Module-5: Maximizing Profit

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Introduction:

The main aim of this assignment is to use linear programming to maximize profit for a hardware company which is planning to set up a distribution centre in southeast. It has four products Pressure washers, Go karts, Generators, and Water pumps, we need to identify the right amount of production quantities of each product to maximize the monthly profit for the entity.

Given Variables:

The data contains the below predominant variables:

Costs of products in dollars	
Item	Cost
Pressure washer	330
Go-kart	370
Generator	410
Case of 5 Water Pumps	635/5

Selling Price of products in dollars	
Item	Cost
Pressure washer	499.99
Go-kart	729.99
Generator	700.99
(Case of 5 Water Pumps)	269.99/5

Constraints	
Shelves	82
Size (Each)	30 ft long and 5 ft wide
Pressure washers	5 ft by 5 ft
Generator	5 ft by 5 ft
Go Kart	8 ft by 5 ft
Case of 5 Water Pumps	5 ft by 5 ft

Decision Variables	
Items	No of Products
Pressure washer	x1
Go-kart	x2
Generator	x3
Case of 5 Water Pumps	x4

Mathematical Formulation:

To solve this problem using linear programming, we need to define decision variables, formulate the objective function, and set up constraints. Let's denote:

x 1= Number of Pressure washers to be purchased

x 2= Number of Go-karts to be purchased

x 3= Number of Generators to be purchased

x 4= Number of Cases of water pumps to be purchased

Maximizing profit can be obtained by , corresponding selling prices of each item multiplied by the quantity sold, and the total cost is the purchase cost of each item multiplied by the quantity purchased.

$Z= (499.99X_1+729.99X_2+700.99\ X_3+269.99/5X_4)-(330X_1+370X_2+410X_3+635/5X_4)$

$Z= 166.99X_1+ 358.99X_2 + 290.99X_3 -73.002X_4$

The variable of X_4 are divided by 5 to derive the cost and selling price of each pump.

Linear Programming Formulation:

The next phase is to identify all constraints to setup a LP formula, subject to the below constraints.

Budget equation: $330x_1+370x_2+410x_3+127x_4 \leq 170{,}000$

Space equation: $5x_1+8x_2+5x_3+5x_4 \leq (82\times30)$

Marketing equation:

$x_1+x_2 \geq 0.3(x_1+x_2+x_3+x_4)$ which is $0.7x_1+0.7x_2-0.3x_3-0.3x_4 \geq 0$

$x_3 \geq 2x4$ which is $x_3-2x4 \geq 0$

Non-negativity constraint: $x_1, x_2, x_3, x_4 \geq 0$

Solver and sensitivity report:

Through excel, I tried to find the maximum profit using objective parameters (x_1, x_2, x_3, x_4), we already have these variables in the objective function Z. Initially the estimated profit is calculated using the SUMPRODUCT function. Now, we construct a constrains table to approximate the profits using the solver. The earlier defined constraints have been populated into the table and LHS constraints are calculated using the same function as earlier and the inequalities of the respective equations have been filled.

Finally, using Solver form the data tab, the objective is set to maximization, changing variables have been assigned and respective constraints have been filled according to the equation. Marked non-variables and using the "Simplex LP" as solving method we solved the objective. Please refer to the Fig-1 and 2 for the findings.

The optimal solutions:

- **Inventory Levels:**
  - Cost x1 (Product 1): 0 units
  - Cost x2 (Product 2): 125.0847458 units
  - Cost x3 (Product 3): 291.8644068 units
  - Cost x4 (Product 4): 0 units
- **Optimal Monthly Profit:**
  - The optimal monthly profit is achieved with the given inventory levels, resulting in a profit of $129,833.80.

This solution ensures that all constraints are satisfied:

- The monthly budget constraint is met with a final value of $165,945.76, which is less than or equal to the constraint of $170,000.
- The space constraints are fully utilized with a final value of 2460, satisfying the limit of 2460 units.
- The allocation constraint for pressure washers and go-karts is also satisfied with a final value close to zero, indicating that the constraint is met.

- The generators constraint is met with a final value of 291.8644068, which is greater than or equal to zero as required.

Overall, the solution achieves the maximum profit possible within the given constraints, ensuring efficient allocation of resources.

Determining the smallest selling price:

Form the below sensitivity report (Fig-3), we can observe that the decision variable X4 has the optimum value of 0. As we set non-constraint variables as non-negative variables the constraint of X4 is greater than or equal to zero, hence the corresponding shadow price is also zero. If the shadow price for the constraint involving Product 4 (x4) is 0, it means that the optimal solution is not sensitive to changes in the constraint RHS (Right-Hand Side). In this case, changing the selling price for Product 4 (x4) would not affect the optimal solution.

We can modify the constraint associated with this variable to ensure that it must be greater than or equal to a certain value. Let's denote this lower bound, which is 0.01, then re-solve the optimization problem.

From Fig-4, we could see the necessary changes and the optimal profit has been derived and sensitivity report has been generated, Fig-5.

- **Variable Cells**:
    - The optimal value of $x4$ is 0.01, indicating that it's now greater than zero.
    - The coefficient associated with $x4$ is 142.99.
- **Constraints**:
    - There's a new constraint labeled "X4 constraint" associated with $x4$.
    - The shadow price of this constraint is -148, indicating that for every unit increase in the right-hand side (R.H. Side), the profit function would decrease by $148.
    - The right-hand constraint (R.H. Side) is 0.01, which is the lower bound you've set for $x4$.
    - The allowable increase for this constraint is 97.27813559, suggesting that you can increase the lower bound by this amount without changing the optimality.

Cost x4 is the smallest selling price, which results in a non-zero value that is approximately 0.01. The shadow price associated with this constraint gives insight into how the objective function changes with respect to changes in the lower bound of $x4$. In this case, for every unit increase in the lower bound, the objective function decreases by $148.

Budget Allocation:

To assess if the entity should allocate excess budget during the first month, we need to analyse the sensitivity report from the Solver, particularly focusing on the "Monthly budget Constraint" which corresponds to the purchasing budget. From Fig-3;

- Cell Name: Q12 (Monthly budget Constraint)
- Final Value: $165,945.7627
- Shadow Price: 0 (indicating no sensitivity to changes in the constraint)
- Constraint RHS (Right-Hand Side): $170,000

Since the shadow price is 0 for the monthly budget constraint, it suggests that the current purchasing budget is not constraining the optimal solution. In other words, the optimal solution is not sensitive to changes in the purchasing budget, and additional investment beyond $170,000 would not directly lead to an increase in monthly profit.

It is pivotal to consider elements such as potential opportunities or risks associated with increased investment. If there are indications of potential growth opportunities or if the company anticipates increased demand for its products, additional investment might be justified. In such cases, a thorough cost-benefit analysis should be conducted to determine the optimal level of additional investment.

If the decision is made to allocate additional money, the company should assess the probable increase monthly profits. This can be estimated by analyzing the changes in the objective function value as a result of the increased investment. However, since the shadow price for the monthly budget constraint is 0, there wouldn't be profits solely due to additional investment in the purchasing budget.

To summarise, by information obtained from the sensitivity report, there is no indication that the company should allocate excess cash beyond the initial budget to the estimated budget during the first month, as current budget is not constraining the optimal solution. However, any decision regarding additional investment should consider other factors and be based on a comprehensive analysis of potential opportunities and risks.

What size should the warehouse be? Small or large:

To determine whether the company should rent a smaller or larger warehouse, we need to analyze the sensitivity report from the Solver, focusing on the "Space Constraints" constraint which corresponds to the size of the warehouse.

From Fig-3:

Cell Name: Q13 (Space Constraints)

Final Value: 2,460 square feet

Shadow Price: 52.7779661

Constraint RHS (Right-Hand Side): 2,460 square feet

The shadow price of 52.7779661 indicates that for one unit increase in square foot, the optimal monthly profit would increase by $52.78, approximately. This suggests that there is a positive sensitivity to changes in size, meaning that increasing the warehouse size is directly proportional to profits.

Therefore, the company should rent a large warehouse to maximize profits.

To determine the ideal size of the warehouse, we must increase in the RHS constraint and its impact on the shadow price. Since the shadow price indicates the marginal increase in profit per square foot, we can estimate the ideal size of the warehouse by dividing the current budget constraint RHS by the shadow price.

Ideal Warehouse Size = Constraint RHS / Shadow Price

Ideal Warehouse Size = 2,460 square feet / 52.7779661 ≈ 46.56 square feet

Thus, 46.56 square feet would be the ideal size.

Now, to estimate its contribution to the monthly profit, we calculate the increase in profit by multiplying the varying size by the shadow price:

Contribution to Monthly Profit = Change in Warehouse Size * Shadow Price

Contribution to Monthly Profit ≈ |(46.56 - 2,460)| square feet * 52.7779661

Contribution to Monthly Profit ≈ $127,378.55

Therefore, the estimated contribution to monthly profit from increasing the warehouse size to the ideal size of approximately 46.56 square feet is approximately $127,378.55.

Conclusion:

Based on the analysis of the project using the Solver sensitivity report, it's evident that optimizing inventory levels, budget allocation, and warehouse size are crucial for maximizing monthly profit. The optimal inventory levels suggest maintaining no inventory for Product 1 and Product 4, while ensuring 125.08 units of Product 2 and 291.86 units of Product 3. Regarding budget allocation, the current purchasing budget of $170,000 appears sufficient as it doesn't constrain the optimal solution. Therefore, no immediate need for additional investment beyond this budget is indicated for the first month. However, continuous evaluation of market conditions and future forecasts is advisable to determine the necessity of adjusting the budget allocation.

Furthermore, the sensitivity analysis indicates a positive impact on monthly profit by increasing the warehouse size. The ideal size of the recommended warehouse is approximately 46.56 square feet, estimated to contribute around $127,378.55 to monthly profit. Therefore, considering expanding the warehouse to this ideal size would be beneficial for the company's profitability. However, it's essential to conduct regular monitoring and adjustment of inventory levels, budget allocation, and warehouse size to adapt to changing market dynamics effectively and optimize overall profitability over time. By implementing these strategies and staying attentive to market trends, the company can achieve its financial objectives while maintaining operational efficiency.

Appendix:

Fig-1: Case-1-Optimal Solution

		x1	x2	x3	x4	166.99x1 + 358.99x2 + 290.99x3 +142x4
		0	125.084746	291.864407	0	$129,833.80
Objective Parameters		166.99	358.99	290.99	-73.002	

Fig-2: Case-1: Constraints

Constraints	x1	x2	x3	x4	**Constranit LHS**	**Inequality**	**Constraint RHS**
Monthly budget	330	370	410	127	165945.7627	<=	170000
Space Constraints	5	8	5	5	2460	<=	2460
Allocation constraint for p	0.7	0.7	-0.3	-0.3	1.42109E-14	>=	0
Generators constraint			1	-2	291.8644068	>=	0

Fig-3: Case-1: Sensitivity Report

Microsoft Excel 16.80 Sensitivity Report
Worksheet: [ALY6050_MOD5Project_PentareddyN.xlsx]Sheet1
Report Created: 2024-03-22 12:29:17 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
M4	Cost x1	0	-33.66610169	166.99	33.66610169	1E+30
N4	Cost x2	125.0847458	0	358.99	106.594	39.726
O4	Cost x3	291.8644068	0	290.99	94.58571429	66.62125
P4	Cost x4	0	-363.992	-73.002	363.992	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
Q12	Monthly budget Constranit LHS	165945.7627	0	170000	1E+30	4054.237288
Q13	Space Constraints Constranit LHS	2460	52.7779661	2460	60.10050251	2460
Q14	Allocation constraint for pressure washers and go-karts Constranit LHS	1.42109E-14	-90.33389831	0	215.25	16.72727273
Q15	Generators constraint Constranit LHS	291.8644068	0	0	291.8644068	1E+30

Fig-4: Case-2: Optimal Solution and Constraints

	x1	x2	x3	x4	166.99x1 + 358.99x2 + 290.99x3 - 508x4		
	0	125.084746	291.854407	0.01	$129,832.32		
Objective Parameters	166.99	358.99	290.99	142.99			
Constraints	x1	x2	x3	x4	**Constranit LHS**	**Inequality**	**Constraint RHS**
Monthly budget	330	370	410	127	165942.9327	<=	170000
Space Constraints	5	8	5	5	2460	<=	2460
Allocation constraint for p	0.7	0.7	-0.3	-0.3	1.70532E-14	>=	0
Generators constraint			1	-2	291.8344068	>=	0
X4 constraint				1	0.01	>=	0.01

Fig-5: Case-2 Sensitivity Report

Microsoft Excel 16.80 Sensitivity Report
Worksheet: [ALY6050_MOD5Project_PentareddyN.xlsx]Sheet3
Report Created: 2024-03-22 12:42:18 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
D5	x1	0	-33.66610169	166.99	33.66610169	1E+30
E5	x2	125.0847458	0	358.99	106.594	39.726
F5	x3	291.8544068	0	290.99	94.58571429	66.62125
G5	x4	0.01	0	142.99	148	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
H13	Monthly budget Constranit LHS	165942.9327	0	170000	1E+30	4057.067288
H14	Space Constraints Constranit LHS	2460	52.7779661	2460	60.14245477	2459.747143
H15	Allocation constraint for pressure washers and go-karts Constranit LHS	1.70532E-14	-90.33389831	0	215.227875	16.73894895
H16	Generators constraint Constranit LHS	291.8344068	0	0	291.8344068	1E+30
H17	X4 constraint Constranit LHS	0.01	-148	0.01	97.27813559	0.01

References:

ALY 6050: Module 5 — Maximizing Profit

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