

# Assignment 2

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**Abstract—This document contains the solution of NCERT class 12 chapter 10 exercise 10.3 question number 11.**

## 1 PROBLEM

Show that  $\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a}$  is perpendicular to  $\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}$ , for any two non zero vectors  $\mathbf{a}$  and  $\mathbf{b}$ .

## 2 SOLUTION

We need to show that vectors,  $\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a}$  and  $\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}$  are perpendicular to each other.

Two vectors are perpendicular if and only if the inner product between them is zero. The inner product between the two given vectors is,

$$(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})^T (\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}) = 0 \quad (2.0.1)$$

Expanding the LHS gives,

$$\|\mathbf{a}\|^2 \mathbf{b}^T \mathbf{b} + \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{a}^T \mathbf{b} - \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{b}^T \mathbf{a} - \|\mathbf{b}\|^2 \mathbf{a}^T \mathbf{a} \quad (2.0.2)$$

$$\|\mathbf{a}\|^2 \|\mathbf{b}\|^2 + \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{a}^T \mathbf{b} - \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{a}^T \mathbf{b} - \|\mathbf{b}\|^2 \|\mathbf{a}\|^2 = 0 \quad (2.0.3)$$

Hence,

$$(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})^T (\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}) = 0 \quad (2.0.4)$$

As the inner products between the two vectors is zero, we can say that  $(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})$  and  $(\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a})$  are perpendicular to each other.

Hence Proved.