

Quiz 12

S Nithish

Abstract—This document contains the solution of the question from NCERT 9th standard chapter 10 exercise 10.6 problem 4

1 EXERCISE 10.6

- 1) Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect the chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

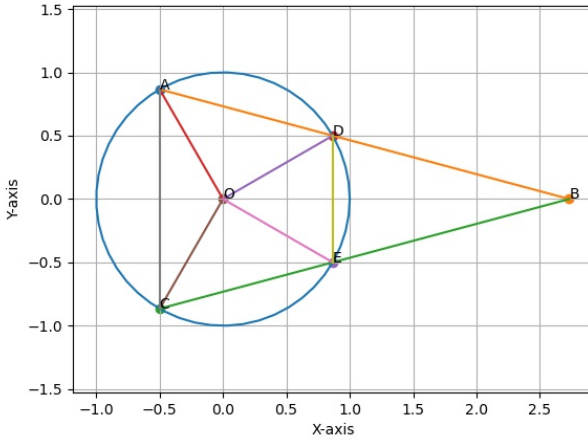


Fig. 1: Circle

Let the circle be unit circle centred at origin,

$$\|\mathbf{x}\|^2 = 1 \quad (1.0.1)$$

Let the points A, B, C be such that,

$$A = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix} \quad (1.0.2)$$

$$C = \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix} \quad (1.0.3)$$

$$D = \begin{pmatrix} \cos \theta_3 \\ \sin \theta_3 \end{pmatrix} \quad (1.0.4)$$

$$E = \begin{pmatrix} \cos \theta_4 \\ \sin \theta_4 \end{pmatrix} \quad (1.0.5)$$

The angle subtended by chord AC is,

$$\phi_1 = |\theta_1 - \theta_2| \quad (1.0.6)$$

The angle subtended by chord DE is,

$$\phi_2 = |\theta_3 - \theta_4| \quad (1.0.7)$$

The direction vector of the line passing through the points A and D is,

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ \frac{\sin \theta_3 - \sin \theta_1}{\cos \theta_3 - \cos \theta_1} \end{pmatrix} \quad (1.0.8)$$

$$= \begin{pmatrix} 1 \\ \frac{2 \cos\left(\frac{\theta_3 + \theta_1}{2}\right) \sin\left(\frac{\theta_3 - \theta_1}{2}\right)}{-2 \sin\left(\frac{\theta_1 + \theta_3}{2}\right) \sin\left(\frac{\theta_3 - \theta_1}{2}\right)} \end{pmatrix} \quad (1.0.9)$$

$$\mathbf{m}_1 = \begin{pmatrix} 1 \\ -\cot\left(\frac{\theta_1 + \theta_3}{2}\right) \end{pmatrix} \quad (1.0.10)$$

$$\|\mathbf{m}_1\| = \sqrt{1 + \cot^2\left(\frac{\theta_1 + \theta_3}{2}\right)} \quad (1.0.11)$$

$$= \operatorname{cosec}\left(\frac{\theta_1 + \theta_3}{2}\right) \quad (1.0.12)$$

The direction vector of the line passing through the points C and E is,

$$\mathbf{m}_2 = \begin{pmatrix} 1 \\ \frac{\sin \theta_4 - \sin \theta_2}{\cos \theta_4 - \cos \theta_2} \end{pmatrix} \quad (1.0.13)$$

$$= \begin{pmatrix} 1 \\ \frac{2 \cos\left(\frac{\theta_4 + \theta_2}{2}\right) \sin\left(\frac{\theta_4 - \theta_2}{2}\right)}{-2 \sin\left(\frac{\theta_2 + \theta_4}{2}\right) \sin\left(\frac{\theta_4 - \theta_2}{2}\right)} \end{pmatrix} \quad (1.0.14)$$

$$= \begin{pmatrix} 1 \\ -\cot\left(\frac{\theta_2 + \theta_4}{2}\right) \end{pmatrix} \quad (1.0.15)$$

$$\|\mathbf{m}_2\| = \sqrt{1 + \cot^2\left(\frac{\theta_2 + \theta_4}{2}\right)} \quad (1.0.16)$$

$$= \operatorname{cosec}\left(\frac{\theta_2 + \theta_4}{2}\right) \quad (1.0.17)$$

Then,

$$\cos \angle ABC = \frac{\mathbf{m}_1^\top \mathbf{m}_2}{\|\mathbf{m}_1\| \|\mathbf{m}_2\|} \quad (1.0.18)$$

$$= \frac{1 + \cot\left(\frac{\theta_1 + \theta_3}{2}\right) \cot\left(\frac{\theta_2 + \theta_4}{2}\right)}{\operatorname{cosec}\left(\frac{\theta_1 + \theta_3}{2}\right) \operatorname{cosec}\left(\frac{\theta_2 + \theta_4}{2}\right)} \quad (1.0.19)$$

$$= \cos\left(\frac{\theta_1 + \theta_3 - \theta_2 - \theta_4}{2}\right) \quad (1.0.20)$$

$$= \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \quad (1.0.21)$$

$$\angle ABC = \frac{\phi_1 - \phi_2}{2} \quad (1.0.22)$$

Hence, $\angle ABC$ is equal to half of the angle subtended by chords AD and CE.

Parameter	Description
θ_1	Angle made by vector OA with X axis
θ_2	Angle made by vector OC with X axis
θ_3	Angle made by vector OD with X axis
θ_4	Angle made by vector OE with X axis

TABLE 1