

Assignment 2

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Abstract—This document contains the solution of NCERT class 12 chapter 10 exercise 10.3 question number 11.

1 PROBLEM

Show that $\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a}$ is perpendicular to $\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}$, for any two non zero vectors \mathbf{a} and \mathbf{b} .

2 SOLUTION

1) Theory

We need to show that vectors, $\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a}$ and $\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}$ are perpendicular to each other.

Two vectors are perpendicular if and only if the inner product between them is zero. The inner product between the two given vectors is,

$$(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})^\top (\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}) = 0 \quad (2.0.1)$$

Expanding the LHS gives,

$$\|\mathbf{a}\|^2 \mathbf{b}^\top \mathbf{b} + \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{a}^\top \mathbf{b} - \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{b}^\top \mathbf{a} - \|\mathbf{b}\|^2 \mathbf{a}^\top \mathbf{a} \quad (2.0.2)$$

$$\|\mathbf{a}\|^2 \|\mathbf{b}\|^2 + \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{a}^\top \mathbf{b} - \|\mathbf{a}\| \|\mathbf{b}\| \mathbf{a}^\top \mathbf{b} - \|\mathbf{b}\|^2 \|\mathbf{a}\|^2 = 0 \quad (2.0.3)$$

Hence,

$$(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})^\top (\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}) = 0 \quad (2.0.4)$$

As the inner products between the two vectors is zero, we can say that $(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})$ and $(\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a})$ are perpendicular to each other.

Hence Proved.

2) Example

Let us take,

$$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (2.0.5)$$

$$\mathbf{b} = \begin{pmatrix} 5 \\ 12 \end{pmatrix} \quad (2.0.6)$$

Then,

$$\|\mathbf{a}\| = \sqrt{3^2 + 4^2} = 5 \quad (2.0.7)$$

$$\|\mathbf{b}\| = \sqrt{5^2 + 12^2} = 13 \quad (2.0.8)$$

$$\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a} = 5 \begin{pmatrix} 5 \\ 12 \end{pmatrix} + 13 \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (2.0.9)$$

$$= \begin{pmatrix} 25 + 39 \\ 60 + 52 \end{pmatrix} = \begin{pmatrix} 64 \\ 112 \end{pmatrix} \quad (2.0.10)$$

$$\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a} = 5 \begin{pmatrix} 5 \\ 12 \end{pmatrix} - 13 \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (2.0.11)$$

$$= \begin{pmatrix} 25 - 39 \\ 60 - 52 \end{pmatrix} = \begin{pmatrix} -14 \\ 8 \end{pmatrix} \quad (2.0.12)$$

$$(\|\mathbf{a}\| \mathbf{b} + \|\mathbf{b}\| \mathbf{a})^\top (\|\mathbf{a}\| \mathbf{b} - \|\mathbf{b}\| \mathbf{a}) = \begin{pmatrix} 64 & 112 \end{pmatrix} \begin{pmatrix} -14 \\ 8 \end{pmatrix} \quad (2.0.13)$$

$$= -896 + 896 \quad (2.0.14)$$

$$= 0 \quad (2.0.15)$$

Hence these two vectors are perpendicular.