

Quiz 4

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Abstract—This document contains the solution of the question from NCERT 11th standard chapter 10 exercise 10.1 problem 6

1 EXERCISE 10.1

- 1) Without using the Pythagoras theorem, show that the points $(4, 4)$, $(3, 5)$ and $(-1, -1)$ are the vertices of a right angled triangle.

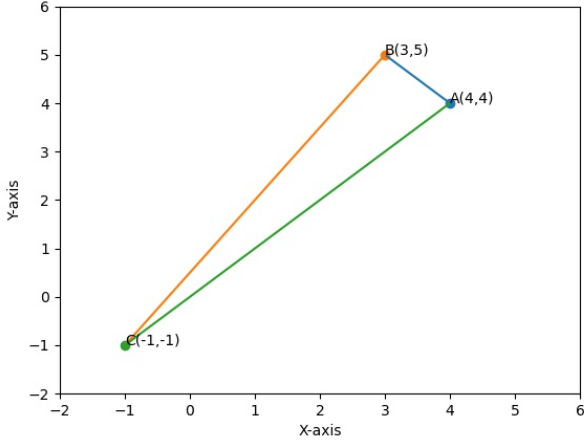


Fig. 1: Triangle ABC

We need to show that points $(4,4)$, $(3,5)$ and $(-1,-1)$ are vertices of a right triangle.

Let,

$$\mathbf{A} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}, \text{ and } \mathbf{C} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} \quad (1.0.1)$$

To check whether the points A, B, C form a triangle we find the rank of the matrix $(\mathbf{A} \ \mathbf{B} \ \mathbf{C})$

$$\begin{pmatrix} 4 & 3 & -1 \\ 4 & 5 & -1 \end{pmatrix} \quad (1.0.2)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - R_1} \quad (1.0.3)$$

$$\begin{pmatrix} 4 & 3 & -1 \\ 0 & 2 & 0 \end{pmatrix} \quad (1.0.4)$$

$$(1.0.5)$$

The rank of the matrix is 2 and the points are in 2-Dimensional space. So the points A, B, C form a triangle.

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (1.0.6)$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -1 \\ -1 \end{pmatrix} - \begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -5 \end{pmatrix} \quad (1.0.7)$$

$$(\mathbf{B} - \mathbf{A})^\top (\mathbf{A} - \mathbf{C}) = \begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} -5 \\ -5 \end{pmatrix} = 5 - 5 = 0 \quad (1.0.8)$$

$(\mathbf{A} - \mathbf{B})^\top (\mathbf{A} - \mathbf{C}) = 0 \Rightarrow$ angle between $\mathbf{A} - \mathbf{B}$ and $\mathbf{B} - \mathbf{C}$ is 90 degrees.

Hence, $\triangle ABC$ is a right triangle with right angle at vertex A $(4, 4)$.