

Quiz 7

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Abstract—This document contains the solution of the question from NCERT 11th standard chapter 10 exercise 10.3 problem 16

1 EXERCISE 10.3

- 1) If p and q are the lengths of perpendicular from the origin to the lines $x \cos (\theta) - y \sin (\theta) = k \cos (2\theta)$ and $x \sec (\theta) + y \operatorname{cosec} (\theta) = k$ respectively, prove that $p^2 + 4q^2 = k^2$.
Equation of line 1 is,

$$\begin{pmatrix} \cos (\theta) & \sin (\theta) \end{pmatrix} \mathbf{x} = k \cos (2\theta) \quad (1.0.1)$$

Equation of line 2 is,

$$\begin{pmatrix} \sec (\theta) & \operatorname{cosec} (\theta) \end{pmatrix} \mathbf{x} = k \quad (1.0.2)$$

$$p = \frac{\left| \begin{pmatrix} \cos (\theta) & \sin (\theta) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - k \cos (2\theta) \right|}{\left\| \begin{pmatrix} \cos (\theta) \\ \sin (\theta) \end{pmatrix} \right\|} \quad (1.0.3)$$

$$p = |k \cos (2\theta)| \quad (1.0.4)$$

$$q = \frac{\left| \begin{pmatrix} \sec (\theta) & \operatorname{cosec} (\theta) \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} - k \right|}{\left\| \begin{pmatrix} \sec (\theta) \\ \operatorname{cosec} (\theta) \end{pmatrix} \right\|} \quad (1.0.5)$$

$$q = \frac{|k|}{\sqrt{\sec^2 (\theta) + \operatorname{cosec}^2 (\theta)}} \quad (1.0.6)$$

$$p^2 = k^2 \cos^2 (2\theta) \quad (1.0.7)$$

$$q^2 = \frac{k^2}{\sec^2 (\theta) + \operatorname{cosec}^2 (\theta)} \quad (1.0.8)$$

$$q^2 = \frac{k^2 \sin^2 (\theta) \cos^2 (\theta)}{\sin^2 (\theta) + \cos^2 (\theta)} \quad (1.0.9)$$

$$4q^2 = k^2 \left(4 \sin^2 (\theta) \cos^2 (\theta) \right) \quad (1.0.10)$$

$$4q^2 = k^2 \sin^2 (2\theta) \quad (1.0.11)$$

$$p^2 + 4q^2 = k^2 \sin^2 (2\theta) + k^2 \cos^2 (2\theta) \quad (1.0.12)$$

$$p^2 + 4q^2 = k^2 \quad (1.0.13)$$