Module 1

Random Experiment: An experiment that can result in different outcomes, even though repeated under the same conditions every time

eg: Tossing a coin, Throwing a die.

Sample Space (S): It is the set of all outcomes of a random experiment

eg: 1) Tossing a coin 2) Throwing a die

$S = \{H, T\}$ $S = \{1, 2, 3, 4, 5, 6\}$

Random Variable:- A variable X that assigns a real number x for every outcome of the sample space

it is a function from sample space to real numbers

$X : S \longrightarrow \mathbb{R}$

eg: Consider an experiment of tossing 2 coins

Here $S = \{HH, HT, TH, TT\}$

Let X denotes the no. of heads,

then X can take values

S	HH	HT	TH	TT
X	2	1	1	0

ie X can take values 0, 1 and 2

2) when a student calls a university help desk

either he/ she will be immediately be able to

speak to some one (S for success)

or will be placed on hold. (F for failure)

Here $S = \{S, F\}$

define random variable by $X(S) = 1$

$X(F) = 0$

Note: A random variable whose only possible values are 0 and 1 is called a Bernoulli random variable.

Two types of random variables

① Discrete random variable :-

whose possible values are either finite set or a countably infinite set

eg) 1) previous 2 examples

2) An experiment in which ~~9 volt~~ 9-volt batteries are tested until one which an acceptable voltage is obtained

$$S = \{ S, FS, FFS, \dots \}$$

Define X = no. of batteries tested before the experiment terminates

$X(S) = 1$

$X(FS) = 2$

$X(FFS) = 3$ etc. so any positive integer is a possible value, set is countably infinite. X is discrete random variable

⋮

② Continuous random variable :-

Random variable taking all the values in an interval say (a, b)

eg: Y = height above sea level at the selected location

Probability mass function / Probability distribution function

The function $p(x) = P[X = x]$ of the discrete random variable is said to be a pmf if it satisfies

1) $P(x) \geq 0$
2) $\sum_{\forall x} P(x) = 1$

Probability Distribution: It is the set of ordered pairs $(x, P(x))$.

eg) 1) Consider the random experiment of throwing a die. Let random variable X denote the no. on the die when it is thrown

X can take values - 1, 2, 3, 4, 5, 6

$p(1) = P[X=1] = \frac{1}{6}$

$p(2) = P[X=2] = \frac{1}{6}$

$\vdots$

$p(6) = P[X=6] = \frac{1}{6}$

Probability distribution is

$p(x) = \frac{1}{6};\ x = 0, 1, 2, \cdots 6$

OR

X	1	2	3	4	5	6
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

2) Six lots of components are ready to be shipped by a certain supplier.

Lot	1	2	3	4	5	6
No. of defectives	0	2	0	1	2	0

One of lots is to be randomly selected for shipment to a customer. Let X denote the no. of defectives in the selected slot. Find the probability distribution?

Ans. X can take values $0, 1, 2$

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then,

$$p(0) = P[X=0] = P[\text{lot } 1 \text{ or } 3 \text{ or } 6] = \frac{3}{6}$$

$$p(1) = P[X=1] = P[\text{lot } 4] = \frac{1}{6}$$

$$p(2) = P[X=2] = P[\text{lot } 2 \text{ or } 5] = \frac{2}{6}$$

(9) Consider whether the next person buying a computer at a certain electronics store buys a laptop or a desktop model. Let

$$X = \begin{cases} 1, & \text{if he purchases a desktop model} \\ 0, & \text{if he purchases a laptop model.} \end{cases}$$

If 20% of all purchasers during that week select a desk top model. Find the probability distribution.

Ans: X can take values 1, 0

$$p(0) = P[X=0] = P\left[\begin{array}{c}\text{next costomer purchases} \\ \text{a laptop model}\end{array}\right] = \frac{80}{100} = .8$$

$$p(1) = P[X=1] = P\left[\begin{array}{c}\text{next costomer purchases} \\ \text{a desktop model}\end{array}\right] = \frac{20}{100} = .2$$

probability distribution

$$p(x) = \begin{cases} .8 & \text{if } x=0 \\ .2 & \text{if } x=1 \end{cases}$$

Note:

In the above example X is a bernoulli r.v

$p(x) = p[X=x] = 0$ for $x \neq 0$ or 1

then Probability Distribution becomes

$$p(x) = \begin{cases} 0.8 & \text{if } x=0 \\ 0.2 & \text{if } x=1 \\ 0 & \text{otherwise} \end{cases}$$

generally by using a parameter α we can write:

$$p(x) = \begin{cases} 1-\alpha, & \text{if } x=0 \\ \alpha, & \text{if } x=1 \\ 0, & \text{otherwise} \end{cases}$$

Here α is called the parameter of the probability distribution

① Check whether the given distribution is a probability distribution

1) $p(x) = x-3$, $x = 1,2,3,4,5$

2) $p(x) = \frac{x^2}{30}$, $x = 0,1,2,3,4$

Ans: i) $p(x) = x-3$

for $x = 1$

$p(1) = 1-3 = -2$ not possible

$\therefore$ not a probability distribution

2) $p(x) = \frac{x^2}{30}$

i) $p(x) \geq 0$

ii) $\sum_x p(x) = 0 + \frac{1}{30} + \frac{4}{30} + \frac{9}{30} + \frac{16}{30}$

$= \frac{30}{30} = 1$

$\therefore$ a probability distribution.

Q) Department of statistics has a lab with six computers reserved for statistics. Let X denote the no. of these computers are in use at a particular time of the day. The probability distribution of X is as given below

X	0	1	2	3	4	5	6
$P(x)$	0.05	0.10	0.15	0.25	0.20	0.15	0.10

Find i) $P(X \leq 2)$ (ii) $P(X \geq 3)$ (iii) $P(2 < X < 5)$

Ans: i) $P(X \leq 2) = P[X = 0, \text{ or } 1 \text{ or } 2]$

$= P[X=0] + P[X=1] + P[X=2]$

$= 0.05 + 0.10 + 0.15$

$= .30$

ii) $P(X \geq 3) = P[X = 3, 4, 5, 6]$

OR $= 1 - P[X \leq 2]$

$= 1 - \{P[X=0] + P[X=1] + P[X=2]\}$

$= 1 - .30$

$= 0.70$

iii) $P(2 < X < 5) = P[X = 3 \text{ or } 4]$

$= P[X=3] + P[X=4]$

$= .25 + .20$

$= \underline{\underline{0.45}}$

6) A random variable X has the following pmf

X	1	2	3	4	5	6	7
$p(x)$	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

Find the following

i) Value of k ii) $P(0 < X < 5)$ (iii) $P(X > 5)$

Ans: i) we know

$$\sum_x p(x) = 1$$

$\Rightarrow k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\Rightarrow 10k^2 + 9k = 1$

$\Rightarrow 10k^2 + 9k - 1 = 0$

$\Rightarrow k = \frac{1}{10}$ or -1

But k cannot be $-ve$ $\therefore k = \underline{\underline{\frac{1}{10}}}$

(ii) $P(0 < X < 5)$

$= P(X = 1, 2, 3, 4)$

$= P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$= k + 2k + 2k + 3k = 8k$

$= \frac{8}{10}$

(iii) $P(X > 5) = P(X = 6, 7)$

$= P(X=6) + P(X=7)$

$= 2k^2 + 7k^2 + k = 9k^2 + k = \frac{9}{100} + \frac{1}{10}$

$= \frac{9}{100} + \frac{10}{100}$

$= \frac{19}{100}$

Q) Consider a group of 5 potential blood donors a, b, c, d and e of whom only a and b have type O+ blood. Five blood samples, one from each individual will be typed in random order until O+ individual is defined. Let the r.v Y = the no. of typings necessary to identify an O+ individual. Find probability distribution?

Ans: Y - no. of typings necessary to identify O^+ individual

$Y \longrightarrow 1, 2, 3, 4$

$P(1) = P[Y=1] = P[$ ~~getting~~ Typing only one time an O^+ individuals $]$

$= P[$ a or b typed first $]$

$= \frac{2}{5} = .4$

$P(2) = P[Y=2] = P[$ Typing 2 times $]$

$= P[$ Typing c, d or e first, and then typing a or b $]$

$= \frac{3}{5} \times \frac{2}{4} = .3$

$P(3) = P[Y=3] = P[$ Typing 3 times $]$

$= P[$ c, d or e first and second, and then a or b $]$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} = .2$

$p(4) = P[Y = 4] = P[\text{Typing 4 times}]$

$= P[\text{a, d, and e typed first and then a or b}]$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \times \frac{2}{2}$

$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} = .1$

Probability distribution

Y	1	2	3	4
P(y)	.4	.3	.2	.1

Q) Starting at a fixed time, we observe the gender of each newborn child at a certain hospital until a boy (B) is born. Let $p = P(B)$. Assume the successive births are independent. Let X be the number of births observed. Find probability distribution.

Ans: X — no. of births observed

$X = 1, 2, 3, 4, 5, \ldots$

given $p = P(B)$

$$P(1) = P[X=1] = P(B) = P$$

$$P(2) = P[X=2] = P(GB) = (1-P)P$$

$$P(3) = P[X=3] = P(GGB) = (1-P)(1-P)P$$
$$= (1-P)^2 P$$

generally we can write

$$P(x) = \begin{cases} (1-P)^{x-1} P, & x = 1,2,3 \ldots \\ 0, & \text{otherwise} \end{cases}$$

here p is a parameter