Quiz 8

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Abstract—This document contains the solution of the question from NCERT 12th standard chapter 11 exercise 11.2 problem 13

1 EXERCISE 11.2

1) Check if the following two planes are parallel or perpendicular and in case they are neither find the angle between them.

$$7x + 5y + 6z + 30 = 0$$
$$3x - y - 10z + 4 = 0$$

The normal vector of the plane

$$7x + 5y + 6z + 30 = 0 \tag{1.0.1}$$

is

$$\mathbf{n_1} = \begin{pmatrix} 7 \\ 5 \\ 6 \end{pmatrix} \tag{1.0.2}$$

The normal vector of the plane

$$3x - y - 10z + 4 = 0 \tag{1.0.3}$$

is

$$\mathbf{n_2} = \begin{pmatrix} 3 \\ -1 \\ -10 \end{pmatrix} \tag{1.0.4}$$

The rank of the matrix

$$\begin{pmatrix} \mathbf{n_1}^\top \\ \mathbf{n_2}^\top \end{pmatrix} = \begin{pmatrix} 7 & 5 & 6 \\ 3 & -1 & -10 \end{pmatrix} \tag{1.0.5}$$

is

$$\begin{pmatrix} \mathbf{n_1}^\top \\ \mathbf{n_2}^\top \end{pmatrix} = \begin{pmatrix} 7 & 5 & 6 \\ 3 & -1 & -10 \end{pmatrix} \tag{1.0.6}$$

$$\xleftarrow{R_2 \leftarrow R_2 - \frac{3}{7}R_1} \tag{1.0.7}$$

$$\begin{pmatrix} 7 & 5 & 6 \\ 0 & \frac{-22}{7} & \frac{-88}{7} \end{pmatrix} \tag{1.0.8}$$

The rank of the matrix is 2. Hence the two planes are not parallel.

$$\mathbf{n_1}^\top \mathbf{n_2} = \begin{pmatrix} 7 & 5 & 6 \end{pmatrix} \begin{pmatrix} 3 \\ -1 \\ -10 \end{pmatrix} \tag{1.0.9}$$

$$= 7\,(3) + 5\,(-1) + 6\,(-10) \tag{1.0.10}$$

$$= 21 - 5 - 60 \tag{1.0.11}$$

$$= -44 \tag{1.0.12}$$

$$\neq 0 \tag{1.0.13}$$

Hence, the planes are not perpendicular.
Let ϕ be the angle between the two planes.

$$\cos\phi = \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\|\,\|\mathbf{n_2}\|} \tag{1.0.14}$$

$$\|n_1\| = \sqrt{7^2 + 5^2 + 6^2} = \sqrt{110} \tag{1.0.15}$$

$$\|n_2\| = \sqrt{3^2 + 1^2 + 10^2} = \sqrt{110} \tag{1.0.16}$$

$$\implies \cos\phi = \frac{-44}{\sqrt{110}\,\sqrt{110}} \tag{1.0.17}$$

$$= \frac{-44}{110} \tag{1.0.18}$$

$$= \frac{-2}{5} \tag{1.0.19}$$

$$\implies \phi = \arccos\left(\frac{-2}{5}\right) \tag{1.0.20}$$