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Quiz 7

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Abstract—This document contains the solution of the question from NCERT 11th standard chapter 10 exercise 10.3 problem 16

1 EXERCISE 10.3

1) If p and q are the lengths of perpendicular from the origin to the lines $x\cos(\theta) - y\sin(\theta) = k\cos(2\theta)$ and $x\sec(\theta) + y\,\mathrm{cosec}\,(\theta) = k$ respectively, prove that $p^2 + 4q^2 = k^2$.
   Equation of line 1 is,

$$\begin{pmatrix} \cos(\theta) & \sin(\theta) \end{pmatrix} \mathbf{x} = k\cos(2\theta) \qquad (1.0.1)$$

Equation of line 2 is,

$$\begin{pmatrix} \sec(\theta) & \mathrm{cosec}\,(\theta) \end{pmatrix} \mathbf{x} = k \qquad (1.0.2)$$

Length of perpendicular from origin to line 1,

$$p = \frac{\left| \begin{pmatrix} \cos(\theta) & \sin(\theta) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - k\cos(2\theta) \right|}{\left\| \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \right\|} \qquad (1.0.3)$$

$$p = |k\cos(2\theta)| \implies p^2 = k^2\cos^2(2\theta) \qquad (1.0.4)$$

Length of perpendicular from origin to line 2,

$$q = \frac{\left| \begin{pmatrix} \sec(\theta) & \mathrm{cosec}\,(\theta) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - k \right|}{\left\| \begin{pmatrix} \sec(\theta) \\ \mathrm{cosec}\,(\theta) \end{pmatrix} \right\|} \qquad (1.0.5)$$

$$q = \frac{|k|}{\sqrt{\sec^2(\theta) + \mathrm{cosec}^2(\theta)}} \qquad (1.0.6)$$

$$\implies q^2 = \frac{k^2}{\sec^2(\theta) + \mathrm{cosec}^2(\theta)} \qquad (1.0.7)$$

$$q^2 = \frac{k^2\sin^2(\theta)\cos^2(\theta)}{\sin^2(\theta) + \cos^2(\theta)} \qquad (1.0.8)$$

$$4q^2 = k^2\left(4\sin^2(\theta)\cos^2(\theta)\right) \qquad (1.0.9)$$

$$4q^2 = k^2\sin^2(2\theta) \qquad (1.0.10)$$

$$\implies p^2 + 4q^2 = k^2\sin^2(2\theta) + k^2\cos^2(2\theta) \qquad (1.0.11)$$

$$p^2 + 4q^2 = k^2 \qquad (1.0.12)$$