Left Inverse of a Matrix

```
A = randi(9,4,3);
```

a)

```
fprintf("The Matrix A:")
```

```
The Matrix A:
```

```
A
```

```
A = 4×3
     5     7     5
     1     2     7
     4     5     8
     1     6     4
```

b)

```
fprintf("The rank of A:")
```

```
The rank of A:
```

```
rank(A)
```

```
ans =
3
```

c)

```
fprintf("The Left Inverse of A:")
```

```
The Left Inverse of A:
```

```
ALI = inv((A')*A)*(A')
```

```
ALI = 3×4
    0.1863   -0.1032    0.1120   -0.2763
    0.0286   -0.0574   -0.0752    0.2151
   -0.0708    0.1331    0.0577   -0.0098
```

d)

```
fprintf("Ali * A: ")
```

```
Ali * A:
```

```
ALI*A
```

```
ans = 3×3
    1.0000         0   -0.0000
   -0.0000    1.0000    0.0000
   -0.0000   -0.0000    1.0000
```

Right Inverse of a Matrix

```
A = randi([0 9],5,7);
```

a)

```
fprintf("The Matrix A:")
```

```
The Matrix A:
```

```
A
```

```
A = 5×7
     1     7     6     4     7     9     5
     2     6     2     5     5     7     8
     5     4     2     1     0     4     4
     8     7     5     8     8     0     1
     9     8     3     2     2     5     2
```

b)

```
fprintf("The rank of A:")
```

```
The rank of A:
```

```
rank(A)
```

```
ans =
5
```

c)

```
fprintf("The Right Inverse of A:")
```

```
The Right Inverse of A:
```

```
ARI = (A')*inv(A*(A'))
```

```
ARI = 7×5
   -0.0472   -0.0318    0.1130    0.0347    0.0116
   -0.0265    0.0796   -0.2356   -0.0477    0.1880
    0.1649   -0.2485    0.3865    0.0826   -0.2416
   -0.0461    0.0660   -0.0061    0.0580   -0.0504
    0.0187    0.0445   -0.1714    0.0193    0.0450
    0.0560    0.0126   -0.0805   -0.0766    0.0884
   -0.0414    0.0552    0.2332    0.0252   -0.1575
```

d)

```
fprintf("Ari * A: ")
```

```
Ari * A:
```

```
A*ARI
```

```
ans = 5×5
    1.0000   -0.0000   -0.0000   -0.0000    0.0000
    0.0000    1.0000   -0.0000   -0.0000    0.0000
    0.0000   -0.0000    1.0000   -0.0000    0.0000
    0.0000    0.0000   -0.0000    1.0000    0.0000
    0.0000    0.0000   -0.0000   -0.0000    1.0000
```

Pseudoinverse of a Matrix

a)

```
A = randi(9,9,9);
fprintf("The Matrix A:")
```

```
The Matrix A:
```

```
A
```

```
A = 9×9
     3     1     5     4     8     4     8     3     5
     1     8     3     1     6     4     9     8     9
     2     8     8     5     5     7     9     3     4
     9     3     5     9     4     4     5     8     2
     1     8     8     4     2     1     7     4     9
     8     7     4     2     3     1     6     4     6
     9     4     8     2     9     5     7     3     2
     5     7     1     1     6     5     5     6     3
     4     6     4     7     3     1     2     3     5
```

```
fprintf("The pinv(A):")
```

```
The pinv(A):
```

```
pinv(A)
```

```
ans = 9×9
    1.9070   -3.6648   -1.1094    0.0047    3.2493   -0.4068   -1.4225    4.1510 ···
   -2.9206    5.4115    1.6971   -0.0443   -4.9421    0.8126    2.1305   -6.1586
    0.7154   -1.5478   -0.5138    0.0293    1.5589   -0.3584   -0.4614    1.7357
   -1.5127    2.9801    0.9672    0.0318   -2.8007    0.4588    1.0957   -3.4814
   -2.7612    5.3800    1.5504   -0.0557   -4.8875    0.6851    2.1920   -6.0956
    7.5814  -14.4765   -4.3014   -0.0229   13.1058   -2.1485   -5.7143   16.6354
   -4.3146    8.4017    2.6573    0.0328   -7.7170    1.4018    3.1935   -9.7083
   -1.0078    1.8269    0.4301    0.1170   -1.4852    0.1214    0.7419   -1.9682
    5.9956  -11.2854   -3.4830   -0.0639   10.2943   -1.6202   -4.4783   12.9494
```

```
fprintf("The inv(A):")
```

```
The inv(A):
```

```
inv(A)
```

```
ans = 9×9
    1.9070   -3.6648   -1.1094    0.0047    3.2493   -0.4068   -1.4225    4.1510 ···
   -2.9206    5.4115    1.6971   -0.0443   -4.9421    0.8126    2.1305   -6.1586
    0.7154   -1.5478   -0.5138    0.0293    1.5589   -0.3584   -0.4614    1.7357
   -1.5127    2.9801    0.9672    0.0318   -2.8007    0.4588    1.0957   -3.4814
   -2.7612    5.3800    1.5504   -0.0557   -4.8875    0.6851    2.1920   -6.0956
```

```
    7.5814  -14.4765   -4.3014   -0.0229   13.1058   -2.1485   -5.7143   16.6354
   -4.3146    8.4017    2.6573    0.0328   -7.7170    1.4018    3.1935   -9.7083
   -1.0078    1.8269    0.4301    0.1170   -1.4852    0.1214    0.7419   -1.9682
    5.9956  -11.2854   -3.4830   -0.0639   10.2943   -1.6202   -4.4783   12.9494
```

b)

```
A = randi(9,5,4);
fprintf("The Matrix A:")
```

```
The Matrix A:
```

```
A
```

```
A = 5×4
     2     3     4     4
     1     2     9     2
     2     2     2     2
     2     3     7     1
     6     5     1     1
```

```
fprintf("The pinv(A):")
```

```
The pinv(A):
```

```
pinv(A)
```

```
ans = 4×5
   -0.5356    0.7977    0.5732   -0.9307    0.3314
    0.6230   -1.0437   -0.7096    1.2176   -0.2031
   -0.1218    0.2115    0.0674   -0.0801    0.0096
    0.1517    0.1517    0.2368   -0.3437   -0.0402
```

```
fprintf("The Left Inverse of A:")
```

```
The Left Inverse of A:
```

```
ALI = inv((A')*A)*(A')
```

```
ALI = 4×5
   -0.5356    0.7977    0.5732   -0.9307    0.3314
    0.6230   -1.0437   -0.7096    1.2176   -0.2031
   -0.1218    0.2115    0.0674   -0.0801    0.0096
    0.1517    0.1517    0.2368   -0.3437   -0.0402
```

c)

```
A = randi(9,3,7);
fprintf("The Matrix A:")
```

```
The Matrix A:
```

```
A
```

```
A = 3×7
     9     9     1     5     8     3     4
     8     5     4     5     7     5     2
     1     9     3     2     5     2     3
```

```
fprintf("The pinv(A):")
```

```
The pinv(A):
```

```
pinv(A)
```

```
ans = 7×3
    0.0885    0.0072   -0.1065
    0.0433   -0.0752    0.0865
   -0.1566    0.1412    0.0785
    0.0017    0.0336   -0.0189
    0.0042    0.0221    0.0116
   -0.0858    0.1052    0.0192
    0.0463   -0.0473    0.0114
```

```
fprintf("The Right Inverse of A:")
```

```
The Right Inverse of A:
```

```
ARI = (A')*inv(A*(A'))
```

```
ARI = 7×3
    0.0885    0.0072   -0.1065
    0.0433   -0.0752    0.0865
   -0.1566    0.1412    0.0785
    0.0017    0.0336   -0.0189
    0.0042    0.0221    0.0116
   -0.0858    0.1052    0.0192
    0.0463   -0.0473    0.0114
```

Practice Questions

1

```
A = [1 4 5 6 9; 3 -2 1 4 -1; -1 0 -1 -2 -1; 2 3 5 7 8;] % no inv?
```

```
A = 4×5
     1     4     5     6     9
     3    -2     1     4    -1
    -1     0    -1    -2    -1
     2     3     5     7     8
```

```
rank(A)
```

```
ans =
2
```

```
B = [3 4;1 8;] % inv
```

```
B = 2×2
     3     4
     1     8
```

```
rank(B)
```

```
ans =
```

```
2
```

```
C = [0 1 0; 0 0 1; 0 4 2;] % no inv?
```

```
C = 3×3
     0     1     0
     0     0     1
     0     4     2
```

```
rank(C)
```

```
ans =
2
```

```
D = [1 2 0 1;0 1 1 0; 2 4 0 2;] % no inv?
```

```
D = 3×4
     1     2     0     1
     0     1     1     0
     2     4     0     2
```

```
rank(D)
```

```
ans =
2
```

```
E = [0 1 4 1;0 0 8 1;0 1 4 0; 1 0 8 0; 0 1 1 0; 1 0 1 0;]  %Left Inv
```

```
E = 6×4
     0     1     4     1
     0     0     8     1
     0     1     4     0
     1     0     8     0
     0     1     1     0
     1     0     1     0
```

```
rank(E)
```

```
ans =
4
```

a)

```
% ONLY B CAN HAVE AN INVERSE (SQUARE AND RANK == N)
inv(B)
```

```
ans = 2×2
    0.4000   -0.2000
   -0.0500    0.1500
```

b)

```
% E CAN HAVE AN LEFT INVERSE (RANK == N)
ELI = inv((E')*E)*(E')
```

```
ELI = 4×6
    0.2549   -0.2549   -0.2745    0.1569    0.0196    0.8431
    0.2680   -0.2680    0.3268   -0.0915    0.4052    0.0915
```

```
   -0.0566    0.0566    0.0610    0.0763   -0.0044   -0.0763
    0.7059    0.2941   -0.5294   -0.4118   -0.1765    0.4118
```

c)

```
% NONE CAN HAVE A RIGHT INVERSE
```

d)

```
pinv(A)
```

```
ans = 5×4
   -0.0188    0.0870   -0.0222    0.0136
    0.0325   -0.0765    0.0172    0.0029
    0.0137    0.0105   -0.0050    0.0165
   -0.0051    0.0976   -0.0271    0.0301
    0.0462   -0.0659    0.0122    0.0194
```

```
pinv(B)
```

```
ans = 2×2
    0.4000   -0.2000
   -0.0500    0.1500
```

```
pinv(C)
```

```
ans = 3×3
         0         0         0
    0.2381   -0.3810    0.1905
   -0.3810    0.8095    0.0952
```

```
pinv(D)
```

```
ans = 4×3
    0.0500   -0.2500    0.1000
    0.0500    0.2500    0.1000
   -0.0500    0.7500   -0.1000
    0.0500   -0.2500    0.1000
```

```
pinv(E)
```

```
ans = 4×6
    0.2549   -0.2549   -0.2745    0.1569    0.0196    0.8431
    0.2680   -0.2680    0.3268   -0.0915    0.4052    0.0915
   -0.0566    0.0566    0.0610    0.0763   -0.0044   -0.0763
    0.7059    0.2941   -0.5294   -0.4118   -0.1765    0.4118
```

2)

```
A = [1 1; 2 -1;];
B = [2; 9;];
X=A\B
```

```
X = 2×1
    3.6667
   -1.6667
```

3)

```
A = [1 1 1; 2 -1 1;3 2 1];
B = [6; 3; 10];
X=pinv(A)*B
```

```
X = 3×1
    1.0000
    2.0000
    3.0000
```

4)

```
A = [1 1 1; 2 1 1;3 2 2];
B = [6; 7; 13];
X=pinv(A)*B
```

```
X = 3×1
    1.0000
    2.5000
    2.5000
```

5)

a)

```
Aa = [3 3 -1; 3 -8 6; 1 1 10];
Ba = [4; 7; 22];
Xa = Aa \ Ba
```

```
Xa = 3×1
     1
     1
     2
```

```
fprintf("This is a Point")
```

```
This is a Point
```

b)

```
Ab = [4 -3 2 5; 9 -2 -3 6; 2 11 3 -6; 8 -3 5 -1];
Bb = [10; 7; 13; 14];
Xb = Ab \ Bb
```

```
Xb = 4×1
    1.0000
    1.0000
    2.0000
    1.0000
```

```
fprintf("This is a Point")
```

```
This is a Point
```

c)

```
Ac = [1 -3 2 5; 2 -2 3 6; 2 11 -3 -6; 5 6 2 5];
Bc = [3; 11; 40; 54];
Xc = pinv(Ac)*Bc
```

```
Xc = 4×1
    3.0238
    4.3228
   -3.2257
    3.8792
```

```
fprintf("This is a Point")
```

```
This is a Point
```

d)

```
Ad = [4 -3 2 5; 9 -2 -3 6; 5 1 -5 1; 8 -6 4 10];
Bd = [10; 7; 13; 20];
% rref(Ad)
fprintf("This is a hyperplane")
```

```
This is a hyperplane
```

e)

```
Ae = [1 1 -1; 2 -2 3; 3 2 -5];
Be = [7; 9; 10];
Xe = pinv(Ae)*Be
```

```
Xe = 3×1
    5.0000
    5.0000
    3.0000
```

```
fprintf("This is a Point")
```

```
This is a Point
```

f)

```
Af = [1 -3 2 5; 2 -2 3 6; 2 11 -3 -6; 5 6 2 5];
Bf = [0; 0; 0; 0];
%Xf = pinv(Af)*Bf
% rref(Af)
fprintf("This is a line")
```

```
This is a line
```

g)

```
Ag = [4 -3 2 5; 9 -2 -3 6; 5 1 -5 1; 8 -6 4 10];
Bg = [0; 0; 0; 0];
%Xg = pinv(Ag)*Bg
% rref(Ag)
fprintf("This is a hyperplane")
```

```
This is a hyperplane
```

h)

```
Ah = [1 1 -2; 2 -3 1; 3 -2 -1];
Bh = [0; 0; 0];
%Xh = pinv(Ah)*Bh
% rref(Ah)
fprintf("This is a Line")
```

```
This is a Line
```

i)

```
Ai = [1 1 -5 3; 2 -3 -10 4; 1 -9 -5 1; 4 -11 -20 8];
Bi = [0; 0; 0; 0];
%Xi = pinv(Ai)*Bi
%rref(Ai)
fprintf("This is a Line")
```

```
This is a Line
```