Tutorial - 1

19MATI15 ~~[illegible]~~ Discrete Mathematics

~~[illegible]~~

1. Express the following in 'if .. then ..' form.

p : you get an A on the final exam.

q : you do every exercise in this book.

r : you get an A in this class.

a) To get an A in this class, it is necessary for you to get an A on the final exam.

b) Getting an A on the final and doing every exercise in this book is sufficient for getting an A in this class.

c) You get an A in this class ~~[illegible]~~ only if you either do every exercise in this book or you get an A on the final.

2. Determine whether the statements are true / false:

a) If $1+1=3$ then UFO exists.

b) If $2+2=4$ then $1+2=3$.

Show using truth tables that the following are tautologies.

a) $\sim(p \to q) \to p$ b) $\sim(p \to q) \to \sim q$.

4) Show without using truth tables.

a) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$.

b) $\sim(p \leftrightarrow q) \equiv \sim p \leftrightarrow q$.

5) If $P(x)$: $x = x^2$ where UOD = $\mathbb{Z}$. What is the truth value of $\forall x\, P(x)$, $\exists x\, Q(x)$, $\exists x; \sim Q(x)$, $\forall x \sim Q(x)$.

6). Translate into logic taking (a) UOD = Set of all students in your class.

(b) UOD = Set of all people.

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Prove

(i) Everyone in your class has a cell phone

(ii) Somebody in your class has seen a foreign movie.

7) Symbolise & Negate :- 1) All dogs have fleas 2) There is a horse that can add.

8) Check the validity of the following arguments :

~~All squares have~~

a) If a person is poor, he is unhappy.
If a person is unhappy, he dies young.

∴ poor dies young.

b). If Ravi studies then he will pass in DM paper.
If Ravi does not play cricket, then he will study
Ravi failed in DM paper

∴ Ravi played cricket.

c). No engineering student of first or second semester studies logic.
Anil is an engineering student who studies logic.

∴ Anil is not in second semester.

d). All squares have four sides.
The quadrilateral ABCD has four sides.

∴ ABCD is a square.

e) All squares have four sides
The quadrilateral ABCD is not a square

∴ ABCD does not have four sides.

f) Some intelligent boys are lazy.
Ravi is an intelligent boy

∴ Ravi is lazy.

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Prove by direct method.

1) product of two even integers is even.
2) If n is odd its square is odd
3) If n is odd then n+11 is even.
4) The sum of two odd integers is even.

i) Prove by method of Contradiction:

1) The sum of a rational and irrational no is irrational.
2) a) b) If the square of a no is even (odd) then the no itself is even (odd).

ii) Prove by method of Contrapositive:

1) If n is an integer, ~~...~~ such that 3n+2 is even, then n is even.
2) For all integers K, l if K+l is even then K, l are both even or both odd.

MAT220 - Tutorial 2. CSE, EAC II Sem

1. Prove by Induction :-

1) $1^2+3^2+5^2 \ldots (2n-1)^2 = n(2n-1)(2n+1)/3$

2) $1.3+2.4+3.5 \ldots + n(n+2) = n(n+1)(2n+7)/6$

3) $1+2+2^2 \ldots 2^n = 2^{n+1}-1, \quad \forall n \geq 1$

4) $3^n < n!$ when $n > 6$.

5) $n! < n^n$ when $n > 1$

6) n^3-n is divisible by 3 $\forall n \geq 1$

7) n^3+2n is divisible by 3, when n is a non-negative integer.

2. Prove by strong induction :-

a) If $n > 23$ then n can be written as sum of 5's and/or 7's.

b) If $n \in Z^+$, $n \neq 1, 3$ then n can be written as sum of 2's and/or 5's.

3. Find $\lceil \frac{3}{4} \rceil$, $\lfloor \frac{1}{2} \cdot \lfloor 5/2 \rfloor \rfloor$, $\lfloor \frac{1}{2} + \lceil \frac{3}{2} \rceil \rfloor$.

4. Are the following functions 1-1, onto or both ?

$f: \{a\ b\ c\ d\} \rightarrow \{a\ b\ c\ d\}$.

1) $f(a)=b$, $f(b)=a$, $f(c)=a$, $f(d)=d$

2) $f(a)=b$, $f(b)=b$, $f(c)=d$, $f(d)=c$

3) $f(a)=d$, $f(b)=b$, $f(c)=c$, $f(d)=d$.

5. Trace the modular exponentiation algorithm with $m=7$, $n=10$, $b=2$ as input. What is the output ? Show all steps.

6. Determine whether the following functions are 1-1 from Z to Z ?

a) $f(n)=n-1$ b) $f(n)=n^2+1$ c) $f(n)=n^3$

7. Let $f(x) = \lfloor x^2/3 \rfloor$ find $f(S)$ if S.

a) $S=\{-2,-1,0,1,2,3\}$ b) $S=\{0,1,2,3,4,5\}$.

8. a) Give a recursive definition of $S_m(n)$, the sum of the integer m and non-negative integer n;

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b) Give a recursive definition of the sequences $\{a_n\}$ $n=1,2,\ldots$ if $a_n = 10^n$.

9. How many different licence plates are possible if a licence no is a sequence of two letters followed by four digits if a) repetitions are allowed b) repetitions not allowed

10. In how many ways can a student choose a project if he is allowed to choose from a list of 20 maths, 35 Comp. Sc and 15 engineering projects.

11. Let $m \in Z^+$ be odd. P.T there exists a +ve integer n such that $2^n - 1$.

12. Prove that if five points are selected from the interior of an equilateral triangle of length 1 unit there are atleast two which are at a distance less than $1/2$.

13. In how many ways can five examinations be scheduled in a week so that no two exams are scheduled on the same day considering Sunday as a holiday

14. A bag contains 6 white marbles and 5 red marbles. Find the no. of ways four marbles can be drawn from the bag if a) they can be of any color b) Two must be white and two red c) they must all be of same color.

15. How many committees of five with a given chairperson can be selected from 12 persons?

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