Assignment 1

RRC summer sessions 2019

May 10, 2019

1 Instructions

- Programming questions in this assignment will provide base for other assignments during the sessions.
- Python or MATLAB are preferable languages.
- Try each question. Discuss with other group-mates or mentors in case of doubts.
- Set a reasonable deadline (2days) to finish the assignment.

2 Linear Algebra

1. Consider the vector space of set of all $n \times n$ matrices:

$$V = A \in \mathbb{R}^{\mathrm{n \times n}}$$

Show that

$$trace(A) = \Sigma_{i=1}^{n} A_{ii}$$

defines an inner product

$$\langle A, B \rangle = trace(B^T A)$$

on V. Here A_{ii} denote the entries on the diagonal of A.

2. Implement RQ decomposition using given's rotation for a matrix A, Where

$$A = RQ$$

R is a upper triangular matrix and Q is a orthogonal matrix.

3 Calculus and Optimization

1. Find the gradient and hessian of the following functions.

   (a)
   $$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = x^3 + 2y$$

   (b)
   $$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = sin(x) + ylog(x)$$

2. In machine learning, we come across different cost functions while training classifiers. In this question we will look at a commonly used classifier known as logistic regression. In logistic regression, the cost is

$$J(\theta) = \frac{1}{m}\Sigma_{i=1}^{m}[-y^{(i)}log(h_\theta(x^{(i)}) - (1-y^{(i)})log(1-h_\theta(x^{(i)}))],$$

Where,

$$h_\theta(x) = g(\theta^T x), g(z) = \frac{1}{1+exp(-z)}, z = \theta^T x \tag{1}$$

also, here $x^{(i)}$ is the i^{th} row of a $m \times n$ matrix, and $y^{(i)}$ is the label for i^{th} entry of label vector y, which is of length m. Here X is called the data, and y is called the label. Here θ is an unknown vector, and it is of length n . Now our goal is to compute the gradient of J with respect t0 θ.

3. Using contour, plot at least 10 level curves of the function $x^2 - y^2$ on the rectangle region $R = \{(x, y)| -3 \leq x \leq 3, -3 \leq y \leq 3\}$. Add a plot of the gradient vectors to the plot of the level curves.

4. Write down $E = ||Ax - b||^2$ as a sum of four squares. Find the derivative equations $\frac{\partial E}{\partial C} = 0$ and $\frac{\partial E}{\partial D} = 0$. Now derive the normal equations $A^T Ax = A^T b$. What do you notice from former equations and normal equations.

$$\begin{aligned} C &= 0 \\ C + D &= 8 \\ C + 3D &= 8 \\ C + 4D &= 20 \end{aligned}$$

5. Solve the unconstrained and constrained problems given below.

   (a)
$$\min_{x,y} \quad 3x^2 \ + 2y^2 \ \ + x \ + 4y \tag{2}$$

   (b)
$$\begin{aligned} &\min_{x,y} && 3x^2 \ + 2y^2 \ \ + x \ + 4y \\ &\text{subject to} && x^2 + y^2 = 1 \end{aligned} \tag{3}$$

4 Rigid body transformations

1. A frame {B} is located initially coincident with a frame {A}. We rotate {B} about $\hat{Z}_B$ by 30 degrees, and then we rotate the resulting frame about $\hat{X}_B$ by 45 degrees. Give the rotation matrix that will change the description from ${}^B P$ to ${}^A P$.

2. A vector must be mapped through three rotation matrices:

$${}^A P = {}^A_B R \, {}^B_C R \, {}^C_D R \, {}^D P$$

   One choice is to first multiply the three rotation matrices together, to form ${}^A_D R$ in the expression

$${}^A P = {}^A_D R \, {}^D P$$

Another choice is to transform the vector through the matrices one at a time—that is,

$$
\begin{aligned}
{}^{A}P &= {}^{A}_{B}R\,{}^{B}_{C}R\,{}^{C}_{D}R\,{}^{D}P \\
{}^{A}P &= {}^{A}_{B}R\,{}^{B}_{C}R\,{}^{C}P \\
{}^{A}P &= {}^{A}_{B}R\,{}^{B}P \\
{}^{A}P &= {}^{A}P
\end{aligned}
$$

If ${}^{D}P$ is changing at 100 Hz, we would have to recalculate ${}^{D}P$ at the same rate. However, the three rotation matrices are also changing, as reported by a vision system that gives us new values for ${}^{A}_{B}R$,${}^{B}_{C}R$ and ${}^{C}_{D}R$ at 30 Hz. What is the best way to organize the computation to minimize the calculation effort (multiplications and additions)?

3. A 3D frame {B} located initially coincident with another 3D frame {A}. Compute the rotation matrix ${}^{B}R_{A}$ that converts a vector ${}^{A}P$ to ${}^{B}P$ for the following sequence of rotations, i.e, ${}^{B}P = {}^{B}R_{A}\,{}^{A}P$

   (a) Rotate {B} by 30 degrees about its $\hat{X}_A$ axis, and then rotate the resulting frame by 20 degrees about the resulting frame's Z axis.

   (b) Rotate {B} by 20 degrees about its $\hat{Y}_A$ axis, and then rotate the resulting frame by 20 degrees about $\hat{Z}_A$.

4. We are interested in plotting a camera $\{C\}$, as a translucent pyramid (fig: 1) with a square base. The pyramid's origin is located at apex with z-axis pointing towards the base. The x and y axis of pyramid are parallel to base sides following right hand coordinate system. Our world coordinate system $\{W\}$ coincides with camera coordinate system $\{C\}$ initially,

   (a) Now, given a rotation matrix ${}^{C}R_{W}$, Plot the rotated camera pyramid at world origin.

   (b) As seen in previous questions, we often give angle of rotation around a particular axis about which a camera is rotated. So, write a function that that takes angles mentioned in Q3 (a),(b) and plot respective camera pyramids.

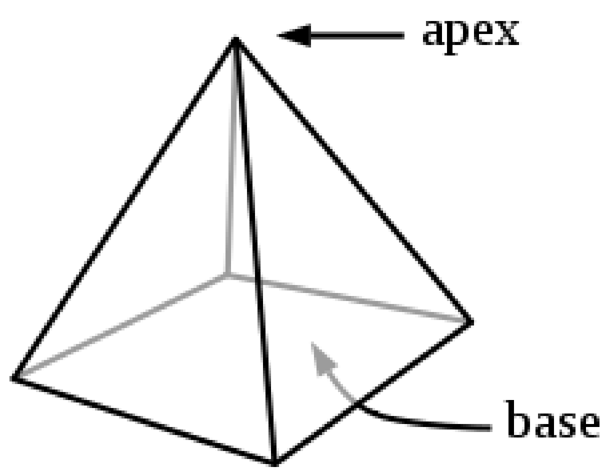

Figure 1: Pyramid

(c) Now generalize the above function to take in $(\alpha, \beta, \gamma, i)$, Where $i \in \{0, 1\}$ indicates fixed or relative rotations performed respectively and $\alpha, \beta and \gamma$ are rotations around x,y and z axis in fixed/relative frames.