Artificial Intelligence in Mathematics Education:

A Comprehensive Review

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Abstract. Artificial intelligence (AI) technology advancements offer an opportunity to assist educators in resolving issues and enhancing teaching and learning outcomes for students. Learning mathematics often poses a significant hurdle for many students. AI offers a way to address this issue by identifying each student's unique learning challenges and giving individualized solutions to maximize their learning outcomes in mathematics courses. This paper explores the role of AI in mathematics education at all educational levels through tailored instructional approaches such as adaptive learning, differentiated instruction, personalized recommendations, assessment, and feedback. This paper provides an overview of the AI enabled technologies that foster mathematics education while addressing implementation considerations.

Keywords: MathematicsEducation, Artificial Intelligence, Personalized Learning, Adaptive Learning

1 Introduction

The research and study of structure, space, quality, and variability is known as mathematics. It includes looking for patterns, making new hypotheses, and establishing truth by rigorous deduction from suitably chosen axioms and definitions. Mathematics originated from the basic acts of counting, measuring, and calculating, alongside the systematic examination of shapes and motions in the physical world. Also, mathematics is a body of knowledge that encompasses the subjects of numbers, formulae, and associated structures, forms and the places in which they exist, and quantities and their variations. In modern mathematics, these concepts are represented by the key branches of number theory, algebra, geometry, and analysis respectively. Mathematics serves as a cornerstone of knowledge, enabling us to delve into the mysteries of the universe, innovate, and navigate the intricate mechanisms of our contemporary world. Its pervasive application spans numerous sectors globally, from natural sciences, engineering, and medicine to social sciences, serving as an indispensable tool in each domain.

Mathematics is the lifeblood of engineering, providing the necessary foundation and tools that enable engineers to conceive, evaluate, and produce modern wonders. Its importance in engineering is immense, infiltrating every aspect of the profession. Mathematics plays an immense role in engineering such as investigation, emphasizing its crucial relevance and invaluable contributions and acting as the core language and toolset for engineers. It is crucial in problem solving because it allows engineers to create and evaluate complicated systems, structures, and machines. Mathematical principles and numerical approaches are essential for solving nonlinear engineering challenges and producing accurate simulations using advanced algorithms. The basic mathematics curriculum incorporated into engineering education, beginning with calculus, and ending with differential equations, provides engineers with the necessary tools for their profession.

Furthermore, mathematics fosters critical thinking and problem-solving abilities, which are vital for engineers when confronted with real-world issues. From civil engineering, which utilizes mathematical concepts to analyses material strength and survey structures, through mechanical engineering, which uses math for geometric calculations and CAD/CAM, and particularly in the field of electrical engineering, where mathematical principles are essential to the study of circuits and signal processing mathematics pervades every aspect of engineering, making it a vital and irreplaceable component of the profession. Hence, mathematics is an essential part of engineering curriculum.

Apart from the obvious benefits, learning mathematics has some indirect benefits as well. Here are some examples of how math instruction might benefit students preparing for the future, developing critical thinking, enhancing learning outcomes, improving overall curriculum, developing life skill.

Figure 1. Indirect benefits of mathematics education.

Mathematics stands as catalyst for enhanced rigor and logical reasoning skills which are essential for developing mental rigor. It also fosters logical and critical thinking, analytical thinking, and thinking skills. Math lays the groundwork for other academic disciplines like science, art, economics, etc. Mathematics make ideas to be taught and simple to understand irrespective of the region. It's clear, unambiguous, and formulaic structure which further enhances its capacity to develop important abilities in children, such as diligence, prudence, and conscientiousness. Students need strong math skills as they go into the future because jobs in science, technology, engineering, and mathematics are expanding swiftly. Every job in the world can benefit from the pattern-seeking and logical thinking skills that math students develop. Also, Mathematics is a useful tool for communicating internationally and understanding different cultures. Students can use it to tackle challenges and make sense of the world. Math is made more relevant and meaningful by being rethought in a global framework, which gives students a new perspective on the standard curriculum. Different mathematical strategies are followed in various nations to approach a problem. Therefore, nations have diverse approaches to math education, with some seeing math as extremely important while others would not. However, every nation values education in mathematics.

1.1 Importance of AI in Mathematics Education

Artificial Intelligence (AI) refers to the scientific and engineering discipline dedicated to imparting computers with the ability to undertake tasks that traditionally necessitate human intelligence, such as sensing, speech recognition, decision-making, and language translation. By offering tailored learning experiences, increasing engagement, improving assessment procedures, democratizing access to education, strengthening evaluation, and offering predictive analytics, AI indeed holds the promise to enhance education significantly. Artificial Intelligence in Education (AIED) denotes the utilization of AI technologies or software applications within educational settings, aimed at bolstering teaching methodologies, facilitating learning processes, and refining assessment procedures.

Artificial intelligence (AI) in mathematics education has the potential to improve students' learning outcomes. Learning may be made more enjoyable and approachable with the help of AI, which can offer tailored learning experiences, real-time feedback, interactive simulations, and gamified learning experiences. Additionally, AI can forecast student performance, recognize at-risk pupils, and offer tailored interventions that can help them become better.

Artificial intelligence (AI) has the budding to completely transform the way mathematics is taught by providing individualized learning experiences and improving students' comprehension and engagement with the topic.

Though AI can be a great aid in the teaching mathematics, it should always be used in conjunction with curriculum-aligned, research-based instructional strategies. To make sure the generated responses from AI technologies are in line with their teaching aims and objectives, teachers should review and modify them.

AI can offer individualized learning experiences for every student by assessing student performance data and customizing the curriculum or content to each learner's requirements and preferences, and AI can increase engagement by delivering interactive simulations, real- time feedback, hands-on learning experiences, and gamified learning opportunities that make learning more fun and approachable. To entirely revolutionize assessment activities, AI can automate grading, provide real-time feedback, and analyze student performance data. AI can anticipate student performance, identify students who are at risk, and provide them with tailored interventions to help them improve. AI can help democratize education access by providing online learning choices, decreasing the cost of education, and expanding its accessibility. Also, AI can enhance evaluation by analyzing massive amounts of data on students' performance and learning patterns.

This research paper aims to examine how AI is transforming mathematics education across all educational levels by utilizing various instructional strategies. It will provide insights into the technologies driving this transformation and discuss important considerations for successfully implementing AI in the classroom.

2. AI-Enhanced Mathematics Education: A Comprehensive Review and Analysis

Tailoring instruction can provide a level of personalization and individualization that would be challenging to attain using only conventional teaching approaches. Therefore, by integrating AI into the educational process could be utilized to benefit the students. When implementing AI technology in education, it is crucial to recognize the benefits of responsible implementation and human engagement. Overall, AI's transformative potential in education lies in its capacity to tailor instruction to the unique requirements of individual students. This adaptability leads to more effective, personalized, and engaging learning experiences, catering to diverse learning styles and pacing, ultimately fostering improved educational outcomes.

AI technologies have been utilized to enhance the mathematical educational experiences of the students worldwide. When focusing on the individual students' needs, customized learning paths could be provided to students with two approaches. They are the adaptation of learning materials and the customized teaching methods or instructions to facilitate a better learning experience. Nonetheless, based on the students' performance evaluations, AI driven mechanism will enable a tailored feedback and recommendation accompanied by comprehensive performance assessment.

AI is essential for adapting educational content to the needs of specific students. By utilizing AI technologies, Educators can use AI to craft lessons that cater precisely to each student's distinct learning needs, preferences, and styles. Using AI to customize instruction can be done in the multiple ways as given in Fig.2: The following subsections will review how AI was utilized effectively to implement personalized learning paths for students and analyzing individual student performance and providing tailored feedback.

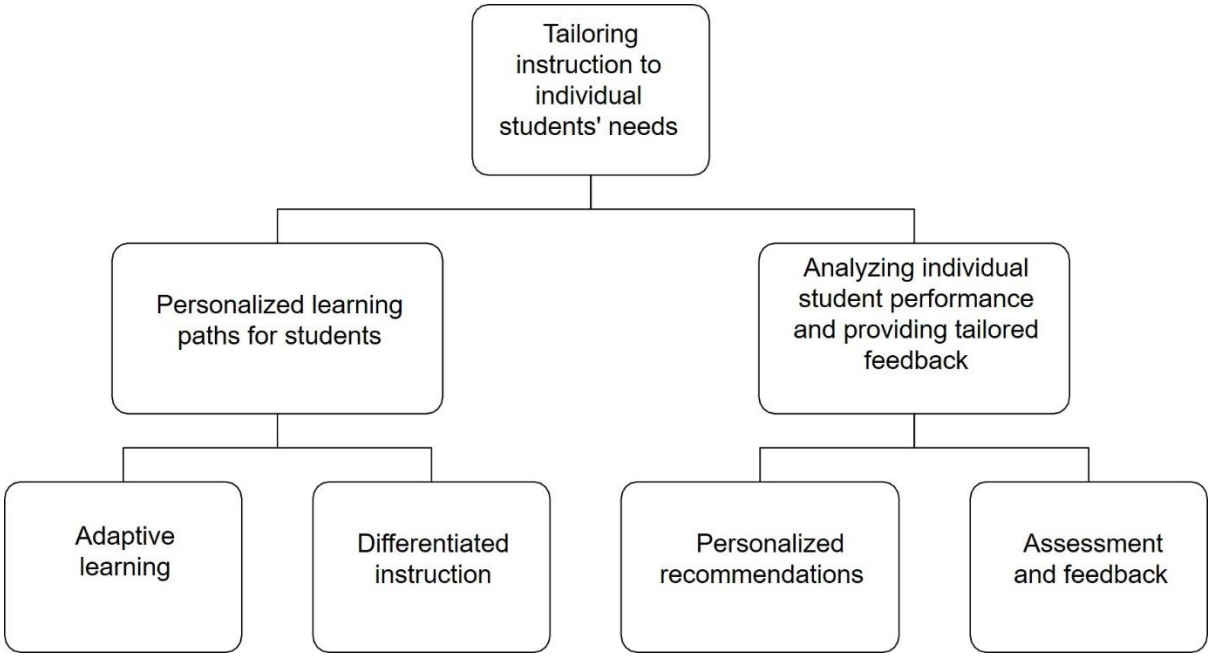

Figure 2. Methods of tailoring instruction by Artificial intelligence

2.1 Personalized learning paths for students

Personalized learning paths for students involve customized educational routes designed to suit the unique requirements, preferences, and learning methodologies of each individual student. In a personalized learning approach, the curriculum, instructional materials, and pace of learning are customized to match the student's abilities and interests, promoting a more effective and engaging learning experience. This method aims to provide students with the right level of challenge and support, helping them reach their full academic potential while addressing their unique learning requirements.

Personalized learning was approached in two different ways by various researchers. One of the popular methods is adaptive learning where students were provided materials based on the inferred results. Other approach was to provide differentiated instruction to students based on the student's needs.

2.1.1 Adaptive Learning: Tutoring programs driven by AI have the potential to deliver specialized aid and custom materials to students. These systems evaluate student comprehension, identify problem areas, and offer specific practice tasks, feedback, and explanations. They adapt course content according to the individual development of each student.

Computer scientists and cognitive researchers have collaborated on creating adaptive learning systems employing artificial intelligence to replicate human tutoring interactions. These systems encompass various functions, including presenting content, posing questions, assigning tasks, offering hints, answering queries, and suggesting improvements in learners' behavior and attitudes [1].

The underlying architecture of adaptive learning systems is unified, gathering learner data to gauge progress, suggest appropriate learning tasks, and offer personalized feedback. These decisions are commonly steered by a domain model outlining the knowledge to be acquired, a student model detailing individual learner traits (such as knowledge level, emotions, and motivation), and a task model defining the attributes of learning activities.

Shuai Wang et al [2] addressed adaptive learning research with the aim to evaluate the learning effects of personalized adaptive learning courseware to two commonly used instructional modalities in China: large-group and small-group classroom instruction. They discuss the findings of two effective experiments on Squirrel AI Learning, one of China's first adaptive learning systems. In both studies, Chinese eighth-grade students from different provinces were randomly assigned to either Squirrel AI Learning or traditional classroom instruction led

by expert teachers. The outcomes indicate that students utilizing Squirrel AI Learning demonstrated more substantial enhancements in mathematics test scores in contrast to those in traditional whole-class or small-group instruction settings. These results advocate for deeper investigations into the selection, integration, and influence of adaptive learning systems within the Chinese education framework.

In reference [3], the authors presented a model for computerized adaptive practice and monitoring, featured in the Maths Garden—an online platform crafted to offer children a stimulating arithmetic practice setting. Their model utilizes an innovative item response method founded on the Elo (1978) rating system and integrates a clear scoring rule. This approach allows for continuous updating of individual ability and item difficulty estimates after each answered question, with consideration of both accuracy and response time. Over a ten-month period, 3,648 children engaged in over 3.5 million arithmetic problems, with a significant portion completed outside of school hours. The study findings highlight enhanced measurement precision, strong validity and reliability, positive pupil satisfaction, and valuable insights into progress monitoring, error diagnosis, and developmental analysis.

In the field of mathematics education, despite an abundance of cognitive science research, there persists a persistent challenge in translating these findings into effective teaching practices. To address this issue, [4] introduce the Knowledge-Learning-Instruction (KLI) framework, specifically tailored to mathematics education. The KLI framework is designed to systematically derive instructional principles with broad applicability while taking into account the intricacies of mathematical knowledge acquisition. Drawing upon research in math education, the article demonstrates how the KLI framework enables the analysis of mathematical knowledge, learning processes, and instructional strategies, offering a set of taxonomies that shed light on different categories of mathematical learning events, ultimately guiding educators in making more effective instructional decisions in mathematics.

Despite the encouraging results, additional research is essential to comprehend the circumstances that optimize the benefits of adaptive learning, especially concerning comparisons with alternative learning conditions. Furthermore, there's a need for more investigation into the impact of adaptive learning specifically on middle and high school students. A systematic review of articles on adaptive and personalized learning revealed that a limited number of studies encompassed middle and high school students, emphasizing the necessity for more research focused on this particular age group.

2.1.2 Differentiated Instruction: AI systems are able to monitor students' progress on tasks, as well as their success rates. Based on this data, the system may aid students who need it and give more difficult assignments to those who are doing well. It can be challenging for teachers to deliver this real-time feedback and adaptive learning environment on an individual level, but AI can make it possible.

The widespread availability of computers and internet access, both in and out of schools, has amplified interest in technology-based learning systems, especially those driven by artificial intelligence. Over the past two decades, there has been growing optimism regarding the potential of artificial intelligence to enhance education, resulting in the increased prevalence of adaptive learning systems in the United States[5], including well-known ones like Knewton[6], ALEKS[7], i-Ready[8], DreamBox Learning[9], Achieve3000[10], and various cognitive tutors.

Adaptive learning systems have been shown to enhance student learning in U.S. schools. According to recent research, out of 37 studies, 86% reported positive effects[11]. In a comparison of 6,400 courses, adaptive courses outperformed non-adaptive ones[12]. A large-scale study of Cognitive Tutor Algebra I[13] in 147 schools across seven states showed an 8 percentile point improvement in student performance after a 2-year intervention [14]. A more recent study found positive effects on student math and reading performance over two years, with significant gains for low-performing students [15].

The authors in [16] aim to investigate the distinct training program within primary school mathematics education. The study contrasts traditional primary school classrooms with those integrating artificial intelligence. It evaluates the viability of integrating primary mathematics education with artificial intelligence concerning training concepts, assessment criteria, and data analysis approaches. Anticipated outcomes of their research include advancements in the mathematics teaching standards at elementary schools and the enhancement of mathematical skills among primary school students.

2.2 Analyzing Individual student performance and providing tailored feedback

Analyzing individual student performance involves assessing their strengths and weaknesses, then offering personalized feedback and guidance to enhance their learning experience and academic growth. This tailored approach fosters continuous improvement and helps students reach their full potential.

2.2.1 Personalized recommendations: AI technology facilitates multimodal learning experiences by integrating various data forms like text, audio, video, and interactive features. This personalized approach makes learning more engaging for students as it allows them to interact with the material in ways that align with their individual learning preferences.

The authors in reference [17] propose a remedy to the absence of personalized course selection in current online education platforms. They introduce an automatic recommendation approach for optimal math courses, utilizing a Bayesian model to enhance the selection process. The results indicate that the Bayesian model effectively models the causal relationships between real-world events using graph theory and probability theory, merging prior and posterior information to encode the causality between knowledge points. The model outperforms user-based collaborative filtering [18], term-based collaborative filtering [19], and the SlopeOne model [20], consistently delivering accurate course recommendations and offering empirical evidence for the enhancement and innovation of professional online math course platforms.

Personalized or precision education (PE) harnesses the power of multimodal technologies to tailor learning experiences to the unique preferences and needs of individual learners. In review of recent educational implementations, [21] examined systems such as Massive Open Online Courses (MOOCs), serious games, AI, learning management systems, mobile applications, augmented and virtual reality, and classroom technologies. The goal was to understand how these technologies impact and enhance personalized education. Their research revealed that Pedagogical Agents (PE) can significantly augment the educational capabilities of online platforms and tools. They play a pivotal role in fostering students' knowledge acquisition and skill enhancement, impacting not just digital learning but also hybrid or traditional in-person learning environments. Importantly, Pedagogical Agents have demonstrated positive outcomes in terms of student efficacy, academic achievement, and general well-being.

The study in [22] demonstrate that The Edu-Metaverse [23], a blend of technologies, is revolutionizing education through embodied, multimodal interaction, immersive teaching scenarios, and AI-driven agents. Smart education, within the Edu-Metaverse, aims to enhance learning outcomes. The research found that the AI driven education model in the Edu-Metaverse improved student scores in various areas compared to traditional instruction. It suggests that this immersive, interactive approach fosters deep learning and critical thinking, helping students become more intelligent individuals in the online learning space. To promote smart learning, the study recommends refining teaching scenarios, emphasizing core literacy-based assessment, and enhancing educators' understanding of the Edu-Metaverse's architecture.

The author in [24] explore the various forms of Computer-Based Assessment (CBA). In higher education, computers are employed in various assessment stages, ranging from self-tests to final exams, particularly within Computer-Aided Learning environments. The utilization of computers in assessment is explored, emphasizing the rationale behind their usage and underscoring the possibility of extending assessment coverage beyond the initial year of study. The paper delves into one specific project and type of CBA: randomly generated open access tests. These tests allow students to practice before taking a graded exam. The study, focusing on a statistics test, utilizes computer logs, questionnaires, and interviews to reveal that this test format motivates students to study, even outside of the computer environment, influencing their revision. The findings suggest that random-based tests offer advantages like longer lifespans, enhanced security, flexibility, increased student motivation, and serve as valuable learning resources. This paper demonstrates that CBA can be a versatile educational tool, not operating in isolation but influencing a student's study approach and enhancing the overall learning environment.

Researchers and educators have responded to concerns about children's mathematics performance in through policy changes and the use of various resources in the classroom. One potential resource is CBAs, which can support primary level mathematics teaching and learning. This study demonstrates how CBAs, by combining traditional error-analysis techniques with digital capabilities, can help teachers better understand and address the

challenges children face in math during their primary school years, emphasizing the need for further research in this area [25].

The author in reference [26] conducted a study aimed at enriching the discussion by offering an extensive review of AI's role in teaching and learning mathematics across all educational levels. A systematic literature review (SLR) was carried out in accordance with well-established and rigorous methodologies, adhering to the suggested reporting guidelines for systematic reviews and meta-analyses. They explore the various forms of CBAs in higher education, from self-tests to final exams, within Computer-Aided Learning environments. It discusses the reasons for using computers in assessment and highlights the potential for expanding assessment coverage beyond the first year of studies. The paper delves into one specific project and type of CBA: randomly generated open access tests. These tests allow students to practice before taking a graded exam. The study, focusing on a statistics test, utilizes computer logs, questionnaires, and interviews to reveal that this test format motivates students to study, even outside of the computer environment, influencing their revision. The findings suggest that random-based tests offer advantages like longer lifespans, enhanced security, flexibility, increased student motivation, and serve as valuable learning resources. The research demonstrates that CBA can be a versatile educational tool, not operating in isolation but influencing a student's study approach and enhancing the overall learning environment.

2.2.2 Personalized Assessment and feedback: AI can examine a lot of student data to offer individualized evaluation and recommendations. AI-based learning systems can provide professors with relevant data on their students' learning preferences, aptitudes, and progress as well as recommendations for tailoring lesson plans to suit specific student requirements. This makes it possible for students to repeat study sessions as necessary and for feedback to be given with better precision.

Authors in [27] addressed the prevalence of Intelligent Tutoring System (ITS)[28], that it is critical to assess the effectiveness of its teaching methods. They looked at the customized and web-based ITS "Lexue 100" for teaching math. Because they are using a quasi-experiment to assess student performance in their study, the experiment class and control class are set up to be compared internally and externally. The examination of test outcomes indicates a significant improvement in math proficiency among the experimental class due to the system. Statistical analysis of student responses highlights the system's overall high teaching quality, encompassing aspects like interface, content, prompt feedback, and a gamified reward mechanism, all of which greatly benefit students' arithmetic learning. The gathered student feedback is both reliable and valid. Additionally, an analysis of word frequency shows that a majority of students perceive this approach as helpful and positive for their learning.

Authors in [29] The study concentrated on deploying a web-based Intelligent Tutoring System specifically crafted for delivering mathematics instruction within lower secondary school settings.. It addresses a growing concern regarding the deficient mathematical skills observed in German lower secondary school students, as indicated by data from educational assessments in North Rhine-Westphalia. To address this issue, several schools introduced the web-based intelligent tutoring system eFit as an intervention. The research aimed to assess whether eFit effectively improved the arithmetic skills of this target group, and the results indicate that, over a 9-month period, the eFit group demonstrated significant improvements in arithmetic performance compared to a non-treatment control groupIt's important to highlight that eFit was specifically designed to address mathematical challenges and was somewhat customized for the assessment, whereas traditional mathematics instruction adhered to the standard curriculum. The implications of this differentiation are examined in relation to established theory and existing research.

The study in [30] presents a cloud-based adaptive learning system that seamlessly integrates personal computer-based e-learning and mobile device-based m-learning within a classroom setting, maintaining individualized assessment and learning across both platforms. While mobile learning offers cost-effective flexibility, the study's pilot results indicate that students can switch between e-learning and m-learning without significant impacts on learning outcomes and engagement, suggesting m-learning can be a valuable alternative or supplement to traditional e-learning.

3. Implementing AI in Mathematics Classroom

AI technology holds the potential to transform the learning process for students and teaching methods for educators, providing personalized learning journeys, instantaneous feedback, and the capability to adjust to each individual's unique needs. To effectively harness the potential of AI in this context, educators and institutions must not only focus on the content and pedagogical aspects but also ensure that the underlying infrastructure is robust and well-prepared. This includes technological, logistical, and policy considerations. A range of techniques and strategies that educators and institutions can consider to lay the foundation for a successful integration of AI in mathematics classrooms is detailed in this section. From upgrading technology infrastructure to implementing data privacy policies, these critical steps will enable a seamless and productive AI-enhanced learning environment for students and teachers alike.

The techniques that can be taken into consideration by educators and institutions to ensure that the infrastructure readiness to enable AI technologies in mathematics classrooms are provided in Fig.3.

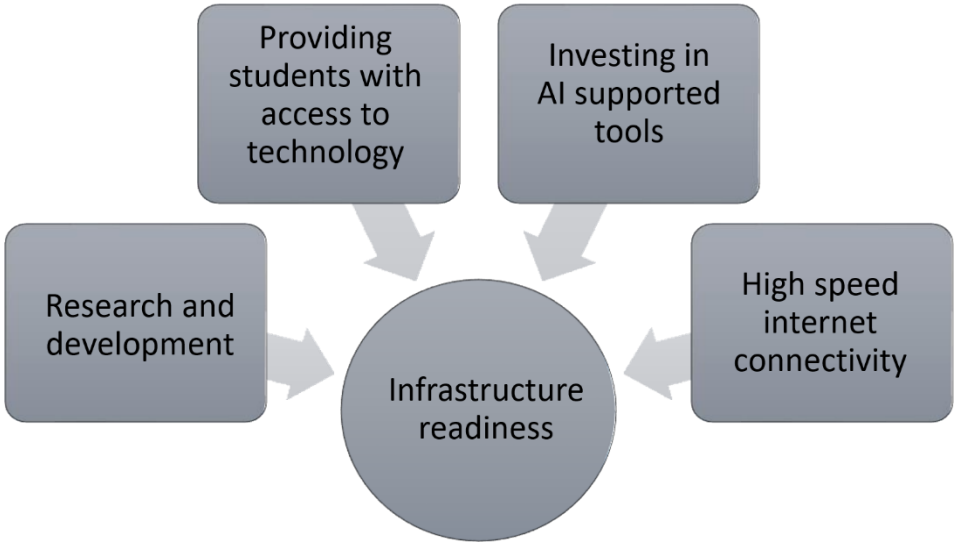

Figure 3. Analysis of Infrastructure readiness in the AI mathematics classrooms

3.1 Investing in AI-supported tools:

To improve the learning environment in mathematics classrooms, educational institutions can allocate funds on AI-supported resources like intelligent tutoring systems, automated feedback systems, and chatbots. These resources can assist students in receiving tailored feedback, enhancing their problem-solving abilities, and interacting with mathematical ideas in fresh and creative ways.

Authors in [31] a tackled a critical societal challenge: the effective implementation of digital technology, specifically artificial intelligence, within educational institutions. They observed the pervasive growth of AI-supported digital services across various societal domains, including schools. The overarching goal of the "Innovative Learning Environments Supported by Digital Technologies" project is to revolutionize education, providing students with enhanced learning opportunities to acquire information and foster digital competencies through the integration of artificial intelligence. To determine whether AI-supported activities are acceptable for attaining their goals and enabling the transformation of teaching, teachers require the knowledge and resources they need and three methods for assisting instructors in integrating AI and digital technologies have been examined as part of the project. They discovered that the RAT scale is the most appropriate scale for teachers in the project "Innovative Learning Environments Supported by ICT" based on the examination of submitted good practice scenarios.

3.2 Providing Students with Access to Technology

Schools and organizations must make sure that students have access to the tools and resources they need, including computers, tablets, and high-speed internet, in order to enable the use of AI in mathematics instruction. This can ensure that all students have an equal chance to gain from learning helped by AI.

The authors in [32] offer an analysis and overview of advancements in utilizing electronic information technology to establish innovative settings for intellectual work in mathematics. Prepared for the survey lecture of theme group T2 at the Sixth International Congress on Mathematical Education in Budapest, the paper is structured into six principal sections. These sections delve into the impacts of various computational aspects, namely: Numerical, Graphic and Symbolic computation, Utilization of Multiple Representations of Information, Programming and Intersections between Computer Science and Mathematics Curricula, Integration of Artificial Intelligence and Machine Tutors.

3.3 Research and Development

Educational institutions can promote research and development in the area of mathematics education supported by AI in order to find fresh and inventive ways to integrate AI technology into the classroom. This can assist instructors in keeping abreast of the most recent advancements in the industry and ensuring that they are utilizing the best tools and techniques to enhance student learning.

The study in [33]explores encompasses diverse facets of AI within mathematics education research, spanning across application domains, participants involved, employed research methods, utilized technologies, prevalent research issues, and the multifaceted roles of AI within this domain. The findings suggest that AI and computer technologies have encouraged diverse AI in Mathematics Education (AIME) studies, and several recommendations are made for future research, including personalized guidance for students, exploring factors affecting learning outcomes, adopting AI in advanced mathematics programs, and considering seldom-adopted sample groups like teachers and senior high school students. Additionally, researchers are encouraged to use both quantitative and qualitative methods and to explore the effectiveness of AI in mathematics learning activities from various perspectives, such as cognitive load and collaboration.

The research in [34] delves into the convergence of artificial intelligence (AI), computational thinking (CT), and mathematics education (ME) among young students (K-8). It underscores three shared fundamental aspects—agency, modeling phenomena, and abstracting concepts—across AI, CT, and ME. Employing a theoretical framework rooted in sociocultural perspectives, it recognizes technology's role as an active element shaping cognitive environments rather than just a tool. The outcomes underscore the substantial overlap among AI, CT, and ME, emphasizing the importance of agency, modeling, and abstraction in math education. Consequently, this offers fresh insights and tools for classroom practices.

3.4 Providing Professional Development:

To enable the successful use of AI technologies in the classroom, educators must get training in their utilization. Professional development programs can give teachers the abilities and information they need to include AI technologies into their instruction, as well as to comprehend how AI might improve students' mathematical learning.

The authors in [35] states that the advent of technology has significantly expanded the array of resources available for mathematics education. However, it has also led to the emergence of a new culture—a virtual culture characterized by distinct paradigms that diverge from earlier cultural forms. This paradigm shift's impact on educational policies, curriculum design, and teacher training is examined. Moreover, the continuous evolution of technology presents both promising opportunities for collaborative learning and potential risks. There's a concern that policy decisions may prioritize technological advancements and accessibility at the expense of fostering students' intellectual growth and teachers' professional development, which should be the more pivotal objectives in mathematics education.

The study in [36] provide an outline for an AI-focused program in data science curriculum and professional development designed to equip high school teachers with the necessary content knowledge for integrating AI methods into their mathematics classrooms. The program also emphasizes educating teachers about the ethical

implications related to bias in AI. The curriculum comprises five-day units covering Data Analytics, Decision Trees, Machine Learning, Neural Networks, and Transfer Learning. These units follow a step-by-step learning approach, integrating lectures using real-world examples, hands-on exercises, interactive web tools, and access to code in Google Collab notebooks for AI model creation, training, and testing. The professional development program engaged secondary school teachers from the Southwest and Northeast regions of the United States, representing diverse STEM disciplines such as Biology, Chemistry, Physics, Engineering, and Mathematics. The authors present insights into teacher outcomes derived from two one-week professional development workshops held in the summer of 2021, accompanied by suggestions provided by teachers for potential program improvements. The researchers wrap up by talking about the opportunities for and difficulties with preparing instructors to incorporate AI education into disciplinary classroom.

3.5 Ensuring Ethical Use of AI: It is critical to make sure that AI technologies are used in mathematics education in an ethical and responsible manner. Establishing rules and procedures for the use of AI in the classroom is important for educational institutions, who also need to make sure that student data is protected and used in accordance with privacy laws and regulations.

The authors in [37]addressed that the AI's linguistic structure heavily incorporates mathematics and they define language as a special structure that exists in both cognitive computing and human cognition. This distinctive structure explains the strong connection between AI and human cognition and opens the door for AI to influence the behavior of individuals who interact with it. The researchers highlight the pivotal involvement of mathematicians in establishing guidelines and protocols for AI development, particularly in crafting algorithms that underpin language processing within AI. Their focus lies in exploring the application of ethical principles to regulate interactions between users and intelligent systems to mitigate this issue. They underscore the significance of this endeavor as AI-driven algorithms often predetermine solutions, impeding users' freedom of choice. The authors argue that integrating ethical standards into the mathematical modeling of algorithms can mitigate manipulation, inequality, and the problem of opaque processes ("black boxes"), thereby safeguarding individual rights. As a result, ethical issues are crucial for individuals using algorithms, highlighting the humanistic nature of mathematics.

3.5 High-speed internet connectivity:

High-speed internet is necessary to facilitate the application of AI technology in mathematics instruction. High-speed broadband access is also necessary. Schools and organizations should make sure that students have dependable and quick internet connections so they may use AI-powered tools, access online materials, and take part in virtual classes.

The authors in [38] provide the relation between technology's arrival and materials available. AI has increased the variety of materials accessible for mathematics teaching and learning, but it has also ushered in a new culture—a virtual culture with new paradigms—that is fundamentally distinct from earlier cultural forms. The effects of this paradigm shift on educational policies, curriculum development, and teacher preparation are covered in their study. Additionally, the pervasive potential for the development of constantly new kinds of technology creates both possible benefits for learning and collaborative work (between students and teachers) as well as potential risks. Policy decisions may prioritize technical advancements and accessibility over the intellectual development of students and the professional growth of teachers, which ought to be the more challenging objectives of mathematics education.

In summary, educators and institutions can enhance the groundwork for AI integration in mathematics classrooms through investments in AI-supported tools, provision of professional development, facilitation of technology access, encouragement of research and development, and the establishment of ethical guidelines for AI use. These strategic measures collectively contribute to the development of a robust infrastructure that empowers AI technologies to enrich the educational experience for both students and educators in the realm of mathematics

4. Approaches for AI integration in mathematics education

The innovative AI approaches have the potential to engage students more effectively, provide personalized learning experiences, and support educators in delivering more tailored instruction. This section delve into a range of approaches and strategies for integrating AI in mathematics education, from AI-powered adaptive learning platforms to data-driven insights that inform pedagogical decisions. These approaches aim to optimize mathematics education, making it more engaging, accessible, and responsive to individual student needs, ultimately fostering a deeper understanding and appreciation of mathematical concepts.

The major approaches used for the studies of AI approach in mathematics education is robotics. The statistics of the methods utilized are provided as shown in Fig.4.

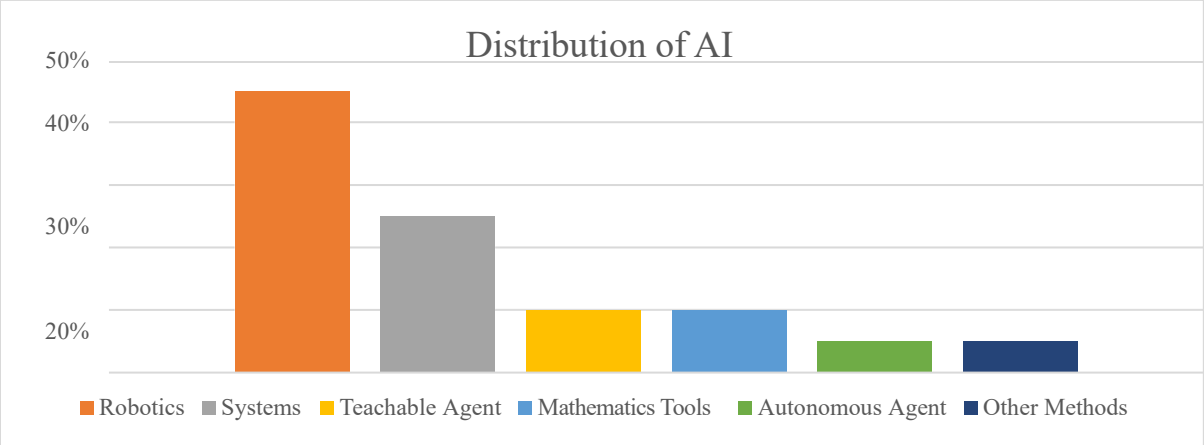

Figure 4. Distribution of artificial intelligence approaches in mathematics.

In the realm of mathematics education, researchers have shown a pronounced focus on harnessing the power of AI to enhance the learning process. The primary AI approach that stands out prominently is robotics, constituting 45% of the research concentration. This is followed closely by systems at 25%. The distribution of AI methods in mathematics education is further delineated, with teachable agents and mathematics tools each occupying 10%, autonomous agents at 5%, and other methods making up the remaining 5%. The preference for robotics and systems may stem from their capacity for customization and perceived effectiveness. These findings illustrate a compelling emphasis on the practical and adaptable applications of robotics and systems in the field of mathematics education, reflecting a commitment to optimizing the teaching and learning experiences for students and educators alike.

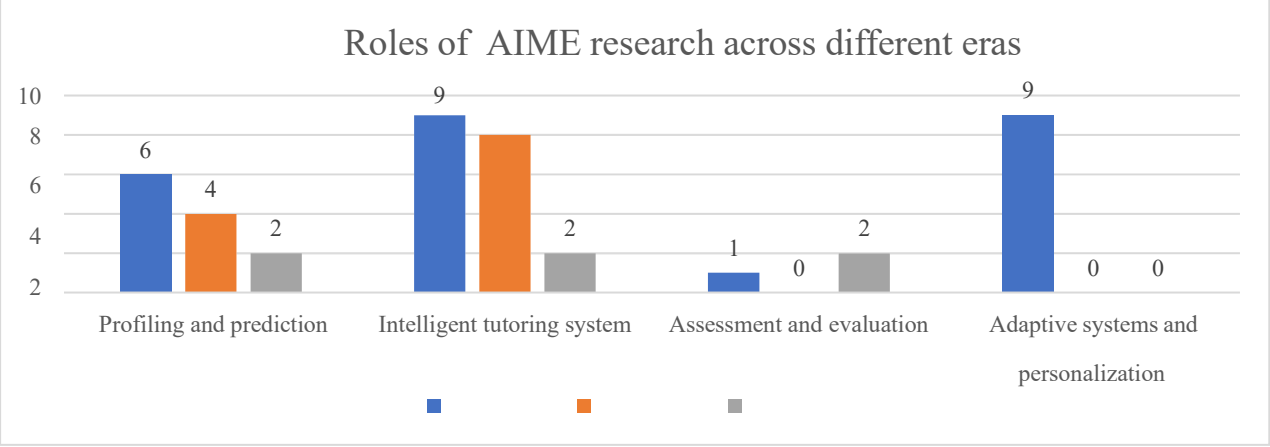

Figure 5. Roles of artificial intelligence in mathematics research in each period

In Fig.5, the significant roles of AI within mathematics education and research are delineated across three distinct periods: 1996-2010, 2011-2015, and 2016-2020. Notably, AI's most prevalent function throughout these periods was as intelligent tutoring systems, accounting for 45.24% of its role. Intelligent tutoring systems encompass AI-driven tools and platforms designed to provide personalized and interactive guidance to students, making the learning of mathematics more effective and engaging. Following closely behind, profiling and prediction constituted 28.57% of AI's role, involving the usage of AI algorithms to understand and forecast students' learning behaviors and performance. Additionally, adaptive systems and personalization played a noteworthy role at 21.43%, demonstrating the use of AI to tailor mathematical content and learning experiences to individual students' needs and preferences. Across all these roles, the number of exhibited a consistent rise from the initial to the third periods, indicating the progressive influence and assimilation of AI within the realm of mathematics education. However, the role of assessment and evaluation remained relatively limited, comprising only 4.75% of AI's involvement during these periods, suggesting a need for further exploration of AI's potential in this area.

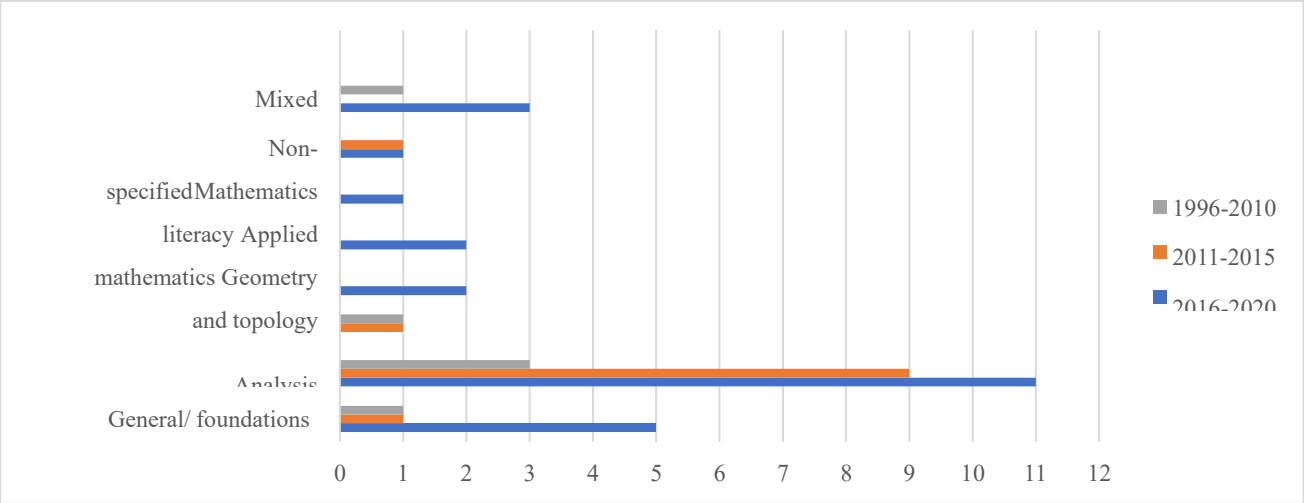

Figure 6. Application domains of artificial intelligence in mathematics education research ineach period.

In Fig.6, the application domains of Artificial Intelligence in Mathematics Education (AIME) research are categorized into three distinct time periods: 1996-2010, 2011-2015, and 2016-2020. During this span, the most prevalent application domain for AIME was discrete mathematics/algebra, accounting for the majority at 53.49%. This domain's dominance reflects the effectiveness of AI-driven approaches in enhancing the teaching and learning of discrete mathematics and algebra, which are foundational components of mathematical education. Furthermore, general/foundations constituted 16.28% of AIME applications, indicating a significant role for AI in addressing fundamental principles and concepts within mathematics. Mixed applications, encompassing a combination of various mathematical domains, represented 9.30%, underlining the versatility of AI in catering to a diverse range of mathematical topics. Interestingly, the overall number of individual application domains increased across the three time periods, demonstrating the growing influence and diversification of AI in mathematics education research.

Over the course of these periods, the expanding diversity of application domains within AIME research is notable. As AI technology advanced, it facilitated its integration into a broader spectrum of mathematical topics. The surge in individual applications illustrates the increasing recognition of AI's potential to improve learning and teaching in mathematics across various domains. This trend also suggests that researchers and educators have recognized the versatility of AI in addressing not only specific mathematical subjects but also broader foundational principles. Overall, these findings underscore the evolving landscape of AI in mathematics education research, with AI increasingly contributing to the enhancement of teaching and learning experiences across a widening range of mathematical application domains.

5. Conclusion

The integration of AI into mathematics education has the potential to bring about a profound transformation in the way students learn and teachers instruct. This comprehensive review and analysis highlight the two crucial aspects of this transformation: personalized learning paths and the analysis of individual student performance. AI-powered personalized learning paths, through adaptive learning and differentiated instruction, offer tailored educational journeys for students, catering to their unique needs and learning styles. These approaches have shown the promise of significantly improving student outcomes in mathematics education, enhancing their comprehension, engagement, and achievement.

Furthermore, AI's role in analyzing individual student performance and providing tailored feedback contributes to a more effective learning experience. Through personalized recommendations and assessment, AI can help students overcome challenges, focus on areas of improvement, and foster a deeper understanding of mathematical concepts.

In essence, AI stands poised to transform learning through tailored experiences, immediate feedback, and customized approaches to each student's requirements, creating an immersive and inclusive educational setting. It has the capacity to elevate students' analytical thinking, problem-solving abilities, and comprehension of mathematical concepts.

However, this exciting venture also raises important ethical considerations. Striking a balance between harnessing AI's potential and preserving the essential role of teachers in guiding and nurturing the learning process is crucial.

Research between 2017 and 2023 has explored the impact of AI on mathematics education, with robotics being one of the most frequently employed strategies. The full potential of AI in education can only be realized through systematic, longitudinal efforts rooted in applied research within educational settings, such as schools and universities. Multidimensional feasibility assessments and empirical studies should accompany these integrations, incorporating adaptive learning materials, innovative assessment methods, and modified tools.

As emerging technologies continue to evolve, the future holds the promise of more sophisticated digital learning tools, precision-based instructional approaches, and improved evaluation measurements. This journey to enhance education through AI, often referred to as "PE" (Personalized Education), presents both promise and challenges.

To achieve these goals, it is essential to address the challenges and shortcomings of current practices, focusing on developing protocols for personalized instruction and integrating corrective measures when needed. Heuristic data collection instruments should be designed with AI in mind, facilitating the development of new frameworks and models to complement data collected through multimodal tools. Long-term efforts and extended experiments are recommended to unlock the full potential of AI in education.

In conclusion, the integration of AI into mathematics classrooms has the potential to reshape the way students learn and teachers instruct, but it must be approached with careful consideration of ethical concerns and a commitment to rigorous research and implementation. This exciting journey represents a dynamic and evolving field that promises to enhance educational outcomes for students in the years to come.

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