Volatility Prediction and Risk Management: An SVR-GARCH Approach

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Increased integration of financial markets has led to a prolonged uncertainty in financial market which in turn stresses the importance of volatility, degree at which values of financial assets changes. Volatility has been used as a proxy of risk that in an among the most important variable in many field such as asset pricing and risk management. Its strong presence and latency make it even compulsory to model. Basel Accord, therefore, came into effect in 1996, and volatility as a risk measure has taken the key role in risk management.

There is a large and growing body of literature regarding the volatility estimation that is a good starting point for assessing the risk. After the ground-breaking study of [7], [2], [13], [14], and [25], this field has got more and more attention. However, the recent fluctuations in financial markets together with the recent development in Machine Learning (ML) make researchers to rethink volatility estimation. For this reason, the process of developing new methods for estimating, predicting and forecasting the volatility in the markets is a top of the agenda. This makes estimating the volatility a challenging task. Thus, a brand new Machine Learning approach called Support Vector Regression- GARCH (hereinafter SVR-GARCH) has been proposed.

In fact, there is a long tradition of GARCH-type models application in that quantitative model-based forecasts can provide financial institutions with a valuable assessment of future market trends. First quantitative model proposed by [15] and called Autoregressive Conditional Heteroskedasticity (ARCH). This model was generalized by [8] and named Generalized Autoregressive Conditional Heteroskedasticity (GARCH). This model assumes a functional form of data generating process and error term and moreover has low forecasting performance. Different variations of GARCH has been introduced to overcome the drawback by proposing improvement on the functional form, volatility proxy, and accuracy metrics [6].

In particular, volatility in financial assets can be affected differently from positive and negative news. Thus, volatility of financial assets can give asymmetrical responses to positive and negative shock and the timing of news might not be a surprise and enable more robust estimation of volatility. ARCH/GARCH models are not sufficient for modelling asymmetric reactions. In order to deal with these, Exponential GARCH (EGARCH) model and Fractionally Integrated GARCH (FIGARCH) model are proposed.

Volatility provides important insights about financial phenomena but also utilizes as an input in risk management along with other areas such as pricing. Therefore, it is quite important to forecast the volatility in order to have solid risk management. As traditional models have empirically low forecast performance, it is expected that SVR-GARCH not only boost the predictive performance but also improve the risk management. To apply volatility models in risk management, volatility predictions are used as an input in Value-at-Risk method and based on the backtesting of Value-at-Risk, it is expected to have improved risk management performance. In a nutshell, volatility prediction with high accuracy provides better risk management by shedding light on the uncertainty of the future path of financial risks as well as increasing awareness about the risks that have yer to come.

To this end, in this study, traditional ARCH-type models in estimating and predicting volatility are employed and compared with Machine Learning-based model. The results are compared using RMSE and MAE metrics. From this point on, these volatility predictions serve as input in the Value-at-Risk as well as its backtesting.

The remainder of this chapter is as follows. In the second section, theoretical introduction of the volatility models are provided. In the third section, Machine Learning models are discussed and algorithms are presented. In the fourth section, volatility estimations with traditional and ML models are provided with the performance comparisons. In the fifth section, VaR applications of these models along with their backtesting are shown and discussed. The final chapter concludes this study.

Traditional Volatility Models

GARCH Model

ARCH model proposed by [15] improved by [8] and [28] by adding p number of delayed past conditional variance and it is called GARCH (p, q) model. GARCH models can be expressed as autoregressive moving average models for conditional variance [9].

Mathematically, ARCH models depends on the realized values of the squared error terms in previous time periods.

The model has the following form:

$$y_t = u_t \tag{1}$$

$$u_t \sim \mathcal{N}(0, \sigma) \tag{2}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 \tag{3}$$

This model is known as ARCH(q) where q amounts to order of the lagged squared errors. As an extension, the structure of the GARCH model implies that conditional variance is determined by historical information implying-market inefficiency. The term u_t in equation 3 is accepted as return and as the amount of news at time t so that positive u_t means good news or vice versa. In the GARCH model, the conditional variance of return is determined by the square of the error term and its lagged values.

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \alpha\epsilon_{t-1}^2 \tag{4}$$

where ω, β, and α are parameters and have restrictions: ω>0, $\beta \geq 0$, and $\alpha \geq 0$. Moreover, in order for GARCH to be consistent, [20] states that $\beta + \alpha < 1$.

The main reasons that GARCH is applied to financial phenomenon are returns are well fitted by GARCH model partly due to the volatility clustering and GARCH does not assume that the returns are independent that allows modeling the leptokurtic property of returns. Despite these useful properties and intuitiveness, GARCH is not able to asymmetric response of the shocks. Therefore, GJR-GARCH, or extended GARCH, is proposed by [17] to account for this asymmetric response to the shocks.Conversely, the effect of a negative shock indicates that the firm is more leveraged [5].

GJR-GARCH Model

As is discussed, GJR-GARCH tries to capture the asymmetry in the new impact function and to do that γ is included into the GARCH equation and it turns out:

$$\sigma_t^2 = \omega + \beta\sigma_{t-1}^2 + \gamma\epsilon_{t-1}^2 I(\epsilon_{t-1} < 0) \tag{5}$$

If γ=0, the response to the past shock is the same. If γ is greater than zero then the response to the past negative shock is stronger than that of a positive one. Thus, GJR-GARCH is designed to capture the asymmetric news impact curve which is a function of ϵ_{t-1}.

EGARCH Model

GJR-GARCH model is not the only one model proposed to capture the asymmetry in the news impact curve. Rather, [24] developed EGARCH model to account for this asymmetry and takes the following form:

$$\log + h_t = \omega + \sum_{j=1}^{p} \beta_j \log(h_{t-j}) + \sum_{i=1}^{q} \alpha_i \frac{|u_{i-1}|}{\sqrt{h_{t-i}}} + \sum_{i=1}^{q} \gamma_i \frac{u_{t-i}}{\sqrt{h_{t-i}}} \tag{6}$$

According to equation 6 , while ω grasps the asymmetric shocks of volatility and α capture the volatility clustering. Thus, similar to GJR-GARCH model asymmetric shocks of volatility and volatility of clustering are overcome by the EGARCH model.

In a nutshell, the reasons why EGARCH is superior models over GARCH model are as follows:

- EGARCH never gets negative values in that the conditional variance equation takes the logarithmic form. Thus, non-negativitiy condition of the GARCH model is no longer needed.
- As EGARCH allows us to take into account the positive and negative shocks, it is possible to examine the leverage effect with EGARCH model as opposed to GARCH model.

GARCH model and its extensions are insufficient to evaluate the long memory property defined as the hyperbolic rate decrease in the autocorrelation functions of the high frequency financial time series, long term dependence and tendency to slow reversion to return. Such time series exhibit hyperbolic decreasing autocorrelations and, if long memory is concerned, the impact of a shock on financial markets persists for a long time. The long memory process is therefore characterized by a fractional degree of integration rather than an integer degree of integration. Fractionally Integrated GARCH (FIGARCH) model is introduced by [3] and discussed in the following part.

FIGARCH Model

[16] first introduced the Integrated GARCH model known as IGARCH to model the long-term persistence. As stated by [26], any shocks to the conditional variance persist into the future and this property can be modeled by Integrated GARCH but it turns out IGARCH without drift converges to zero. Thus, the IGARCH model is considered as short-term volatility model.

In literature, many researchers, among others, [1], [18], and [19], have demonstrated that long memory features can be modeled by extending an integrated process into a fractional integrated process. If the stock market returns volatility has long memory characteristics, it is not a random process and prices can be estimated by using past prices.

The derivation of FIGARCH is provided below. To start with, GARCH (p,q)can be written as:

$$\sigma_t^2 = \omega + \beta(L)\sigma_{t-1}^2 + \alpha(L)\epsilon_{t-1}^2 \tag{7}$$

where $\alpha(L) = \alpha_1 L^2 + \alpha_2 L^2 + ... + \alpha_q L^q$ and$\beta(L) = \beta_1 L^2 + \beta_2 L^2 + ... + \beta_p L^p$

Once $\upsilon \equiv \epsilon_t^2 - \sigma_t^2$, then it is possible to define GARCH(p,q) as follows:

$$[1 - \alpha(L) - \beta(L)]\epsilon_t^2 = \omega + [1 - \beta(L)]\upsilon \tag{8}$$

The FIGARCH model can be written as:

$$\phi(L)(1-L)^d \epsilon_t^2 = \omega + [1 - \beta(L)]\upsilon \tag{9}$$

where $\phi(L) = [1 - \alpha(L) - \beta(L)](1-L)^{-1}$

Equation 10 can be converted to the following equation which is the standard representation of FIGARCH model:

$$[1 - \beta(L)]\sigma_t^2 = \omega + [1 - 1 - \beta(L) - \phi(L)(1-L)^d]\epsilon_t^2 \tag{10}$$

SVR Model

Support Vector Machines (SVM) are machine learning algorithms based on convex optimization that operate according to the structural risk minimization principle and it is distribution-free learning algorithm [27]. Besides, support vector machine is a controlled learning method used in classification and regression analysis that analyzes data and learns from samples. The support vector machine was first proposed by [29].

Both linearly distinguishable and non-distinguishable data sets can be classified with SVM. The disticntion feature of SVM lies in its n-dimensional application. With a nonlinear mapping, the n-dimensional data set is converted to a new d-size data set with d>n. With a suitable transformation, the data can always be divided into two classes with a hyperplane.

In this study, Support Vector Regression-based GARCH (SVR-GARCH) modeling is applied. SVR nonlinearly maps the input space into a high dimensional feature space and employs the linear regression in high dimensional feature space. To show the theoretical background of SVR, let x_t and y_t be training dataset where $x_t \in \mathbb{R}^p$, $y_t \in \mathbb{R}^1$.

As the dataset used has a time-series structure, x_t can be considered as lagged values of y_t. Data is generated from a function

$$y_t = f(x_t) + \epsilon_t \tag{11}$$

At this point, it is needed to define a decision function f(x) as follows:

$$f(x_t) = w^T \phi(x_t) + b = \sum_{i=1}^{n} w_i \phi_i(x) + b \tag{12}$$

where $\phi(x) = [\phi_1(x), ..., \phi_n(x)]^T$ and $w = [w_1, ..., w_n]^T$ is a non-linear transformation to a higher dimension space. [29] suggests a ϵ-insensitive loss function, $L_\epsilon(x, y, f(x))$ and it is defined by:

$$L_\epsilon = \begin{cases} |y = f(x)| - \epsilon, & |y = f(x)| \geq \epsilon \\ 0, & \text{otherwise} \end{cases}$$

As [11] denotes the loss function does not penalize errors below ϵ. In this case, error is ignored and no loss occurs. This implies that f(x) is obtained through datapoint located on or outside the ϵproximity. This is called ϵ-insensitivity.

At this point, ξ and $\xi*$ are introduced as slack variable to describe the ϵ-insensitivit loss and ϵ-SVR is provided as:

$$\min_{w,b,\xi,\xi*} [\frac{1}{2}||w||^2 + C \sum_{t=1}^{n} (\xi + \xi^*)] \tag{13}$$

subject to

$$y_t - w'\phi(x_t) - b \leq \epsilon + \xi_t^* \tag{14}$$

$$w'\phi(x_t) + b - y_t \leq \epsilon + \xi_t^* \tag{15}$$

$$\xi_t, \xi_t^* \geq 0 \tag{16}$$

In equation 13, $\frac{1}{2}||w||^2$ measure the function flatness and the second term relates to ϵ-insensitive loss function. This problem can be solved using Langrangian approach.

$$L_p = \frac{1}{2}||w||^2 + C\sum_{t=1}^{n}(\xi + \xi^*) \tag{17}$$

$$-\sum_{t=1}^{n}\alpha_t(\epsilon + \xi_t - y_t + w'\phi(x_t) + b) - \sum_{t=1}^{n}\mu_t + \xi_t \tag{18}$$

$$-\sum_{t=1}^{n}\alpha_t^*(\epsilon + \xi_t^* - y_t + w'\phi(x_t) + b) - \sum_{t=1}^{n}\mu_t^* + \xi_t^* \tag{19}$$

Karush-Kuhn-Tucker condition makes it possible to have following equations:

$$\frac{\partial dL_p}{\partial dw} = w - \sum_{t=1}^{n}(\alpha_t - \alpha_t^*)\phi(x_t) = 0 \tag{20}$$

$$\frac{\partial dL_p}{\partial db} = \sum_{t=1}^{n}(\alpha_t - \alpha_t^*) = 0 \tag{21}$$

$$\frac{\partial dL_p}{\partial d\xi_t} = C - \alpha_t - \mu_t = 0 \tag{22}$$

$$\frac{\partial dL_p}{\partial d\xi_t^*} = C - \alpha_t^* - \mu_t^* = 0 \tag{23}$$

where the parameters $\alpha_t, \mu_t, \alpha_t^*, \mu_t^* \geq 0$. Additionally the kernel function is of considerable importance role in forecasting performance of the SVR model. There are three types of kernel function which are:

- Linear Kernel: $x_t x$
- Polynomial: $(x_t x + 1)^d$
- Gaussian: $e^{\frac{-||x - x_t||^2}{2\sigma^2}}$

SVR-GARCH Model

As it is discussed, SVM is a state-of-the-art method and can be applied in a wide range of areas. In this study, SVM is employed to model the volatility using GARCH model. The main motivation of this rest upon the fact that SVR there is no probability density function over returns [29].

The SVR-GARCH equips researcher with a very powerful tool which is forecasting the volatility and in turn employing it in modeling risk. Traditional risk models such as VaR, for instance, uses standard deviation to account for the volatility in returns and then feed the model. However, aside from the traditional volatility model, there is no many other tools to forecast volatility to be utilized in forecasting the risk. SVR-GARCH model produces better-suited approach and provides more robust results.

Following structure specifies the SVR-GARCH process:

$$r_t = \log(P_t / P_{t-1}) \tag{24}$$

where P_t and r_t are price and return at time t, respectively. In order to find the squared residuals, the conditional mean estimation is used and it is given by:

$$r_t = g(r_{t-1}) + a_t \tag{25}$$

where g is the estimation function for mean equation estimated by SVR as suggested by [6].
Following [10], a volatility proxy is used due to the unobservability of the volatility.

$$\sigma_t^2 = (r_t - \overline{r})^2 \tag{26}$$

where σ_t is the conditional variance.

$$\sigma_t^2 = f(r_{t-1}^2, \sigma_{t-1}^2) \tag{27}$$

where f is the SVR decision function.

Empirical Application

In this part, empirical application is conducted over 30 stocks listed in S&P-500 between 2010/01-2019/01 corresponding to 2268 days. The first 90% of the data is allocated to the training set and the rest 10% is reserved for test set. The stock employed in this analysis is shown in the Table **??**.

Exhibit-1: Companies

Companies	Tickers	Companies	Tickers
Amgen Inc.	AMG	Lockheed Martin Corporation	LMT
Amazon.com Inc.	AMZN	Macy's Inc	M
AutoZone Inc.	AZO	Mettler-Toledo International Inc.	MTD
Booking Holdings Inc.	BKNG	Mylan N.V.	MYL
BlackRock Inc.	BLK	NIKE Inc. Class B	NKE
Duke Realty Corporation	DUK	O'Reilly Automotive Inc.	ORLY
Consolidated Edison Inc.	ED	People's United Financial Inc.	PBCT
Ford Motor Company	F	Prologis Inc.	PLD
Freeport-McMoRan Inc.	FCX	Regions Financial Corporation	RF
General Electric Company	GE	Raymond James Financial Inc.	RJF
Alphabet Inc. Class C	GOOG	Sherwin-Williams Company	SHW
Gap Inc.	GPS	TransDigm Group Incorporated	TDG
Huntington Bancshares Incor.	HBAN	Under Armour Inc.	UAA
Intercontinental Exchange Inc.	ICE	V.F. Corporation	VFC
Leidos Holdings Inc.	LDOS	WEC Energy Group Inc.	WEC

Using this data, return volatilities are modeled by GARCH, GJR-GARCH, E-GARCH, FIGARCH, and SVR-GARCH.

In SVR-GARCH application process, grid search method is applied. To do that C, ϵ, γ parameters spanned between 0 and 10. Based on the Akaike Information Criteria (AIC), the best model is chosen so that volatility is modeled by these parameters. Then, RMSE and MAE are calculated to compare the performance of the model.

Before moving forward, the returns of the stock considered along with the histograms and autocorrelation functions are depicted in Figure **??**, **??**, and **??**, respectively. Return plots of the stocks included in the study are provided in Figure **??**, by eyeballing, it indicates that returns oscillate around zero as anticipated though some stocks such as WEC Energy, Regions Financial Corporation, BlackRock show greater extent of volatility.

In the Figure **??**, it is checked whether the returns are normally distributed and moreover normality test is run and its result revealed in the Table **??**. Even if some of the histograms shows asymmetry and fat tail, normality test confirms that stocks returns are normally distributed at 1% level.

In the Table **??**, p-values suggest that all stock returns are stationary in that p-values are less than 0.05. In the volatility estimation, autocorrelation function (ACF) plays an important role as it exhibits volatility clustering. No autocorrelation does not require that linear increases or decreases in returns be independent of each other. Rather, independence of any non-linear function of returns requires autocorrelation. In practice, simple nonlinear functions of returns show significant autocorrelation or persistency. This is the so-called volatility clustering. In Figure **??**, ACF of all stocks are provided, it is observed that autocorrelation coefficients, in almost all cases, remain outside the confidence interval implying that there exists volatility clustering.

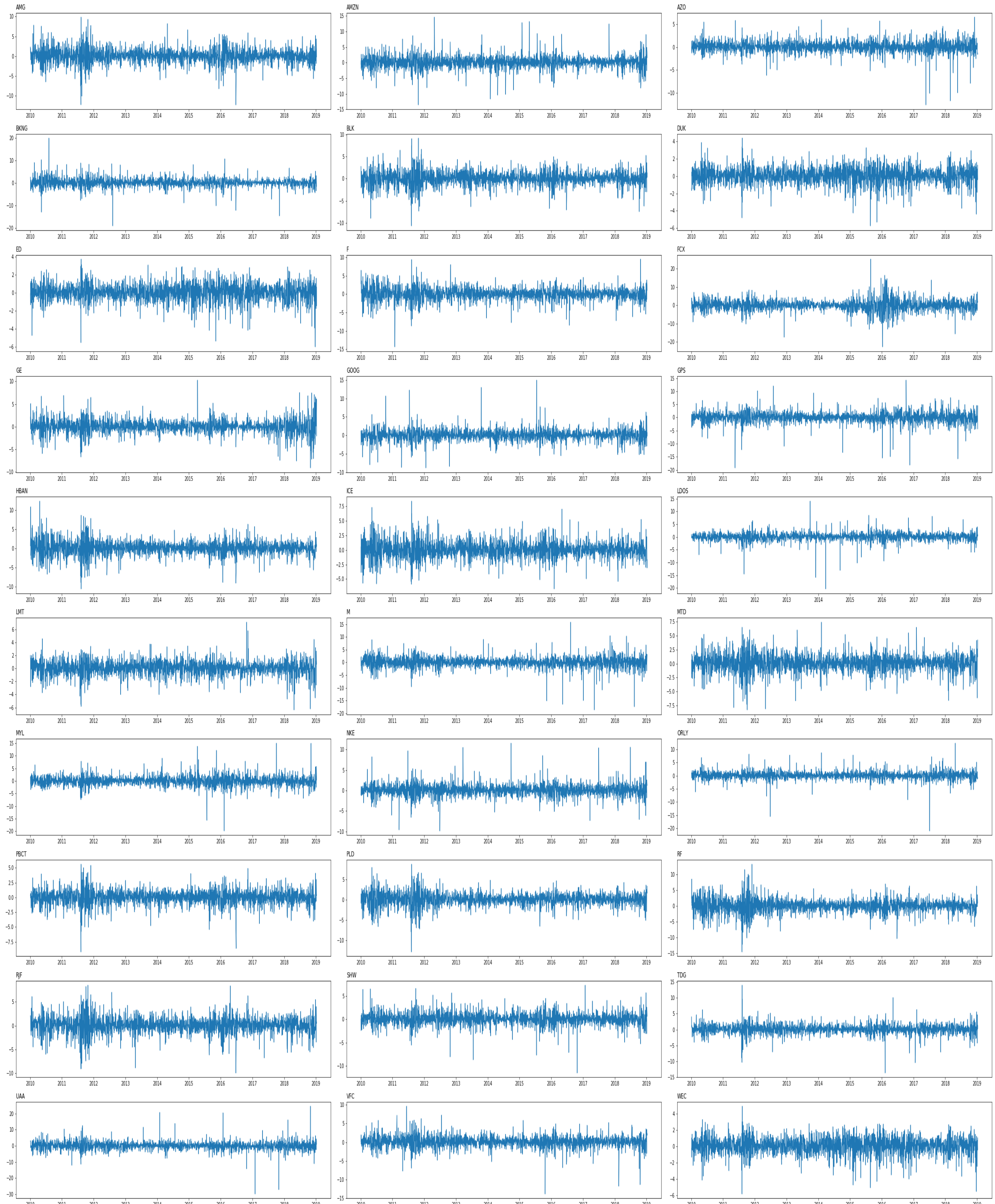

Exhibit-2: Returns

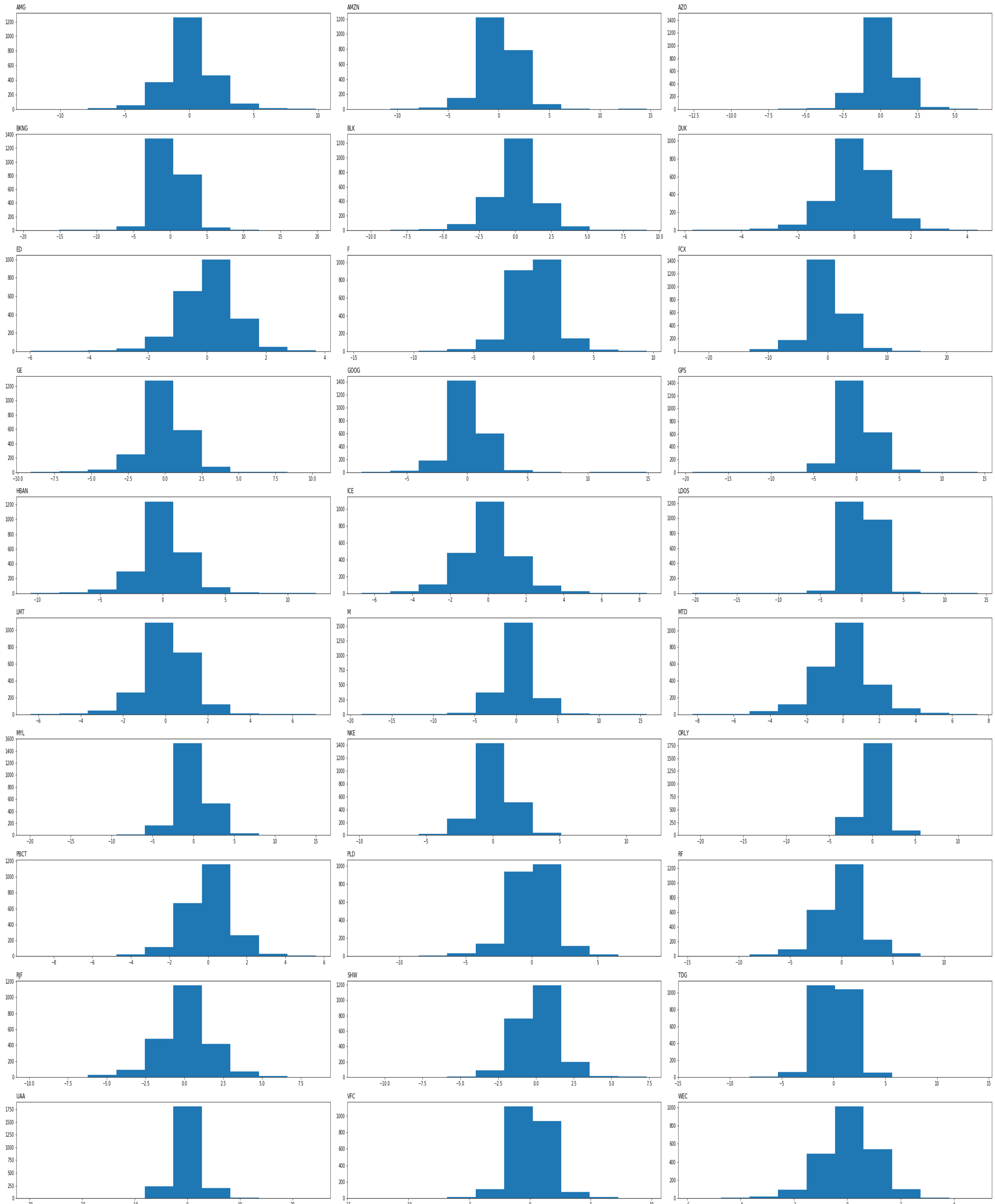

Exhibit-3: Histograms

Exhibit-4: The Results of the Normality Test

Companies	p-values	Companies	p-values
AMG	4.77e-47	LMT	8.85e-50
AMZN	1.159e-74	M	4.32e-137
AZO	1.9e-173	MTD	5.79e-45
BKNG	3.37e-105	MYL	2.43e-91
BLK	5.69e-49	NKE	6.74e-99
DUK	1.77e-45	ORL	6.76e-213
ED	2.41e-62	PBCT	2.54e-66
F	3.49e-64	PLD	4.50e-69
FCX	9.75e-67	RF	9.99e-53
GE	2.13e-54	RJF	6.16e-51
GOOG	1.58e-119	SHW	1.21e-77
GPS	1.10e-183	TGD	6.05e-99
HBAN	2.02e-45	UAA	2.59e-129
ICE	2.97e-30	VFC	1.23e-108
LDOS	1.35e-251	WEC	9.26e-49

Exhibit-5: Stationarity Test

Companies	p-values	Companies	p-values
AMG	2.66e-17	LMT	0.0000
AMZN	0.0000	M	0.0000
AZO	0.0000	MTD	0.0000
BKNG	0.0000	MYL	0.0000
BLK	0.0000	NKE	0.0000
DUK	0.0000	ORLY	0.0000
ED	0.0000	PBCT	0.0000
F	0.0000	PLD	0.0000
FCX	4.21e-16	RF	0.0000
GE	4.66e-13	RJF	4.42e-21
GOOG	0.0000	SHW	0.0000
GPS	0.0000	TDG	6.45e-22
HBAN	1.08e-17	UAA	0.0000
ICE	0.0000	VFC	0.000000e+0
LDOS	0.0000	WEC	0.0000

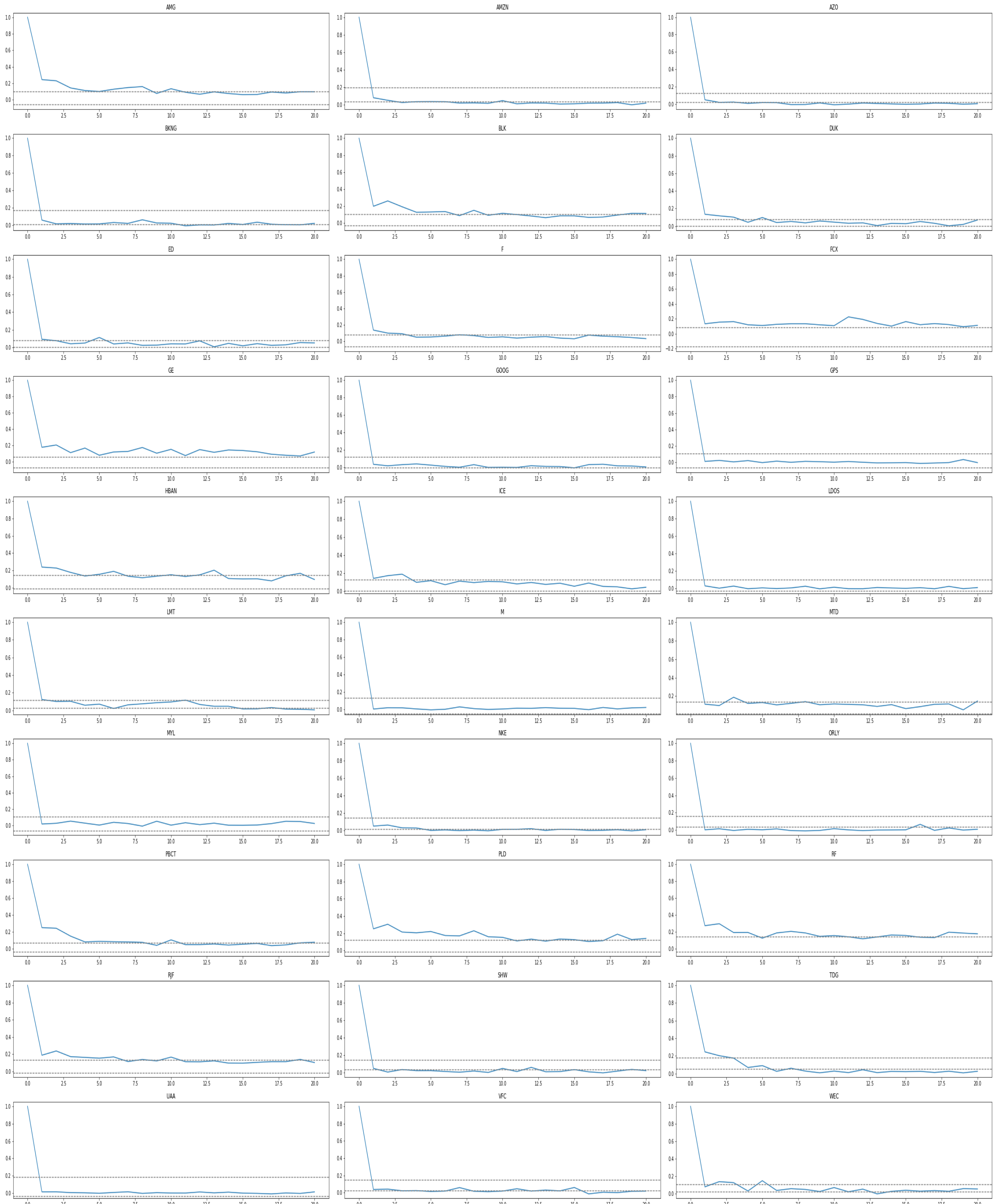

Exhibit-6: Autocorrelation Function

Volatility Prediction Assessment

In this part of the study, volatility estimation via proposed models are conducted and these results are used as an input in the Value-at-Risk estimation. The proposed volatility models used in the out of sample evaluation are GARCH, GJR-GARCH, EGARCH, FIGARCH, and SVR-GARCH. Aside from SVR-GARCH, normal distribution, student-t and skewed distributions are employed in the model. In the SVR-GARCH, linear, Gaussian, and Polynomial extensions are utilized.

Data used are extracted from yahoo finance for the period of 01/01/2010-09/01/2019. The returns are calculated by using closing price and 90% of the total reserved for training data and the remaining 10%, corresponding to 253 data points, belongs to test data. According to the performance metrics of MAE and RMSE, SVR-GARCH model produce the best results compared to other traditional volatility models. To be interpret, SVR-GARCH-linear model has an MAE of 0.009 which is nearly one-fifth of the remaining traditional models. For instance, GARCH, GJR-GARCH, EGARCH, and FIGARCH with normal distribution have an MAE of 0.0052, 0.0054, 0.0057, and 0.0057, respectively. Similarly, SVR-GARCH-linear and EGARCH have the lowest and highest RMSE, respectively.

Besides, the second and third best models in terms of performance metrics is SVR-GARCH-RBF and SVR-GARCH-Polynomial. These findings highlights that SVR-GARCH model outperforms the GARCH models at every confidence level. Visualization based on these findings can be found in the Appendix .

Exhibit-7: Out-of-Sample Evaluation

Models	MAE	RMSE
SVR-GARCH-linear	0.0009	0.0013
SVR-GARCH-RBF	0.0013	0.0025
SVR-GARCH-Polynomial	0.0014	0.0029
GARCH-Normal	0.0052	0.0068
GARCH-Student t	0.0052	0.0068
GARCH-Skewed	0.0052	0.0068
GJR-GARCH-Normal	0.0054	0.0070
GJR-GARCH-Student t	0.0054	0.0070
GJR-GARCH-Skewed	0.0053	0.0070
EGARCH-Normal	0.0057	0.0075
EGARCH-Student t	0.0053	0.0071
EGARCH-Skewed	0.0053	0.0071
FIGARCH-Normal	0.0057	0.0075
FIGARCH-Student t	0.0053	0.0071
FIGARCH-Skewed	0.0053	0.0071

Risk Management

Risk management is an indispensable of the financial management in that it makes it possible to properly price the assets and helps instutions prepare the unexpected. In financial institutions, risk management is conducted by these four risks types: Market risk, liquidity risk, credit risk and operational risk.

In recent years, financial globalization, subsequent increase in competition, diversity in financial products and increasing transaction volumes, have led institutions to form a complex and interconnected financial system. In such an environment, especially financial institutions have become more open to the risks arising from price movements in the market. To this end, institutions have intensified their efforts to identify, measure, control and update their risks and have been in search of an effective risk management system.

Thus, these developments have even increased the importance of risk management and its tools. Over decades, financial risk measurements process which started with basic risk measurements, continued with the development of an internal model for the measurement of risks by many financial institutions. In a higly evolving and complex risk environment, the fact that institutions are exposed to diverse and intensified risks has led regulatory authorities to establish a standard and intuitive methodology. JP Morgan address this need introducing the Riskmetrics system that enables to calculate of Value at Risk (VaR) in 1994.

Value-at-Risk Application

VaR is defined as a financial instrument that measures the expected loss and unexpected loss that can occur in a given time period in a given confidence interval as shown in the Figure [**?**] [21]. Value at risk method can be measured on a securities basis as well as on a portfolio basis. Risks arising from different positions and risk factors may occur in the portfolio and it is calculated in terms of monetary basis.

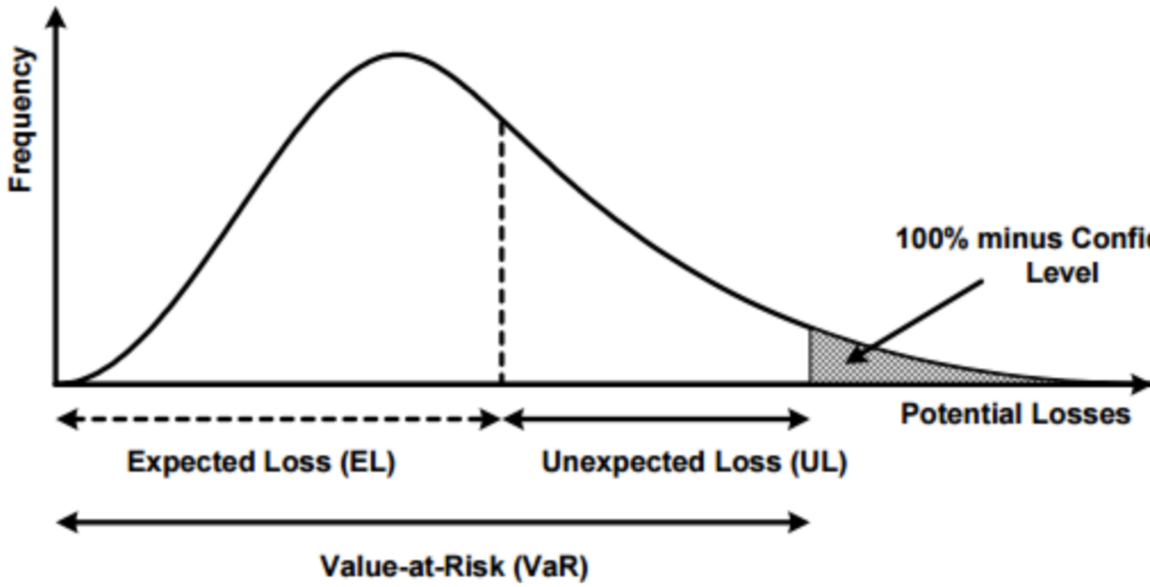

Exhibit-8: Value-at-Risk Representation

VaR can be computed as follows:

$$\text{VaR} = V_t * \sigma * \sqrt{t} * \alpha \tag{28}$$

whereV_t is the value of the portfolio, σ is the standard devation of portfolio, t is the holding period, α is the confidence interval. In this regard, it can be concluded that in estimation the VaR, four parameters are important which are:

- Portfolio Diversification
- Distribution of the portfolio returns
- Holding time period
- Confidence interval

Differently, the standard VaR definition is [4]:

$$\mathbb{P}(P(T) - P(t) < -VaR_{\alpha}^{P/L}(t,T)) = 1 - \alpha \tag{29}$$

where P(T) represents the price of the portfolio. Using CDF of log-returns, it turns out:

$$\mathbb{P}(r(t,T) < -VaR_{\alpha}^{r}(t,T)) = 1 - \alpha \tag{30}$$

Given that $P(T) = P(t)e^{r(t,T)}$, VaR^r and $\text{VaR}^{P/L}$ can be redefined as:

$$\text{VaR}^r_\alpha(t,T)) = -\ln(1 - \frac{\text{VaR}^{P/L}_\alpha(t,T)}{P(t)}) \tag{31}$$

$$\text{VaR}^{P/L}_\alpha(t,T)) = P(t)(1 - e^{-\text{VaR}^r_\alpha(t,T)}) \tag{32}$$

Thus,

$$\text{VaR}^{P/L}_\alpha(t,T) \approx P(t)\text{VaR}^r_\alpha(t,T) \tag{33}$$

In this thesis, a parametric method of VaR called Variance-Covariance VaR methods is used. Using this method, VaR is calculated by multiplying the significance level, α value corresponding to the confidence level and the standard deviation (σ) by the market value (M) of the portfolio. This method is advantageous in terms of ease of calculation and computational efficiency compared to other methods.

Backtesting

Several models have been developed to calculate VaR estimates. Due to the high diversity of the VaR models, it is of considerable importance to test the validity of the method applied. The main reason why VaR is questioned is the various shortcomings of VaR models. VaR models are only strong when they predict future in a proper way. Thus, it has become common practice to test the performance of VaR models called backtesting.

Backtesting is a statistical procedure where the deviations between the losses realized and the estimated losses during the backtesting process are calculated. For instance, if the confidence level set for calculating daily VaR is 90%, it is expected to occur ten exceptions in every 100 days on average. In addition, if the confidence level is 99%, one exception in every 100 days is expected on average. In a nutshell, in backtesting, it is statistically checked whether the frequency of exceptions over some specified time interval agrees with the related confidence level. This procedure is called as unconditional coverage tests [22]. Some well-known backtests can be listed as:

- Binomial test
- Traffic light test
- Kupiec's tests
- Christoffersen's tests
- Haas's tests

In this thesis, POF-Test (Proportion of Failures) proposed by [23] is applied and it measure whether the number of exceptions is in line with the confidence level. So, the test hypotheses can be stated as:

$$H_0 : p = \hat{p} = \frac{E}{T}$$

$$H_0 : p \neq \hat{p} = \frac{E}{T}$$

where E is the number of exception and T is the number of observations. This test assumes that the number of exceptions follows binomial distribution given below:

$$f(E) = \binom{T}{E} p^E (1-p)^{T-E} \tag{34}$$

Hence, POF-test tries to find out whether there is statistically significance between observed failure, $\hat{f}$, and failure rate proposed by confidence level and likelihood ratio (LR) test is employed to decide the correcteness of the model.

$$LR_{POF} = -2\ln\left[\frac{(1-p)^{T-E}p^E}{(1-\frac{E}{T})^{T-E}(\frac{E}{T})^E}\right] \tag{35}$$

LR test asymptotically follows χ^2 with one degree of freedom and model is true under null hypothesis of this test. The Table **??** shows the acceptance and rejection regions of the POF test.

Probability Level p	VaR Confidence Level	Nonrejection Region for Number of Failures N		
		T = 255 days	T = 510 days	T = 1000 days
0.01	99 %	N < 7	1 < N < 11	4 < N < 17
0.025	97.5 %	2 < N < 12	6 < N < 21	15 < N < 36
0.05	95 %	6 < N < 21	16 < N < 36	37 < N < 65
0.075	92.5 %	11 < N < 28	27 < N < 51	59 < N < 92
0.1	90 %	16 < N < 36	38 < N < 65	81 < N < 120

Exhibit-9: Acceptance-Rejection Regions for POF Test

Based on the number of fails obtained via POF test, it is possible to run Traffic Light backtest which is suggested by Basel Committee and this backtest is used as robustness test. According to the Traffic Light backtest, accuracy of a VaR forecast is assessed based on the number of VaR breaches using POF test. For example, according to the recommendations of the Basel Committee in Basel II, VaR values at 99% confidence level of the previous year are compared daily with actual losses. The Basel Committee tolerates up to 4 deviations per year and defines this level as a green zone. If the VaR violation exceeds 4 and remain lower than 10, it corresponds to yellow zone and finally violations at and over 10 defines red zone.

The VaR violation can be defined as follows:

$$E_{\mathrm{VaR}}^{i}(\alpha) := \mathbb{1}_{L_i \leq \mathrm{VaR}_i(\alpha)} = \begin{cases} 1 & \text{if } L_i \leq \mathrm{Var}_i(\alpha) \\ 0 & \text{if } L_i > \mathrm{Var}_i(\alpha) \end{cases} \tag{36}$$

where $E_{VaR}^{i}(\alpha) : [0,1] \to \{0,1\}$. Here, in this setup, X stores the violations happened within a trading day i. When it is generalized over the period under examination, it turns out [12]:

$$E_{\mathrm{VaR}}^{N}(\alpha) := \sum_{i=1}^{N} \mathbb{1}_{L_i \leq \mathrm{VaR}_i(\alpha)} \tag{37}$$

where $E_{VaR}^{N}(\alpha) : [0,1] \to \{0,1,2,3,...,N\}$. In addition, $\mathbb{E}[E_{\mathrm{VaR}}^{N}] = N\alpha$. At this point, it is necessary to introduce cumulative probability to count the number of violation over a defined period of time. For given α and N, the cumulative probability of having e number of violation can be defined as [12] :

$$\Phi_{\mathrm{VaR}}^{\alpha,N}(e) := \mathbb{P}(E_{\mathrm{VaR}}^{N}(\alpha) \leq e) \tag{38}$$

In summary, three color zones can be formulated via cumulative probability given below:

- Green zone if $\Phi_{\mathrm{VaR}}^{\alpha,N}(e) < 95\%$
- Yellow zone if $95\% < \Phi_{\mathrm{VaR}}^{\alpha,N}(e) < 99\%$
- Red zone if $\Phi_{\mathrm{VaR}}^{\alpha,N}(e) < 99.99\%$

Figure **??** shows these zone and number of corresponding violations [12]. Based on these violation, the performance of the VaR application and the corresponding volatility method is decided. Accordingly, if the number of VaR violations is less than 5, then backtesting suggest that VaR application performs well. If the VaR violations are greater than 4 and less than 10, it implies that the VaR approach should be treated with caution. Eventually, when the VaR violation is greater then 10, it implies that VaR method does not working well.

Basel Traffic Light Approach to VaR		
Zone	Breach Value	Cumulative Probability
Green	0	8.11 %
	1	28.58 %
	2	54.32 %
	3	75.81 %
	4	89.22 %
Yellow	5	95.88 %
	6	98.63 %
	7	99.60 %
	8	99.89 %
	9	99.97 %
Red	more than 10	99.99 %

Exhibit-10: Basel Traffic Light Approach

In the light of these two approaches, Kupiec's POF test and Basel Traffic Light, the validity of the VaR result incorporating volatility estimated via different models are discussed in the next part. First, failure rate and total number of violation are compared and then these violations are assessed considering Basel's Traffic Light approach.

Interpreting the Backtesting Result

After calculating VaR based on variance-covariance method, Kupiec's POF backtesting and Basel's Traffic Light Approach are embraced so that it is possible to compare the performance of the VaR application which varies depending on the volatility model incorporated.

Table **??** shows the POF test and the corresponding number of VaR violations based on the models used. To save space, only the name of the volatility models are kept in the Table **??**. As is known, the hypothesis testing of the POF test is conducted by LR test which can be found in Appendix .

Table **??** reveals that VaR applications based on the SVR-GARCH models outperforms those with traditional models. To be interpret, when SVR-GARCH models are incorporated into the VaR application as a volatility model, it turns out that failure rate and corresponding number of violation significantly diminish compared to the traditional models. LR test also suggests that null hypotheses empirically determined probability ($\hat{p}$)is equal to the expected probability (p) for every single stocks included are accepted at 1% level. This observation confirms the fact that Machine Learning models not solely increase the volatility prediction performance but also provide more consistent and reliable risk management application.

As for the VaR violation of traditional models, even if VaR estimation with FIGARCH is appeared to have better performance, LR test result of 10.17 in skewed and Student-t distributions tell that empirical probability is not equal to expected one. However, LR test suggests accepting the null hypothesis for the rest of the models at 1% level. In this case, Based on the failure rate, VaR application with EGARCH-Student-t distribution and with EGARCH-skewed distribution are second best performing model.

Exhibit-11: Failure Test and Number of Violations

Volatility Models	Failure Rate	Total Number of Violations
SVR-GARCH-linear	0.0000	0
SVR-GARCH-RBF	0.0027	21
SVR-GARCH-Polynomial	0.0026 20	
GARCH-Normal	0.0111	85
GARCH-Student t	0.0111	85
GARCH-Skewed	0.0114	87
GJR-GARCH-Normal	0.0090	69
GJR-GARCH-Student t	0.0125	95
GJR-GARCH-Skewed	0.0122	93
EGARCH-Normal	0.0092	70
EGARCH-Student t	0.0084	64
EGARCH-Skewed	0.0086	66
FIGARCH-Normal	0.0092	70
FIGARCH-Student t	0.0000	0
FIGARCH-Skewed	0.0000	0

Table **??** indicates the VaR performance based on the Regulatory Framework which is called Basel Traffic Light Approach. According to this approach, if a company falls into a green zone, it is highly unlikely that inaccurate model is accepted. Again, results imply that VaR application with SVR-GARCH volatility performs much better performance that those with traditional models. Because, 21 VaR violations of the total 30 stocks fall into green zone whereas there is no other VaR violation falling into other categories, namely yellow and red.

Conversely, there are violations both in the green and yellow zones in the rest of the models. For instance, VaR violations in the green zone with GARCH-normal, Student-t, and Skewed distrubutions are 77, 76, and 80, respectively and the violations in the yellow zone of the some models are 8, 9, and 7, respectively. Similarly VaR violation in the green zone when GJR-GARCH models are applied as volatility model are 65, 86, and 84, respectively and violations in the yellow zone are 4, 9, and 90, respectively. Based on the results, there is no VaR violations falling into the red zone.

Thus, considering the VaR violations falling into the yellow categories, Basel Committee would suggest monitoring and the model is considered to be more inaccurate than being accurate and moreover VaR measures need to be more heavily weighted in calculating required capital. Even if there is no violation falling into the red category, it provides a clear signal to the Regulatory Committe that the model is not working well and needs to be improved and investigated.

Exhibit-12: Assesing the Violations Based on Basel Traffic Light Approach

Models	Violations in Green Zone	Violations in Yellow Zone	Violations in Red Zone
SVR-GARCH-linear	0	0	0
SVR-GARCH-RBF	21	0	0
SVR-GARCH-Polynomial	20	0	0
GARCH-Normal	77	8	0
GARCH-Student t	76	9	0
GARCH-Skewed	80	7	0
GJR-GARCH-Normal	65	4	0
GJR-GARCH-Student t	86	9	0
GJR-GARCH-Skewed	84	9	0
EGARCH-Normal	67	3	0
EGARCH-Student t	63	1	0
EGARCH-Skewed	64	2	0
FIGARCH-Normal	67	3	0
FIGARCH-Student t	0	0	0
FIGARCH-Skewed	0	0	0

Conclusion

As the volatility is a phenomenon that partly explains the value of financial assets, it is an indispensable part of pricing and risk management. However, it is equally important to have prediction as accurate as possible. This study is an attempt to improve the volatility prediction technique so that more accurate volatility prediction and forecasting can be incorporated which in turn enable researchers/practitioners conduct solid risk management as well as value an asset.

To do that SVR-GARCH approach is introduced as a Machine Learning algorithm and compare its performance with the traditional GARCH-type volatility models, namely GARCH, GJR-GARCH, EGARCH, and FIGARCH. Results suggest that SVR-GARCH outperforms the traditional models in terms of RMSE and MAE performance metrics.

Then, volatility prediction obtained from the above-given models are incorporated into the Value-at-Risk model which is a frequently used financial risk management tool. As a VaR model, variance-covariance method is applied and the prediction obtained from volatility models are replaced by standard deviation of return. As a final step, in order to check the validity of the VaR results backtesting is conducted. Kupiec's POF test and Basel Traffic Light Approach are the test applied as backtesting. Again, the findings suggest that VaR applications with SVR-GARCH volatility model performs much better than those with traditional models in that failure rate of SVR-GARCH is quite lower and the VaR violations fall into the green zone in Basel's Traffic Light Approach.

Thus, this study hightlights the importance of application of Machine Learning model to further increase the performance of the volatility prediction as well as risk management. As the volatility gives insight about many phenomena in the field of finance, it is of considerable importance to have sound volatility prediction and the method introduced here is an attempt to accomplish this task.

Appendix

Visualization of Prediction Results

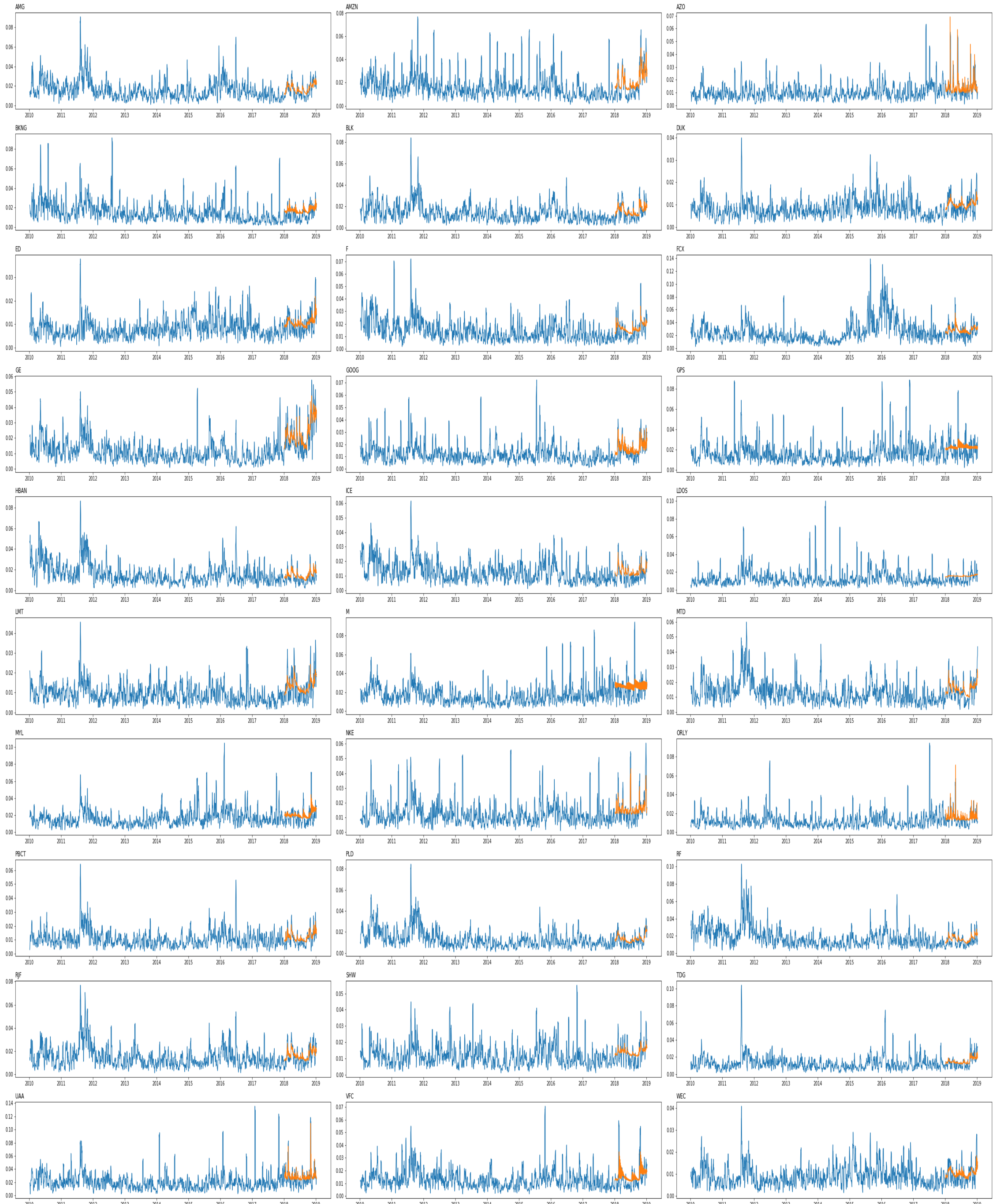

Exhibit-13: GARCH-Normal Prediction Results

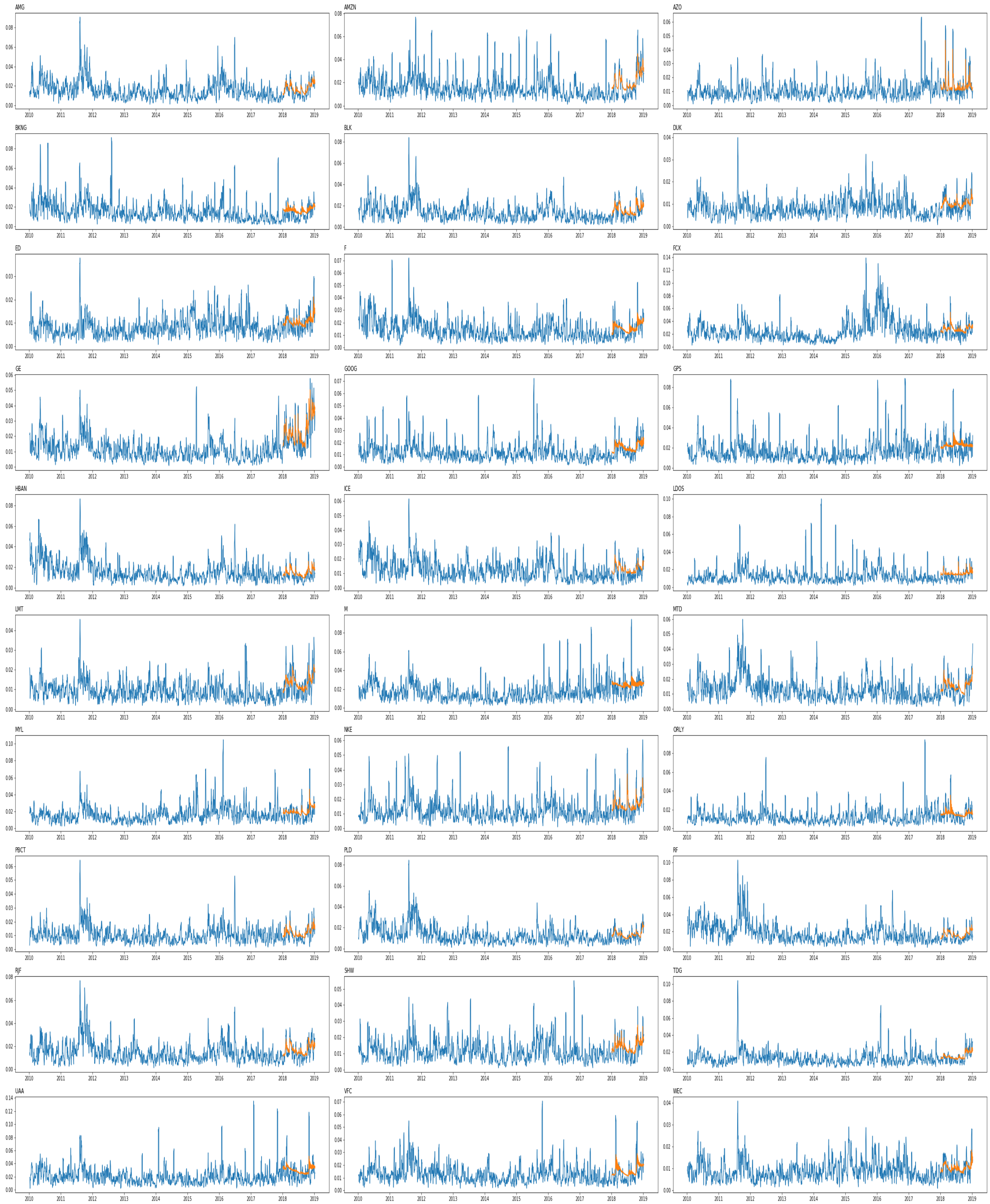

Exhibit-14: GARCH-Student t Prediction Results

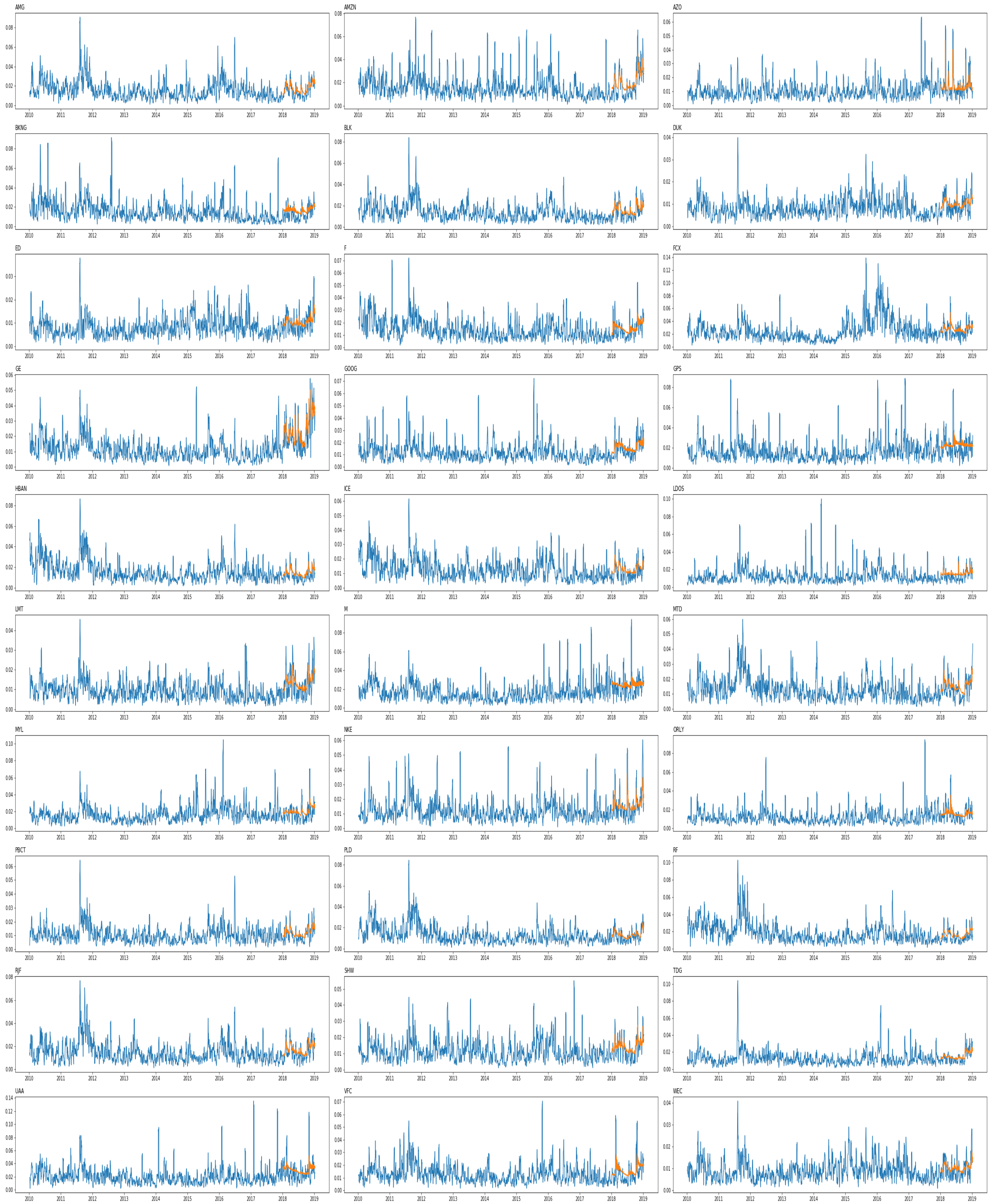

Exhibit-15: GARCH-Skewed Prediction Results

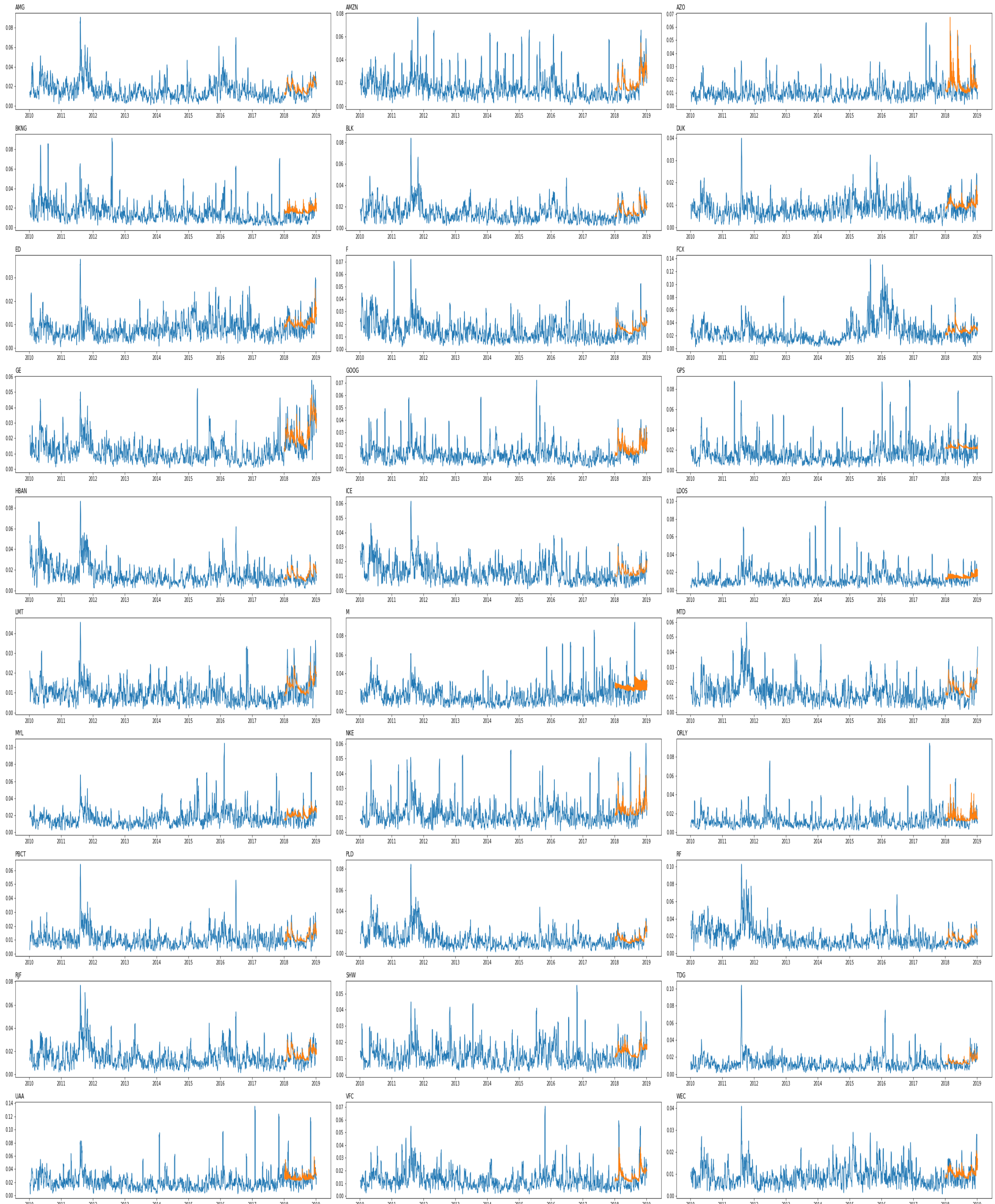

Exhibit-16: GJR-GARCH-Normal Prediction Results

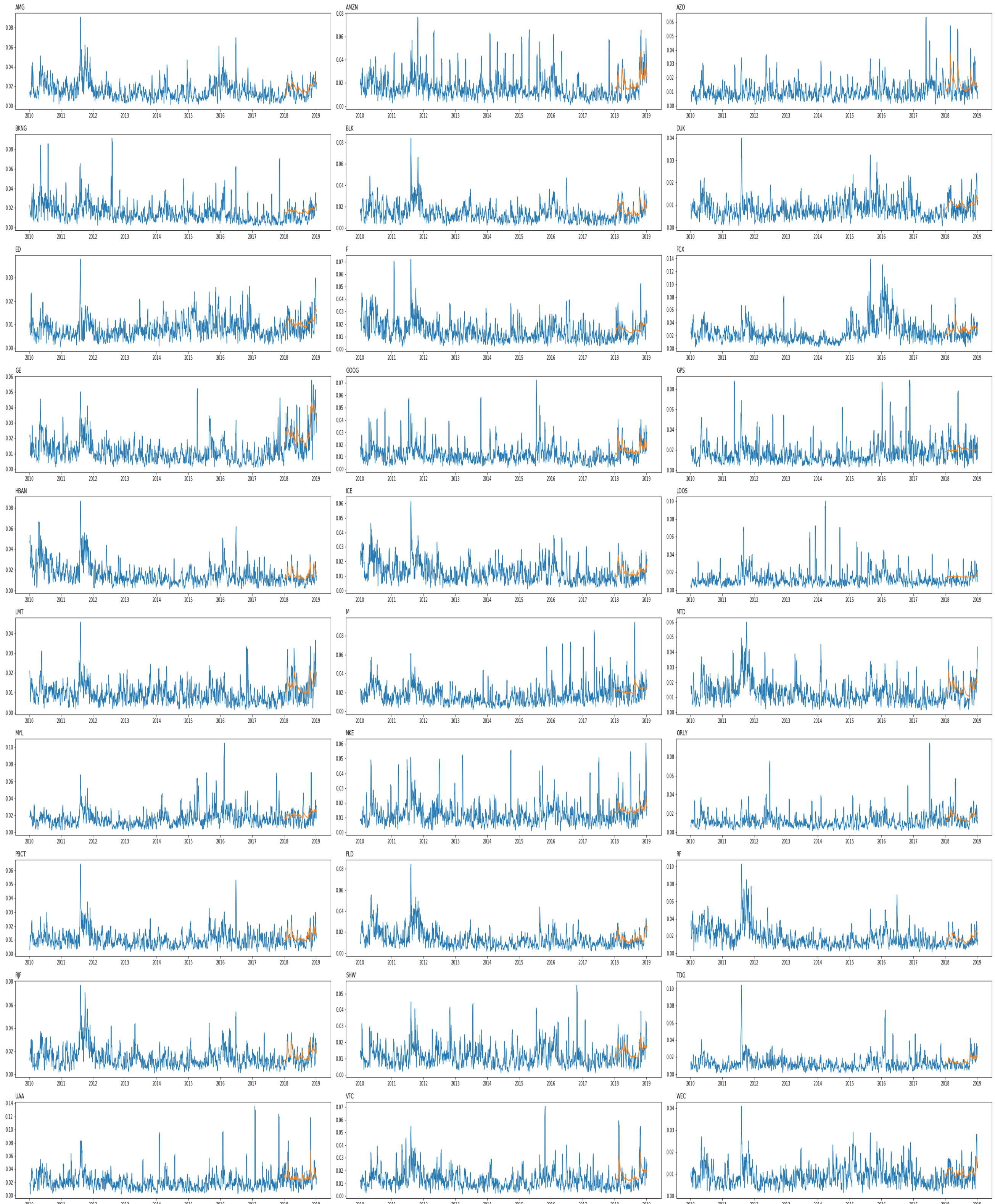

Exhibit-17: GJR-GARCH-Student t Prediction Results

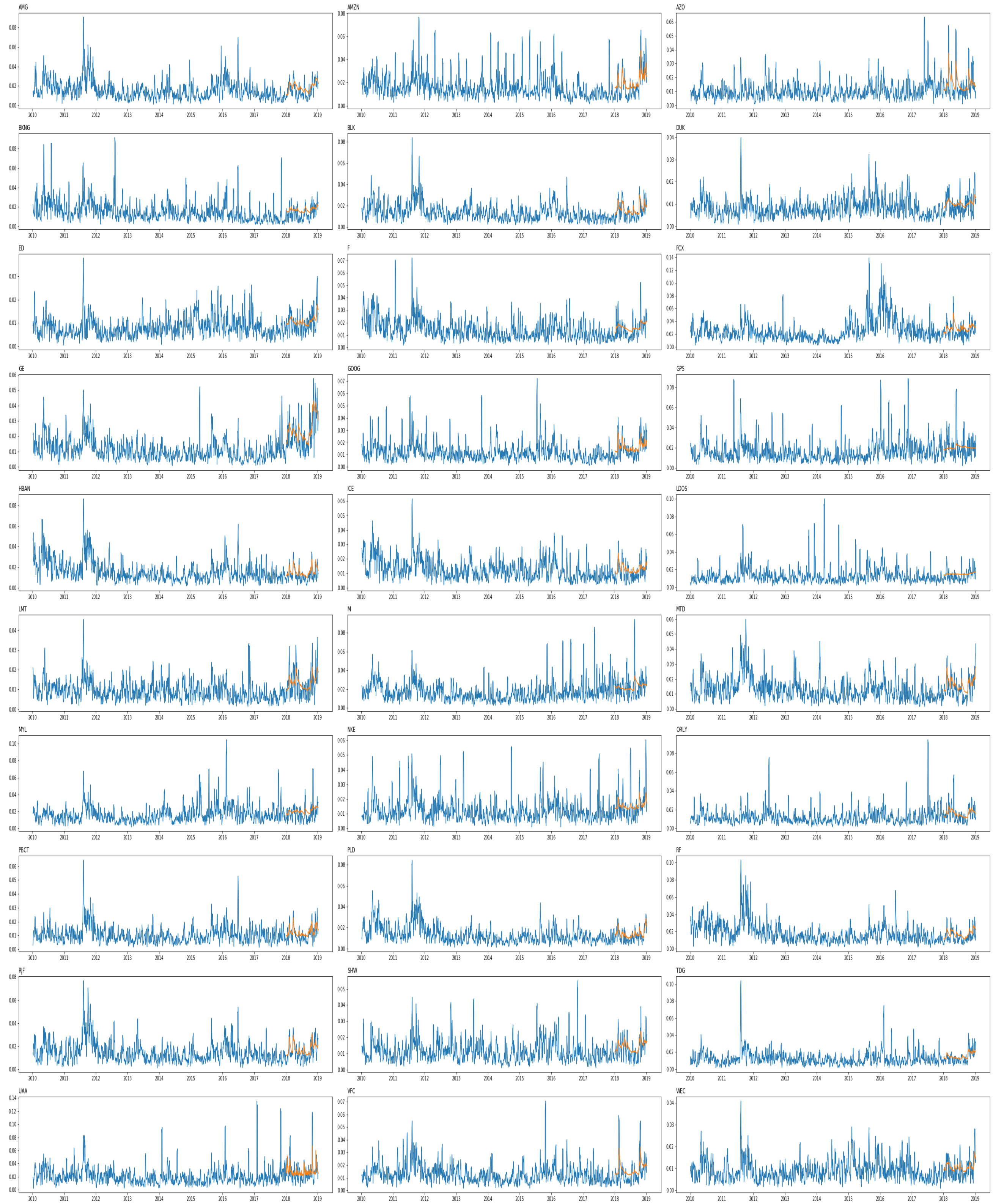

Exhibit-18: GJR-GARCH-Skewed Prediction Results

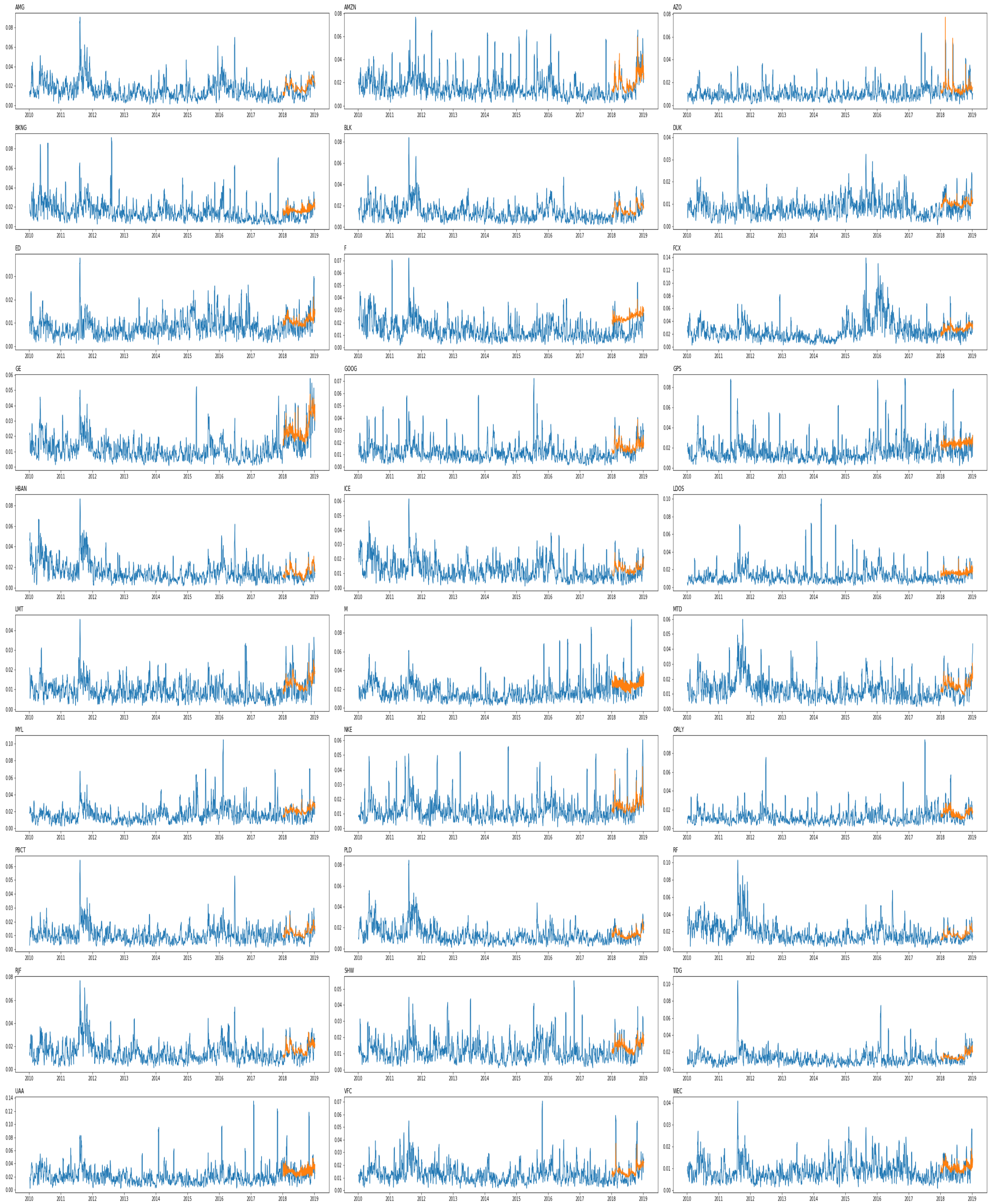

Exhibit-19: EGARCH-Normal Prediction Results

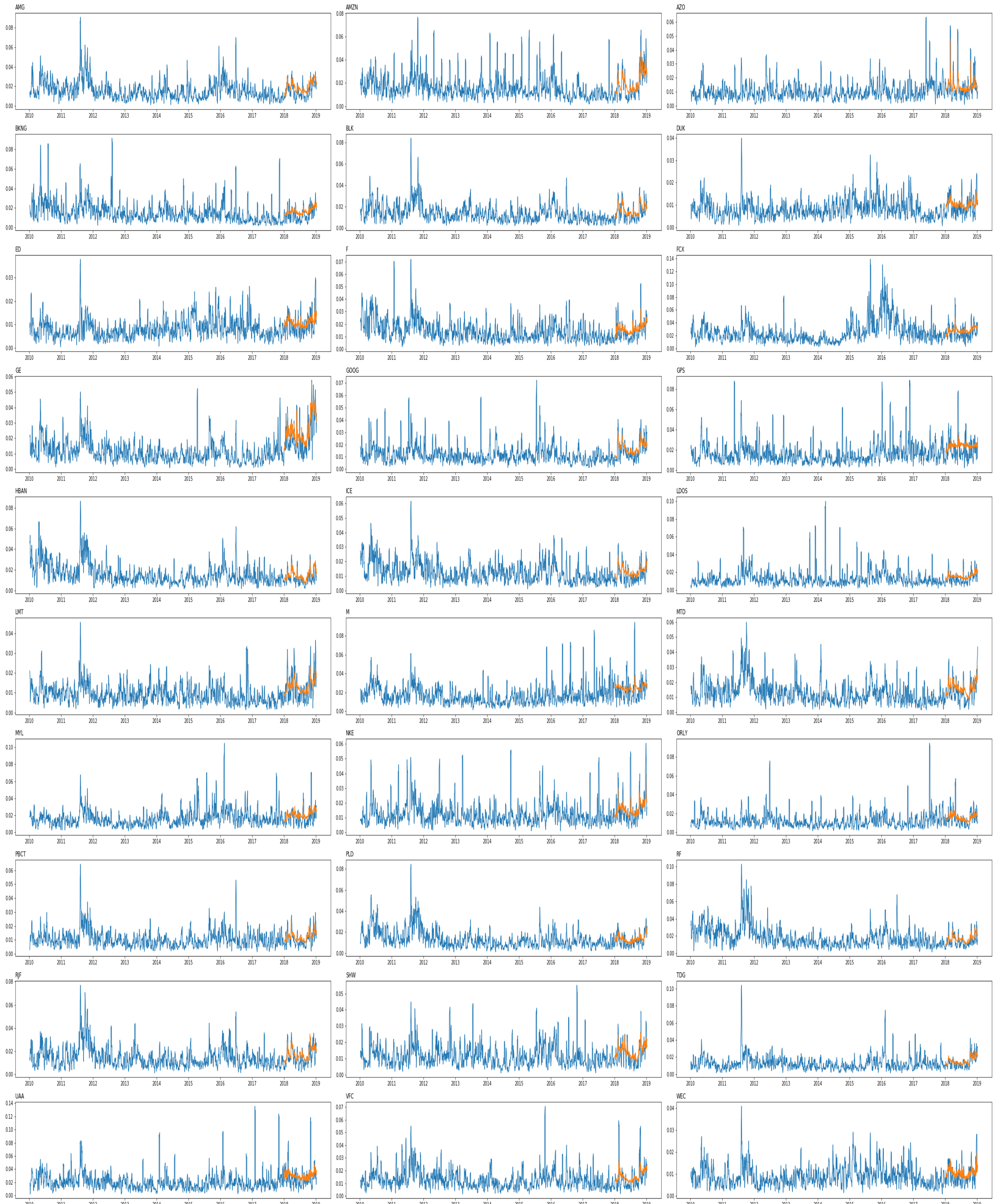

Exhibit-20: EGARCH-Student t Prediction Results

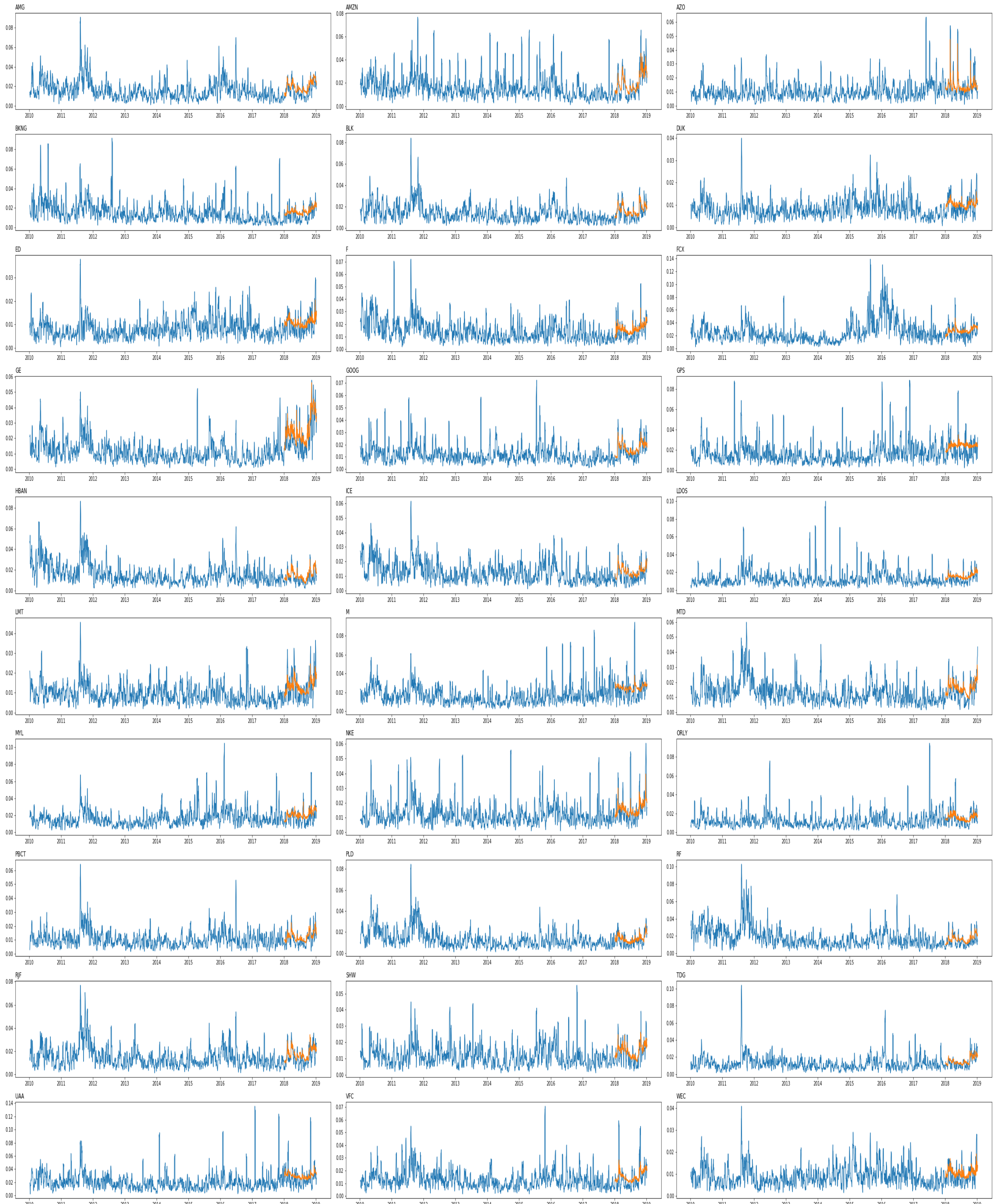

Exhibit-21: EGARCH-Skewed Prediction Results

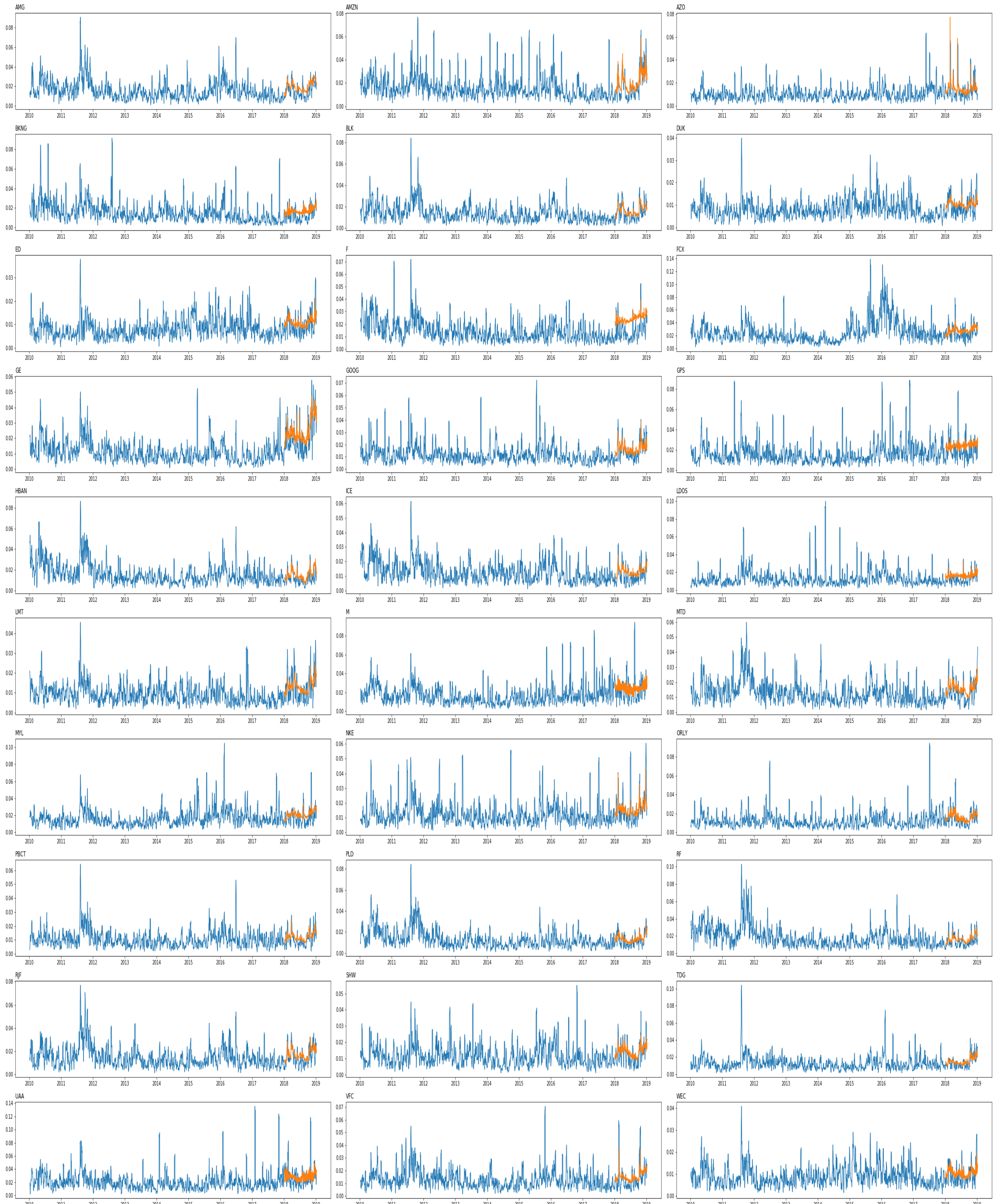

Exhibit-22: FIGARCH-Normal Prediction Results

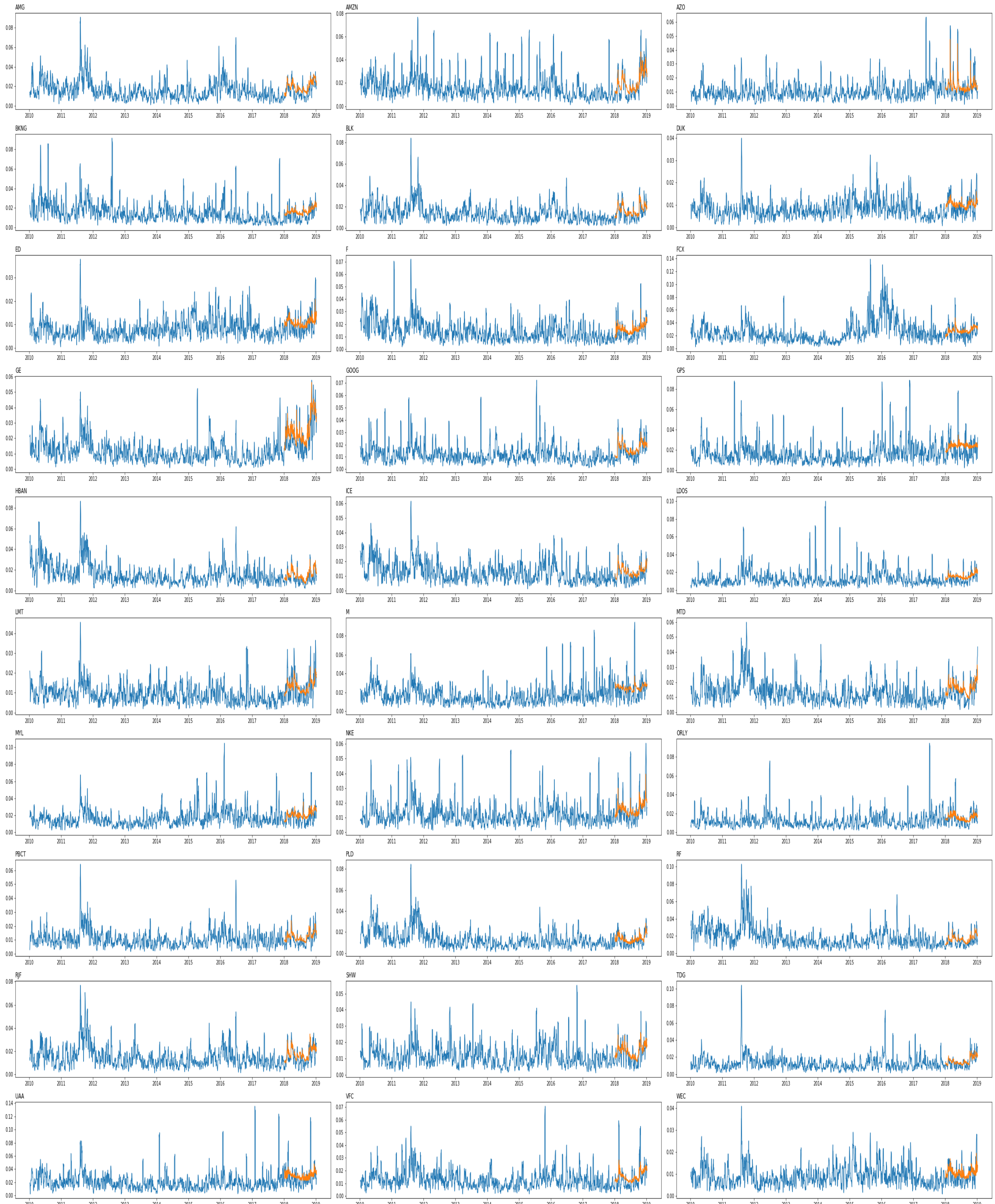

Exhibit-23: FIGARCH-Student t Prediction Results

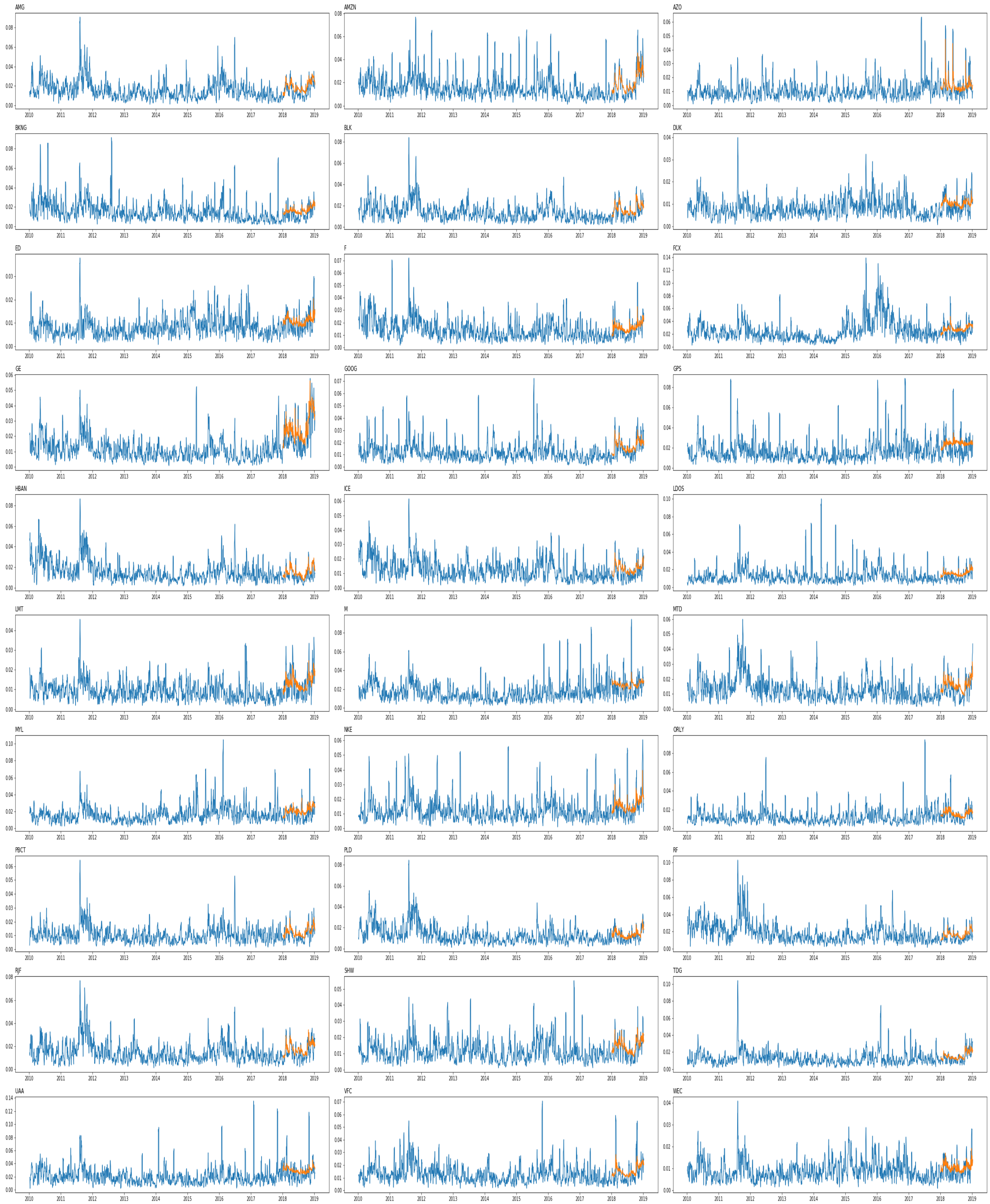

Exhibit-24: FIGARCH-Skewed Prediction Results

Exhibit-25: SVR-GARCH-Linear Prediction Results

Exhibit-26: SVR-GARCH-RBF Prediction Results

POF LR Test Results

Exhibit-27: LR Test Result for GARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	5.085470	5.085470	5.085470
1	AMZN	5.085470	1.212888	1.212888
2	AZO	5.085470	0.083240	0.083240
3	BKNG	0.733245	1.212888	1.212888
4	BLK	0.083240	0.083240	0.083240
5	DUK	0.083240	0.083240	0.083240
6	ED	0.733245	0.733245	0.733245
7	F	0.120832	1.896624	3.470779
8	FCX	1.896624	1.896624	1.896624
9	GE	5.085470	5.085470	5.085470
10	GOOG	3.470779	3.470779	3.470779
11	GPS	1.896624	1.896624	0.733245
12	HBAN	5.085470	5.085470	5.085470
13	ICE	5.085470	1.212888	1.212888
14	LDOS	0.733245	5.085470	5.085470
15	LMT	0.083240	1.896624	0.733245
16	M	0.083240	0.083240	0.083240
17	MTD	3.470779	1.896624	1.896624
18	MYL	0.083240	0.120832	0.120832
19	NKE	1.212888	0.083240	0.083240
20	ORLY	1.896624	1.896624	1.896624
21	PBCT	1.896624	0.083240	0.083240
22	PLD	5.085470	5.085470	0.120832
23	RF	1.896624	1.896624	1.896624
24	RJF	0.083240	0.083240	0.733245
25	SHW	0.083240	5.085470	5.085470
26	TDG	1.212888	1.212888	1.212888
27	UAA	0.120832	0.733245	0.733245
28	VFC	0.733245	0.733245	0.733245
29	WEC	1.896624	1.896624	1.896624

Exhibit-28: LR Test Result for GJR-GARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	1.212888	5.085470	5.085470
1	AMZN	1.212888	5.085470	5.085470
2	AZO	5.085470	0.120832	0.120832
3	BKNG	3.470779	0.120832	0.120832
4	BLK	0.083240	0.083240	0.083240
5	DUK	0.083240	0.083240	0.083240
6	ED	0.083240	0.733245	0.733245
7	F	1.212888	0.083240	0.083240
8	FCX	3.470779	0.733245	1.896624
9	GE	1.212888	5.085470	5.085470
10	GOOG	0.733245	3.470779	3.470779
11	GPS	1.896624	0.733245	0.083240
12	HBAN	5.085470	5.085470	5.085470
13	ICE	5.085470	5.085470	5.085470
14	LDOS	1.212888	0.120832	0.120832
15	LMT	0.083240	0.733245	0.733245
16	M	0.733245	0.120832	0.120832
17	MTD	0.083240	0.083240	0.083240
18	MYL	1.212888	5.085470	5.085470
19	NKE	1.212888	1.212888	1.212888
20	ORLY	0.733245	0.083240	0.083240
21	PBCT	0.120832	1.212888	0.083240
22	PLD	5.085470	5.085470	5.085470
23	RF	0.083240	0.733245	0.733245
24	RJF	0.733245	0.733245	0.733245
25	SHW	1.212888	5.085470	5.085470
26	TDG	5.085470	5.085470	5.085470
27	UAA	1.212888	1.212888	1.212888
28	VFC	0.733245	0.733245	0.733245
29	WEC	0.733245	0.733245	0.733245

Exhibit-29: LR Test Result for EGARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	5.085470	5.085470	5.085470
1	AMZN	5.085470	5.085470	5.085470
2	AZO	0.083240	1.896624	1.896624
3	BKNG	0.120832	0.083240	0.083240
4	BLK	0.083240	0.083240	0.083240
5	DUK	0.083240	0.083240	0.083240
6	ED	1.896624	3.470779	3.470779
7	F	0.733245	3.470779	3.470779
8	FCX	1.896624	1.896624	1.896624
9	GE	1.212888	0.120832	0.120832
10	GOOG	1.212888	0.733245	0.733245
11	GPS	0.083240	7.599894	7.599894
12	HBAN	5.085470	5.085470	5.085470
13	ICE	5.085470	5.085470	5.085470
14	LDOS	0.733245	0.733245	0.733245
15	LMT	1.896624	3.470779	3.470779
16	M	0.120832	1.896624	1.896624
17	MTD	0.083240	0.083240	0.083240
18	MYL	0.120832	1.896624	1.896624
19	NKE	5.085470	0.733245	0.733245
20	ORLY	1.212888	0.083240	0.083240
21	PBCT	1.212888	5.085470	5.085470
22	PLD	5.085470	5.085470	5.085470
23	RF	0.733245	1.896624	1.896624
24	RJF	0.120832	0.120832	0.120832
25	SHW	0.733245	0.120832	0.120832
26	TDG	5.085470	1.212888	1.212888
27	UAA	0.120832	0.120832	5.085470
28	VFC	0.733245	0.733245	0.733245
29	WEC	1.896624	0.733245	0.733245

Exhibit-30: LR Test Result for FIGARCH

	Companies	Normal Dist.	Student- t Dist.	Skewed Dist.
0	AMG	1.212888	10.17094	10.17094
1	AMZN	1.212888	10.17094	10.17094
2	AZO	5.085470	10.17094	10.17094
3	BKNG	3.470779	10.17094	10.17094
4	BLK	0.083240	10.17094	10.17094
5	DUK	0.083240	10.17094	10.17094
6	ED	0.083240	10.17094	10.17094
7	F	1.212888	10.17094	10.17094
8	FCX	3.470779	10.17094	10.17094
9	GE	1.212888	10.17094	10.17094
10	GOOG	0.733245	10.17094	10.17094
11	GPS	1.896624	10.17094	10.17094
12	HBAN	5.085470	10.17094	10.17094
13	ICE	5.085470	10.17094	10.17094
14	LDOS	1.212888	10.17094	10.17094
15	LMT	0.083240	10.17094	10.17094
16	M	0.733245	10.17094	10.17094
17	MTD	0.083240	10.17094	10.17094
18	MYL	1.212888	10.17094	10.17094
19	NKE	1.212888	10.17094	10.17094
20	ORLY	0.733245	10.17094	10.17094
21	PBCT	0.120832	10.17094	10.17094
22	PLD	5.085470	10.17094	10.17094
23	RF	0.083240	10.17094	10.17094
24	RJF	0.733245	10.17094	10.17094
25	SHW	1.212888	10.17094	10.17094
26	TDG	5.085470	10.17094	10.17094
27	UAA	1.212888	10.17094	10.17094
28	VFC	0.733245	10.17094	10.17094
29	WEC	0.733245	10.17094	10.17094

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