

# An Improved Personal Space Model for Robot Socially-aware Navigation

Yifan Xu, Nitish Kumar and Qianwei Wang

**Abstract**—We are dedicated to solving the problem of how robots can safely and effectively navigate through dynamic crowds. Balancing safety and efficiency has always been a critical concern in social navigation. Inspired by the Gestalt theory from psychology, some have considered group structures unfolding in crowded scenes to enhance the safety of robot navigation. Building upon the foundation of prior work, we aim to make improvements by drawing inspiration from the Simplex Model in psychology. Our goal is to dynamically model individual pedestrian spaces by considering multiple factors (e.g., density) and inducing dynamic changes in group spatial arrangements. Our ultimate objective is to develop a solution that simultaneously prioritizes both safety and efficiency. To achieve this, we will leverage the velocity and density around moving pedestrians to determine the shape of personal space. Subsequently, we will construct a Model Predictive Control (MPC) framework for path planning.

## I. INTRODUCTION

Navigating through densely populated environments presents a significant challenge for autonomous robots. To avoid collisions with pedestrians, robots must consider not only the current state of individuals in their vicinity but also estimate their future trajectories to generate paths that are both safe and efficient. Challenges arise from the absence of standardized traffic regulations, the close proximity of multiple agents, and the intricate interactions between these agents, as discussed in the context of group model predictive control (MPC) [1]. Efficient and safe navigation in crowded spaces is pivotal for various applications, ranging from service robots in public areas to autonomous vehicles negotiating busy streets [2]. In practical terms, it is essential for robots to maintain a safe distance from humans during navigation in crowded environments, ensuring adaptability to dynamic surroundings [3].

Recent models [4, 5, 6, 7] focus on anticipating individual agent interactions, enabling robots to identify collision-free potential paths. However, they tend to overlook the structured nature of interactions in group scenarios [1]. In many instances, people tend to walk and gather in groups [8]. Without considering grouping behaviors, it becomes challenging for robots to accurately predict pedestrian trajectories. In [1], the authors constructed social groups based on adaptive personal space derived from robot velocity, achieving positive results in social navigation testing. Notably, personal space construction in most social groups is solely based on robot velocity, neglecting the influence of pedestrian density on the size of the comfort zone [9]. Moreover, group-based socially aware navigation often lacks extensive testing in specific scenarios, such as bottleneck situations or extremely dense crowds [10].

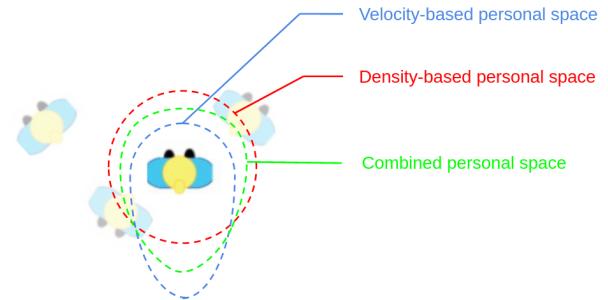


Fig. 1: The illustration of the combined and improved personal space. Red circle is density based personal space. Blue circle is velocity-based personal space and green circle is the combined personal space.

In these cases, an overly simplistic group structure may lead to excessively large groups and compromise prediction accuracy due to the cumbersome nature of the group arrangement. Thus, there exists a research gap between socially aware navigation and group-based navigation.

To enhance the prediction of pedestrians' walking intentions, as illustrated in Fig. 1, we have introduced an advanced personal space model for group-based navigation. This model incorporates both the velocity and density surrounding pedestrians. Subsequently, we employ group-based Model Predictive Control (MPC) motion planning algorithms that leverage the improved personal space. This approach facilitates the identification of safe and efficient pathways within the complex network of crowd movements for the robot. Through our evaluation, the results demonstrate that our proposed improved personal space-based group-aware navigation empowers robots not only to detect and circumvent obstacles in their path but also to comprehend and predict the intentions and behaviors of individuals in their proximity.

In particular, this work has the following contribution:

- 1) We have proposed an innovative adaptive representation of individuals' personal space, incorporating considerations for speed, directions, and the density surrounding the person.
- 2) This enhanced personal space representation has been seamlessly integrated into the group-based Model Predictive Control (MPC) motion planning framework.
- 3) The evaluation results clearly demonstrate that our combined personal space approach outperforms solely

velocity- and density-based personal space models.

## II. RELATED WORK

### A. Personal space

Personal space, as defined, encompasses the region within which individuals feel discomfort if encroached upon by others [11]. Existing studies [1, 9] predominantly utilize walking speed and direction as the primary criteria for defining personal space. Xu et al. [9] introduce density around pedestrians as an additional factor for personal space construction, while [1] employs the velocity and direction of pedestrians. Despite the importance of these individual criteria in personal space construction, the current approaches tend to consider them in isolation. This unimodal consideration can result in personal space being either excessively small or too large, thereby compromising navigation safety and efficiency.

### B. Social Groups

'Social Group' pertains to the creation of shared group spaces accommodating multiple pedestrians based on their personal space. In the work by [1], personal spaces are constructed using pedestrians' velocity information, and subsequently, social groups are identified using DBSCAN [12]. While this method improves navigation safety by considering pedestrian velocity, its real-world efficiency may be compromised. In contrast, [3] introduces the simplex model for constructing personal spaces, advocating that personal space should not be a fixed, singular variable. Instead, it should be determined by multiple factors and adapt to diverse scenarios.

### C. MPC

The authors of the research [13] discuss different uncertainty-unaware and uncertainty-aware MPC designs that use a Covariance Net-based approach for pedestrian trajectory prediction. In the paper[1], the authors used Model Predictive Control (MPC) as a local planner and incorporated the predictions from S-GAN into the cost function, which had a positive impact in the safety of navigation.Meanwhile, another paper[14] also introduced MPC using S-GAN for prediction in social navigation, but unlike the former, the authors not only predicted pedestrian trajectories but also made predictions for the robot itself. They added the discrepancy between the robot's self-prediction and its actual motion to the cost function, referred to as the 'inconsistency cost'.

## III. METHODOLOGY

The methodology focuses on novel personal space formula for a robot navigating in a workspace containing dynamic agents. The robot aims to reach a destination while being unaware of other agents' destinations and policies. Two key components, Personal Space and Agent Characteristic Extraction, are explored to enhance the robot's navigation in the presence of other agents.

### A. Assumption

Consider a robot navigating among  $n$  other dynamic agents in a workspace  $W \subseteq \mathbb{R}^2$ . The robot's state is denoted by  $s \in W$ , while the state of agent  $i \in N = \{1, \dots, n\}$  is indicated by  $s^i \in W$ . By implementing a policy  $\pi : W^{n+1} \times U \rightarrow U$ , which transfers the presumptive fully observable world state  $S = s \cup_{i=1:n} s^i$  to a control action  $u \in U$ , taken from a set of controls  $U \subseteq \mathbb{R}^2$ , the robot is navigating from a state  $s_0$  towards a destination  $s_T^i$ .

Assumed to be unaware of agents' destinations  $s_T^i$  and policies  $\pi_i : W^{n+1} \times U^i \rightarrow U^i$ ,  $i \in N$ , of the robot. The robot's policy  $\pi$  is designed to map the combined state of the robot and other agents to a control action  $u \in U$ . Mathematically, this can be expressed as:

$$\pi : W^{n+1} \times U \rightarrow U \quad (1)$$

Each agent  $i$  has its own policy  $\pi_i$  and destination  $s_T^i$ . The policy  $\pi_i$  maps the state of agent  $i$  and its controls to a new control action:

$$\pi_i : W^{n+1} \times U^i \rightarrow U^i \quad (2)$$

In this scenario, the robot operates with a policy that guides its navigation in the presence of other dynamic agents. The unawareness of other agents' destinations and policies adds complexity to the robot's decision-making process.

### B. Personal Space

Personal space is subjective and varies among individuals and cultures.[3] The combination of velocity- and density-based methods for tackling personal space prediction and management during group navigation by robots is explored in this part.

1) *Velocity-based Personal Space*: Velocity-based personal space predicts future positions of persons in a group based on their speed and direction, and updates the robot's trajectory appropriately. This method allows the robot to proactively update its course to avoid collisions and respect individual preferences by acknowledging that individuals often maintain particular velocities and trajectories while moving around.

2) *Density-based Personal Space*: Density-based personal space considers how people are arranged in space within a group. It acknowledges that the estimated comfort zone around an individual might change based on the crowd's overall density. People tend to tolerate smaller personal spaces in densely populated regions, whereas greater personal spaces are more typical in less congested settings[15]. Robot navigation systems that use density-based models allow the robot to dynamically adapt its behaviour to the surrounding group's density, hence respecting different requirements for personal space.

In previous works[3], to simplify things, uncomfortable distance was set fixed which would cause freezing robot problem when the crowds grow. The uncomfortable distance is defined as the minimum distance from a central person to all the surrounding persons who are within in a circle scope of

2m radius. The uncomfortable distance changes depending on pedestrian density (the number of persons in the specific area). Each individual in the dataset has been traversed as a central person. The formula of uncomfortable distance ( $d_{uncomfy}$ ) and pedestrian density ( $p_{den}$ ) is obtained by ‘Curve Fitting’ the Asia and Pacific Trade Center (ATC) dataset[16].

$$d_{uncomfy} = \frac{1.577}{(p_{den} - 0.8824)^{0.215}} - 0.967 \quad (3)$$

### C. Extracting Social Group Spaces

The Agent State and Extracting Group Membership components aim to create a comprehensive representation of agent states and their group memberships, facilitating the robot’s navigation in a crowded environment by considering the social structure and motion characteristics of groups.

1) *Agent State*: Define  $\theta_i \in [0, 2\pi]$  as the orientation of agent  $i \in N$ , assumed to be aligned with the direction of its velocity  $u_i$ , extracted via finite differencing of its position over a time step  $dt$ . Denote by  $v_i = \|u_i\| \in \mathbb{R}^+$  its speed. We define an augmented state for agent  $i$  as  $q_i = (s_i, \theta_i, v_i)$ . Similarly the augmented state for Robot  $R$  as  $q_R = (s_R, \theta_R, r_R)$ .

We treat a social group as a set of agents who are in close proximity and share similar motion characteristics. Assume that a set of  $J$  groups,  $J = \{1, \dots, J\}$  navigate in a scene. Defined by  $g_i \in J$  a label indicating the group membership of agent  $i$ . We then define a group  $j \in J$  as a set  $G_j = \{i \in N \mid g_i = j\}$  and collect the set of all groups in a scene into a set  $G = \{G_j \mid j \in J\}$ .

2) *Extracting Group Membership*: Extracting group memberships is performed using the Density-Based Spatial Clustering of Applications with Noise algorithm (DBSCAN) [15] on agent states:

$$G \leftarrow \text{DBSCAN}(q \mid \epsilon_s, \epsilon_\theta, \epsilon_v) \quad (4)$$

Where  $\epsilon_s, \epsilon_\theta, \epsilon_v$  are respectively threshold values on agent distances, orientation, and speeds.

For each group  $G_j, j \in J$ , define a social group space as a geometric enclosure  $G_j$  around agents of the group. For each agent  $i \in G_j$ , define a personal space  $P_i$  as a two-dimensional asymmetric Gaussian model for velocity-based personal space [17] and a basic concentric circle model for density based personal space.

Given the personal spaces  $P_i, i \in G_j$ , of all agents in a group  $j$ , extract the social group space of the whole group as a convex hull [18]:

$$G_j = \text{Convexhull}(\{P_i \mid i \in G_j\}) \quad (5)$$

The shape described by  $G_j$  represents an obstacle space representation of a group containing agents in close proximity with similar motion characteristics. For convenience, let us collect the spaces of all groups in a scene into a set  $G = \{G_j \mid j \in J\}$ .

### D. Group Space Prediction Oracle

Based on the group-space representation discussed earlier, we describe a prediction oracle that outputs an estimate of the future spaces occupied by a set of groups  $G_{t:t_f}$  up to a time  $t_f = t + f$ , where  $f$  is a future horizon given a past sequence of group spaces  $G_{t_h:t}$  from time  $t_h = t - h$  where  $h$  is a window of past observations:

$$G_{t:t_f} \leftarrow O(G_{t_h:t}) = \bigcup_{j=1}^J O_j(G_j^{th:t}) \quad (6)$$

where  $O_j$  is a model generating a group space prediction for group  $G_j$ . We implement the oracle  $O_j$  of eq. (6) using a simple encoder-decoder network. The encoder follows the 3D convolutional architecture based on [1]

### E. G-MPC

Model Predictive Control (MPC) with group-based prediction [13] is an advanced control strategy that combines elements of MPC with the use of predictions based on grouped data. In traditional MPC, a mathematical model of the system is used to predict future behavior, and control actions are computed to optimize a performance criterion over a finite prediction horizon [1]. In the context of group-based prediction, the system is divided into groups based on certain characteristics or similarities.

In MPC with group-based prediction, predictive models are developed for each group, allowing for a more tailored and efficient control strategy. This approach acknowledges that different parts of a system may exhibit distinct behaviors and responds by adapting the control strategy accordingly. By grouping similar elements, the predictive models can be more accurate, leading to improved control performance.

## IV. EVALUATION

Our evaluation can be divided into three sections. Sec. IV-A introduces the simulation environment, data, metrics for all the experiments. In the Sec. IV-B we conduct a pair of experiments aimed at assessing the impact of personal space dimensions. These dimensions are determined using two different formulations: one based on density, as outlined in the density-based formula [19], and the other grounded in velocity, as described in the velocity-based formula [1]. In Sec. IV-C, following these initial experiments, we select the most effective group from each for further analysis. These selected groups then form the foundation for our third section. In this subsequent section, we integrate both the density-based and velocity-based formulas, applying varying weights to each. The objective is to evaluate and compare their combined performance under these new conditions. In Sec. IV-D, we have analyzed the limitation of our experiment and outlined directions for future improvements.

### A. Experiment Setup

Referring to the experimental setup in [1], we also employed a set of real-world datasets, sourced from the ETH[20] and UCY[21] and in each scenario. We established two tasks: flow

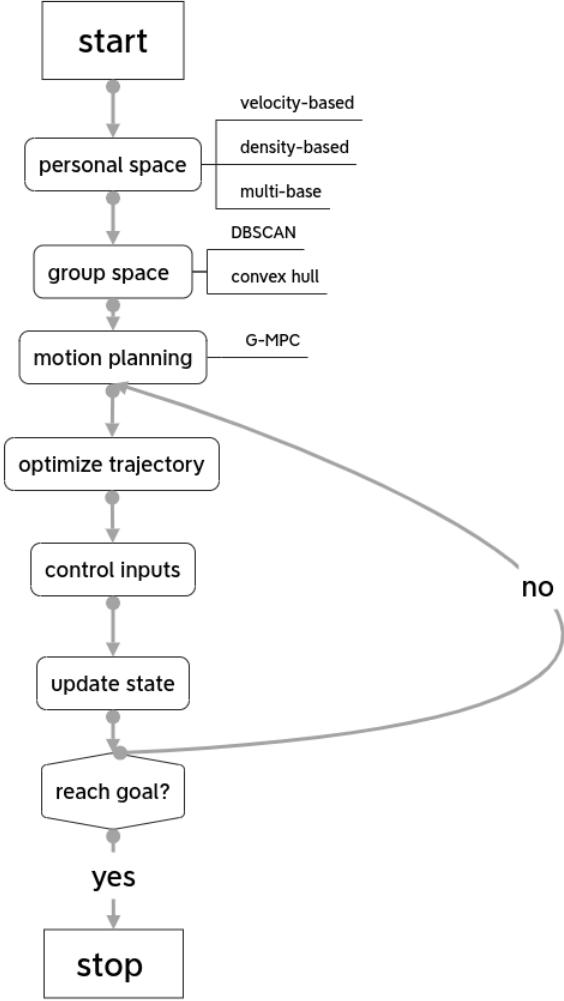


Fig. 2: Framework

and cross and one experiment per tasks . Throughout all trials, we consistently utilized ORCA[4] to ensure that pedestrians remained reactive. For the MPC controller, from the predictive network to the parameter settings of the MPC, we maintained complete consistency with G-MPC[1]. The only variable was the model of personal space, which differed.

To analyze the performance of navigation, we established three metrics: a) Safety, representing the normalized value of the minimum distance between the robot and pedestrians during navigation; b) Efficiency, denoting the length of the robot's path, which is normalized and then inverted with an addition of 1; c) Rationality, indicating the total sum of angular changes between consecutive points on the robot's path, which, after normalization, is inverted and increased by 1; Additionally, to measure the overall performance of navigation, we define a variable called 'performance', which is obtained by directly summing the previous three metrics.

### B. Evaluation of single-based personal space

In these two experiments, we reduced the size of the personal space based on the original formula[19][1] and tested the navigation performance of each group. The results shown in Fig. 3a and Fig. 3b reveal that both density-based and velocity-based personal space models exhibit similar trends when their sizes are proportionally reduced. Regarding safety, the decrease in personal space size leads to shorter distances between the robot and pedestrians, thereby reducing navigation safety. As for efficiency, a smaller personal space allows the robot to choose shorter paths, thus enhancing efficiency. Finally, concerning rationality, its trend is akin to a quadratic curve; both excessively large or small personal spaces can negatively impact the rationality of the path.

In conclusion, based on the experiment, the size of personal space, whether velocity-based or density-based, has a significant impact on the performance of navigation. In these series of experiments, after summing the three metrics with equal weights in Fig. 3c Fig. 3d, we found that 50% $D_{original}$  and 75% $V_{original}$  demonstrated relatively optimal overall performance, and we have chosen them as the baseline groups for the next experiment.

### C. Multi-based personal space

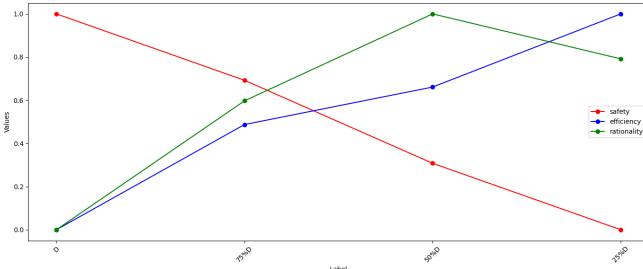
In this experiment, we used 50% $D_{original}$  from the previous experiments as  $D_{original}$  and 75% $V_{original}$  as  $V_{original}$  for this study. After combining these two through weights, we obtained the experimental results as shown in Fig 4a Fig 4b. From analyzing these figures, we can observe the following:

1) *Safety*: There is not much variation in safety across the different groups, with only original density-based personal space and a personal space which combine 95% density distance and 5% velocity showing relatively better performance. This suggests that the differences in personal space models do not significantly impact safety, except in specific combinations.

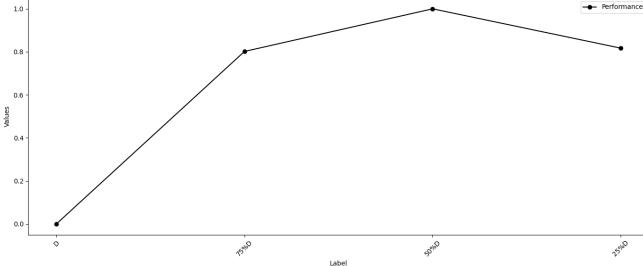
2) *Efficiency*: The performance in terms of efficiency shows considerable fluctuation with changes in weights. However, it can be generally summarized that efficiency is highest when personal space is solely determined by velocity. Excluding the  $V_{original}$  group, in other groups, when the weight difference is either too large or too small, there is a significant decrease in efficiency. This indicates that while a velocity-based personal space model can enhance efficiency, an imbalance in the weighting of density and velocity factors can lead to low efficiency.

3) *Rationality*: The overall performance for rationality presents an M-shaped trend, meaning that when the weight difference is too small or too large, there is a decrease in rationality. However, with appropriate weighting, performance can surpass the baseline. This suggests that a balanced approach in weighting density and velocity factors can optimize the rationality of the navigation path, avoiding extremes in either direction.

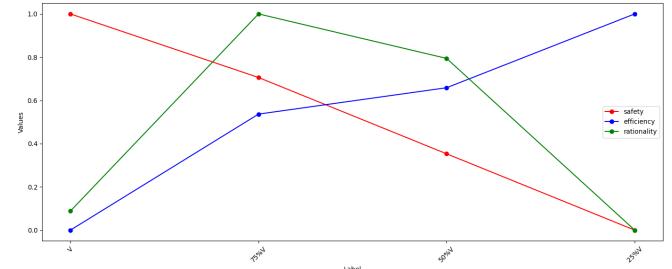
4) *Performance*: For overall performance, both purely velocity-based and density-based approaches can achieve relatively good results. However, a multi-based personal space



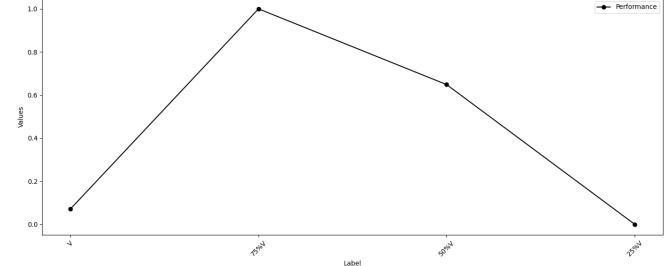
(a) Experiment for the density-based personal space. The numbers following the underscore in the group names represent the percentage by which the size was reduced. For instance,  $50\%D$  indicates a reduction of fifty percent from the original size.



(c) The performance of density-based personal space with varying sizes.  $Performance = \text{norm}(\text{Safety} + \text{Efficiency} + \text{Rationality})$ .

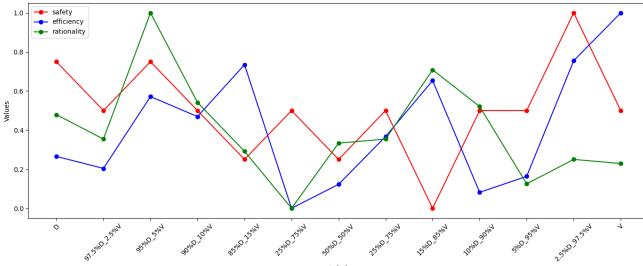


(b) Experiment for the velocity-based personal space. The numbers following the underscore in the group names represent the percentage by which the size was reduced. For instance,  $50\%V$  indicates a reduction of fifty percent from the original size.

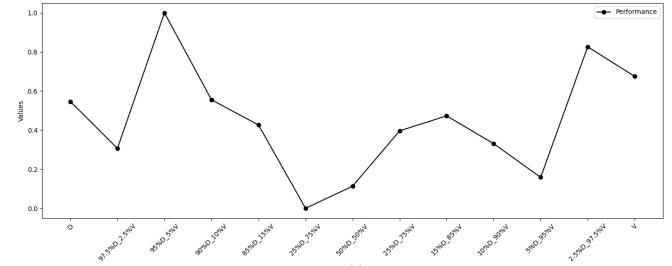


(d) The performance of velocity-based personal space with varying sizes.  $Performance = \text{norm}(\text{Safety} + \text{Efficiency} + \text{Rationality})$ .

Fig. 3: Result for single-based personal space



(a) The safety, efficiency and rationality of personal space that combines density and velocity factors changes with variations in density and velocity. In this context,  $50\%D\_50\%V$  represents a configuration where the weight of density is 50 percent and the weight of velocity is also 50 percent. The baseline quantities D and V for this experiment are derived from  $50\%D$  and  $75\%V$  of the first two experiments.



(b) The performance of multi-based personal space caculated by:  $Performance = \text{norm}(\text{Safety} + \text{Efficiency} + \text{Rationality})$ .

model like the combination of 95% density and 5%velocity or 2.5%density and 97.5%velocity performs better, provided that the weights are correctly allocated. As can be seen from the graph, the optimal effect is achieved when one factor plays a dominant role which is from 90% to 97.5% while the other factor remains secondary. This indicates that the most effective navigation performance is realized when there is a balanced yet distinct prioritization of factors. Regarding the cause of this phenomenon, we believe it may be similar to the process of designing a cost function. Here, the two factors, density

and velocity, can be considered as two constraints. When used individually, they can yield satisfactory results. However, if they are incorporated into a cost function with equal weights, it may lead the optimization towards a point with very average performance.

These findings highlight the importance of a balanced approach in determining personal space for navigation algorithms, where both density and velocity factors, when appropriately weighted, can optimize overall performance in terms of safety, efficiency, and rationality.

#### *D. Discussion*

At the same time, our experiments have limitations. The shape of the velocity-based personal space is an asymmetric Gaussian model, whereas the density-based personal space is circular. Therefore, the final shape obtained by directly adding weights is irregular, which may negatively impact navigation. Hence, in the future, it would be beneficial to explore better methods of combining these models like using learning-based technique to learn the shape of personal space by combining density and velocity.

#### V. CONCLUSION

In this paper, we combined the density-based personal space [19] and velocity-based personal space [1] by distributing different weights. Through our experiments, we first confirmed that the size of personal space, whether density-based or velocity-based, significantly impacts navigation performance. Subsequently, we discovered that personal space, which combines both velocity and density factors through weighted integration, also known as multi-based personal space, can yield very positive results if the weights are correctly and reasonably allocated. This approach outperforms the individual density-based or velocity-based models.

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