Module-4: A Prescriptive Model for Strategic Decision-making: An Inventory Model

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Introduction:

In this assignment, we are about to build a prescriptive model to resolve an inventory problem to minimize total inventory cost and determine reorder point. In Part-1, we will find the variables using excel and R, later we will use triangular probability distribution to find the minimum total cost, expected order quantity and the expected annual number of orders by constructing a 95% confidence interval and determine the probability distribution.

Part-1:

The data contains the below pivotal variables:

Model Parameters

- Ordering costs given as $220
- unit costs it is $80
- Holding costs which is sated as 18%; that is $80*0.18= $14.4

Uncontrollable Variables

- Annual Demand is 15000 units
- Unit Cost
- Holding cost

Controllable Variables

- Production Level
- Units Ordered
- Inventory levels

Decision Variables:

- Economic Order Quantity (EOQ)
- Reorder Point (ROP)

Model Objective

Here we will determine the minimum total inventory cost with respect to reorder point, provides sufficient quantity to meet the demand until the orders are shipped as well as received.

Mathematical functions that compute the annual ordering cost and annual holding cost based on average inventory:

Economic Order Quantity: Please refer to Fig-1

The EOQ formula is given by:

EOQ= Sqrt(2*(D*S)/H)

Where:

D = Annual demand

S = Ordering cost per order

H = Holding cost per unit per year

Given:

D = 15,000 units

S = $220

H = 18% of the unit value = 0.18 * $80 = $14.40

Therefore EOQ is 677

Reorder Point: To determine the reorder point (ROP), I'll need to calculate the lead time and safety stock. Lead time refers to the duration it takes for the supplier's order to be shipped and received. Considering our policy to order twice as many units, I'll calculate the demand during this lead time and add the necessary safety stock to ensure uninterrupted inventory availability.

Lead time:

Since the policy is to order twice as many units, lead time is the time it takes to receive an order. Assuming lead time is constant, it would be the time it takes to sell the EOQ amount of units and then receive the next order. Let's say it's one month.

Safety Stock:

Safety stock is to cover for any unexpected fluctuations in demand or lead time. It's often expressed in terms of a certain number of days or units of demand. For simplicity, let's assume a safety stock of one month's demand.

Demand per month = Annual demand / 12=1,250

Safety stock = Demand per month = 1,250 units

Reorder Point (ROP) = Lead time demand + Safety stock

ROP=1,250+1,250=2,500

Therefore, the company should order the key engine component in batches of approximately 677 units, and when the inventory level reaches 2,500 units, they should place a reorder to maintain optimal inventory levels. Please refer to Fig-2.

Functions for Costs: Please refer to Fig-3

Annual Holding Cost (Holding Cost per Unit * Average Inventory):

Holding Cost=H× EOQ/2= 4874.42

Annual Ordering Cost (Ordering Cost per Order * Number of Orders):

Ordering Cost= EOQ/D×S = 4874.42

Total Inventory Cost (Sum of Holding and Ordering Costs):

Total Inventory Cost = Holding Cost + Ordering Cost = 9748.85

Data tables to find an approximate order quantity:

In order to approximate the min inventory cost we have consider random ranging from 550 to 800 with an increment of 10 and calculated the number of orders for each quantity by dividing the order by demand. Then we calculated the holding cost by taking the average of the demand and multiplied it by the holding cost, the order cost is obtained by multiplying the orders by order cost. The total inventory cost is the sum of annual holding cost and order cost. Please refer to the Fig-4. From the table we can observe that at order quantity 680, we have minimum total inventory cost 9748. Form Fig-5, we can observe the cost curve as well, which shows the same.

Solver to Verify the results:

To cross check the observations we have used solver. By setting the Total Inventory Cost as objective to minimum and EOQ as changing variable with EOQ as integer as constraint, the output shows that at EOQ 677 we have the minimum TIC which is 9748. The results confirm our approximation. Please refer to Fig-6.

What-IF Analysis of Inventory Cost vs Reorder Point and Economic Order Quantity

The analysis is conducted by using TIC as reference variable for several EOQ's raging form 654-729 and several ROP's varying form 2400 to 3100. By using the Data Table option under "What IF analysis", we estimated the Inventory cost for each intersection of EOQ and ROP and identified that we have the minimum cost at order quantity of 678. From Fig-7 and 8, we can observe the findings form the table and the cost curve.

Part-1 in R:

The same analysis is conducted in R, we identified the Total Inventory cost based on the given variables and observed that the TIC is 9748.84, which is similar to that of excel values, please refer to Fig-9.

Additonally, in order to calculate the TIC over a range of random entries, we created a function called "cal_total_cost" which calculated the summation of holding and order cost for different order quantities raging form 100 to 5000.

Total inventory cost for each order quantity is calculated by using sapply function and passing order_quantities, function(Q) cal_total_cost(Q, D, S, H, C) as input variables. Secondly, we found the optimal reorder quantity which is the minimal total cost of order quantities. Lastly, we plotted the cost curve and identities that minimum TIC is with order quantity of 700. Please refer to Fig-10 and Fig-11.

Part-II: Triangular probability distribution between 13000 and 17000 units with a mode of 15000 units.

It is given that annual demand follows a triangular probability distribution between 13000 and 17000 units, with a mode of 15000 units and the unit cost, holding cost rate, and ordering cost per order are defined earlier.

Please refer to Fig-12: Next, the code runs a simulation 1000 times to mimic different scenarios of annual demand. Each time, it generates a random value for the demand based on the triangular distribution. Then, it calculates the optimal order quantity and the total cost associated with that demand.

Further, We set the seed for reproducibility and specify the number of occurrences for the simulation. Using the rtriangle() function, we generate random annual demand values following a triangular distribution. We initialize empty vectors to store order quantities and total costs for each occurrence.

In a loop over each occurrence:

- We calculate the order quantity based on the demand using the economic order quantity (EOQ) formula.
- We calculate the total cost using the cal_total_cost() function and store the results.

Estimate Expected Minimum Total Cost: Please refer to Fig-13:

- We compute the mean and standard deviation of the total costs.
- Using the t-distribution, we calculate the 95% confidence interval for the mean total cost.
- We plot a histogram of the total costs and overlay the confidence interval (Fig-16).
- Expected Minimum Total Cost: 44115.69
- 95% Confidence Interval for Minimum Total Cost: 44041.44 - 44189.94

Estimate Expected Order Quantity: Please refer to Fig-14:

- We compute the mean and standard deviation of the order quantities.

- Using the t-distribution, we calculate the 95% confidence interval for the mean order quantity.
- Expected Order Quantity: 14992.31
- 95% Confidence Interval for Order Quantity: 14941.91 - 15042.72

Estimate Expected Annual Number of Orders: Please refer to Fig-15:

- We calculate the ratio of annual demand to order quantity for each occurrence to determine the number of orders.
- We compute the mean and standard deviation of the number of orders.
- Using the t-distribution, we calculate the 95% confidence interval for the mean number of orders.
- Expected Annual Number of Orders: 1
- 95% Confidence Interval for Annual Number of Orders: 0.9379453 - 1.062055

From the Histograms in Fig-16:

- The true population mean of Expected Minimum Total Cost within the confidence interval has high probability (95% in this case), it suggests that our estimation is reliable.
- The true population mean of Expected Order Quantity within the confidence interval has high probability (95% in this case), it suggests that the interval encompasses plausible values for the order quantity
- The true population mean of annual number of orders within the confidence interval probability (95% in this case) is skewed to left. The leftward skewness of the histogram and the lower end of the confidence interval align with the expectation of a lower number of orders per year, which is consistent with the company's inventory management approach aiming to minimize ordering frequency.

Conclusion:

In Part 1, we utilized both Excel Solver and R to determine optimal inventory management decisions for a key engine component. By calculating the Economic Order Quantity (EOQ) and Total Inventory Cost (TIC), we identified that an EOQ of 680 units resulted in the minimum TIC, consistent with the Excel Solver's findings. However, our R analysis identified a slightly different optimal order quantity of 700 units, indicating a nuanced difference in the cost curve. This discrepancy underscores the importance of considering multiple analytical approaches to ensure robust decision-making in inventory management.

Moving to Part 2, we conducted a simulation to estimate the expected minimum total cost, expected order quantity, and expected annual number of orders. Through statistical analysis, including calculation of means, standard deviations, and confidence intervals, we provided insights into the reliability and validity of these estimations. Our histogram visualizations, accompanied by confidence intervals, showcased the distribution of total costs, order quantities, and annual number of orders, affirming the accuracy and consistency of our estimations.

Overall, by integrating both analytical and simulation-based approaches, we have equipped the manufacturing company with comprehensive insights into inventory management strategies for their key engine component. Our findings not only validate traditional analytical methods but also highlight the importance of incorporating simulation and statistical analysis to ensure robust decision-making in complex business scenarios. These insights enable the company to optimize inventory levels, minimize costs, and enhance operational efficiency, ultimately driving towards sustained business success.

Appendix:

Fig-1: EOQ

C17 | =SQRT(2*(G4*C5)/D4)

	A	B	C	D
13				
14	1	Economic Order Quantity		
15				
16				
17		EOQ	677	
18				

Fig-2: ROP

C23 | =C21+C22

	A	B	C
18			
19	2	Calculate Reorder Point:	
20		Reorder Point (ROP) is calculated as the demand during the lead time (time between placing an order and receiving it) plus safety stock.	
21		Demand per month	1250
22		Safety stock	1250
23		Reorder Point	2500

Fig-3: Cost Functions:

3	Functions for Costs	
	Annual Holding Cost (Holding Cost per Unit * Average Inventory)	
	Average Inventory	338.5016
	Holding costs	14.4
	Annual Holding Cost	4874.42
	Annual Ordering Cost (Ordering Cost per Order * Number of Orders):	
	No of Orders per year	22.16
	Annual Ordering Cost	4874.42
4	Total Inventory Cost	
	Total Inventory Cost=Holding Cost+Ordering Cost	
	Total Total Inventory Cost`	9748.85

Fig-4: Approximation :

Order Quantity	No. of Times to Order	Annual Holdings Cost	Annual Ordering Cost	Total Inventory Cost	Min Cost
550	27.27272727	$ 3,960.00	$ 6,000.00	9960	9748.94118
560	26.78571429	$ 4,032.00	$ 5,892.86	9924.857143	
570	26.31578947	$ 4,104.00	$ 5,789.47	9893.473684	
580	25.86206897	$ 4,176.00	$ 5,689.66	9865.655172	
590	25.42372881	$ 4,248.00	$ 5,593.22	9841.220339	
600	25	$ 4,320.00	$ 5,500.00	9820	
610	24.59016393	$ 4,392.00	$ 5,409.84	9801.836066	
620	24.19354839	$ 4,464.00	$ 5,322.58	9786.580645	
630	23.80952381	$ 4,536.00	$ 5,238.10	9774.095238	
640	23.4375	$ 4,608.00	$ 5,156.25	9764.25	
650	23.07692308	$ 4,680.00	$ 5,076.92	9756.923077	
660	22.72727273	$ 4,752.00	$ 5,000.00	9752	
670	22.3880597	$ 4,824.00	$ 4,925.37	9749.373134	
680	22.05882353	$ 4,896.00	$ 4,852.94	9748.941176	
690	21.73913043	$ 4,968.00	$ 4,782.61	9750.608696	
700	21.42857143	$ 5,040.00	$ 4,714.29	9754.285714	
710	21.12676056	$ 5,112.00	$ 4,647.89	9759.887324	
720	20.83333333	$ 5,184.00	$ 4,583.33	9767.333333	
730	20.54794521	$ 5,256.00	$ 4,520.55	9776.547945	
740	20.27027027	$ 5,328.00	$ 4,459.46	9787.459459	
750	20	$ 5,400.00	$ 4,400.00	9800	
760	19.73684211	$ 5,472.00	$ 4,342.11	9814.105263	
770	19.48051948	$ 5,544.00	$ 4,285.71	9829.714286	
780	19.23076923	$ 5,616.00	$ 4,230.77	9846.769231	
790	18.98734177	$ 5,688.00	$ 4,177.22	9865.21519	
800	18.75	$ 5,760.00	$ 4,125.00	9885	

Fig-5: Cost Curve

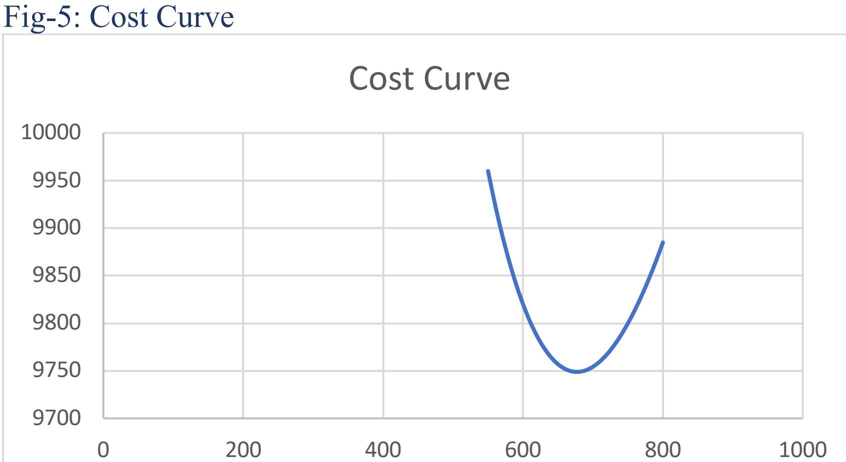


Fig-6: Solver

Solver	
Total Invento	9748.85
ROP	2500
EOQ	677

Fig-7: What-If Analysis

What-IF Analysis of Inventry Cost vs Reorder Point and Economic Order Quantity

	TIC	ROP							
	9749	2400.00	2500.00	2600.00	2700.00	2800.00	2900.00	3000.00	3100.00
EOQ	654	9754.67	9754.67	9754.67	9754.67	9754.67	9754.67	9754.67	9754.67
	657	9753.23	9753.23	9753.23	9753.23	9753.23	9753.23	9753.23	9753.23
	660	9752.00	9752.00	9752.00	9752.00	9752.00	9752.00	9752.00	9752.00
	663	9750.98	9750.98	9750.98	9750.98	9750.98	9750.98	9750.98	9750.98
	666	9750.15	9750.15	9750.15	9750.15	9750.15	9750.15	9750.15	9750.15
	669	9749.54	9749.54	9749.54	9749.54	9749.54	9749.54	9749.54	9749.54
	672	9749.11	9749.11	9749.11	9749.11	9749.11	9749.11	9749.11	9749.11
	675	9748.89	9748.89	9748.89	9748.89	9748.89	9748.89	9748.89	9748.89
	678	9748.86	9748.86	9748.86	9748.86	9748.86	9748.86	9748.86	9748.86
	681	9749.01	9749.01	9749.01	9749.01	9749.01	9749.01	9749.01	9749.01
	684	9749.36	9749.36	9749.36	9749.36	9749.36	9749.36	9749.36	9749.36
	687	9749.89	9749.89	9749.89	9749.89	9749.89	9749.89	9749.89	9749.89
	690	9750.61	9750.61	9750.61	9750.61	9750.61	9750.61	9750.61	9750.61
	693	9751.50	9751.50	9751.50	9751.50	9751.50	9751.50	9751.50	9751.50
	696	9752.58	9752.58	9752.58	9752.58	9752.58	9752.58	9752.58	9752.58
	699	9753.83	9753.83	9753.83	9753.83	9753.83	9753.83	9753.83	9753.83
	702	9755.25	9755.25	9755.25	9755.25	9755.25	9755.25	9755.25	9755.25
	705	9756.85	9756.85	9756.85	9756.85	9756.85	9756.85	9756.85	9756.85
	708	9758.62	9758.62	9758.62	9758.62	9758.62	9758.62	9758.62	9758.62
	711	9760.55	9760.55	9760.55	9760.55	9760.55	9760.55	9760.55	9760.55
	714	9762.65	9762.65	9762.65	9762.65	9762.65	9762.65	9762.65	9762.65
	717	9764.91	9764.91	9764.91	9764.91	9764.91	9764.91	9764.91	9764.91
	720	9767.33	9767.33	9767.33	9767.33	9767.33	9767.33	9767.33	9767.33
	723	9769.92	9769.92	9769.92	9769.92	9769.92	9769.92	9769.92	9769.92
	726	9772.65	9772.65	9772.65	9772.65	9772.65	9772.65	9772.65	9772.65
	729	9775.55	9775.55	9775.55	9775.55	9775.55	9775.55	9775.55	9775.55

Fig-8: Inventory Cost Curve

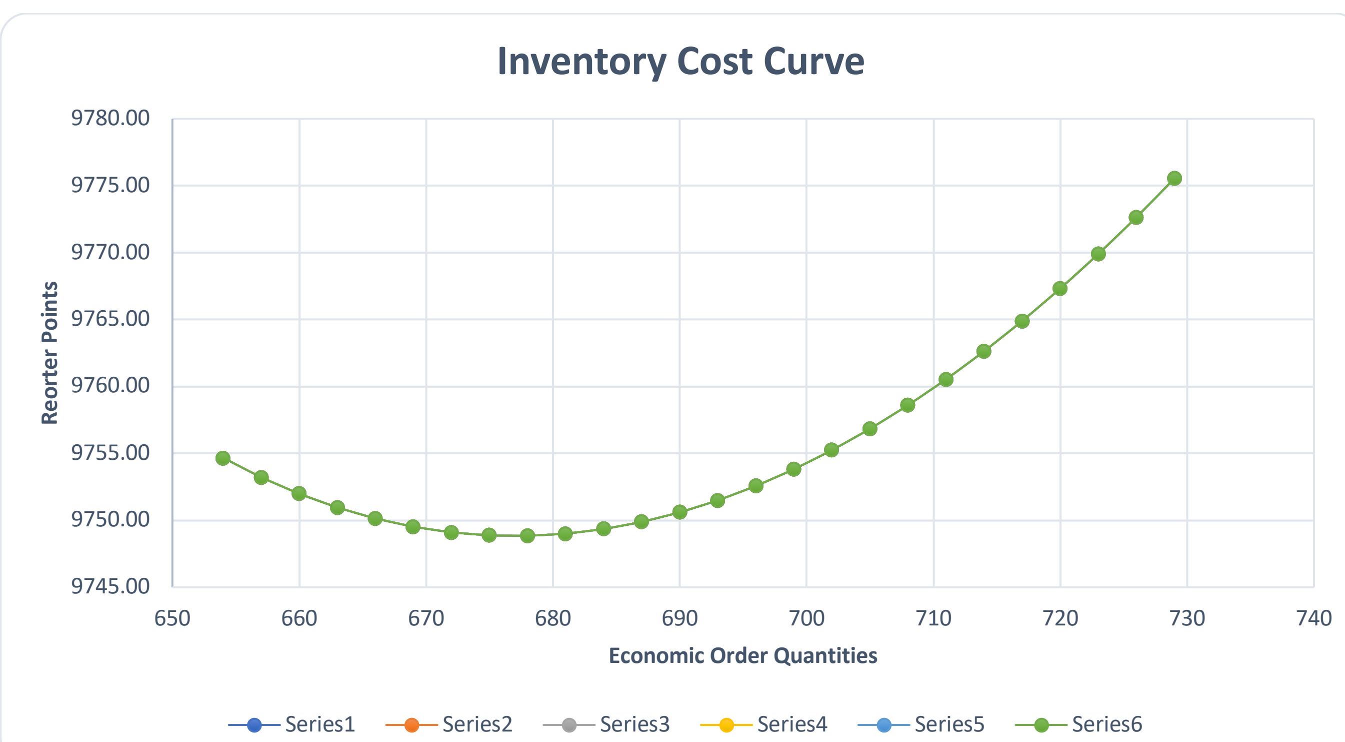


Fig-9: TIC in R

```
> ####ALY 6070- Assignment-4
>
> # the parameters and data
> D <- 15000 # Annual demand
> C <- 80    # Unit cost
> H <- 0.18  # Holding cost rate (18%)
> Holding_cost_rate <- H * C
> S <- 220   # Ordering cost per order
>
> #  Economic Order Quantity (EOQ)
> EOQ <- sqrt((2 * D * S) / Holding_cost_rate)
> EOQ
[1] 677.0032
> #  Reorder point (ROP)
> ROP <- EOQ / 2
> ROP
[1] 338.5016
> #  number of orders per year
> N <- D / EOQ
>
> #  total ordering cost per year
> Ordering_Cost <- N * S
> Ordering_Cost
[1] 4874.423
> #  average inventory
> Avg_Inventory <- EOQ / 2
>
> #  holding cost per year
> Holding_Cost <- Avg_Inventory * H * C
> Holding_Cost
[1] 4874.423
> #  total inventory cost per year
> Total_Inventory_Cost <- Ordering_Cost + Holding_Cost
> Total_Inventory_Cost
[1] 9748.846
```

Fig-10: Function

```
> #  a function to calculate total inventory cost
>
> cal_total_cost <- function(Q , D , S , H, C ) {
+   N <- D/Q
+   Ordering_Cost <- N * S
+   Avg_Inventory <- Q / 2
+   Holding_Cost <- Avg_Inventory * H * C
+   return(Ordering_Cost + Holding_Cost)
+ }
>
> #  sequence of order quantities
> order_quantities <- seq(100, 5000, by = 100)
>
> #  total inventory cost for each order quantity
> total_costs <- sapply(order_quantities, function(Q) cal_total_cost(Q, D, S, H, C))
> total_costs
 [1] 33720.000 17940.000 13160.000 11130.000 10200.000  9820.000  9754.286  9885.000 10146.667 10500.000
[11] 10920.000 11390.000 11898.462 12437.143 13000.000 13582.500 14181.176 14793.333 15416.842 16050.000
[21] 16691.429 17340.000 17994.783 18655.000 19320.000 19989.231 20662.222 21338.571 22017.931 22700.000
[31] 23384.516 24071.250 24760.000 25450.588 26142.857 26836.667 27531.892 28228.421 28926.154 29625.000
[41] 30324.878 31025.714 31727.442 32430.000 33133.333 33837.391 34542.128 35247.500 35953.469 36660.000
> optimal_reorder_quantity <- order_quantities[which.min(total_costs)]
> optimal_reorder_quantity
[1] 700
> plot(order_quantities, total_costs, type = "l", ylab = "Order Quantity", xlab = "Total Cost",
+      main = "Total Cost vs. Order Quantity")
```

Fig-11: Total Cost vs. Order Quantity

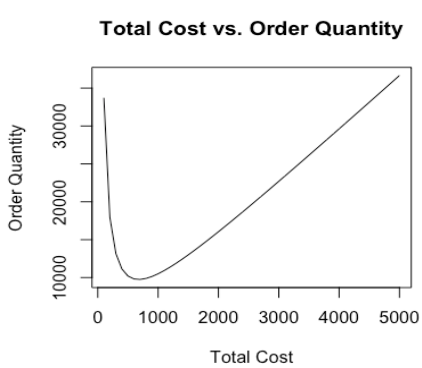


Fig-12: Parameters for Part-2

```
> # Set seed for reproducibility
> set.seed(123)
>
> # Defined parameters
> S <- 220    # Ordering cost per order
> H <- 0.18   # Holding cost rate (18%)
> C <- 80     # Unit cost
> Holding_cost_rate <- H * C
> num_simulations <- 1000
>
> #  Finding Annual demand using triangular distribution
> annual_demand <- rtriangle(num_simulations, a = 13000, b = 17000, c = 15000)
>
>
> #  Simulating and calculating minimum total cost for each occurrence
> min_total_costs <- numeric(num_simulations)
> for (i in 1:num_simulations) {
+   D <- annual_demand[i]
+   # Calculate optimal order quantity using EOQ formula
+   EOQ <- sqrt((2 * D * S) / H)
+   Q <- round(EOQ)  # Round to nearest integer
+   min_total_costs[i] <- cal_total_cost(Q, D, S, H, C)
+ }
```

Fig-13: Estimate expected minimum total cost and construct 95% confidence interval

```
> # (i) Estimate expected minimum total cost and construct 95% confidence interval
> mean_min_total_cost <- mean(min_total_costs)
> sd_min_total_cost <- sd(min_total_costs)
> n <- length(min_total_costs)
> lower_ci_min_total_cost <- mean_min_total_cost - qt(0.975, df = n - 1) * (sd_min_total_cost / sqrt(n))
> upper_ci_min_total_cost <- mean_min_total_cost + qt(0.975, df = n - 1) * (sd_min_total_cost / sqrt(n))
> cat("Expected Minimum Total Cost:", mean_min_total_cost, "\n")
Expected Minimum Total Cost: 44115.69
> cat("95% Confidence Interval for Minimum Total Cost:", lower_ci_min_total_cost, "-", upper_ci_min_total_cost, "\n")
95% Confidence Interval for Minimum Total Cost: 44041.44 - 44189.94
```

Fig-14: Estimate expected order quantity and construct 95% confidence interval

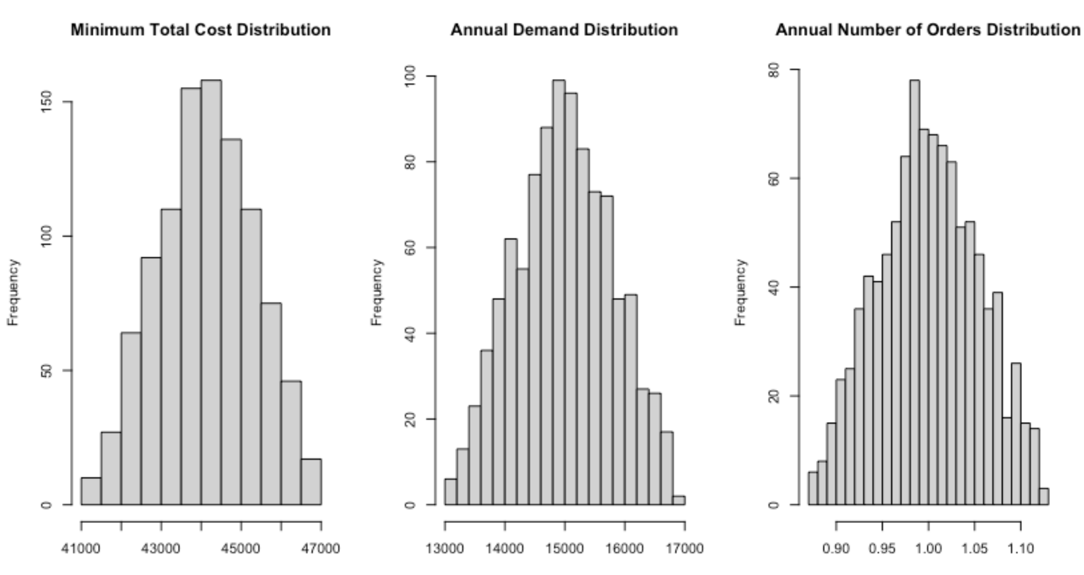
```
> # (ii) Estimate expected order quantity and construct 95% confidence interval
> mean_order_quantity <- mean(annual_demand)
> sd_order_quantity <- sd(annual_demand)
> n <- length(annual_demand)
> lower_ci_order_quantity <- mean_order_quantity - qt(0.975, df = n - 1) * (sd_order_quantity / sqrt(n))
> upper_ci_order_quantity <- mean_order_quantity + qt(0.975, df = n - 1) * (sd_order_quantity / sqrt(n))
> cat("Expected Order Quantity:", mean_order_quantity, "\n")
Expected Order Quantity: 14992.31
> cat("95% Confidence Interval for Order Quantity:", lower_ci_order_quantity, "-", upper_ci_order_quantity, "\n")
95% Confidence Interval for Order Quantity: 14941.91 - 15042.72
```

Fig-15: Estimate expected annual number of orders and construct 95% confidence interval

```
> # (iii) Estimate expected annual number of orders and construct 95% confidence interval
> mean_annual_orders <- mean(annual_demand) / mean_order_quantity
> sd_annual_orders <- sd(annual_demand) / sd_order_quantity
> n <- length(annual_demand)
> lower_ci_annual_orders <- mean_annual_orders - qt(0.975, df = n - 1) * (sd_annual_orders / sqrt(n))
> upper_ci_annual_orders <- mean_annual_orders + qt(0.975, df = n - 1) * (sd_annual_orders / sqrt(n))
> cat("Expected Annual Number of Orders:", mean_annual_orders, "\n")
Expected Annual Number of Orders: 1
> cat("95% Confidence Interval for Annual Number of Orders:", lower_ci_annual_orders, "-", upper_ci_annual_orders, "\n")
95% Confidence Interval for Annual Number of Orders: 0.9379453 - 1.062055
```

Fig-16: Histograms to visualize the distributions

```
> # Determine the probability distributions that best fit the data
> # For simplicity, let's use histograms to visualize the distributions
>
> par(mfrow = c(1, 3))
> hist(min_total_costs, breaks = 20, main = "Minimum Total Cost Distribution", xlab = "Total Cost")
> hist(annual_demand, breaks = 20, main = "Annual Demand Distribution", xlab = "Demand")
> hist(annual_demand / mean_order_quantity, breaks = 20, main = "Annual Number of Orders Distribution", xlab = "Number of Orders")
```



References:

ALY 6050: Module 4 — Perspective Analysis

Steve Crow: Excel What If Analysis: Data Table Two Variable (Excel 2016); [online]: https://www.youtube.com/watch?v=6I6fpfbkge8

Mark Keith: MS Excel Solver Examples; [online]: https://www.youtube.com/watch?v=MU5r2ceGbxg&t=1145s

Microsoft: Define and solve a problem by using Solver; [online]: Define and solve a problem by using Solver - Microsoft Support