Quiz 10

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***Abstract*—This document contains the solution of the question from NCERT 9th standard chapter 10 exercise 10.5 problem 4**

1 Exercise 10.4

1) In the below figure $\angle ABC = 69°$, $\angle ACB = 31°$, find $\angle BDC$

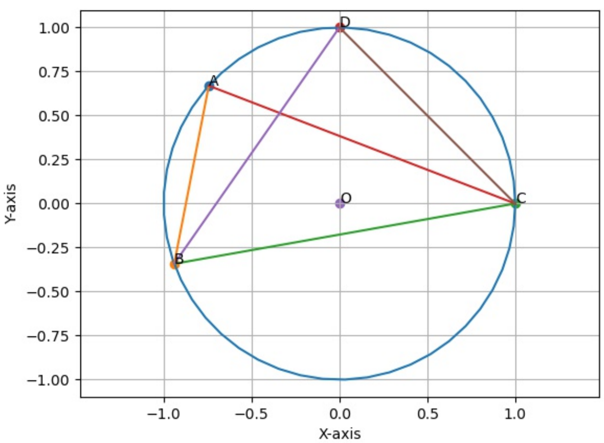

Fig. 1: Circle

Let the circle be unit circle centred at origin,

$$\|\mathbf{x}\|^2 = 1 \tag{1.0.1}$$

Let the points A, B, C be such that,

$$C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.0.2}$$

$$A = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \tag{1.0.3}$$

$$B = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \tag{1.0.4}$$

$$BA = \begin{pmatrix} \cos\theta_1 - \cos\theta_2 \\ \sin\theta_1 - \sin\theta_2 \end{pmatrix} \tag{1.0.5}$$

$$BA = \begin{pmatrix} -2\sin\left(\frac{\theta_1-\theta_2}{2}\right)\sin\left(\frac{\theta_1+\theta_2}{2}\right) \\ 2\cos\left(\frac{\theta_1+\theta_2}{2}\right)\sin\left(\frac{\theta_1-\theta_2}{2}\right) \end{pmatrix} \tag{1.0.6}$$

$$BA = 2\sin\left(\frac{\theta_1-\theta_2}{2}\right)\begin{pmatrix} -\sin\left(\frac{\theta_1+\theta_2}{2}\right) \\ \cos\left(\frac{\theta_1+\theta_2}{2}\right) \end{pmatrix} \tag{1.0.7}$$

$$BC = \begin{pmatrix} 1 - \cos\theta_2 \\ -\sin\theta_2 \end{pmatrix} \tag{1.0.8}$$

$$BC = \begin{pmatrix} 2\sin^2\left(\frac{\theta_2}{2}\right) \\ -2\sin\frac{\theta_2}{2}\cos\frac{\theta_2}{2} \end{pmatrix} \tag{1.0.9}$$

$$BC = 2\sin\frac{\theta_2}{2}\begin{pmatrix} \sin\frac{\theta_2}{2} \\ -\cos\frac{\theta_2}{2} \end{pmatrix} \tag{1.0.10}$$

$$AC = \begin{pmatrix} 1 - \cos\theta_1 \\ -\sin\theta_1 \end{pmatrix} \tag{1.0.11}$$

$$AC = \begin{pmatrix} 2\sin^2\left(\frac{\theta_1}{2}\right) \\ -2\sin\frac{\theta_1}{2}\cos\frac{\theta_1}{2} \end{pmatrix} \tag{1.0.12}$$

$$AC = 2\sin\frac{\theta_1}{2}\begin{pmatrix} \sin\frac{\theta_1}{2} \\ -\cos\frac{\theta_1}{2} \end{pmatrix} \tag{1.0.13}$$

$$\angle ABC = 69° \tag{1.0.14}$$

$$\implies \cos 69° = \frac{\mathbf{BA}^\top \mathbf{BC}}{\|\mathbf{BA}\|\,\|\mathbf{BC}\|} \tag{1.0.15}$$

$$= -\cos\left(\frac{\theta_1+\theta_2}{2} - \frac{\theta_2}{2}\right) \tag{1.0.16}$$

$$= -\cos\frac{\theta_1}{2} \tag{1.0.17}$$

$$\implies \frac{\theta_1}{2} = (2k+1)\,180° + 69° \tag{1.0.18}$$

$$\theta_1 = (2k+1)\,360° + 138° \tag{1.0.19}$$

$$\theta_1 = -360° + 138° \text{ (for k=-1)} \tag{1.0.20}$$

$$= -222° \tag{1.0.21}$$

$$\angle ACB = 31^\circ \tag{1.0.22}$$

$$\implies \cos 31^\circ = \frac{\mathbf{BC}^\top \mathbf{AC}}{\|\mathbf{BC}\| \, \|\mathbf{AC}\|} \tag{1.0.23}$$

$$= \begin{pmatrix} \sin\left(\frac{\theta_2}{2}\right) & -\cos\left(\frac{\theta_2}{2}\right) \end{pmatrix} \begin{pmatrix} \sin\frac{\theta_1}{2} \\ -\cos\frac{\theta_1}{2} \end{pmatrix} \tag{1.0.24}$$

$$= \cos\left(\frac{\theta_2 - \theta_1}{2}\right) \tag{1.0.25}$$

$$\implies \left(\frac{\theta_2 - \theta_1}{2}\right) = (k)\, 360^\circ + 31^\circ \tag{1.0.26}$$

$$\theta_2 - \theta_1 = (2k)\, 360^\circ + 62^\circ \tag{1.0.27}$$

$$\theta_2 - \theta_1 = 62^\circ \text{ (for k=0)} \tag{1.0.28}$$

$$\theta_2 + 222^\circ = 62^\circ \tag{1.0.29}$$

$$\theta_2 = -160^\circ \tag{1.0.30}$$

$$A = \begin{pmatrix} \cos 222 \\ -\sin 222 \end{pmatrix} \tag{1.0.31}$$

$$B = \begin{pmatrix} \cos 160 \\ -\sin 160 \end{pmatrix} \tag{1.0.32}$$

$$C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.0.33}$$

Let us take point D such that,

$$D = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.0.34}$$

Then,

$$DC = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \tag{1.0.35}$$

$$DB = \begin{pmatrix} \cos 160 \\ -\sin 160 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.0.36}$$

$$= \begin{pmatrix} \cos 160 \\ -1 - \sin 160 \end{pmatrix} \tag{1.0.37}$$

$$DB = \begin{pmatrix} -\sin 70 \\ -1 - \cos 70 \end{pmatrix} = -2\cos 35 \begin{pmatrix} \sin 35 \\ \cos 35 \end{pmatrix} \tag{1.0.38}$$

$$\tag{1.0.39}$$

$$\cos \angle BDC = \frac{DC^\top DB}{\|DC\| \, \|DB\|} \tag{1.0.40}$$

$$= \frac{-2\cos 35\,(\sin 35 - \cos 35)}{(2\cos 35)\left(\sqrt{2}\right)} \tag{1.0.41}$$

$$= \frac{\cos 35 - \sin 35}{\sqrt{2}} \tag{1.0.42}$$

$$= \cos(35 + 45) = \cos 80 \tag{1.0.43}$$

$$\implies \angle DBC = 80^\circ \tag{1.0.44}$$