

Quiz 9

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Abstract—This document contains the solution of the question from NCERT 9th standard chapter 10 exercise 10.4 problem 4

1 EXERCISE 10.4

- 1) If a line intersects two concentric circles with centre O at A,B,C and D, prove that AB=CD. Let the inner circle be of radius 3 and the outer circle be of radius 5. Then the equation of the two circles are,

$$\|\mathbf{x}\| = 9 \quad (1.0.1)$$

$$\|\mathbf{x}\| = 25 \quad (1.0.2)$$

Let the equation of the line be,

$$\mathbf{x} = \begin{pmatrix} 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (1.0.3)$$

$$\mathbf{x} = \begin{pmatrix} \lambda \\ \lambda - 2 \end{pmatrix} \quad (1.0.4)$$

The points of intersection of the line and the inner circle are,

$$\|\mathbf{x}\|^2 = \lambda^2 + (\lambda - 2)^2 \quad (1.0.5)$$

$$\|\mathbf{x}\|^2 = 9 \quad (1.0.6)$$

$$\lambda^2 + (\lambda - 2)^2 = 9 \quad (1.0.7)$$

$$2\lambda^2 - 4\lambda + 4 = 9 \quad (1.0.8)$$

$$\lambda^2 - 2\lambda - \frac{5}{2} = 0 \quad (1.0.9)$$

$$\Rightarrow \lambda_c = 1 + \sqrt{\frac{7}{2}}, \lambda_b = 1 - \sqrt{\frac{7}{2}} \quad (1.0.10)$$

The points of intersection of the line and the outer circle are,

$$\|\mathbf{x}\|^2 = \lambda^2 + (\lambda - 2)^2 \quad (1.0.11)$$

$$\|\mathbf{x}\|^2 = 25 \quad (1.0.12)$$

$$\lambda^2 + (\lambda - 2)^2 = 25 \quad (1.0.13)$$

$$2\lambda^2 - 4\lambda + 4 = 25 \quad (1.0.14)$$

$$\lambda^2 - 2\lambda - \frac{21}{2} = 0 \quad (1.0.15)$$

$$\Rightarrow \lambda_d = 1 + \sqrt{\frac{23}{2}}, \lambda_a = 1 - \sqrt{\frac{23}{2}} \quad (1.0.16)$$

$$\lambda_a = 1 - \sqrt{\frac{23}{2}} \Rightarrow \mathbf{x}_a = \begin{pmatrix} 1 - \sqrt{\frac{23}{2}} \\ -1 - \sqrt{\frac{23}{2}} \end{pmatrix} \quad (1.0.17)$$

$$\lambda_b = 1 - \sqrt{\frac{7}{2}} \Rightarrow \mathbf{x}_b = \begin{pmatrix} 1 - \sqrt{\frac{7}{2}} \\ -1 - \sqrt{\frac{7}{2}} \end{pmatrix} \quad (1.0.18)$$

$$\lambda_c = 1 + \sqrt{\frac{7}{2}} \Rightarrow \mathbf{x}_c = \begin{pmatrix} 1 + \sqrt{\frac{7}{2}} \\ -1 + \sqrt{\frac{7}{2}} \end{pmatrix} \quad (1.0.19)$$

$$\lambda_d = 1 + \sqrt{\frac{23}{2}} \Rightarrow \mathbf{x}_d = \begin{pmatrix} 1 + \sqrt{\frac{23}{2}} \\ -1 + \sqrt{\frac{23}{2}} \end{pmatrix} \quad (1.0.20)$$

The distance between points A and B is,

$$AB = \|\mathbf{x}_a - \mathbf{x}_b\| = \left\| \left(\frac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \frac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \right) \right\| = \sqrt{23} - \sqrt{7} \quad (1.0.21)$$

$$CD = \|\mathbf{x}_c - \mathbf{x}_d\| = \left\| \left(\frac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \frac{\sqrt{7}-\sqrt{23}}{\sqrt{2}} \right) \right\| = \sqrt{23} - \sqrt{7} \quad (1.0.22)$$

$$\Rightarrow AB = CD \quad (1.0.23)$$