Quiz 6

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***Abstract*—This document contains the solution of the question from NCERT 12th standard chapter 10 exercise 10.4 problem 11**

1 Exercise 10.4

1) Let the vectors $\mathbf{a}$ and $\mathbf{b}$ be such that $\|\mathbf{a}\| = 3$ and $\|\mathbf{b}\| = \frac{\sqrt{2}}{3}$, then $\mathbf{a} \times \mathbf{b}$ is a unitvector, if the angle between $\mathbf{a}$ and $\mathbf{b}$ is

Let the angle between $\mathbf{a}$ and $\mathbf{b}$ be ϕ.
$\mathbf{a} \times \mathbf{b}$ is unit vector $\Rightarrow \|\mathbf{a} \times \mathbf{b}\| = 1$

$$\|\mathbf{a}\| = 3 \text{ and } \|\mathbf{b}\| = \frac{\sqrt{2}}{3} \tag{1.0.1}$$

$$1 = 3 \times \frac{\sqrt{2}}{3} \times \sin\phi \tag{1.0.2}$$

$$\sin\phi = \frac{1}{\sqrt{2}} \Rightarrow \phi = \frac{\pi}{4} \tag{1.0.3}$$

Hence, the angle between $\mathbf{a}$ and $\mathbf{b}$ is $\frac{\pi}{4}$.
Example: Let,

$$\mathbf{a} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} \frac{1}{3} \\ \frac{1}{3} \end{pmatrix} \tag{1.0.4}$$

$$\|\mathbf{a}\| = 3 \tag{1.0.5}$$

$$\|\mathbf{b}\| = \sqrt{\frac{1}{3}^2 + \frac{1}{3}^2} = \frac{\sqrt{2}}{3} \tag{1.0.6}$$

$$\mathbf{a} \times \mathbf{b} = \left(3 \times \frac{1}{3}\right)\hat{n} = 1\hat{n} \tag{1.0.7}$$

$$\|\mathbf{a} \times \mathbf{b}\| = \|1\hat{n}\| = 1 \tag{1.0.8}$$

$$\tag{1.0.9}$$

If ϕ is the angle between vector $\mathbf{a}$ and $\mathbf{b}$.

$$\cos\phi = \frac{\mathbf{a}^\top \mathbf{b}}{\|\mathbf{a}\| \|\mathbf{b}\|} = \frac{3 \times \frac{1}{3} + 0 \times \frac{1}{3}}{3 \times \frac{\sqrt{2}}{3}} = \frac{1}{\sqrt{2}} \tag{1.0.10}$$

$$\cos\phi = \frac{1}{\sqrt{2}} \Rightarrow \phi = \frac{\pi}{4} \tag{1.0.11}$$