# Quiz 9

S Nithish

***Abstract*—This document contains the solution of the question from NCERT 9th standard chapter 10 exercise 10.4 problem 4**

## 1 Exercise 10.4

1) If a line intersects two concentric circles with centre O at A,B,C and D, prove that AB=CD.

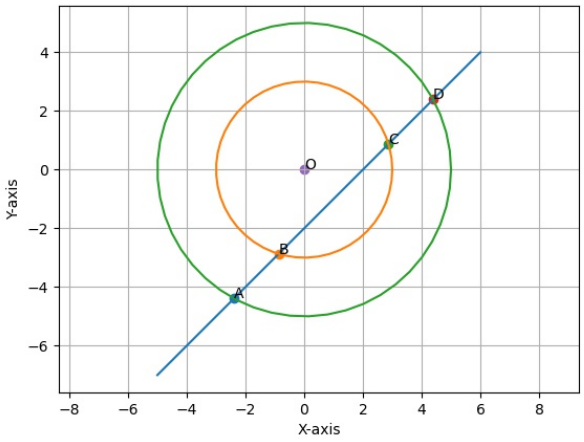


Fig. 1: Circle

Let the inner circle be of radius 3 and the outer circle be of radius 5. Then the equation of the two circles are,

$$\|\mathbf{x}\| = 9 \tag{1.0.1}$$

$$\|\mathbf{x}\| = 25 \tag{1.0.2}$$

Let the equation of the line be,

$$\mathbf{x} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{1.0.3}$$

$$\mathbf{x} = \begin{pmatrix} \lambda \\ \lambda - 2 \end{pmatrix} \tag{1.0.4}$$

The points of intersection of the line and the inner circle are,

$$\|\mathbf{x}\|^2 = \lambda^2 + (\lambda - 2)^2 \tag{1.0.5}$$

$$\|\mathbf{x}\|^2 = 9 \tag{1.0.6}$$

$$\lambda^2 + (\lambda - 2)^2 = 9 \tag{1.0.7}$$

$$2\lambda^2 - 4\lambda + 4 = 9 \tag{1.0.8}$$

$$\lambda^2 - 2\lambda - \frac{5}{2} = 0 \tag{1.0.9}$$

$$\implies \lambda_c = 1 + \sqrt{\frac{7}{2}},\ \lambda_b = 1 - \sqrt{\frac{7}{2}} \tag{1.0.10}$$

The points of intersection of the line and the outter circle are,

$$\|\mathbf{x}\|^2 = \lambda^2 + (\lambda - 2)^2 \tag{1.0.11}$$

$$\|\mathbf{x}\|^2 = 25 \tag{1.0.12}$$

$$\lambda^2 + (\lambda - 2)^2 = 25 \tag{1.0.13}$$

$$2\lambda^2 - 4\lambda + 4 = 25 \tag{1.0.14}$$

$$\lambda^2 - 2\lambda - \frac{21}{2} = 0 \tag{1.0.15}$$

$$\implies \lambda_d = 1 + \sqrt{\frac{23}{2}},\ \lambda_a = 1 - \sqrt{\frac{23}{2}} \tag{1.0.16}$$

$$\lambda_a = 1 - \sqrt{\frac{23}{2}} \implies \mathbf{x_a} = \begin{pmatrix} 1 - \sqrt{\frac{23}{2}} \\ -1 - \sqrt{\frac{23}{2}} \end{pmatrix} \tag{1.0.17}$$

$$\lambda_b = 1 - \sqrt{\frac{7}{2}} \implies \mathbf{x_b} = \begin{pmatrix} 1 - \sqrt{\frac{7}{2}} \\ -1 - \sqrt{\frac{7}{2}} \end{pmatrix} \tag{1.0.18}$$

$$\lambda_c = 1 + \sqrt{\frac{7}{2}} \implies \mathbf{x_c} = \begin{pmatrix} 1 + \sqrt{\frac{7}{2}} \\ -1 + \sqrt{\frac{7}{2}} \end{pmatrix} \tag{1.0.19}$$

$$\lambda_d = 1 + \sqrt{\frac{23}{2}} \implies \mathbf{x_d} = \begin{pmatrix} 1 + \sqrt{\frac{23}{2}} \\ -1 + \sqrt{\frac{23}{2}} \end{pmatrix} \tag{1.0.20}$$

The distance between points A and B is,

$$AB = \|\mathbf{x_a} - \mathbf{x_b}\| = \left\| \left( \tfrac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \tfrac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \right) \right\| = \sqrt{23} - \sqrt{7} \tag{1.0.21}$$

$$CD = \|\mathbf{x_c} - \mathbf{x_d}\| = \left\| \left( \tfrac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \tfrac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \right) \right\| = \sqrt{23} - \sqrt{7} \tag{1.0.22}$$

$$\implies AB = CD \tag{1.0.23}$$