Amrita School of Engineering,

Bangalore-3523MAT117-Linear Algebra

Lab Session Sheet -5

(Fundamental Subspaces of a matrix)

- **Obtaining the basis for rowspace, columnspace, nullspace, and the nullity of a given matrix.**

A=randi([0,9],5,2)*randi([0,9],2,5)

[RR, ic]=rref(A);

% RR is the row-reduced echelon form of A,

% ic is a row vector that tells the column position of the leading pivotal element.

r = length(ic);

R=RR(1:r,:)

% gives a matrix, whose row vectors forms the basis of row space of A

C=A(:,ic)

% gives a matrix, whose column vectors forms the basis of the column-space of A

N=null(A)

% gives a matrix, whose column vectors forms the basis of null-space of A

S=size(N);

Nullity=S(:,2)

% gives the nullity of the matrix

- ✓ Another way to find nullity: S=size(A); NC=S(:,2); Nullity=NC-rank(A)
- ✓ Another way to find the basis of column space is to find the basis of row space of A^T
- ✓ Basis for left null space can be found similarly using the command null (A^T)

- **3D scatterplot of Row space and column space of any random 3×3 matrix with rank 2.**

A = randi([-3, 3], 3, 2) * randi([-3, 3], 2, 3);

[RR, ic] = rref(A);

r = length(ic);

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R = RR(1:r, :);
RSB1 = R(1, :)';
RSB2 = R(2, :)';
[RR, ic] = rref(A');
r = length(ic);
C = RR(1:r, :);
CSB1 = C(1, :)';
CSB2 = C(2, :)';
RSpts = [];
CSpts = [];
for i = 1:10000
   k1 = -1 + 2 * rand(1);
   k2 = -1 + 2 * rand(1);
   a1 = -1 + 2 * rand(1);
   a2 = -1 + 2 * rand(1);
   RSpts = [RSpts, k1 * RSB1 + k2 * RSB2];
   CSpts = [CSpts, a1 * CSB1 + a2 * CSB2];
end
         scatter3(RSpts(1, :), RSpts(2, :), RSpts(3, :), 1);
hold on
scatter3(CSpts(1, :), CSpts(2, :), CSpts(3, :), 1);
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- **How to check if a given vector y is in any of the subspaces generated by a matrix A ?**
  - If rank of $[\mathrm{A} \mid \mathbf{y}] = \mathrm{rank}(\mathrm{A})$, then y in column space of A
    where $[\mathrm{A} \mid \mathbf{y}]$ is matrix with y vector appended as last column of A
  - If rank of $\begin{bmatrix} \mathrm{A} \\ \mathbf{y}^{\mathrm{T}} \end{bmatrix} = \mathrm{rank}(\mathrm{A})$, then y is in row space of A
    where $\begin{bmatrix} \mathrm{A} \\ \mathbf{y}^{\mathrm{T}} \end{bmatrix}$ is matrix with y^{T} vector appended as last row of A.
  - If $A\boldsymbol{y} = \mathbf{0}$**,** then **y** is in the null space of A.
  - If $\boldsymbol{y}^T A = \mathbf{0}$**,** then **y** is in the left null space of A.
  - If $\boldsymbol{y}^T A = \mathbf{0}$**,** then **y** is in the left null space of A.

Practice Questions

1. Find the null space and row space of $M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Explain what each of them represents geometrically and plot them in MATLAB using a scatter plot.
2. Find the null space of the following matrices manually. Also, find the scatterplot of these null spaces

   $A = \begin{bmatrix} 1 & 4 \\ 0 & 5 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 0 & 5 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
3. Generate a random 3× 3 matrix of rank 2. Provide the scatter plot of the row-space and the column space of the matrix in the same figure.
4. Generate a random 3× 3 matrix of rank 1. Provide the scatter plot of the column-space and the left null space of the matrix in the same figure.
5. Given, a matrix, $A = \begin{pmatrix} 1 & 3 & 4 & 7 \\ 2 & 4 & 6 & 10 \\ 3 & 5 & 8 & 13 \\ 4 & 6 & 10 & 16 \end{pmatrix}$. Find out the subspace associated with A in which each of the following vectors lie.

(i) $u = \begin{pmatrix} -2 \\ -3 \\ 1 \\ 1 \end{pmatrix}$, (ii)$v = \begin{pmatrix} 5 \\ 8 \\ 11 \\ 14 \end{pmatrix}$ (iii) $w = \begin{pmatrix} 1 \\ 1 \\ 2 \\ 3 \end{pmatrix}$ (iv) $y = \begin{pmatrix} 1 \\ 2 \\ 0 \\ -1 \end{pmatrix}$ (v) $m = \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix}$

6. Given, a matrix, $A = \begin{pmatrix} 1 & -1 & 2 & 3 \\ 0 & 2 & 1 & 4 \\ 1 & 1 & 3 & 1 \\ 2 & 0 & 5 & 4 \end{pmatrix}$. Find out the subspace associated with A in which each of the following vectors lie?

(i) $v = \begin{pmatrix} 5 \\ 1 \\ -2 \\ 0 \end{pmatrix}$, (ii) $w = \begin{pmatrix} 0 \\ 2 \\ 2 \\ 2 \end{pmatrix}$, (iii) $u = \begin{pmatrix} -1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$ (iv) $m = \begin{pmatrix} 3 \\ -1 \\ 7 \\ 7 \end{pmatrix}$