

# Quiz 7

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**Abstract**—This document contains the solution of the question from NCERT 11th standard chapter 10 exercise 10.3 problem 16

## 1 EXERCISE 10.3

- 1) If p and q are the lengths of perpendicular from the origin to the lines  $x \cos(\theta) - y \sin(\theta) = k \cos(2\theta)$  and  $x \sec(\theta) + y \operatorname{cosec}(\theta) = k$  respectively, prove that  $p^2 + 4q^2 = k^2$ .  
Equation of line 1 is,

$$\begin{pmatrix} \cos(\theta) & \sin(\theta) \end{pmatrix} \mathbf{x} = k \cos(2\theta) \quad (1.0.1)$$

Equation of line 2 is,

$$\begin{pmatrix} \sec(\theta) & \operatorname{cosec}(\theta) \end{pmatrix} \mathbf{x} = k \quad (1.0.2)$$

Length of perpendicular from origin to line 1,

$$p = \frac{\left| \begin{pmatrix} \cos(\theta) & \sin(\theta) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - k \cos(2\theta) \right|}{\left\| \begin{pmatrix} \cos(\theta) \\ \sin(\theta) \end{pmatrix} \right\|} \quad (1.0.3)$$

$$\Rightarrow p = |k \cos(2\theta)| \quad (1.0.4)$$

$$\Rightarrow p^2 = k^2 \cos^2(2\theta) \quad (1.0.5)$$

Length of perpendicular from origin to line 2,

$$q = \frac{\left| \begin{pmatrix} \sec(\theta) & \operatorname{cosec}(\theta) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - k \right|}{\left\| \begin{pmatrix} \sec(\theta) \\ \operatorname{cosec}(\theta) \end{pmatrix} \right\|} \quad (1.0.6)$$

$$\Rightarrow q = \frac{|k|}{\sqrt{\sec^2(\theta) + \operatorname{cosec}^2(\theta)}} \quad (1.0.7)$$

$$\Rightarrow q^2 = \frac{k^2}{\sec^2(\theta) + \operatorname{cosec}^2(\theta)} \quad (1.0.8)$$

$$\Rightarrow q^2 = \frac{k^2 \sin^2(\theta) \cos^2(\theta)}{\sin^2(\theta) + \cos^2(\theta)} \quad (1.0.9)$$

$$\Rightarrow 4q^2 = k^2 (4 \sin^2(\theta) \cos^2(\theta)) \quad (1.0.10)$$

$$\Rightarrow 4q^2 = k^2 \sin^2(2\theta) \quad (1.0.11)$$

$$\Rightarrow p^2 + 4q^2 = k^2 \sin^2(2\theta) + k^2 \cos^2(2\theta) \quad (1.0.12)$$

$$p^2 + 4q^2 = k^2 (\sin^2(2\theta) + \cos^2(2\theta)) \quad (1.0.13)$$

$$\Rightarrow p^2 + 4q^2 = k^2 \quad (1.0.14)$$