Module-6: Optimizing

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Introduction:

The purpose of the assignment is to implement non-liner techniques to optimize the decision variables. In Part-1; we optimize the allocation of waste disposal between disposal sites and collection sites, whereas in Part-2 we derive the maximum return percentage to allocate weightage between the stocks and observe the trend between the expected return and variance.

Part-1: Rockhill Shipping & Transport Company

Given Variables:

The data contains the below predominant variables:

	Waste Proposal Site		
Plant:	*Orangeburg*	*Florence*	*Macon*
Denver	$12	$15	$17
Morganton	14	9	10
Morrisville	13	20	11
Pineville	17	16	19
Rockhill	7	14	12
Statesville	22	16	18

Plant:	*Waste per Week (bbl)*
Denver	45
Morganton	26
Morrisville	42
Pineville	53
Rockhill	29
Statesville	38

	Plant					
Plant:	Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville
Denver	$---	$3	$4	$9	$5	$4
Morganton	6	---	7	6	9	4
Morrisville	5	7	---	3	4	9
Pineville	5	4	3	---	3	11
Rockhill	5	9	5	3	---	14
Statesville	4	7	11	12	8	---

	Waste Proposal Site		
Waste Disposal Site:	Orangeburg	Florence	Macon
Orangeburg	$---	$12	$10
Florence	12	---	15
Macon	10	15	---

Mathematical Formulation:

To solve this problem using non-linear programming, we need to define decision variables, objective function as well as constraints. The objective function is to minimize the total shipping cost, which includes direct shipments and shipments through intermediate points. Therefore, the objective is:

Minimize $Z = i\sum j\sum(cij \cdot xij) + i\sum k\sum(cik \cdot yik) + j\sum k\sum(cjk \cdot zjk)$

Where *cij*, *cik*, and cjk represent the shipping costs per barrel between plants and disposal sites, plants and plants (intermediate points), and between disposal sites, respectively.

Subject to the following constraints:

Supply constraints: The waste generated should not exceed its capacity:

$\sum xij + k\sum yik >=$ Waste per Week (bbl)

Demand constraints: Waste sent to each disposal site should not exceed its capacity:

$\sum xij + k\sum zjk \leq$ Disposal Site Capacity (bbl)

The amount of waste disposed from between the sites should be equal to the waste disposed to the site

Non-negativity constraints: xij, yik, and $zjk \geq 0$

Solver and sensitivity report:

I tried to minimize the transportation cost using objective function Z. It is initially calculated by using SUMPRODUCT function, of cost and amount of waste transported between the sites. Further, the constraints and inequalities have been set up; we should now calculate the amount that is shipped from and shipped to the disposal sites. This would be the sum of all variables I,e wasted disposed from Denver to remaining sites, in the similar way the sum for the remaining constraints is also calculated.
Through solver form data tab, the objective function is set to minimum, and changing varies I,e quantity shipped is assigned and respective constraints have been assigned. Marked non-variables and using the "Simplex LP" method we solved the objective. Please refer to the Fig-1 and 2 for the findings.

The optimal solutions:

- **Optimal cost:**
    - The minimal cost achieved is $2743.

- **Total barrels transported each week:**

Total barrels transported each week = 36 + 9 + 0 + 0 + 0 + 0 + 0 + 26 + 42 + 29 + 37 + 0 + 63 = 242 barrels

- 9 barrels from Denver to Morganton
- 36 barrels from Denver to Orangeburg
- 26 barrels from Morganton to Florence
- 42 barrels from Morrisville to Macon
- 29 barrels from Rockhill to Orangeburg
- 37 barrels from Orangeburg to Florence
- 63 barrels from Orangeburg to Macon

Therefore, 242 barrels would be transported each week from the sources to the destination and the corresponding total cost is $2743.

Part 2: Investment Allocations

Given Variables:

Expected Returns	
Bonds	7%
High tech stocks	12%
Foreign stocks	11%
Call options	14%
Put options	14%
Gold	9%

	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006
Foreign stocks			0.008	0.0015	-0.0055	-0.0007
Call options				0.012	-0.0005	0.0008
Put options					0.012	-0.0008
Gold						0.005

Table 1: The Covariance matrix of assets' returns

(i)To determine the how much money should the investor invest in each stock (total investment of $10,000), with minimum expected return of 11% and with the minimum risk.

Objective:

The objective is to invest $10,000 with minimum risk and return greater than 11%.

Constraints:

The sum of parts of investment should be equal to $10,000.

Expected return should be greater than or equal to 11%.

Calculation Variables:

Invested Amount: We need to derive total investment which is sum of all investments in stocks.

Return: We have the probability of expected retunes in each stock, hence the total return can be calculated by using SUMPRODUCT function among probabilities and variables covariance matrix

Expected Return: is the sum product of probability and return

Variance (min risk): is the product of respective probabilities with the square of difference between individual returns and expected return.

Now we have all the requited variables to calculate the amount invested, find the expected return and variance.

Using GRG Nonlinear method in solver, the objective is set to minimize the variance, the changing variables are assigned I,e investments in different stocks, and constraints, that invested amount is 10,000 and expected return greater than qual to 0.11 has been assigned.

From Fig-4; we can observe the optimal allocation.

Asset Type	Invested Amount
Bonds	$7,264.19
High tech stocks	$253.28
Foreign stocks	$844.17

Asset Type	Invested Amount
Call options	$15.93
Put options	$1003.23
Gold	$619.20

This allocation minimizes portfolio variance while meeting the constraints of a minimum expected return of 11% and a total investment of $10,000.

(ii) Use successive values of 10%, 10.5%, 11%, 11.5%, 12%, 12.5%, 13% and 13.5% as the baseline return values to obtain eight pairs of solutions (r, e). Plot "e" versus "r".

The variance (r) and expected return (e) for 11% have been calculated and using the similar approach, I derived the results for the remaining expected returns. Please refer to Fig-5 to Fig-11, for the respective values.

Form the results we obtained the below values:

	e	r
1	3.22573996	2.17188455
2	3.22573998	2.17188456
3	3.22573912	2.17188456
4	3.22573998	2.17188456
5	3.22573912	2.17188456
6	0.51017556	3.34764471
7	0.51017556	3.34764471
8	0.51017556	3.34764471

The line plot for these values have been plotted, please refer to Fig-12. From the plot we observe that:

- As the expected portfolio return increases, the minimized risk tends to increase: For lower expected returns (around 3.2%), the minimized risk remains relatively constant at approximately 2.17. However, for higher expected returns (around 0.51%), the minimized risk increases to approximately 3.35.
- There is a nonlinear relationship between risk and return: The plot does not exhibit a simple linear relationship between risk and return. Instead, it seems to follow a more complex pattern, possibly exhibiting a convex shape.

- Diminishing marginal returns: At higher expected returns, the increase in minimized risk becomes more pronounced compared to the increase in expected return. This suggests diminishing marginal returns to risk reduction as the expected return increases.

Based on these observations, it appears that the relationship between risk and return is nonlinear and exhibits diminishing marginal returns.

Conclusion:

The analysis of the optimal investment allocation, its corresponding risk-return relationship, and the observed patterns provide a nuanced understanding of portfolio management dynamics. The non-linear relationship between risk and return, as evidenced by the plot exhibiting a possible convex shape, underscores the intricate interplay between these two crucial factors. Notably, as the expected portfolio return increases, the minimized risk tends to escalate, indicating the inherent trade-off inherent in investment decisions. This observation aligns with conventional wisdom in finance, highlighting the need for investors to carefully balance their risk appetite with desired return objectives. Moreover, the diminishing marginal returns to risk reduction as expected return rises underscores the complexity of optimizing portfolios for risk-adjusted returns. Ultimately, the optimal investment allocation, meticulously crafted to minimize portfolio variance while adhering to constraints such as minimum expected return and total investment, underscores the significance of portfolio optimization techniques in navigating the intricate landscape of financial markets. This analysis accentuates the importance of informed decision-making in portfolio management, where investors must weigh the potential rewards against associated risks to achieve their investment goals effectively.

Appendix:

Fig-1: Case-1-Minimum Cost

										Cost Function
	Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	Z
Denver	1000	3	4	9	5	4	12	15	17	2743
Morganton	6	1000	7	6	9	4	14	9	10	
Morrisville	5	7	1000	3	4	9	13	20	11	
Pineville	5	4	3	1000	3	11	17	16	19	
Rockhill	5	9	5	3	1000	14	7	14	12	
Statesville	4	7	11	12	8	1000	22	16	18	
Orangeburg	1000	1000	1000	1000	1000	1000	1000	12	10	
Florence	1000	1000	1000	1000	1000	1000	12	1000	15	
Macon	1000	1000	1000	1000	1000	1000	10	15	1000	

Fig-2: Case-1: Constraints

	Denver	Morganton	Morrisville	Pineville	Rockhill	Statesville	Orangeburg	Florence	Macon	Shipped from	inequality	supply	
Denver	0	9	0	0	0	0	36	0	0	45	>=	45	
Morganton	0	0	0	0	0	0	0	26	0	26	>=	26	
Morrisville	0	0	0	0	0	0	0	0	42	42	>=	42	
Pineville	0	0	53	0	0	0	0	0	0	53	>=	53	
Rockhill	0	0	0	0	0	0	29	0	0	29	>=	29	
Statesville	38	0	0	0	0	0	0	0	0	38	>=	38	
Orangeburg	0	0	0	0	0	0	0	37	63	100			
Florence	0	0	0	0	0	0	0	0	0	0			total Supply
Macon	0	0	0	0	0	0	0	0	0	0			233
Shipped TO	38	9	53	0	0	0	65	63	105				
							<=	<=	<=				
							65	80	105				
							Total Shipped						
							233						

Fig-3: Part-1: Sensitivity Report

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
X20	Denver Denver	0	997	1000	1E+30	997
Y20	Denver Morganton	9	0	3	0	1
Z20	Denver Morrisville	0	1	4	1E+30	1
AA20	Denver Pineville	0	6	9	1E+30	6
AB20	Denver Rockhill	0	2	5	1E+30	2
AC20	Denver Statesville	0	1	4	1E+30	1
AD20	Denver Orangeburg	36	0	12	1	4
AE20	Denver Florence	0	0	15	1E+30	0
AF20	Denver Macon	0	4	17	1E+30	4
X21	Morganton Denver	0	9	6	1E+30	9
Y21	Morganton Morganton	0	1003	1000	1E+30	1003
Z21	Morganton Morrisville	0	10	7	1E+30	10
AA21	Morganton Pineville	0	9	6	1E+30	9
AB21	Morganton Rockhill	0	12	9	1E+30	12
AC21	Morganton Statesville	0	7	4	1E+30	7
AD21	Morganton Orangeburg	0	8	14	1E+30	8
AE21	Morganton Florence	26	0	9	3	9
AF21	Morganton Macon	0	3	10	1E+30	3
X22	Morrisville Denver	0	4	5	1E+30	4
Y22	Morrisville Morganton	0	6	7	1E+30	6
Z22	Morrisville Morrisville	0	999	1000	1E+30	999
AA22	Morrisville Pineville	0	2	3	1E+30	2
AB22	Morrisville Rockhill	0	3	4	1E+30	3
AC22	Morrisville Statesville	0	8	9	1E+30	8
AD22	Morrisville Orangeburg	0	3	13	1E+30	3
AE22	Morrisville Florence	0	7	20	1E+30	7
AF22	Morrisville Macon	42	0	11	2	13
X23	Pineville Denver	0	2	5	1E+30	2
Y23	Pineville Morganton	0	1	4	1E+30	1
Z23	Pineville Morrisville	53	0	3	0	15
AA23	Pineville Pineville	0	997	1000	1E+30	997
AB23	Pineville Rockhill	0	0	3	1E+30	0
AC23	Pineville Statesville	0	8	11	1E+30	8
AD23	Pineville Orangeburg	0	5	17	1E+30	5
AE23	Pineville Florence	0	1	16	1E+30	1
AF23	Pineville Macon	0	6	19	1E+30	6
X24	Rockhill Denver	0	7	5	1E+30	7
Y24	Rockhill Morganton	0	11	9	1E+30	11
Z24	Rockhill Morrisville	0	7	5	1E+30	7
AA24	Rockhill Pineville	0	5	3	1E+30	5
AB24	Rockhill Rockhill	0	1002	1000	1E+30	1002
AC24	Rockhill Statesville	0	16	14	1E+30	16
AD24	Rockhill Orangeburg	29	0	7	4	10
AE24	Rockhill Florence	0	4	14	1E+30	4
AF24	Rockhill Macon	0	4	12	1E+30	4
X25	Statesville Denver	38	0	4	0	16
Y25	Statesville Morganton	0	3	7	1E+30	3
Z25	Statesville Morrisville	0	7	11	1E+30	7
AA25	Statesville Pineville	0	8	12	1E+30	8
AB25	Statesville Rockhill	0	4	8	1E+30	4
AC25	Statesville Statesville	0	996	1000	1E+30	996
AD25	Statesville Orangeburg	0	9	22	1E+30	9
AE25	Statesville Florence	0	0	16	1E+30	0
AF25	Statesville Macon	0	4	18	1E+30	4
X26	Orangeburg Denver	0	1000	1000	1E+30	1000
Y26	Orangeburg Morganton	0	1000	1000	1E+30	1000
Z26	Orangeburg Morrisville	0	1000	1000	1E+30	1000
AA26	Orangeburg Pineville	0	1000	1000	1E+30	1000
AB26	Orangeburg Rockhill	0	1000	1000	1E+30	1000
AC26	Orangeburg Statesville	0	1000	1000	1E+30	1000
AD26	Orangeburg Orangeburg	0	991	1000	1E+30	991
AE26	Orangeburg Florence	37	0	12	0	2
AF26	Orangeburg Macon	63	0	10	2	2
X27	Florence Denver	0	1000	1000	1E+30	1000
Y27	Florence Morganton	0	1000	1000	1E+30	1000
Z27	Florence Morrisville	0	1000	1000	1E+30	1000
AA27	Florence Pineville	0	1000	1000	1E+30	1000
AB27	Florence Rockhill	0	1000	1000	1E+30	1000
AC27	Florence Statesville	0	1000	1000	1E+30	1000
AD27	Florence Orangeburg	0	3	12	1E+30	3
AE27	Florence Florence	0	988	1000	1E+30	988
AF27	Florence Macon	0	5	15	1E+30	5
X28	Macon Denver	0	1000	1000	1E+30	1000
Y28	Macon Morganton	0	1000	1000	1E+30	1000
Z28	Macon Morrisville	0	1000	1000	1E+30	1000
AA28	Macon Pineville	0	1000	1000	1E+30	1000
AB28	Macon Rockhill	0	1000	1000	1E+30	1000
AC28	Macon Statesville	0	1000	1000	1E+30	1000
AD28	Macon Orangeburg	0	1	10	1E+30	1
AE28	Macon Florence	0	3	15	1E+30	3
AF28	Macon Macon	0	990	1000	1E+30	990

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
AD29	Shipped TO Orangeburg	65	-3	65	9	17
AE29	Shipped TO Florence	63	0	80	1E+30	17
AF29	Shipped TO Macon	105	-2	105	37	17
AD34	Total Shipped	233	12	0	17	37
AG20	Denver Shipped from	45	15	45	17	9
AG21	Morganton Shipped from	26	9	26	17	26
AG22	Morrisville Shipped from	42	13	42	17	37
AG23	Pineville Shipped from	53	15	53	17	37
AG24	Rockhill Shipped from	29	10	29	17	9
AG25	Statesville Shipped from	38	16	38	17	37

Fig-4: Part-2: Minimizing variance

	at 11%						
	Question-1						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	7264.19365
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	253.281938
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	844.16747
Call options				0.012	-0.0005	0.0008	15.9255684
Put options					0.012	-0.0008	1003.2331
Gold						0.005	619.198275
Probability	0.07	0.12	0.11	0.14	0.14	0.09	10000
Return	7.26419365	4.45879553	4.67539444	4.4050769	4.440194436	4.77287474	
Expected Return	3.22573912						
Min expected Return	0.11						
Variance	2.17188456						

Fig-5: Expected Return at 10%

	at 10%						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	7264.19182
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	253.281621
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	844.168479
Call options				0.012	-0.0005	0.0008	15.925818
Put options					0.012	-0.0008	1003.23339
Gold						0.005	619.198844
Probability	0.07	0.12	0.11	0.14	0.14	0.09	9999.99997
Return	7.26419182	4.45879214	4.67540294	4.40508027	4.440193412	4.77287593	
Expected Return	3.22573996						
Min expected Return	0.1						
Variance	2.17188455						

Fig-6: Expected Return at 10.5%

	at 10.5%						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	7264.19181
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	253.281629
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	844.168481
Call options				0.012	-0.0005	0.0008	15.9258222
Put options					0.012	-0.0008	1003.23339
Gold						0.005	619.198853
Probability	0.07	0.12	0.11	0.14	0.14	0.09	9999.99999
Return	7.26419181	4.4587922	4.67540295	4.40508033	4.440193443	4.77287597	
Expected Return	3.22573998						
Min expected Return	0.105						
Variance	2.17188456						

Fig-7: Expected Return at 11.5%

	at 11.5%						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	1666.5
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	1666.5
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	1666.5
Call options				0.012	-0.0005	0.0008	1667.5
Put options					0.012	-0.0008	1666.5
Gold						0.005	1666.5
Probability	0.07	0.12	0.11	0.14	0.14	0.09	10000
Return	7.26419181	4.4587922	4.67540295	4.40508033	4.440193443	4.77287597	
Expected Return	3.22573998						
Min expected Return	0.115						
Variance	2.17188456						

Fig-8: Expected Return at 12%

	at 12%						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	1666.5
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	1666.5
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	1666.5
Call options				0.012	-0.0005	0.0008	1667.5
Put options					0.012	-0.0008	1666.5
Gold						0.005	1666.5
Probability	0.07	0.12	0.11	0.14	0.14	0.09	10000
Return	7.26419365	4.45879553	4.67539444	4.4050769	4.440194436	4.77287474	
Expected Return	3.22573912						
Min expected Return	0.12						
Variance	2.17188456						

Fig-9: Expected Return at 12.5%

	at 12.5%						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	1666.5
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	1666.5
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	1666.5
Call options				0.012	-0.0005	0.0008	1667.5
Put options					0.012	-0.0008	1666.5
Gold						0.005	1666.5
Probability	0.07	0.12	0.11	0.14	0.14	0.09	10000
Return	7.26419365	0	0	0.012	-0.0005	0.0008	
Expected Return	0.51017556						
Min expected Return	0.125						
Variance	3.34764471						

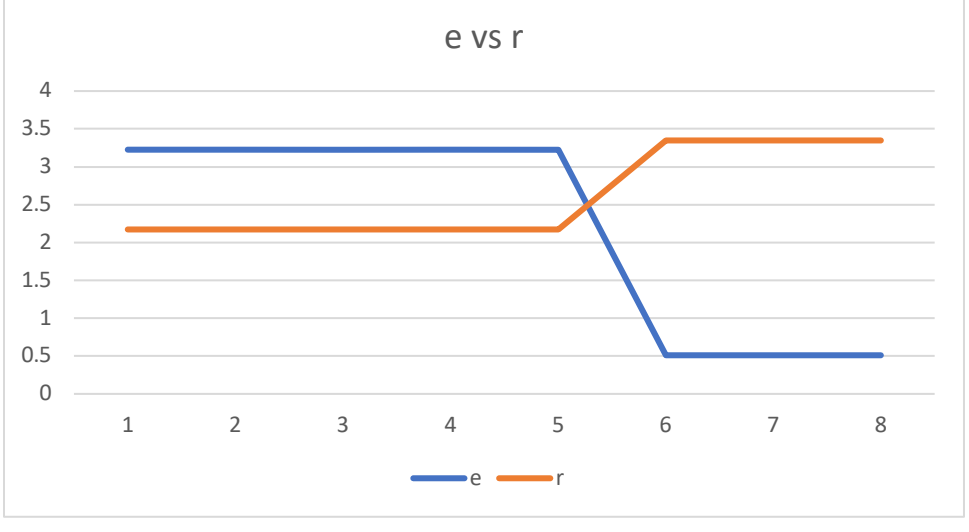
Fig-10: Expected Return at 13%

	at 13%						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	1666.5
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	1666.5
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	1666.5
Call options				0.012	-0.0005	0.0008	1667.5
Put options					0.012	-0.0008	1666.5
Gold						0.005	1666.5
Probability	0.07	0.12	0.11	0.14	0.14	0.09	10000
Return	7.26419365	0	0	0.012	-0.0005	0.0008	
Expected Return	0.51017556						
Min expected Return	0.13						
Variance	3.34764471						

Fig-11: Expected Return at 13.5%

	at 13.5%						
	Bonds	High tech stocks	Foreign stocks	Call options	Put options	Gold	Invested Amount
Bonds	0.001	0.0003	-0.0003	0.00035	-0.00035	0.0004	1666.5
High tech stocks		0.009	0.0004	0.0016	-0.0016	0.0006	1666.5
Foreign stocks			0.008	0.0015	-0.0055	-0.0007	1666.5
Call options				0.012	-0.0005	0.0008	1667.5
Put options					0.012	-0.0008	1666.5
Gold						0.005	1666.5
Probability	0.07	0.12	0.11	0.14	0.14	0.09	10000
Return	7.26419365	0	0	0.012	-0.0005	0.0008	
Expected Return	0.51017556						
Min expected Return	0.135						
Variance	3.34764471						

Fig-11: e vs r plot



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