Amrita School of Engineering, Bangalore-35
23MAT117-Linear Algebra
Lab Session Sheet - 6
(Orthogonal Spaces)

❖ **Orthogonal Vectors**: - Two non-zero vectors u and v in R^n are said to be orthogonal (or perpendicular) if $u \cdot v = 0$.

➢ Orthogonality of the given two vectors $u = (-2, 3, 1, 4)$ and $v = (1, 2, 0, -1)$.

```
u = [-2; 3; 1; 4]; v = [1; 2; 0; -1]; % Define the vectors
dot_product = dot(u, v);   % Calculate the dot product

% Check if the dot product is zero (orthogonal)
if dot_product == 0
   disp('The vectors are orthogonal.');
else
   disp('The vectors are not orthogonal.');
end
```

➢ Generation of two orthogonal vectors of a given 3D vector.

```
A = [1,2,3];
N = null(A);
OrthVec1 = N(:,1)
OrthVec2 = N(:,2)
```

- **NOTE:** How can two orthogonal vectors be derived from a given vector in 3D (or R3) space?
  - ✓ Consider, $A = [1\ 2\ 3]$ and determine $null(A)$.
  - ✓ The returned matrix has two column vectors that are perpendicular to A's row vector.
  - ✓ $Null(A)$ returns a matrix containing basis vectors for A's null space.

❖ **Orthogonal Complement**: - Let W be a subspace of R^n. Its orthogonal complement is the subspace $W^\perp$ ={v in R^n | $v.w = 0$ for all w in W}.

- **NOTE:** Taking the orthogonal complement is an operation that is performed on subspaces.
  - ✓ null(A) for a matrix A, gives a matrix whose column vectors give the basis of null space for A. i.e., the column vectors of null(A) will be only the basis of the orthogonal complement of the space spanned by the row vectors of A. If we span these column vectors then we will get the orthogonal complement of the row space of A.
  - ✓ Row space and null space of a matrix are orthogonal complements to each other.

✓ Column space and left null space of a matrix are orthogonal complements to each other.

➢ Orthogonal complement of the space spanned by $[1,7,2]$ and $[-2,3,1]$.

```
V1 = [1,7,2]; V2 = [-2,3,1]; % Define the set of vectors
A = [V1; V2];
% Compute the null space of A to find vectors orthogonal to both v1 and v2.
N = null(A)
[RR,ic]=rref(A)
r=length(ic)
R=RR(1:r,:) % gives a matrix, whose row vectors forms the basis of row-space
of A.
% To check whether column vectors of null(A) and the space spanned by the two
vectors are orthogonal complement of each other.
for i=1:size(R,1);
    for j=1:size(N,2);
        if round(dot(R(i,:),N(:,j)))==0
            disp('They are orthogonal complements of each other')
        else
            disp('They are not orthogonal complements of each other')
        end
    end
end
```

Practice Questions

1) Find two orthogonal vectors to the vector $[3,1,-2]$.
2) Find the orthogonal complement of the space spanned by the vectors $[2,0,-1]$ and $[1,1,1]$.
3) Given V is the solution of the system $x+y+3z=0$ and $2x-3y+z=0$. Find the orthogonal complement of V.
4) Find the orthogonal complement of the subspace spanned by the vectors $\{(1,0,1),(-2,0,0),(0,0,3)\}$.
5) Find the orthogonal complement of the space spanned by the vectors, $\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix}$.
6) Find all the vectors in $\mathbb{R}^4$ that are orthogonal to $\begin{pmatrix} 4 \\ 1 \\ -2 \\ 0 \end{pmatrix}$and $\begin{pmatrix} -1 \\ 2 \\ -1 \\ 1 \end{pmatrix}$.
7) Find the row space and null space of the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & -1 & 0 \\ 3 & 1 & 8 \end{bmatrix}$ and prove that they are orthogonal complement of one another.