Optimization

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Abstract—**This document contains the solution of the question from NCERT 11th standard chapter 10 exercise 10.4 problem 4**

1 EXERCISE 10.4

1) What are the points on y axis whose distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units.

   The given line is,

$$\begin{pmatrix} 4 & 3 \end{pmatrix} \mathbf{x} = 12 \qquad (1.0.1)$$

Let the required point on y axis be $\begin{pmatrix} 0 \\ y \end{pmatrix}$, then the distance of this point from the given line is,

$$d = \frac{\left| \begin{pmatrix} 4 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ y \end{pmatrix} - 12 \right|}{\sqrt{3^2 + 4^2}} \qquad (1.0.2)$$

$$d = \frac{|3y - 12|}{5} \qquad (1.0.3)$$

$$d = 4 \implies \frac{|3y - 12|}{5} = 4 \qquad (1.0.4)$$

$$|3y - 12| = 20 \qquad (1.0.5)$$

$$y = 4 + \frac{20}{3} = \frac{32}{3} \text{ or } y = 4 - \frac{20}{3} = -\frac{8}{3} \qquad (1.0.6)$$

$$(1.0.7)$$

The foot of perpendicular to the line from the point $(0, \frac{32}{3})$ is,

$$\mathbf{x}_0 = \min_{\mathbf{x}} \left\| \mathbf{x} - \begin{pmatrix} 0 \\ \frac{32}{3} \end{pmatrix} \right\| \qquad (1.0.8)$$

$$\text{s.t} \quad \begin{pmatrix} 4 & 3 \end{pmatrix} \mathbf{x} = 12 \qquad (1.0.9)$$

Solving the above optimization with cvxpy gives,

$$\mathbf{x}_0 = \begin{pmatrix} -3.2 \\ 8.2667 \end{pmatrix} \qquad (1.0.10)$$

The foot of perpendicular to the line from the point $(0, -\frac{8}{3})$ is,

$$\mathbf{x}_1 = \min_{\mathbf{x}} \left\| \mathbf{x} - \begin{pmatrix} 0 \\ -\frac{8}{3} \end{pmatrix} \right\| \qquad (1.0.11)$$

$$\text{s.t} \quad \begin{pmatrix} 4 & 3 \end{pmatrix} \mathbf{x} = 12 \qquad (1.0.12)$$

Solving the above optimization with cvxpy gives,

$$\mathbf{x}_1 = \begin{pmatrix} 3.2 \\ -0.2667 \end{pmatrix} \qquad (1.0.13)$$