010\ 0000\ 0110.0100 \\ 0100\ 1001\ 0010.0100 \\ \hline \underbrace{0110}_{6}\ \underbrace{1001}_{9}\ \underbrace{1000}_{8}.\underbrace{1000}_{8} \end{array}$$

$$\begin{array}{r} \overset{0999\ 1}{\cancel{1}000.0} \\ 698.8 \\ \hline 301.2 \end{array}$$

Ans: $-(301.2)_{10}$

$= (-15.6815)_{10}$

Topic: Hexadecimal Addition

Hex	dec
Hex: 0 – 9	
A	10
B	11
C	12
D	13
E	14
F	15

① $23_{16} + 16_{16}$

$$\begin{array}{r} 23+ \\ 16 \\ \hline (39)_{16} \end{array}$$

②

$$\begin{array}{r} 58+ \\ 22 \\ \hline (7A)_{16} \end{array}$$

③ $2B_{16} + 84_{16}$

$$\begin{array}{r} 2B+ \\ 84 \\ \hline (AF)_{16} \end{array}$$

④ $(DF)_{16} + (AC)_{16}$

$$\begin{array}{r} \overset{1}{D}F+ \\ AC \\ \hline (18B)_{16} \end{array}$$

15+ 12 = 27   16 | 27 B, 1

$(27)_{10} \rightarrow (1B)_{16}$

⑤ $(ADD)_{16} + (DAD)_{16}$

$$\begin{array}{r} \overset{1}{A}\overset{1}{D}D+ \\ DAD \\ \hline (188A)_{16} \end{array}$$

⑥ $(5689)_{16} + (4574)_{16}$

$$\begin{array}{r} 5689+ \\ 4574 \\ \hline (9BFD)_{16} \end{array}$$

14+ 10 = 24   16 | 24 8, 1

13+ 13 = 26   16 | 26 A, 1

10 14 = 24 $(24)_{10} \rightarrow (18)_{16}$

⑦

$$\begin{array}{r} 9\overset{1}{8}\overset{1}{F}.A2+ \\ B11.94 \\ \hline (14A1.36)_{16} \end{array}$$

11+ 13 = 24   16 | 19 3, 1

16 | 17 1, 1

⑧ $(3F8)_{16} + (5B3)_{16}$

$$\begin{array}{r} \overset{1}{3}F8+ \\ 5B3 \\ \hline (9AB)_{16} \end{array}$$

11+ 9 = 20   16 | 20 4, 1

15+ 11 = 26   16 | 26 A, 1

Topic: Hexadecimal Subtraction

a) $(84)_{16} - (2A)_{16}$ using 2's complement

$$\begin{array}{r} {}^{7}\cancel{8}{}^{16}4 - \\ 2A \\ \hline (5A)_{16} \end{array}$$

$$\begin{array}{r} 00101010 \\ 11010101 + \\ 1 \\ \hline \underbrace{1101}_{D}\ \underbrace{0110}_{6} \end{array}$$

$(D6)_{16}$

$$\begin{array}{r} 84 + \\ D6 \\ \hline \boxed{1}\ (5A)_{16} \end{array}$$

discard carry

$$\begin{array}{r} 13 + \\ 8 \\ \hline 21 \end{array}$$

$16 \lfloor 21 \ 5$

1

(b) $(C3)_{16} - (0B)_{16}$

$$\begin{array}{r} 00001011 \\ 11110100 + \\ 1 \\ \hline \underbrace{1111}_{F}\ \underbrace{0101}_{5} \end{array}$$

$$\begin{array}{r} C3 + \\ F5 \\ \hline \boxed{1}\ B8 \end{array}$$

Ans: $(B8)_{16}$

$$\begin{array}{r} 15 + \\ 12 \\ \hline 27 \end{array}$$

$16 \lfloor 27 \ B$

1

$$\begin{array}{r} {}^{B}\cancel{C}{}^{16}3 - \\ 0B \\ \hline (B8)_{16} \end{array}$$

$$\begin{array}{r} 19 - \\ 11 \\ \hline 08 \end{array}$$

V.Q

1) $A5C4_{16} + 39A5_{16}$

2) $A96B_{16} - 9F2C_{16}$

Topic: Hexadecimal Addition

Hex: 0 – 9

Hex	dec
A	10
B	11
C	12
D	13
E	14
F	15

① $23_{16} + 16_{16}$

$$\begin{array}{r} 23 + \\ 16 \\ \hline (39)_{16} \end{array}$$

② $58 +$

$$\begin{array}{r} 58 + \\ 22 \\ \hline (7A)_{16} \end{array}$$

③ $2B_{16} + 84_{16}$

$$\begin{array}{r} 2B + \\ 84 \\ \hline (AF)_{16} \end{array}$$

④ $(DF)_{16} + (AC)_{16}$

$$\begin{array}{r} \overset{1}{D}F + \\ AC \\ \hline (18B)_{16} \end{array}$$

$$\begin{array}{r} 15 + \\ 12 \\ \hline 27 \end{array} \qquad 16 \,\underline{|27}\; B, \; 1$$

$(27)_{10} \to (1B)_{16}$

⑤ $(ADD)_{16} + (DAD)_{16}$

$$\begin{array}{r} \overset{1}{A}\overset{1}{D}D + \\ DAD \\ \hline (188A)_{16} \end{array}$$

⑥ $(5689)_{16} + (4574)_{16}$

$$\begin{array}{r} 5689 + \\ 4574 \\ \hline (9BFD)_{16} \end{array}$$

$$\begin{array}{r} 14 + \\ 10 \\ \hline 24 \end{array} \qquad 16 \,\underline{|24}\; 8, \; 1$$

$$\begin{array}{r} 13 + \\ 13 \\ \hline 26 \end{array} \qquad 16 \,\underline{|26}\; A, \; 1$$

$$\begin{array}{r} 10 \\ 14 \\ \hline 24 \end{array} \qquad (24)_{10} \to (18)_{16}$$

⑦

$$\begin{array}{r} 9\overset{1}{8}\overset{1}{F}.A2 + \\ B11.94 \\ \hline (14A1.36)_{16} \end{array}$$

$$\begin{array}{r} 11 + \\ 13 \\ \hline 24 \end{array} \qquad 16 \,\underline{|19}\; 3, \; 1$$

$16 \,\underline{|17}\; 1, \; 1$

$$\begin{array}{r} 11 + \\ 9 \\ \hline 20 \end{array} \qquad 16 \,\underline{|20}\; 4, \; 1$$

⑧ $(3F8)_{16} + (5B3)_{16}$

$$\begin{array}{r} \overset{1}{3}F8 + \\ 5B3 \\ \hline (9AB)_{16} \end{array}$$

$$\begin{array}{r} 15 + \\ 11 \\ \hline 26 \end{array} \qquad 16 \,\underline{|26}\; A, \; 1$$

Topic: Hexadecimal Subtraction

a) $(84)_{16} - (2A)_{16}$ using 2's complement

$$\begin{array}{r} \overset{7}{\cancel{8}}\overset{16}{4} - \\ 2A \\ \hline (5A)_{16} \end{array}$$

$$\begin{array}{r} 00101010 \\ 11010101 + \\ 1 \\ \hline \underbrace{1101}_{(D}\ \underbrace{0110}_{6)_{16}} \end{array}$$

$$\begin{array}{r} {}^{1}13 + \\ 8 \\ \hline 21 \end{array}$$

$$16 \underline{|21}\ 5 \quad 1$$

$$\begin{array}{r} 84 + \\ D6 \\ \hline \boxed{1}\ (5A)_{16} \end{array}$$

discard carry

(b) $(C3)_{16} - (0B)_{16}$

$$\begin{array}{r} 00001011 \\ 11110100 + \\ 1 \\ \hline \underbrace{1111}_{F}\ \underbrace{0101}_{5} \end{array}$$

$$\begin{array}{r} 15 + \\ 12 \\ \hline 27 \end{array}$$

$$16 \underline{|27}\ B \quad 1$$

$$\begin{array}{r} C3 + \\ F5 \\ \hline \boxed{1}B8 \end{array}$$

Ans: $(B8)_{16}$

$$\begin{array}{r} \overset{B}{\cancel{C}}\overset{16}{3} - \\ 0B \\ \hline (B8)_{16} \end{array}$$

$$\begin{array}{r} 19 - \\ 11 \\ \hline 08 \end{array}$$

V.Q

1) $A5C4_{16} + 39A5_{16}$

2) $A96B_{16} - 9F2C_{16}$

Topic: Octal Addition

0 - 7

dec	octal
8	10
9	11
10	12
11	13
12	14
13	15
14	16

(a) $(147)_8 + (261)_8$

$$\begin{array}{r} 147\,+ \\ 261 \\ \hline (420)_8 \end{array}$$

(b) $(366.23)_8 + (243.62)_8$

$$\begin{array}{r} 366.23\,+ \\ 243.62 \\ \hline (632.05)_8 \end{array}$$

(c) $(567)_8 + (243)_8$

$$\begin{array}{r} 567\,+ \\ 243 \\ \hline (1032)_8 \end{array}$$

(d) $(243)_8 + (212)_8$

$$\begin{array}{r} 243\,+ \\ 212 \\ \hline (455)_8 \end{array}$$

(e) $(766)_8 + (774)_8$

$$\begin{array}{r} 766\,+ \\ 774 \\ \hline (1762)_8 \end{array}$$

Topic: Octal Subtraction

a) Using 8's complement, $372_8 - 144_8$

Direct

$$\begin{array}{r} 3\cancel{7}^{6}2^{8} - \\ 144 \\ \hline (226)_8 \end{array}$$

8's:

$$\begin{array}{r} \cancel{1}000 \\ 144 \\ \hline 634 \end{array}$$

$$\begin{array}{r} 372 + \\ 634 \\ \hline \boxed{1}\,226_8 \end{array}$$

discard carry

Ans: 226_8

(b) $144_8 - 372_8$, using 7's complement

7's:

$$\begin{array}{r} 777 \\ 372 \\ \hline (405)_8 \end{array}$$

$$\begin{array}{r} 144 + \\ 405 \\ \hline 551_8 \end{array}$$

7's:

$$\begin{array}{r} 777 - \\ 551 \\ \hline (226)_8 \end{array}$$

Ans: $-(226)_8$

U.Q Perform the following operations:

(i) $(E39)_{16} + (3F9)_{16}$ (ii) $(721)_8 - (32)_8$

(i)

$$\begin{array}{r} E39 + \\ 3F9 \\ \hline (1232)_{16} \end{array}$$

$16 \lfloor 18 \quad 2$
1

$16 \lfloor 19 \quad 3$
1

0-9	
10	A
11	B
12	C
13	D
14	E
15	F

(ii)

$$\begin{array}{r} 6\cancel{7}21 - \\ 32 \\ \hline 667_8 \end{array}$$

$= (-15.6875)_{10}$

$(721)_8 - (32)_8$ using 7's complement

$$\begin{array}{r} 777 \\ 032 \\ \hline (745)_8 \end{array} \qquad \begin{array}{r} 721+ \\ 745 \\ \hline \boxed{1}\,666+ \\ 1 \\ \hline 667_8 \end{array} \qquad 8\underline{|14}\ 6 \quad 1$$

Ans: $(667)_8$

Topic: Floating point Number Representation

2 parts

① Mantissa

② Exponent.

eg: $\underbrace{3.62}_{\text{mantissa}} \times 10^{5}$ → exponent

Addition & Subtraction

1. Check if the exponent of 2 nos. are same or not.
2. If both are same, directly perform addition or subtraction
3. If they are different, adjust mantissa part to make them same and perform addition or subtraction.

eg: a) $3.26 \times 10^5 + 5.72 \times 10^5 = \underline{8.98 \times 10^5}$

b) $7.13 \times 10^3 + 2.13 \times 10^5$

Either make, first no. exponent to 5 or second to 3.

$0.0713 \times 10^5 + 2.13 \times 10^5$

$= \underline{2.2013 \times 10^5}$

Multiplication

1) Directly multiply the mantissa parts
2) Add the exponent parts

$$1.34 \times 10^2 \times 2.2 \times 10^5$$

$$= 2.948 \times 10^7$$

Division

1) Directly divide the mantissa part
2) Subtract the exponent parts.

$$5.36 \times 10^3 \div 2.12 \times 10^2$$

$$= 2.53 \times 10^1$$

Floating Point Numbers

→ A floating point numbers consists of two parts plus a sign

* Mantissa is the part of floating point numbers that represents the magnitude of the number and is between 0 and 1.
* Exponent is the part of floating point number that represents the number of places that the decimal point is to be moved.

$F \times r^{E}$ (F = fraction, r = radix, E = exponent)

eg: $10110 = 1.0110 \times 2^4 = \underbrace{0.10110}_{\text{mantissa}} \times 2^{\overset{\text{exponent}}{5}}$

For binary floating point numbers, the format is defined by ANSI/IEEE std 754-1985 in 3 forms:

* Single Precision (S.P)
* Double Precision (D.P)
* Extended Precision (E.P)

→ These all have the same basic formats except for the no. of bits.

S.P - 32 bits
D.P - 64 bits
E.P - 80 bits

Single precision Floating point Binary Nos:

The sign bit (S) is the left most bit, the exponent (E) includes the next

$= (-15.6875)_{10}$

eight bits and the matissa or fraction part(F) includes the remaining 23 bits, as shown next.

← 32 bits →

S	Exponent (E)	Mantissa
1 bit	8 bits	23 bits

→ The 8 bits in the exponent represent a biased exponent, which is obtained by adding 127 to the actual exponent.

→ The biased exponent allows a range of actual exponent values from −126 to +128.

Example

Convert the decimal number 3.248×10^4 to IEEE 754 std ~~single~~ precision floating point binary number.

$3.248 \times 10^4 = (32480)_{10}$

$= (111111011100000)_2$

Normalizing $= 1.11111011100000 \times 2^{14}$

$S = 0 \quad E = 14 \quad$ Biased $= 127 + e = 127 + 14$

$= 141$

$= (10001101)_2$

$M = 11111011100000$

0	10001101	1111101110000000000000	0000

Topic: Floating point Representation Example

Q) ② Explain the format of single precision floating point number representation and find the decimal value corresponding to the given floating point number.

$(1,10000010,11110110000000000000000\ 00)_2$

1	10000010	11110110000000000000000

$(10000010)_2$

$2^7\ 2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0 \quad = 2^7 + 2^1$

$= 128 + 2$

biased exponent $= (130)_{10}$

actual exponent $= 130 - 127$

$= \underline{\underline{3}}$

Mantissa $= 1111011000000000000000000$
(-1 -2 -3 -4 -5 -6 -7)

$= 2^{-1} + 2^{-2} + 2^{-3} + 2^{-4} + 2^{-6} + 2^{-7}$

$= 0.5 + 0.25 + 0.125 + 0.0625 + 0.015625 + 0.0078125 = (0.9609375)_{10}$

$(-1)^s \times (1+m) \times 2^e = (-1)^1 \times (1 + 0.9609375) \times 2^3$

$= -1.9609375 \times 2^3$

$= (-15.6875)_{10}$

3) A = 110010001,10001100010

1	10010001	10001100 01.000000000000000

Biased Exp = 10010001
76543210

$= 2^7 + 2^4 + 2^0$

$= 128 + 16 + 1 = 145$

$e = 145 - 127 = \underline{\underline{18}}$

Mantissa = 1000110001
-1 -2 -3 -4 -5 -6 -7 -8 -9 -10

$= 2^{-1} + 2^{-5} + 2^{-6} + 2^{-10}$

$= 0.5 + 0.03125 + 0.015625 +$
0.00097656

$= 0.54787856$

$(-1)^s * (1+m) * 2^e = (-1)^1 \times (1 + .54787856) \times 2^{18}$

$= -1.5478 \times 2^{18}$

$= -(405746.483)_{10}$

4) A = (1000 1000 1000 1000 1000 0000 0000 0000)$_2$ into decimal no.

1	00010001	00010001000000000000000

S = 1

Biased Exponent = 00010001
= 17

e = 17 − 127 = −110

Mantissa = (00010001000000000000000)
$\quad$ -1 -2 -3 -4 -5 -6 -7 -8

$= 2^{-4} + 2^{-8}$

$= 0.06640625_{10}$

$(-1)^S \times (1+m) \times 2^e$

$= (-1)^1 \times (1+0.06640625)\, 2^{-110}$

$= \underline{-1.06640625 \times 2^{-110}}$

5) Convert the decimal number 3.248×10^4 to IEEE 754 std double (64 bit) precision floating point binary number.

$3.24 \times 10^4 = (32480)_{10}$

$= 111111011100000$

Normalizing ⇒ $1.11111011100000 \times 2^{14}$

S = 0

E = 14

Biased exponent = 14 + 1023
= $(1037)_{10}$

$= (10000001101)_{12}$

$M = 11111011100000$

0	10000001101	11111011100000 0000
← 1 →	← 11 bit →	← 52 bit →

Q6) $(5EC.2CD)_{16}$

$(010111101100.001011001101)_2$

Normalizing

$= 1.0111101100001011001101 \times 2^{10}$

$S = 0 \quad E = 10$

Biased exponent $= 10 + 1023 = 1033$

$= (10000001001)_2$

$M = 0111101100001011001101$

0	10000001001	0111101100001011001101..
← 1 →	← 11 bit →	← 52 bit - - →