AMRITA VISWA VIDYAPEETHAM

AMRITA SCHOOL OF ENGINEERING, BENGALURU

LINEAR ALGEBRA

Academic year: **2024-25 Even** Semester: **Second (CSE)**

Course Code: 23MAT117

Course Title: **LINEAR ALGEBRA**

Course Objectives

Understand the basic concepts of vector space, subspace, basis and dimension. Also, to understand the orthogonality concepts and apply to various problems computer science.

Course Outcomes

CO1: To understand the basic concepts of vector space, subspace, basis and dimension.
CO2: To understand the basic concepts of inner product space, norm, angle, Orthogonality and projection and implementing the Gram-Schmidt process, to compute least square solution.
CO3: To understand and compute the linear transformations.
CO4: To compute the eigenvalues and eigenvectors and apply to transformation problems.
CO5: To perform case studies on least square and image transformations.

Course Articulation CO-PO Mapping

PO/PSO CO	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO10	PO11	PO12	PSO 1	PSO 2
CO1	2	2	-	-	3	-	-	-	-	-	-	-		
CO2	2	2	-	-	2	-	-	-	-	-	-	-		
CO3	3	3	-	-	2	-	-	-	-	-	-	-		
CO4	2	2	-	-	1	-	-	-	-	-	-	-		
CO5	3	2	-	-	2	-	-	-	-	-	-	-		

Syllabus

Unit 1: Vector Spaces: Vector spaces - Sub spaces - Linear independence - Basis – Dimension; Inner Product Spaces: Inner products - Orthogonality - Orthogonal basis - Gram Schmidt Process - Change of basis - Orthogonal complements - Projection on subspace - Least Square Principle. QR- Decomposition.

Unit 2: Linear Transformations: Linear transformation - Relation between matrices and linear transformations - Kernel and range of a linear transformation - Change of basis - Nilpotent transformations. Symmetric and Skew Symmetric Matrices, Adjoint and Hermitian Adjoint of a Matrix, Hermitian, Unitary and Normal Transformations, Self-Adjoint and Normal Transformations.

Unit 3: Eigen values and Eigen vectors: Eigen Values and Eigen Vectors, Diagonalization, Orthogonal Diagonalization, Quadratic Forms, Diagonalizing Quadratic Forms, Conic Sections. Similarity of linear transformations - Diagonalization and its applications - Jordan form and rational canonical form. Case Studies: Applications on least square and image transformations.

Textbook(s)

1. *Howard Anton and Chris Rorres, "Elementary Linear Algebra", Eleventh Edition, John Wiley & Sons, 2010.*
2. *Gilbert Strang, "Linear Algebra for Learning Data", Cambridge press, 2019.*

Reference(s)

- *Nabil Nassif, Jocelyne Erhel, Bernard Philippe, "Introduction to Computational Linear Algebra", CRC press, 2015.*
- *Sheldon Axler, "Linear Algebra Done Right", Springer, 2014.*
- *Kenneth Hoffmann and Ray Kunze, "Linear Algebra", Second Edition, Prentice Hall, 1971.*
- *Mike Cohen, "Practical Linear Algebra for Data Science", Oreilly Publisher, 2022*.

Lab Experiments

- Matrix operations, Generation of random matrices with given rank
- Solution to linear system of equations, Left Inverse, Right Inverse, Pseudo Inverse
- Revision of curve and surface plots using parametric representations
- Span of a set (scatter plots for span of different sets)
- Finding basis for row space, column space, null space and left null space
- Finding orthogonal compliment of a given vector space
- QR decomposition
- Projections onto subspaces, Least Square Approximation, Linear Regression
- Eigenvalues, Eigenvectors, characteristic polynomial.
- Similar matrices, diagonalization, Cayley Hamilton Theorem.
- Scaling, Shifting, Rotation of images using Linear Transformations

Evaluation Pattern: 70:30 (Internal: External)

Assessment	Internal	External
Midterm	20	
*Continuous Assessments (CA)	50	
**End Semester		30 (50 Marks; 2 hours exam)

Sub:	23MAT117 Linear Algebra									
	Evaluation	**Max. Marks**	**Weightage**	**Tentative dates**	**CO-Mapping**	**CO1 Marks**	**CO2 Marks**	**CO3 Marks**	**CO4 Marks**	**CO5 Marks**
Internal	Quiz-1	10	10	2nd week of Feb	CO1, CO2	5	5	0	0	0
	Lab-Evaluation-1	30	15	4th week of Feb	CO1, CO2	15	15	0	0	0
	Mid Sem	50	20	March 03-07, 2025	CO1,2,3	15	17	18	0	0
	Quiz-2	10	10	4th week of April	CO3, CO4	0	0	5	5	0
	Lab-Evaluation-2	30	15	5th week of April	CO3, CO4, CO5	0	0	5	15	10
External	End Sem	50	30	08 May 2025	CO1,2,3,4,5	7	7	15	15	6
	Total					42	44	43	35	16

Course Plan:

Lecture No.	Topics	Key-words	Objectives	CO
Unit 1: Vector Spaces: Vector spaces - Sub spaces - Linear independence - Basis – Dimension; Inner Product Spaces: Inner products - Orthogonality - Orthogonal basis - Gram Schmidt Process - Change of basis - Orthogonal complements - Projection on subspace - Least Square Principle. QR- Decomposition.				
1-3	Review: Matrices and rank of a matrix	Operations on matrices, Rank	To find the rank of the matrix	CO1
4-5	Linear System of Equations	Consistent and inconsistent solution	To find the solution of the given system of equations	CO1
LAB 1				
6-8	General Vector Spaces	Vector spaces, vector addition, scalar multiplication, Field	To understand the concept of a vector space and its axioms, including addition and scalar multiplication	CO1
LAB 2				
9-11	Subspaces	Subspaces	To understand the definition of a subspace and the necessary conditions for a subset to qualify as a subspace. Visualize subspaces in R^2 and R^3 with geometric connections.	CO1
LAB 3				
12	Linear Independence	Linear combination, independent vectors,	To distinguish between linearly independent and dependent sets of vectors.	CO1

		dependent vectors		
13-14	Basis and Dimension	Span, dimension	To understand the concept of span and how a set of vectors generates a vector space. To identify the basis of a vector space using concept of linear independence and span and to calculate its dimension.	CO1
LAB 4				
15-16	Change of Basis	Standard basis, coordinates	To find the matrix that transforms coordinates of a vector from one basis to another and to apply the change of basis matrix to find the coordinates of a vector in a new basis.	CO1
17-18	Inner Product Spaces	Inner product, norm	To explore inner products, norms, and their properties.	CO2
LAB 5				
QUIZ 1 (2nd week of Feb)				
19-20	Orthogonality and projections	Orthogonal spaces, projection	To apply the concepts of orthogonality and orthogonal projections in vector spaces. To learn how to project vectors onto a subspace and its role in Gram-Schmidt.	CO2
21-22	Orthonormal Spaces – Gram Schmidt Process	Inner Product, norm, Orthonormal spaces, orthogonal vectors	To use the Gram-Schmidt process to construct orthonormal bases from a set of linearly independent vectors.	CO2
LAB EVALUATION 1 (4th week of Feb)				
23-24	QR Decomposition	Orthogonal matrix, triangular matrix, dimensionality reduction	To understand the factorization of a matrix A into an orthogonal matrix Q and an upper triangular matrix R and to apply QR decomposition for dimensionality reduction.	CO2
25-26	Best Approximation – Least square method	Residuals, curve fitting	To understand the concept of fitting a model to data by minimizing the sum of squared residuals.	CO2
LAB 6				
Unit 2 Linear Transformations: Linear transformation - Relation between matrices and linear transformations - Kernel and range of a linear transformation - Change of basis - Nilpotent transformations. Symmetric and Skew Symmetric Matrices, Adjoint and Hermitian Adjoint of a Matrix, Hermitian, Unitary and Normal Transformations, Self-Adjoint and Normal Transformations.				
27-28	Linear Transformation, Inverse Linear Transformation Kernel and Range	Linear Transformation, Kernel, Range, Rank-Nullity Theorem	To understand the linear transformations, their properties, and the conditions for invertibility. To learn to compute the inverse when it exists. And to analyze the kernel and range of a transformation and their connection to the rank-nullity theorem.	CO3
MID TERM (March 03-07, 2025)				
29-30	Matrices of general Linear Transformation	Matrix representations, transformation	To explore the matrix representation of linear transformations, understand their effects on vectors, and apply them to solve practical problems such as rotations, reflections, scaling, and projections.	CO3

			Additionally, to analyze the relationship between transformations and their representations in different bases.	
LAB 7				
Unit 3 Eigen values and Eigen vectors: Eigen Values and Eigen Vectors, Diagonalization, Orthogonal Diagonalization, Quadratic Forms, Diagonalizing Quadratic Forms, Conic Sections. Similarity of linear transformations - Diagonalization and its applications - Jordan form and rational canonical form. Case Studies: Applications on least square and image transformations.				
31-33	Eigen-values and eigen-vectors	Eigen values, eigen vectors	To learn the definitions and significance of eigenvalues and eigenvectors in understanding linear transformations. Properties.	CO4
LAB 8				
34-36	Similarity Transformation, Diagonalization	Diagonalization, Similar matrices, Eigen values and Eigen vectors	To use eigenvalues and eigenvectors to diagonalize matrices and simplify computations.	CO4
LAB 9				
QUIZ 2 (4th week of April)				
37-38	Quadratic forms, Diagonalizing Quadratic forms	Quadratic forms	To understand diagonalization as a process of finding a change of variables that simplifies a quadratic form by eliminating cross-product terms.	CO4
39-40	Symmetric, skew-symmetric, adjoint, Hermitian adjoint of a matrix. Positive definite matrices.	Symmetric, Skew-Symmetric, adjoint, Hermitian, Adjoint of a matrix, Positive definite matrices.	To understand the definition and properties of these matrices	CO4
LAB 10				
41-42	Conic sections. Jordan Form and Rational canonical forms	Jordan form, Rational canonical forms, block matrices	To find Jordan form of a given matrix. To recognize that every matrix is similar to a unique rational canonical form, which provides a canonical representation for linear transformations.	CO4
43-44	Case studies: Applications on least square and image transformations.			CO5
LAB 11				
LAB EVALUATION 2 (5th week of April)				
END SEMESTER EXAMINATION (08 May 2025)				

Assessment	Internal	External
*Continuous Assessment (CA) CA1- Quiz 1: Before Midterm (10 Marks) CA2- Lab 1: Before Midterm (30 Marks) CA3- Quiz 2: After Midterm (10 Marks) CA4- Lab 2: After Midterm (30 Marks)	50	
Mid term (50)	20	
End Semester(50)		30

***CA – Can be Quizzes and Lab Practice.**

Proposed Evaluation Scheme

Component	**Event Type**	**Weightage**
Internal	**Quizzes: Two (1 Before Mid Term; 1 After Mid Term) – 20 M**	**20%**
	Lab Evaluation 1- 30 M Lab Evaluation 2- 30 M	**30%**
	Mid Term Exam- 50	**20%**
External	**End Sem Exam- 50**	**30%**

Evaluations- CO Mapping/ Course Articulation Matrix

	Quiz 1 (10)	**Quiz 2 (10)**	**Lab Eval 1 (15)**	**Lab Eval 2 (15)**	**Mid Term Exam (20)**	**End Sem (30)**
CO1	**50%**		**50%**		**30%**	**14%**
CO2	**50%**		**50%**		**34%**	**14%**
CO3		**50%**		**40%**	**36%**	**30%**
CO4		**50%**		**40%**		**30%**
CO5				**20%**		**12%**