Quiz 12

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Abstract—**This document contains the solution of the question from NCERT 9th standard chapter 10 exercise 10.6 problem 4**

1 Exercise 10.6

1) Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect the chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

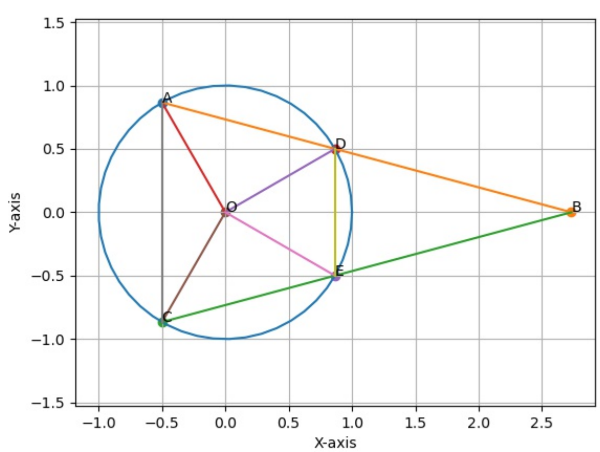

Fig. 1: Circle

Let the circle be unit circle centred at origin,

$$\|\mathbf{x}\|^2 = 1 \tag{1.0.1}$$

Let the points A, B, C be such that,

$$A = \begin{pmatrix}\cos\theta_1\\ \sin\theta_1\end{pmatrix} \tag{1.0.2}$$

$$C = \begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix} \tag{1.0.3}$$

$$D = \begin{pmatrix}\cos\theta_3\\ \sin\theta_3\end{pmatrix} \tag{1.0.4}$$

$$E = \begin{pmatrix}\cos\theta_4\\ \sin\theta_4\end{pmatrix} \tag{1.0.5}$$

The angle subtended by chord AC is,

$$\phi_1 = |\theta_1 - \theta_2| \tag{1.0.6}$$

The angle subtended by chord DE is,

$$\phi_2 = |\theta_3 - \theta_4| \tag{1.0.7}$$

The direction vector of the line passing through the points A and D is,

$$\mathbf{m_1} = \begin{pmatrix}1\\ \frac{\sin\theta_3 - \sin\theta_1}{\cos\theta_3 - \cos\theta_1}\end{pmatrix} \tag{1.0.8}$$

$$= \begin{pmatrix}1\\ \frac{2\cos\left(\frac{\theta_3+\theta_1}{2}\right)\sin\left(\frac{\theta_3-\theta_1}{2}\right)}{-2\sin\left(\frac{\theta_1+\theta_3}{2}\right)\sin\left(\frac{\theta_3-\theta_1}{2}\right)}\end{pmatrix} \tag{1.0.9}$$

$$\mathbf{m_1} = \begin{pmatrix}1\\ -\cot\left(\frac{\theta_1+\theta_3}{2}\right)\end{pmatrix} \tag{1.0.10}$$

$$\|\mathbf{m_1}\| = \sqrt{1+\cot^2\left(\frac{\theta_1+\theta_3}{2}\right)} \tag{1.0.11}$$

$$= \operatorname{cosec}\left(\frac{\theta_1+\theta_3}{2}\right) \tag{1.0.12}$$

The direction vector of the line passing through the points C and E is,

$$\mathbf{m_2} = \begin{pmatrix}1\\ \frac{\sin\theta_4 - \sin\theta_2}{\cos\theta_4 - \cos\theta_2}\end{pmatrix} \tag{1.0.13}$$

$$= \begin{pmatrix}1\\ \frac{2\cos\left(\frac{\theta_4+\theta_2}{2}\right)\sin\left(\frac{\theta_4-\theta_2}{2}\right)}{-2\sin\left(\frac{\theta_2+\theta_4}{2}\right)\sin\left(\frac{\theta_4-\theta_2}{2}\right)}\end{pmatrix} \tag{1.0.14}$$

$$= \begin{pmatrix}1\\ -\cot\left(\frac{\theta_2+\theta_4}{2}\right)\end{pmatrix} \tag{1.0.15}$$

$$\|\mathbf{m_2}\| = \sqrt{1+\cot^2\left(\frac{\theta_2+\theta_4}{2}\right)} \tag{1.0.16}$$

$$= \operatorname{cosec}\left(\frac{\theta_2+\theta_4}{2}\right) \tag{1.0.17}$$

Then,

$$\cos \angle ABC = \frac{\mathbf{m_1}^\top \mathbf{m_2}}{\|\mathbf{m_1}\|\mathbf{m_2}\|\|\|} \tag{1.0.18}$$

$$= \frac{1 + \cot\left(\frac{\theta_1+\theta_3}{2}\right)\cot\left(\frac{\theta_2+\theta_4}{2}\right)}{\operatorname{cosec}\left(\frac{\theta_1+\theta_3}{2}\right)\operatorname{cosec}\left(\frac{\theta_2+\theta_4}{2}\right)} \tag{1.0.19}$$

$$= \cos\left(\frac{\theta_1 + \theta_3 - \theta_2 - \theta_4}{2}\right) \tag{1.0.20}$$

$$= \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \tag{1.0.21}$$

$$\angle ABC = \frac{\phi_1 - \phi_2}{2} \tag{1.0.22}$$

Hence, $\angle ABC$ is equal to half of the angle subtended by chords AD and CE.