

Quiz 10

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Abstract—This document contains the solution of the question from NCERT 9th standard chapter 10 exercise 10.5 problem 4

1 EXERCISE 10.4

- 1) In the below figure $\angle ABC = 69^\circ$, $\angle ACB = 31^\circ$, find $\angle BDC$

Let the circle be unit circle centred at origin,

$$\|\mathbf{x}\|^2 = 1 \quad (1.0.1)$$

Let the points A, B, C be such that,

$$C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (1.0.2)$$

$$A = \begin{pmatrix} \cos \theta_1 \\ \sin \theta_1 \end{pmatrix} \quad (1.0.3)$$

$$B = \begin{pmatrix} \cos \theta_2 \\ \sin \theta_2 \end{pmatrix} \quad (1.0.4)$$

$$BA = \begin{pmatrix} \cos \theta_1 - \cos \theta_2 \\ \sin \theta_1 - \sin \theta_2 \end{pmatrix} \quad (1.0.5)$$

$$BA = \begin{pmatrix} -2 \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \sin \left(\frac{\theta_1 + \theta_2}{2} \right) \\ 2 \cos \left(\frac{\theta_1 + \theta_2}{2} \right) \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \end{pmatrix} \quad (1.0.6)$$

$$BA = 2 \sin \left(\frac{\theta_1 - \theta_2}{2} \right) \begin{pmatrix} -\sin \left(\frac{\theta_1 + \theta_2}{2} \right) \\ \cos \left(\frac{\theta_1 + \theta_2}{2} \right) \end{pmatrix} \quad (1.0.7)$$

$$BC = \begin{pmatrix} 1 - \cos \theta_2 \\ -\sin \theta_2 \end{pmatrix} \quad (1.0.8)$$

$$BC = \begin{pmatrix} 2 \sin^2 \left(\frac{\theta_2}{2} \right) \\ -2 \sin \frac{\theta_2}{2} \cos \frac{\theta_2}{2} \end{pmatrix} \quad (1.0.9)$$

$$BC = 2 \sin \frac{\theta_2}{2} \begin{pmatrix} \sin \frac{\theta_2}{2} \\ -\cos \frac{\theta_2}{2} \end{pmatrix} \quad (1.0.10)$$

$$AC = \begin{pmatrix} 1 - \cos \theta_1 \\ -\sin \theta_1 \end{pmatrix} \quad (1.0.11)$$

$$AC = \begin{pmatrix} 2 \sin^2 \left(\frac{\theta_1}{2} \right) \\ -2 \sin \frac{\theta_1}{2} \cos \frac{\theta_1}{2} \end{pmatrix} \quad (1.0.12)$$

$$AC = 2 \sin \frac{\theta_1}{2} \begin{pmatrix} \sin \frac{\theta_1}{2} \\ -\cos \frac{\theta_1}{2} \end{pmatrix} \quad (1.0.13)$$

$$\angle ABC = 69^\circ \quad (1.0.14)$$

$$\Rightarrow \cos 69^\circ = \frac{\mathbf{BA}^\top \mathbf{BC}}{\|\mathbf{BA}\| \|\mathbf{BC}\|} \quad (1.0.15)$$

$$= -\cos \left(\frac{\theta_1 + \theta_2}{2} - \frac{\theta_2}{2} \right) \quad (1.0.16)$$

$$= -\cos \frac{\theta_1}{2} \quad (1.0.17)$$

$$\Rightarrow \frac{\theta_1}{2} = (2k + 1) 180^\circ + 69^\circ \quad (1.0.18)$$

$$\theta_1 = (2k + 1) 360^\circ + 138^\circ \quad (1.0.19)$$

$$\theta_1 = -360^\circ + 138^\circ \text{ (for } k=-1) \quad (1.0.20)$$

$$= -222^\circ \quad (1.0.21)$$

$$\angle ACB = 31^\circ \quad (1.0.22)$$

$$\Rightarrow \cos 31^\circ = \frac{\mathbf{BC}^\top \mathbf{AC}}{\|\mathbf{BC}\| \|\mathbf{AC}\|} \quad (1.0.23)$$

$$= \left(\sin \left(\frac{\theta_2}{2} \right) - \cos \left(\frac{\theta_2}{2} \right) \right) \begin{pmatrix} \sin \frac{\theta_1}{2} \\ -\cos \frac{\theta_1}{2} \end{pmatrix} \quad (1.0.24)$$

$$= \cos \left(\frac{\theta_2 - \theta_1}{2} \right) \quad (1.0.25)$$

$$\Rightarrow \left(\frac{\theta_2 - \theta_1}{2} \right) = (k) 360^\circ + 31^\circ \quad (1.0.26)$$

$$\theta_2 - \theta_1 = (2k) 360^\circ + 62^\circ \quad (1.0.27)$$

$$\theta_2 - \theta_1 = 62^\circ \text{ (for } k=0) \quad (1.0.28)$$

$$\theta_2 + 222^\circ = 62^\circ \quad (1.0.29)$$

$$\theta_2 = -160^\circ \quad (1.0.30)$$

$$A = \begin{pmatrix} \cos 222 \\ -\sin 222 \end{pmatrix} \quad (1.0.31)$$

$$B = \begin{pmatrix} \cos 160 \\ -\sin 160 \end{pmatrix} \quad (1.0.32)$$

$$C = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (1.0.33)$$

Let us take point D such that,

$$D = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.0.34)$$

Then,

$$DC = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad (1.0.35)$$

$$DB = \begin{pmatrix} \cos 160 \\ -\sin 160 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (1.0.36)$$

$$= \begin{pmatrix} \cos 160 \\ -1 - \sin 160 \end{pmatrix} \quad (1.0.37)$$

$$DB = \begin{pmatrix} -\sin 70 \\ -1 - \cos 70 \end{pmatrix} = -2 \cos 35 \begin{pmatrix} \sin 35 \\ \cos 35 \end{pmatrix} \quad (1.0.38)$$

$$\begin{aligned} \cos \angle BDC &= \frac{DC^T DB}{\|DC\| \|DB\|} \quad (1.0.39) \\ &= \frac{-2 \cos 35 (\sin 35 - \cos 35)}{(2 \cos 35) (\sqrt{2})} \quad (1.0.40) \end{aligned}$$

$$= \frac{\cos 35 - \sin 35}{\sqrt{2}} \quad (1.0.41)$$

$$= \cos (35 + 45) = \cos 80 \quad (1.0.42)$$

$$\Rightarrow \angle DBC = 80^\circ \quad (1.0.43)$$