Quiz 12

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***Abstract*—This document contains the solution of the question from NCERT 9th standard chapter 10 exercise 10.6 problem 4**

1 Exercise 10.6

1) Let the vertex of an angle ABC be located outside a circle and let the sides of the angle intersect the chords AD and CE with the circle. Prove that $\angle ABC$ is equal to half the difference of the angles subtended by the chords AC and DE at the centre.

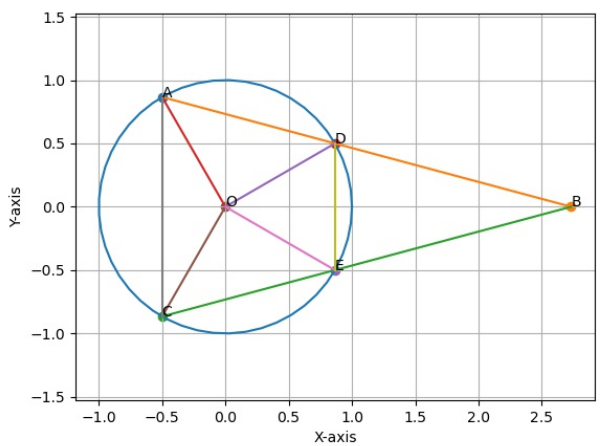

Fig. 1: Circle

Let the circle be unit circle centred at origin,

$$\|\mathbf{x}\|^2 = 1 \tag{1.0.1}$$

Let the points A, B, C be such that,

$$A = \begin{pmatrix} \cos\theta_1 \\ \sin\theta_1 \end{pmatrix} \tag{1.0.2}$$

$$C = \begin{pmatrix} \cos\theta_2 \\ \sin\theta_2 \end{pmatrix} \tag{1.0.3}$$

$$D = \begin{pmatrix} \cos\theta_3 \\ \sin\theta_3 \end{pmatrix} \tag{1.0.4}$$

$$E = \begin{pmatrix} \cos\theta_4 \\ \sin\theta_4 \end{pmatrix} \tag{1.0.5}$$

The angle subtended by chord AC is,

$$\phi_1 = |\theta_1 - \theta_2| \tag{1.0.6}$$

The angle subtended by chord DE is,

$$\phi_2 = |\theta_3 - \theta_4| \tag{1.0.7}$$

The direction vector of the line passing through the points A and D is,

$$\mathbf{m_1} = \begin{pmatrix} \cos\theta_3 - \cos\theta_1 \\ \sin\theta_3 - \sin\theta_1 \end{pmatrix} \tag{1.0.8}$$

$$= \begin{pmatrix} -2\sin\left(\frac{\theta_1+\theta_3}{2}\right)\sin\left(\frac{\theta_3-\theta_1}{2}\right) \\ 2\cos\left(\frac{\theta_3+\theta_1}{2}\right)\sin\left(\frac{\theta_3-\theta_1}{2}\right) \end{pmatrix} \tag{1.0.9}$$

$$\mathbf{m_1} = -2\sin\left(\frac{\theta_3-\theta_1}{2}\right)\begin{pmatrix} \sin\left(\frac{\theta_1+\theta_3}{2}\right) \\ -\cos\left(\frac{\theta_1+\theta_3}{2}\right) \end{pmatrix} \tag{1.0.10}$$

$$\|\mathbf{m_1}\| = 2\sin\left(\frac{\theta_3-\theta_1}{2}\right) \tag{1.0.11}$$

The direction vector of the line passing through the points C and E is,

$$\mathbf{m_2} = \begin{pmatrix} \cos\theta_4 - \cos\theta_2 \\ \sin\theta_4 - \sin\theta_2 \end{pmatrix} \tag{1.0.12}$$

$$= \begin{pmatrix} -2\sin\left(\frac{\theta_2+\theta_4}{2}\right)\sin\left(\frac{\theta_4-\theta_2}{2}\right) \\ 2\cos\left(\frac{\theta_4+\theta_2}{2}\right)\sin\left(\frac{\theta_4-\theta_2}{2}\right) \end{pmatrix} \tag{1.0.13}$$

$$\mathbf{m_1} = -2\sin\left(\frac{\theta_4-\theta_2}{2}\right)\begin{pmatrix} \sin\left(\frac{\theta_2+\theta_4}{2}\right) \\ -\cos\left(\frac{\theta_2+\theta_4}{2}\right) \end{pmatrix} \tag{1.0.14}$$

$$\|\mathbf{m_2}\| = 2\sin\left(\frac{\theta_4-\theta_2}{2}\right) \tag{1.0.15}$$

Then,

$$\cos \angle ABC = \frac{\mathbf{m_1}^\top \mathbf{m_2}}{\|\mathbf{m_1}\|\mathbf{m_2}\|\|} \tag{1.0.16}$$

$$= \begin{pmatrix} \sin\left(\frac{\theta_1+\theta_3}{2}\right) & -\cos\left(\frac{\theta_1+\theta_3}{2}\right) \end{pmatrix} \begin{pmatrix} \sin\left(\frac{\theta_2+\theta_4}{2}\right) \\ -\cos\left(\frac{\theta_2+\theta_4}{2}\right) \end{pmatrix} \tag{1.0.17}$$

$$= \cos\left(\frac{\theta_1 + \theta_3 - \theta_2 - \theta_4}{2}\right) \tag{1.0.18}$$

$$= \cos\left(\frac{\phi_1 - \phi_2}{2}\right) \tag{1.0.19}$$

$$\angle ABC = \frac{\phi_1 - \phi_2}{2} \tag{1.0.20}$$

Hence, $\angle ABC$ is equal to half of the angle subtended by chords AD and CE.

Parameter	**Description**
θ_1	Angle made by vector OA with X axis
θ_2	Angle made by vector OC with X axis
θ_3	Angle made by vector OD with X axis
θ_4	Angle made by vector OE with X axis

TABLE 1