

Quiz 4

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Abstract—This document contains the solution of the question from NCERT 11th standard chapter 10 exercise 10.1 problem 6

1 EXERCISE 10.1

- 1) Without using the Pythagoras theorem, show that the points $(4, 4)$, $(3, 5)$ and $(-1, -1)$ are the vertices of a right angled triangle.
We need to show that points $(4,4)$, $(3,5)$ and $(-1,-1)$ are vertices of a right triangle.
Let,

$$\mathbf{A} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \text{ and } \mathbf{C} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad (1.0.1)$$

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (1.0.2)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \end{pmatrix} \quad (1.0.3)$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{C} - \mathbf{A}) = \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} -5 \\ -5 \end{pmatrix} = 5 - 5 = 0 \quad (1.0.4)$$

First, let us prove that the three points form triangle, i.e the three points are not collinear.
The three points form a triangle when two side vectors are linearly independent. Hence, it is enough to show that the rank of the following matrix,

$$\begin{pmatrix} -1 & 1 \\ -5 & -5 \end{pmatrix} \quad (1.0.5)$$

is 2. We will use row reduction method to solve the problem.

$$R_2 \rightarrow R_2 - 5R_1 \Rightarrow \begin{pmatrix} -1 & 1 \\ 0 & -10 \end{pmatrix} \quad (1.0.6)$$

The matrix is reduced to row echelon form and the number of non zero rows is 2. Hence, the

rank of the matrix is 2, and the three points form a triangle.

$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{C}) = 0 \Rightarrow$ angle between $\mathbf{A} - \mathbf{B}$ and $\mathbf{B} - \mathbf{C}$ is 90 degrees.

Hence, $\triangle ABC$ is a right angled triangle with right angle at vertex $A(4, 4)$.