Assignment 2

S Nithish

***Abstract*—This document contains the solution of NCERT class 12 chapter 10 exercise 10.3 question number 11.**

1 Problem

Show that $\|\mathbf{a}\|\,\mathbf{b}+\|\mathbf{b}\|\,\mathbf{a}$ is perpendicular to $\|\mathbf{a}\|\,\mathbf{b}-\|\mathbf{b}\|\,\mathbf{a}$, for any two non zero vectors $\mathbf{a}$ and $\mathbf{b}$.

2 Solution

We need to show that vectors, $\|\mathbf{a}\|\,\mathbf{b}+\|\mathbf{b}\|\,\mathbf{a}$ and $\|\mathbf{a}\|\,\mathbf{b}-\|\mathbf{b}\|\,\mathbf{a}$ are perpendicular to each other.

Two vectors are perpendicular if and only if the inner product between them is zero. The inner product between the two given vectors is,

$$\left(\|\mathbf{a}\|\,\mathbf{b}+\|\mathbf{b}\|\,\mathbf{a}\right)^\top\left(\|\mathbf{a}\|\,\mathbf{b}-\|\mathbf{b}\|\,\mathbf{a}\right)=0 \tag{2.0.1}$$

Expanding the LHS gives,

$$\|\mathbf{a}\|^2\,\mathbf{b}^\top\mathbf{b}+\|\mathbf{a}\|\,\|\mathbf{b}\|\,\mathbf{a}^\top\mathbf{b}-\|\mathbf{a}\|\,\|\mathbf{b}\|\,\mathbf{b}^\top\mathbf{a}-\|\mathbf{b}\|^2\,\mathbf{a}^\top\mathbf{a} \tag{2.0.2}$$

$$\|\mathbf{a}\|^2\,\|\mathbf{b}\|^2+\|\mathbf{a}\|\,\|\mathbf{b}\|\,\mathbf{a}^\top\mathbf{b}-\|\mathbf{a}\|\,\|\mathbf{b}\|\,\mathbf{a}^\top\mathbf{b}-\|\mathbf{b}\|^2\,\|\mathbf{a}\|^2=0 \tag{2.0.3}$$

Hence,

$$\left(\|\mathbf{a}\|\,\mathbf{b}+\|\mathbf{b}\|\,\mathbf{a}\right)^\top\left(\|\mathbf{a}\|\,\mathbf{b}-\|\mathbf{b}\|\,\mathbf{a}\right)=0 \tag{2.0.4}$$

As the inner products between the two vectors is zero, we can say that $\left(\|\mathbf{a}\|\,\mathbf{b}+\|\mathbf{b}\|\,\mathbf{a}\right)$ and $\left(\|\mathbf{a}\|\,\mathbf{b}-\|\mathbf{b}\|\,\mathbf{a}\right)$ are perpendicular to each other.

Hence Proved.

3 Examples

Let us take,

$$\mathbf{a}=\begin{pmatrix}3\\4\end{pmatrix} \tag{3.0.1}$$

$$\mathbf{b}=\begin{pmatrix}5\\12\end{pmatrix} \tag{3.0.2}$$

Then,

$$\|\mathbf{a}\|=\sqrt{3^2+4^2}=5 \tag{3.0.3}$$

$$\|\mathbf{b}\|=\sqrt{5^2+12^2}=13 \tag{3.0.4}$$

$$\|\mathbf{a}\|\,\mathbf{b}+\|\mathbf{b}\|\,\mathbf{a}=5\begin{pmatrix}5\\12\end{pmatrix}+13\begin{pmatrix}3\\4\end{pmatrix} \tag{3.0.5}$$

$$=\begin{pmatrix}25+39\\60+52\end{pmatrix}=\begin{pmatrix}64\\112\end{pmatrix} \tag{3.0.6}$$

$$\|\mathbf{a}\|\,\mathbf{b}-\|\mathbf{b}\|\,\mathbf{a}=5\begin{pmatrix}5\\12\end{pmatrix}-13\begin{pmatrix}3\\4\end{pmatrix} \tag{3.0.7}$$

$$=\begin{pmatrix}25-39\\60-52\end{pmatrix}=\begin{pmatrix}-14\\8\end{pmatrix} \tag{3.0.8}$$

$$\left(\|\mathbf{a}\|\,\mathbf{b}+\|\mathbf{b}\|\,\mathbf{a}\right)^\top\left(\|\mathbf{a}\|\,\mathbf{b}-\|\mathbf{b}\|\,\mathbf{a}\right)=\begin{pmatrix}64 & 112\end{pmatrix}\begin{pmatrix}-14\\8\end{pmatrix} \tag{3.0.9}$$

$$=-896+896 \tag{3.0.10}$$

$$=0 \tag{3.0.11}$$

Hence these two vectors are perpendicular.