Multiple-view Geometry 3

Triangulation, Resection, Bundle Adjustment

RRC Summer Sessions

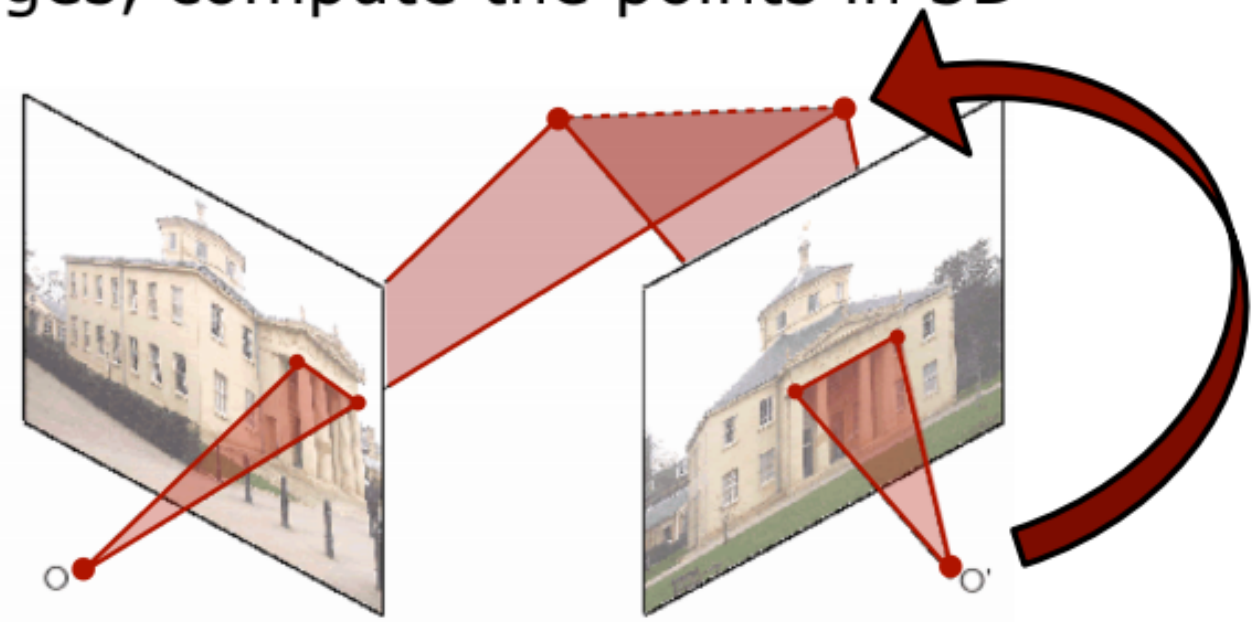
Last class

- Elements of epipolar geometry - epipolar plane, epipoles, epipolar axis, epipolar lines
- Given an image point in the first view, how does it constrain the position of the corresponding point in the second view
- Essential matrix as a special case of the fundamental matrix, and its decomposition into R, S_b
- Estimation of the fundamental matrix using the normalized 8-point algorithm
- Robust estimation using RANSAC

Triangulation

Triangulation

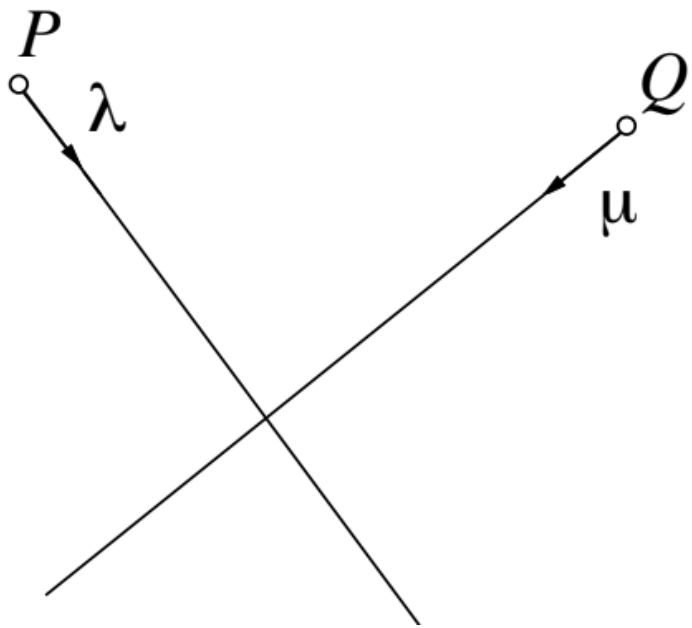
Given the relative orientation of two images, compute the points in 3D



Courtesy: Stachniss

Triangulation

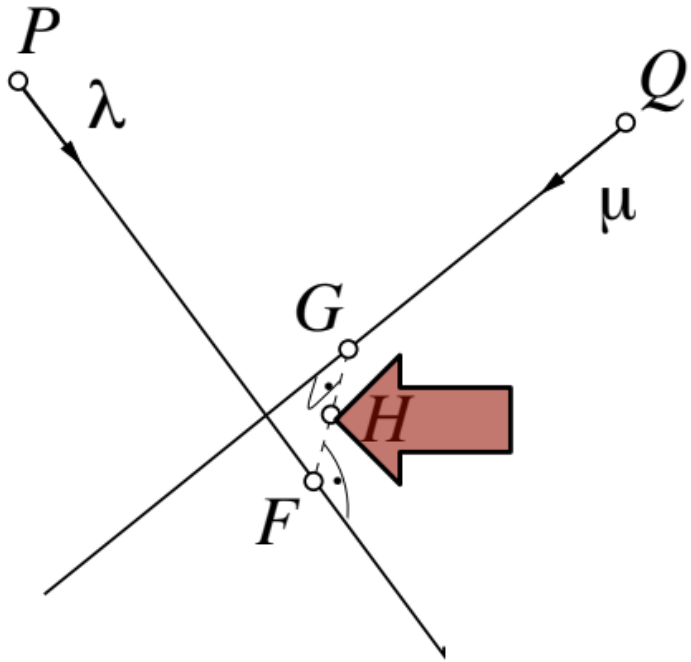
Geometric



Courtesy: Stachniss

Triangulation

Geometric



Courtesy: Stachniss

Triangulation

Geometric

- Equation for two lines in 3D

$$\boldsymbol{f} = \boldsymbol{p} + \lambda \boldsymbol{r} \qquad \boldsymbol{g} = \boldsymbol{q} + \mu \boldsymbol{s}$$

- with the points $\boldsymbol{p} = \boldsymbol{X}_{O'} \quad \boldsymbol{q} = \boldsymbol{X}_{O''}$
- and the directions (calibrated camera)

$$\boldsymbol{r} = R'^{\mathsf{T}}\, {}^{k}\mathbf{x}' \qquad \boldsymbol{s} = R''^{\mathsf{T}}\, {}^{k}\mathbf{x}''$$

- with ${}^{k}\mathbf{x}' = (x', y', c)^{\mathsf{T}} \quad {}^{k}\mathbf{x}'' = (x'', y'', c)^{\mathsf{T}}$

Courtesy: Stachniss

Triangulation

Geometric

Geometric Solution

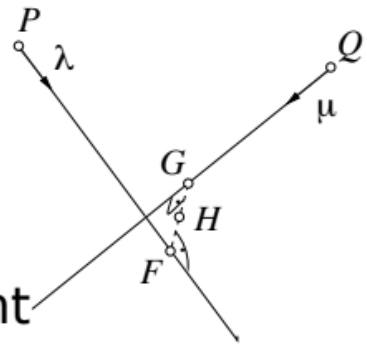
- The shortest connection requires that FG is orthogonal to both lines
- This leads to the constraint

$$(\boldsymbol{f}-\boldsymbol{g})\cdot\boldsymbol{r}=0 \qquad (\boldsymbol{f}-\boldsymbol{g})\cdot\boldsymbol{s}=0$$

which directly leads to

$$\begin{aligned}(\boldsymbol{q}+\lambda\boldsymbol{s}-\boldsymbol{p}-\mu\boldsymbol{r})\cdot\boldsymbol{s} &= 0\\ (\boldsymbol{q}+\lambda\boldsymbol{s}-\boldsymbol{p}-\mu\boldsymbol{r})\cdot\boldsymbol{r} &= 0\end{aligned}$$

- Two equations, two unknowns



Courtesy: Stachniss

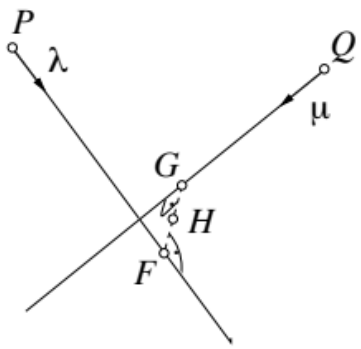
Triangulation

Geometric

- By solving the equations

$$(\boldsymbol{q} + \lambda \boldsymbol{s} - \boldsymbol{p} - \mu \boldsymbol{r}) \cdot \boldsymbol{s} = 0$$
$$(\boldsymbol{q} + \lambda \boldsymbol{s} - \boldsymbol{p} - \mu \boldsymbol{r}) \cdot \boldsymbol{r} = 0$$

  we obtain λ, μ
- λ, μ directly yield F and G
- We compute H as the middle of the line connecting F and G



Courtesy: Stachniss

Triangulation

Algebraic

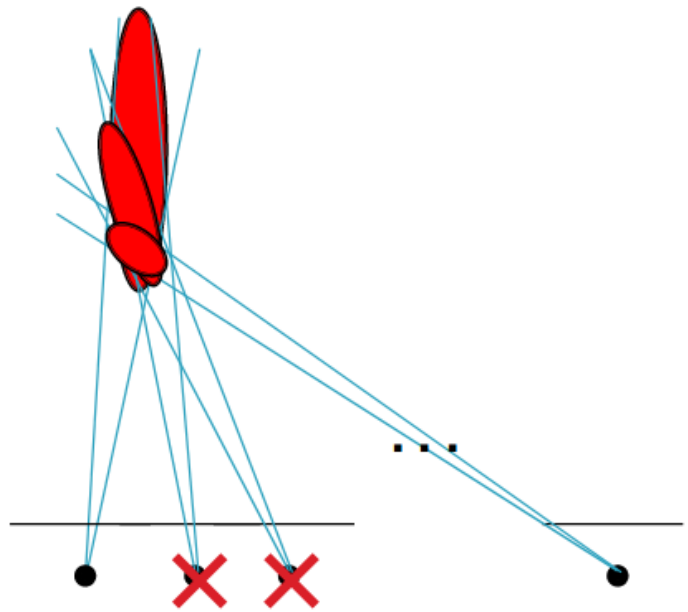
We use a linear triangulation method that is analogous to DLT.

- Given a pair of corresponding points $x = PX$ and $x^{'} = P^{'}X$, we form a system of linear equations $AX = 0$.
- We get three equations for each image point, of which two are linearly independent, by taking the cross product $x \times PX = 0$.
- Taking two equations from each image, we get a total of four equations. $A = \begin{bmatrix} xp^{3T} - p^{1T} \\ yp^{3T} - p^{2T} \\ x^{'}p^{'3T} - p^{'1T} \\ y^{'}p^{'3T} - p^{'2T} \end{bmatrix}$
- The solution for X is the singular vector corresponding to the smallest singular value of A.

Triangulation

Keyframe selection

- The std. deviation of the distances of the triangulated 3D point from all rays gives an idea of the quality of the 3D point.
- When frames are taken at nearby intervals, the 3D point typically exhibit large uncertainty.
- To avoid this, we skip adjacent frames until the average uncertainty of the 3D points decreases below a certain threshold. The selected frames are called *keyframes*.



Courtesy: Scaramuzza

Computing relative scale

- To estimate the trajectory of the agent, different transformations (with different scales) need to be concatenated together.
- To do this, the relative scales for the subsequent transformations are computed.
- One way to do this is to triangulate 3D points X_{k-1}, X_k from two subsequent image pairs. The relative scale is then,

$$r = \frac{\|X_{k-1,i} - X_{k-1,j}\|}{\|X_{k,i} - X_{k,j}\|}$$

- For robustness, the scale ratios for many point pairs are computed and their mean (or median) is used.
- The translation vector t_k is then scaled with this ratio.

Recap

Visual odometry

Algorithm 1. VO from 2-D-to-2-D correspondences.

1) Capture new frame I_k
2) Extract and match features between I_{k-1} and I_k
3) Compute essential matrix for image pair I_{k-1}, I_k
4) Decompose essential matrix into R_k and t_k, and form T_k
5) Compute relative scale and rescale t_k accordingly
6) Concatenate transformation by computing $C_k = C_{k-1}T_k$
7) Repeat from 1).

Courtesy: Scaramuzza

Recall

Visual Odometry

We saw motion estimation from two sets of corresponding features f_k, f_{k-1} can be done in three different methods,

- 2D-2D - Both f_{k-1}, f_k are in 2D
- 3D-2D - f_{k-1} are in 3D, f_k are their corresponding reprojections in image I_k
- 3D-3D - Both f_{k-1}, f_k are in 3D

Recall

Visual Odometry

We saw motion estimation from two sets of corresponding features f_k, f_{k-1} can be done in three different methods,

- 2D-2D - Both f_{k-1}, f_k are in 2D
- **3D-2D - f_{k-1} are in 3D, f_k are their corresponding reprojections in image I_k**
- 3D-3D - Both f_{k-1}, f_k are in 3D

Resection

Resection

PnP

Camera 3D Registration

Perspective-n-Point Algorithm

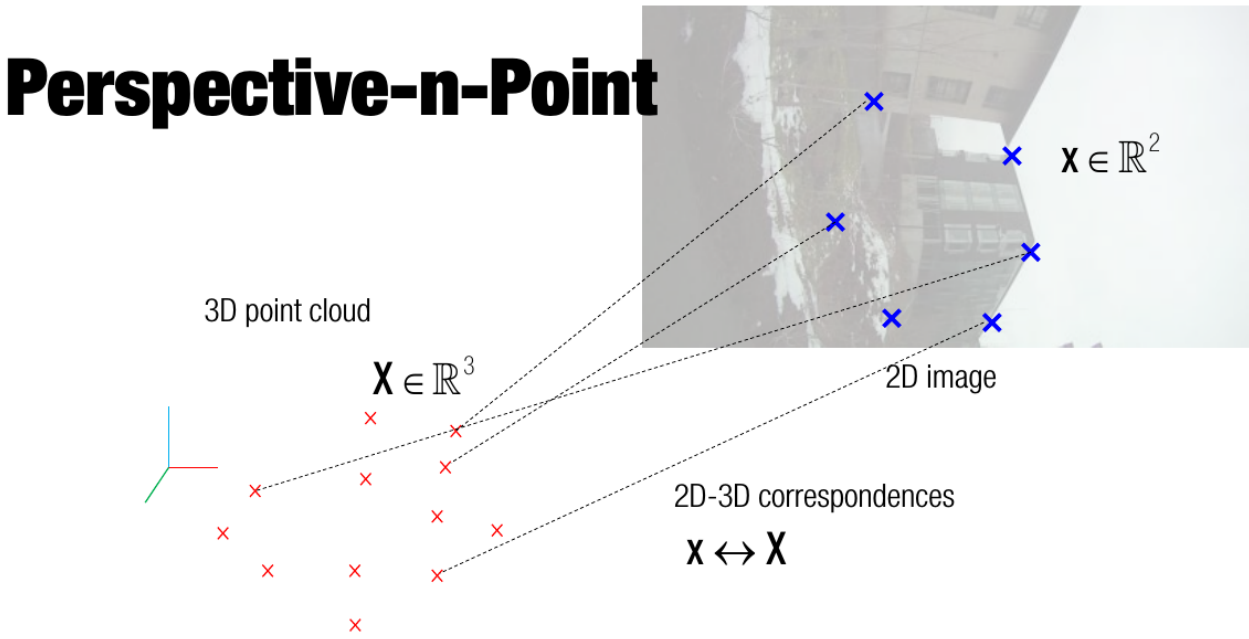
3D point cloud via triangulation

Where?

Courtesy: Jianbo Shi

Resection

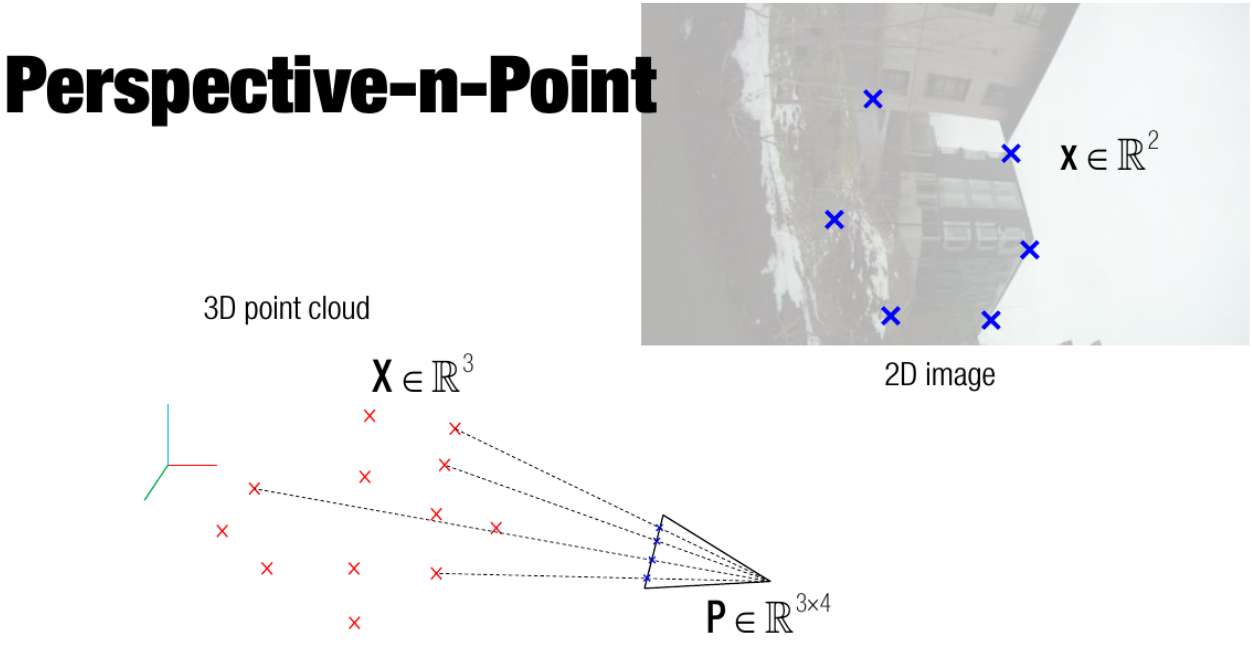
PnP



Courtesy: Jianbo Shi

Resection

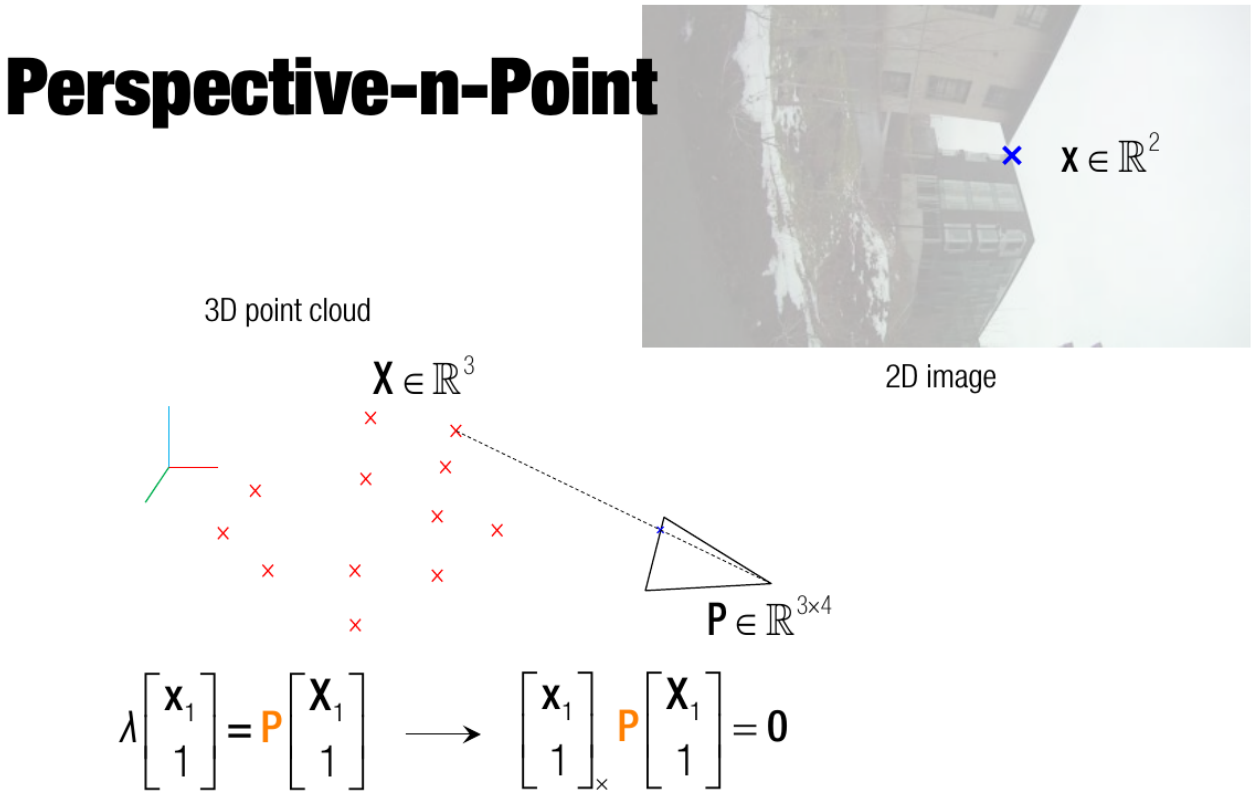
PnP



Courtesy: Jianbo Shi

Resection

PnP

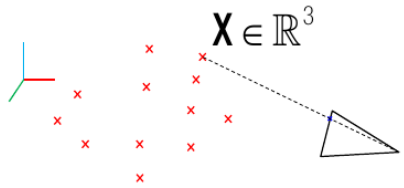


Courtesy: Jianbo Shi

Resection

PnP

Perspective-n-Point



2D image

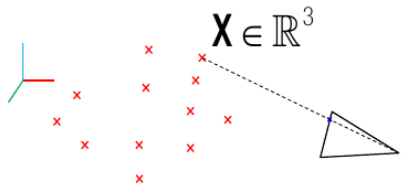
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_{\times} \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}}$$

Courtesy: Jianbo Shi

Resection

PnP

Perspective-n-Point



2D image

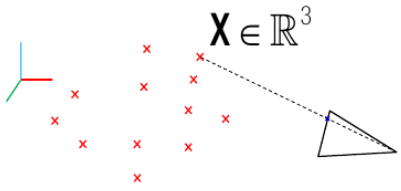
$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_{\times} \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_{\times} \begin{bmatrix} \mathbf{P}_1 \tilde{\mathbf{X}} \\ \mathbf{P}_2 \tilde{\mathbf{X}} \\ \mathbf{P}_3 \tilde{\mathbf{X}} \end{bmatrix}$$

Courtesy: Jianbo Shi

Resection

PnP

Perspective-n-Point



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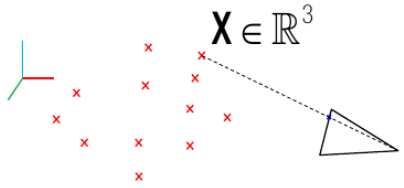
Courtesy: Jianbo Shi

Resection

PnP

Perspective-n-Point



2D image

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \tilde{\mathbf{X}} \\ \mathbf{P}_2 \tilde{\mathbf{X}} \\ \mathbf{P}_3 \tilde{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} =$$

$$\begin{bmatrix} \mathbf{0}_{1\times4} & -\tilde{\mathbf{X}}^\mathsf{T} & v\tilde{\mathbf{X}}^\mathsf{T} \\ \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & -u\tilde{\mathbf{X}}^\mathsf{T} \\ -v\tilde{\mathbf{X}}^\mathsf{T} & u\tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} = \mathbf{0}$$

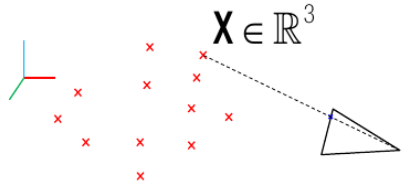
3x12 matrix

Courtesy: Jianbo Shi

Resection

PnP

Perspective-n-Point



2D image

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \tilde{\mathbf{X}} \\ \mathbf{P}_2 \tilde{\mathbf{X}} \\ \mathbf{P}_3 \tilde{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} =$$

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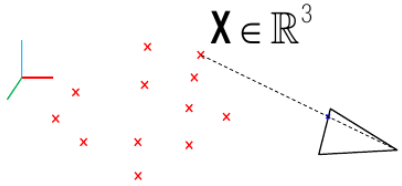
3x12 matrix

Courtesy: Jianbo Shi

Resection

PnP

Perspective-n-Point



2D image

$$\begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \end{bmatrix} \tilde{\mathbf{X}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}_\times \begin{bmatrix} \mathbf{P}_1 \tilde{\mathbf{X}} \\ \mathbf{P}_2 \tilde{\mathbf{X}} \\ \mathbf{P}_3 \tilde{\mathbf{X}} \end{bmatrix} = \begin{bmatrix} 0 & -1 & v \\ 1 & 0 & -u \\ -v & u & 0 \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \\ \mathbf{0}_{1\times4} & \mathbf{0}_{1\times4} & \tilde{\mathbf{X}}^\mathsf{T} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} =$$

$$\begin{bmatrix} \mathbf{0}_{1\times4} & -\tilde{\mathbf{X}}^\mathsf{T} & v\tilde{\mathbf{X}}^\mathsf{T} \\ \tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} & -u\tilde{\mathbf{X}}^\mathsf{T} \\ -v\tilde{\mathbf{X}}^\mathsf{T} & u\tilde{\mathbf{X}}^\mathsf{T} & \mathbf{0}_{1\times4} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1^\mathsf{T} \\ \mathbf{P}_2^\mathsf{T} \\ \mathbf{P}_3^\mathsf{T} \end{bmatrix} = \mathbf{0}$$

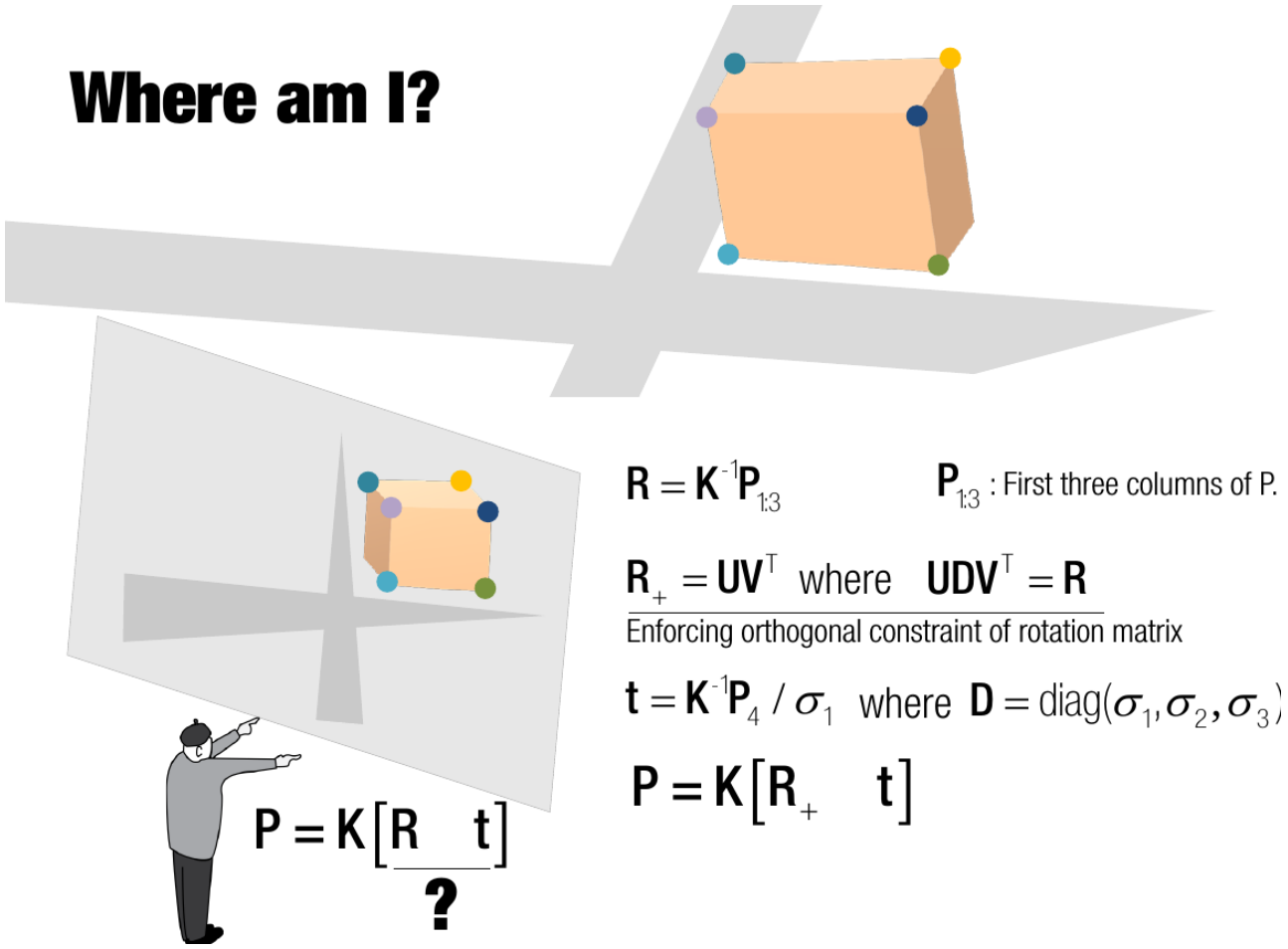
3x12 matrix (rank 2)

$$\begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \\ \vdots \\ \mathbf{A}_6 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Courtesy: Jianbo Shi

Resection

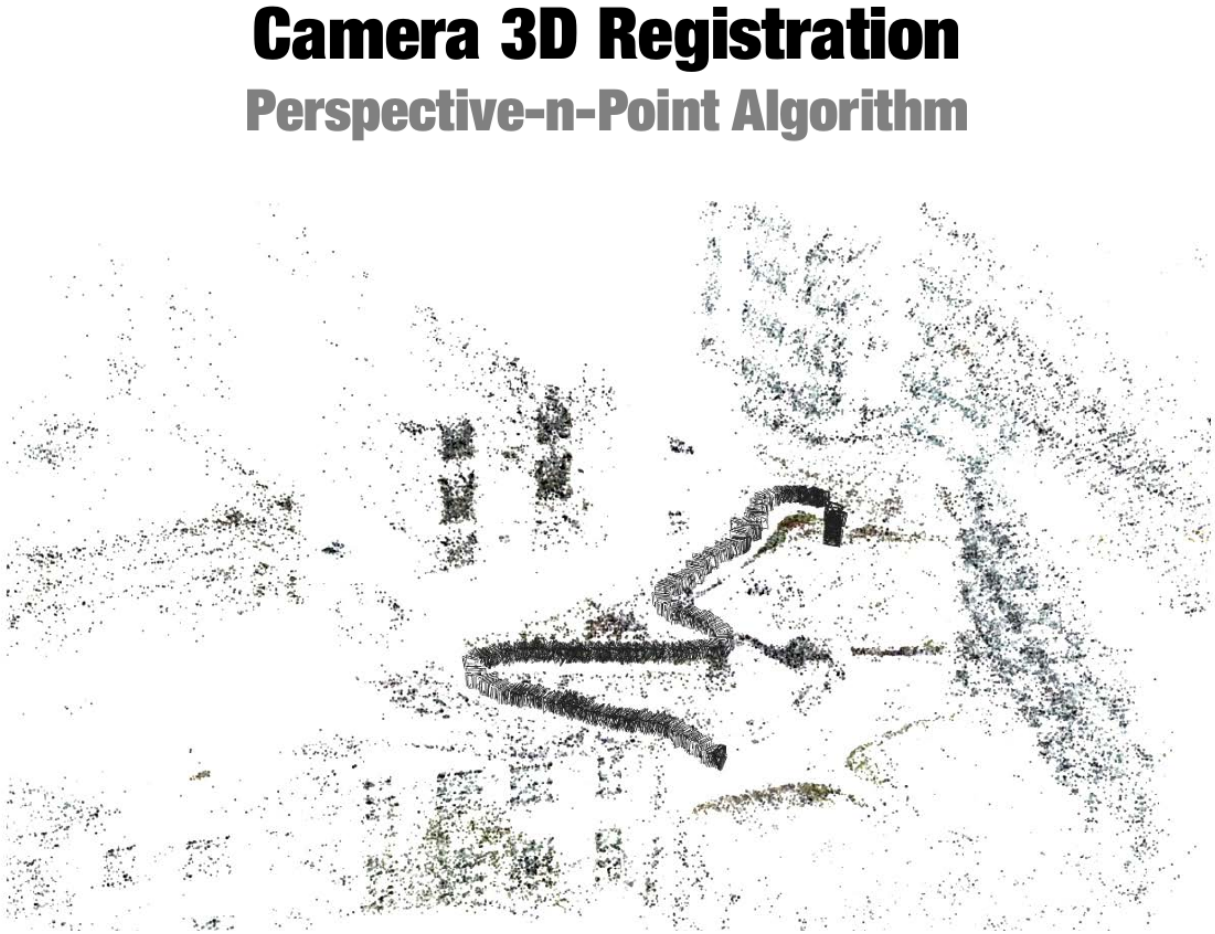
PnP



Courtesy: Jianbo Shi

Resection

PnP



Courtesy: Jianbo Shi

Resection

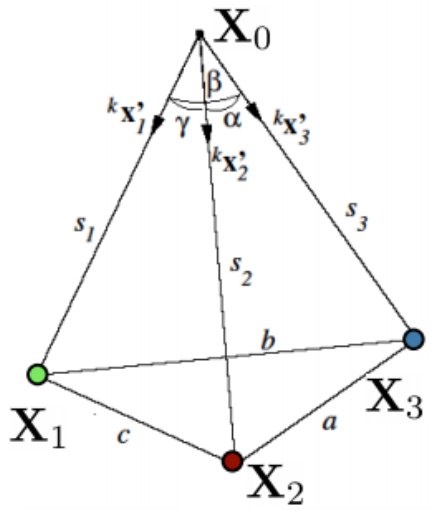
P3P

- For robust estimation, *perspective-from-three-points* is the standard method, and can be seen as the minimal case of the PnP method.
- The method typically returns four solutions that can be disambiguated using one or more additional points.
- Like PnP, various approaches have been proposed. Current state of the art is *Lambda Twist P3P*. We'll look at the earliest approach (Grunert 1841).

Resection

P3P

- We start with the direct solution
  - Grunert 1841
  - Analogous to planar resection
  - 2 steps
    1. Estimate length of projection rays
    2. Estimate orientation



Courtesy: Stachniss

Resection

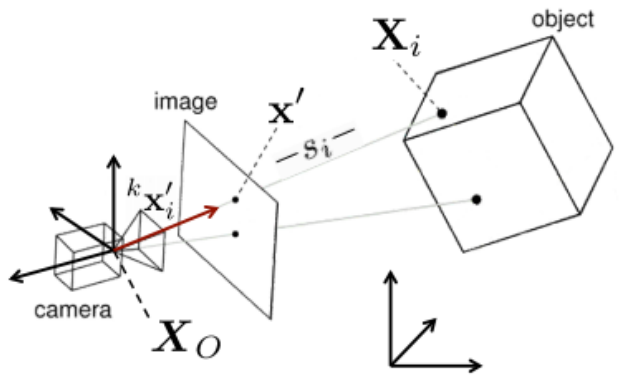
P3P

- Coordinates of object points within the camera system are given by

$$s_i \mathsf{N}({}^k\mathbf{x}'_i) = R(\boldsymbol{X}_i - \boldsymbol{X}_O) \qquad i = 1, 2, 3$$

spherical normalization

$$\mathsf{N}(\mathbf{x}) = \frac{\mathbf{x}}{|\mathbf{x}|}$$



Courtesy: Stachniss

Resection

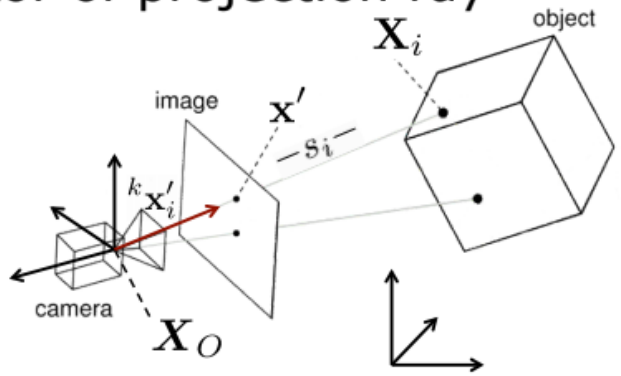
P3P

- Coordinates of object points within the camera system are given by

$$s_i \mathsf{N}({}^k\mathbf{x}'_i) = R(\boldsymbol{X}_i - \boldsymbol{X}_O) \qquad i = 1, 2, 3$$

- From image coordinates we obtain the directional vector of projection ray

$${}^k\mathbf{x}'_i = \mathsf{K}^{-1}\mathbf{x}'_i$$

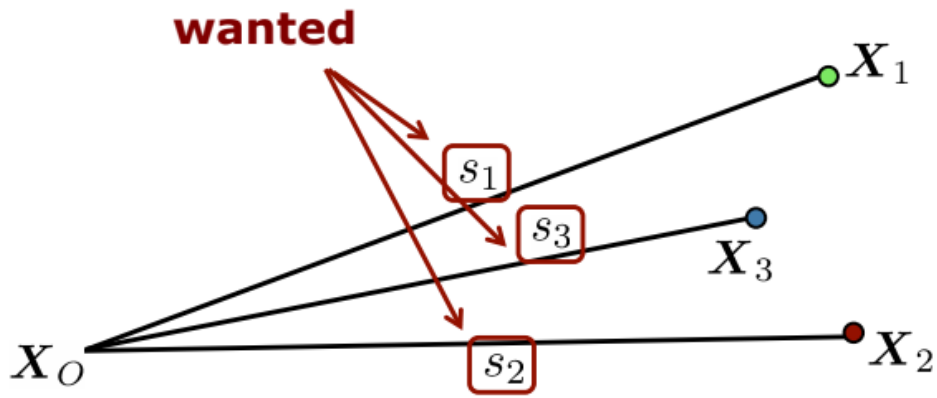


Courtesy: Stachniss

Resection

P3P

1. Get length of projection rays



Courtesy: Stachniss

Resection

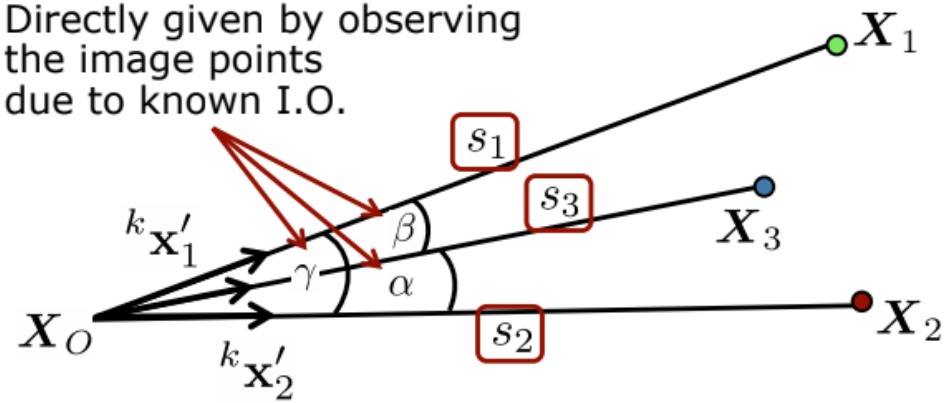
P3P

1. Get length of projection rays

$$ {}^k\mathbf{x}'_i = \mathsf{K}^{-1}\mathbf{x}'_i $$

$$
\begin{aligned}
\alpha &= \arccos\left({}^k\mathbf{x}'_2, {}^k\mathbf{x}'_3\right) \\
\beta &= \arccos\left({}^k\mathbf{x}'_3, {}^k\mathbf{x}'_1\right) \\
\gamma &= \arccos\left({}^k\mathbf{x}'_1, {}^k\mathbf{x}'_2\right)
\end{aligned}
$$

Directly given by observing the image points due to known I.O.



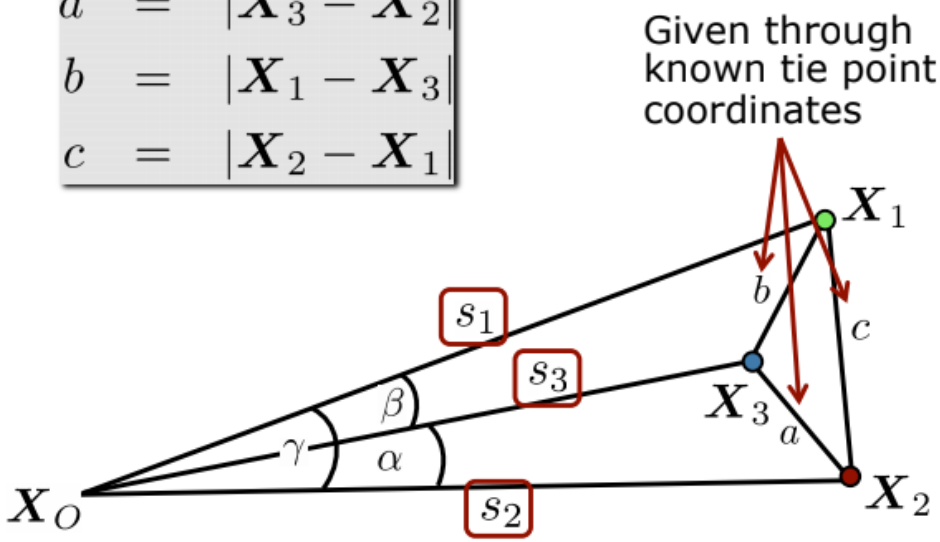
Courtesy: Stachniss

Resection

P3P

1. Get length of projection rays

$$
\begin{aligned}
a &= |\boldsymbol{X}_3 - \boldsymbol{X}_2| \\
b &= |\boldsymbol{X}_1 - \boldsymbol{X}_3| \\
c &= |\boldsymbol{X}_2 - \boldsymbol{X}_1|
\end{aligned}
$$



Courtesy: Stachniss

Resection

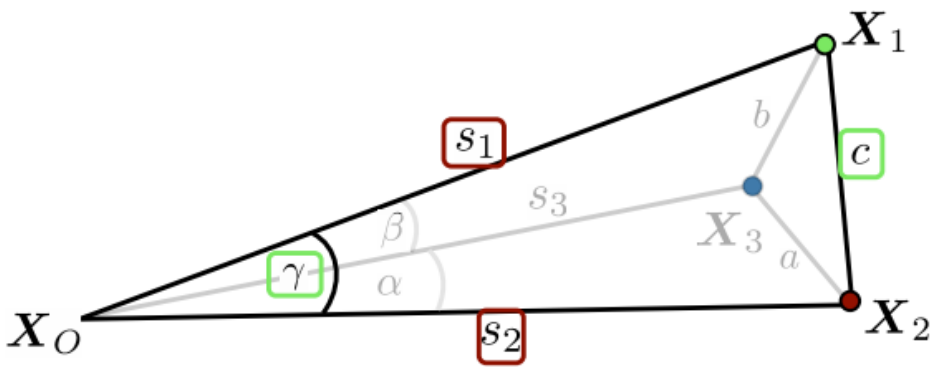
P3P

Use the Law of Cosines

In triangle $\mathbf{X}_0, \mathbf{X}_1, \mathbf{X}_2$

$$s_1^2 + s_2^2 - 2 s_1 s_2 \cos\gamma = c^2$$

wanted known



Courtesy: Stachniss

Resection

P3P

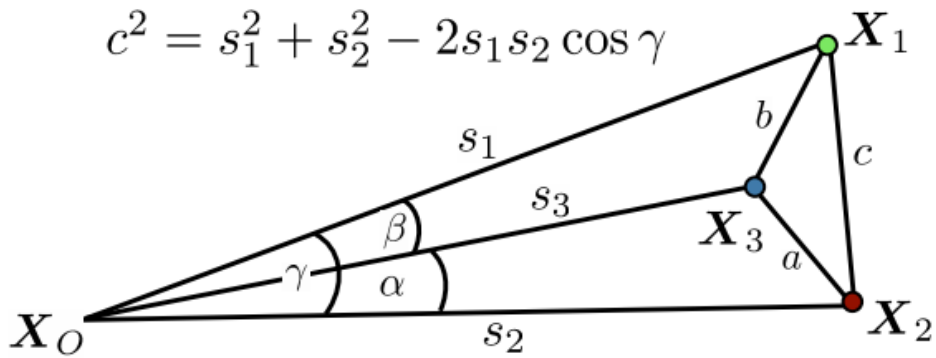
Use the Law of Cosines

analogous solutions

$$a^2 = s_2^2 + s_3^2 - 2 s_2 s_3 \cos\alpha$$

$$b^2 = s_1^2 + s_3^2 - 2 s_1 s_3 \cos\beta$$

$$c^2 = s_1^2 + s_2^2 - 2 s_1 s_2 \cos\gamma$$



Courtesy: Stachniss

Resection

P3P

Estimate distances

- We start from:

$$a^2 = s_2^2 + s_3^2 - 2 s_2 s_3 \cos\alpha$$

- Define: $u = \frac{s_2}{s_1}$ $v = \frac{s_3}{s_1}$
- Substitute: $a^2 = s_1^2(u^2 + v_2^2 - 2uv\cos\alpha)$
- Rearrange to get the distance:

$$s_1^2 = \frac{a^2}{u^2 + v_2^2 - 2uv\cos\alpha}$$

Courtesy: Stachniss

Resection

P3P

Rearrange again

- Solve one equation for u put into the other

$$s_1^2 = \frac{a^2}{u^2+v_2^2-2uv\cos\alpha}$$

$$s_1^2 = \frac{b^2}{1+v^2-2v\cos\beta}$$

$$s_1^2 = \frac{c^2}{1+u^2-2u\cos\gamma}$$

Results in an fourth degree polynomial

$$A_4v^4 + A_3v^3 + A_2v^2 + A_1v + A_0 = 0$$

Courtesy: Stachniss

Resection

P3P

Forth degree polynomial

$$A_4 v^4 + A_3 v^3 + A_2 v^2 + A_1 v + A_0 = 0$$

$$A_4 = \left(\frac{a^2-c^2}{b^2} - 1\right)^2 - \frac{4c^2}{b^2}\cos^2\alpha$$

$$\begin{aligned} A_3 &= 4\left[\frac{a^2-c^2}{b^2}\left(1 - \frac{a^2-c^2}{b^2}\right)\cos\beta \right. \\ &\quad \left. - \left(1 - \frac{a^2+c^2}{b^2}\right)\cos\alpha\cos\gamma + 2\frac{c^2}{b^2}\cos^2\alpha\cos\beta\right] \end{aligned}$$

Courtesy: Stachniss

Resection

P3P

Forth degree polynomial

$$A_4 v^4 + A_3 v^3 + A_2 v^2 + A_1 v + A_0 = 0$$

$$\begin{aligned} A_2 = \; & 2\Bigg[\left(\frac{a^2 - c^2}{b^2}\right)^2 - 1 + 2\left(\frac{a^2 - c^2}{b^2}\right)^2 \cos^2\beta \\ & + 2\left(\frac{b^2 - c^2}{b^2}\right)\cos^2\alpha \\ & - 4\left(\frac{a^2 + c^2}{b^2}\right)\cos\alpha\cos\beta\cos\gamma \\ & + 2\left(\frac{b^2 - a^2}{b^2}\right)\cos^2\gamma \Bigg] \end{aligned}$$

Courtesy: Stachniss

Resection

P3P

Forth degree polynomial

$$A_4 v^4 + A_3 v^3 + A_2 v^2 + A_1 v + A_0 = 0$$

$$\begin{aligned} A_1 = 4\Bigg[& -\left(\frac{a^2-c^2}{b^2}\right)\left(1+\frac{a^2-c^2}{b^2}\right)\cos\beta \\ & + \frac{2a^2}{b^2}\cos^2\gamma\cos\beta \\ & - \left(1-\left(\frac{a^2+c^2}{b^2}\right)\right)\cos\alpha\cos\gamma\Bigg] \end{aligned}$$

$$A_0 = \left(1+\tfrac{a^2-c^2}{b^2}\right)^2 - \tfrac{4a^2}{b^2}\cos^2\gamma$$

Courtesy: Stachniss

Resection

P3P

Forth degree polynomial

$$A_4 v^4 + A_3 v^3 + A_2 v^2 + A_1 v + A_0 = 0$$

- Solve for v to get s_1, s_2, s_3

$$s_1^2 = \frac{b^2}{1 + v^2 - 2v\cos\beta}$$

$$s_3 = v\ s_1$$

$$a^2 = s_2^2 + s_3^2 - 2 s_2 s_3 \cos\alpha \Rightarrow s_2 = \cdots$$

Problem:

Up to 4 possible solutions !

$$\{s_1, s_2, s_3\}_{1\ldots4}$$

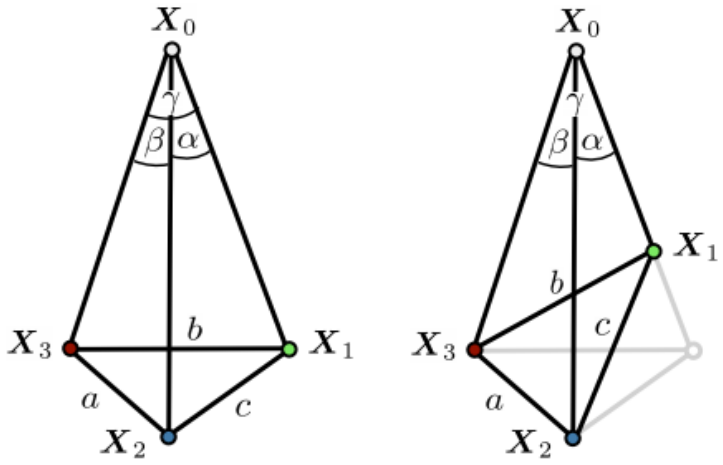
Courtesy: Stachniss

Resection

P3P

Four Solutions

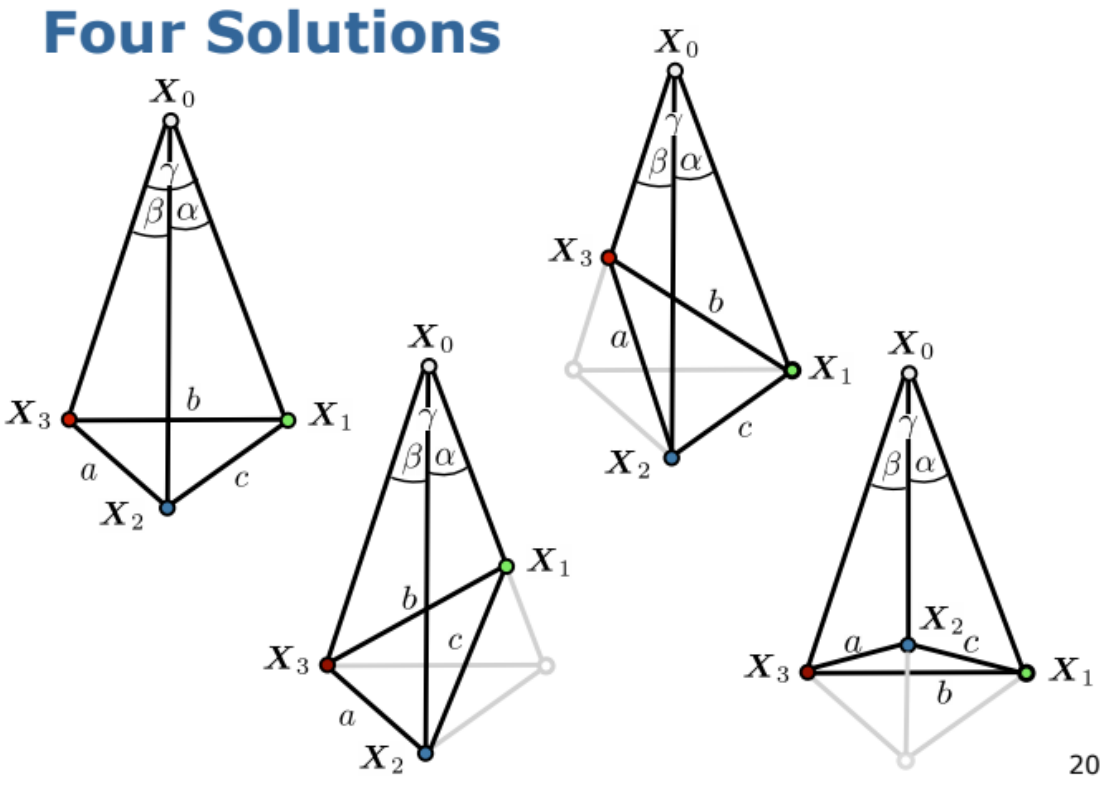
- Example:
  - Assume $a = b = c \quad \alpha = \beta = \gamma$
  - Tilting the rectangle $\Delta(\boldsymbol{X}_1, \boldsymbol{X}_2, \boldsymbol{X}_3)$ has no effect on (a, b, c) and (α, β, γ)



Courtesy: Stachniss

Resection

P3P



Courtesy: Stachniss

Resection

P3P

How to get rid of ambiguity?

- Approximate solution, e.g. from GPS
- Use 4th points to confirm the right solution

Unique solution for

$$s_1, s_2, s_3$$

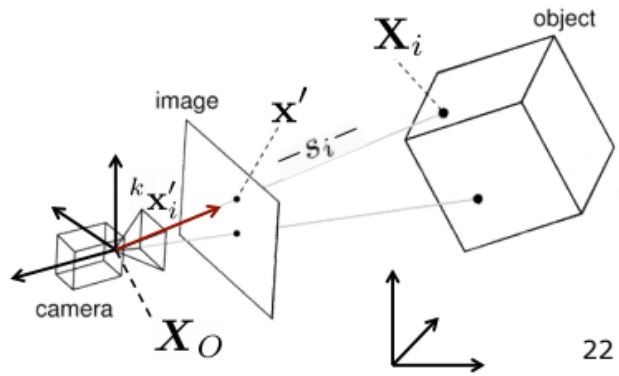
Courtesy: Stachniss

Resection

P3P

2. Orientation of the Camera

- Given:
  - Distances to tie points
- Task:
  - Estimate 6 parameters of E.O.



Courtesy: Stachniss

Resection

P3P

- We now have two sets of 3 corresponding 3-D points. We want to find the transformation that best aligns the two sets.

$$arg \min_{T_k} \sum_i \|X_k^i - T_k X_{k-1}^i\|$$

- $R_k = VU^T whereUSV^T = svd((X_{k-1} - \overline{X_{k-1}})(X_k - \overline{X_k})^T)$
- $t_k = \overline{X_k} - R\overline{X_{k-1}}$
- where $\bar{\cdot}$ stands for the mean value
- Details: Least-squares fitting of two 3D point-sets by Arun et al.

Bundle Adjustment

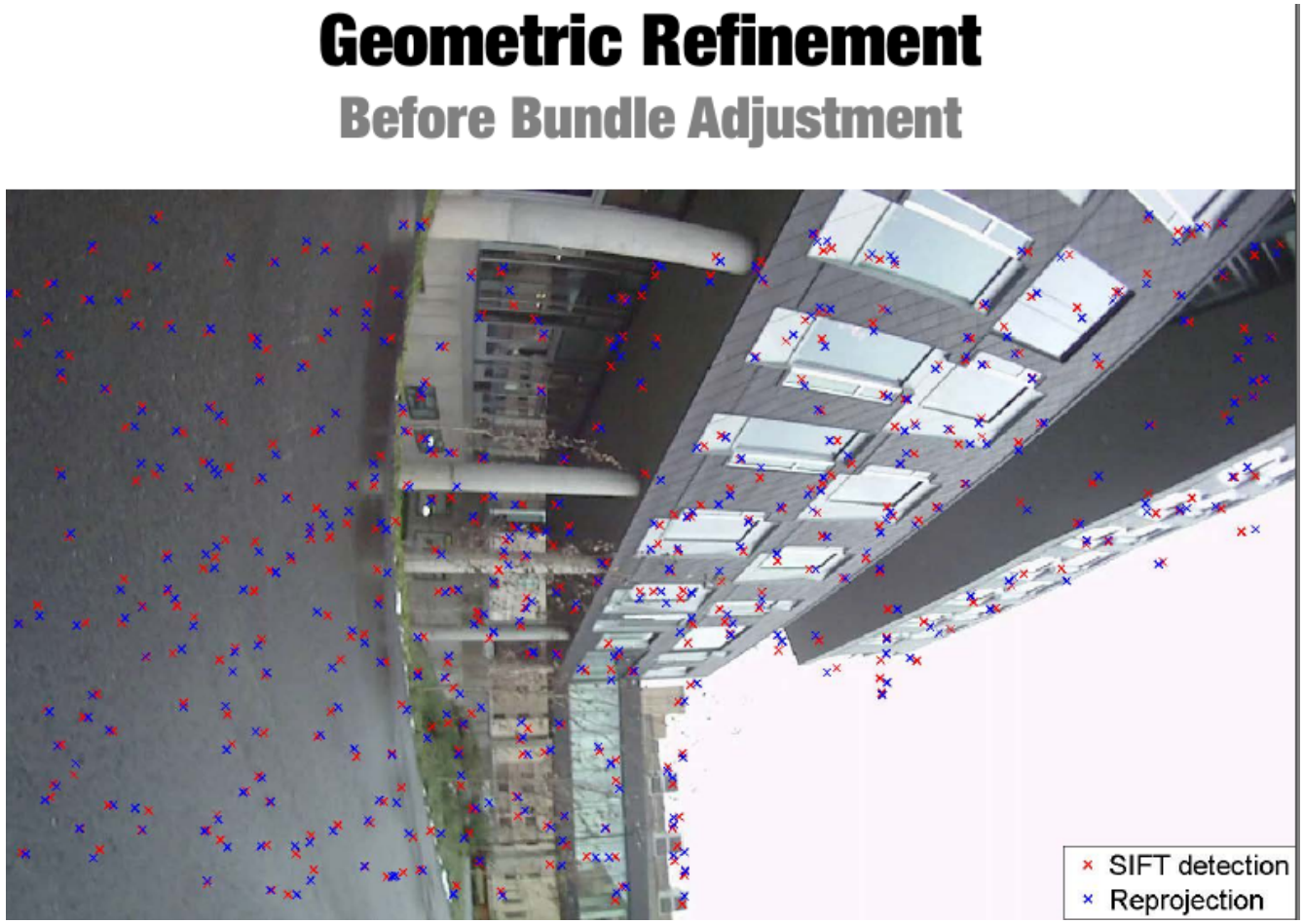
Bundle Adjustment

- To reduce the drift and improve the accuracy of the local trajectory in VO, we run an optional iterative refinement step, called bundle adjusment.
- This is equivalent to building a local map in SLAM, but only to improve the local consistency of the trajectory.
- This is a non-linear optimization problem where we minimize the sum of the squared reprojection errors of the reconstructed 3D points over the last m images. This is technically called *windowed-bundle adjustment*.

$$arg \min_{X_j, P_i} \sum_{i=1}^{M} \sum_{j=1}^{N} \|x_{ij} - P_i X_j\|^2$$

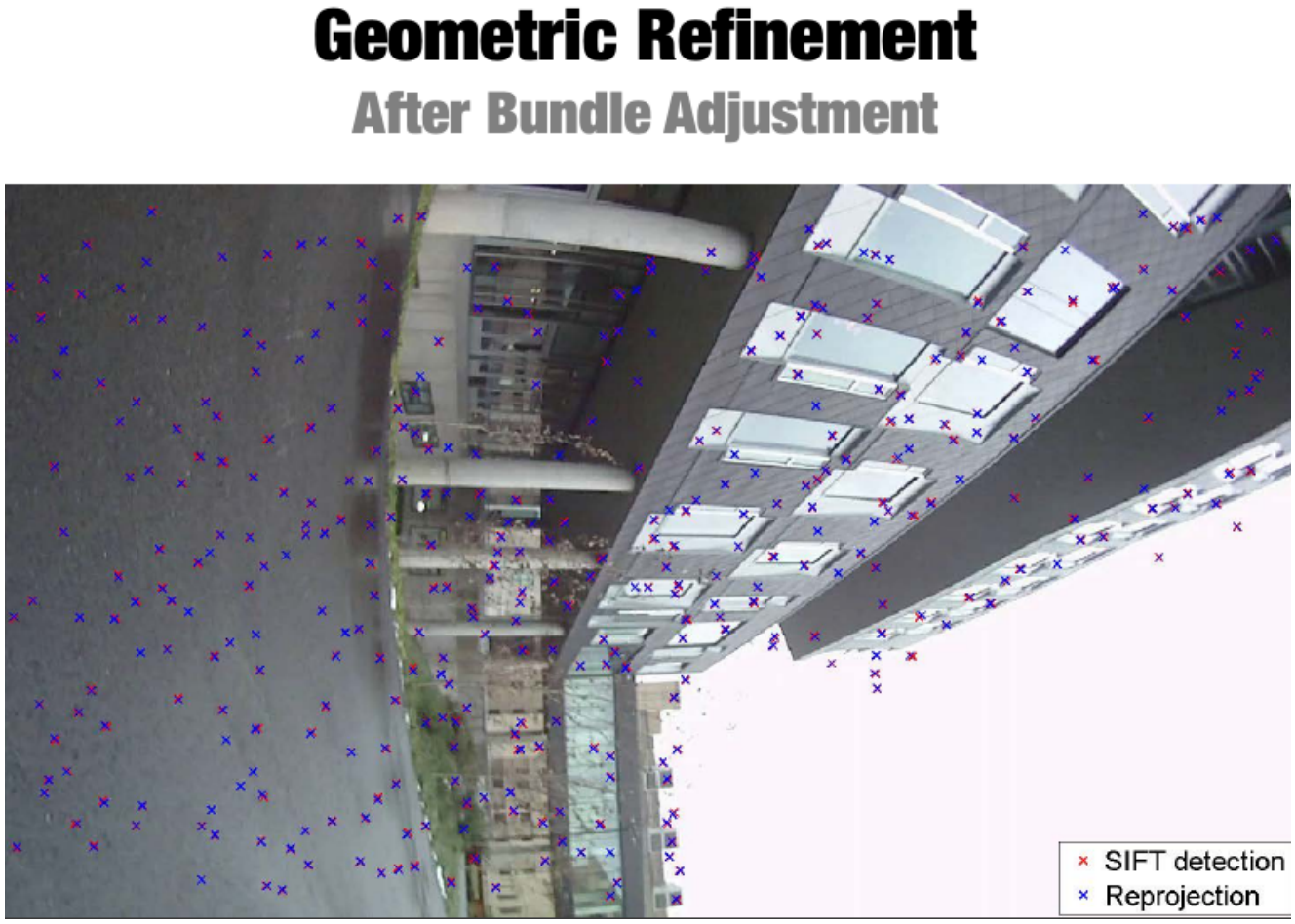
where P_i is the ith view, x_{ij} is the image of the jth point in the ith view.

Bundle adjustment



Courtesy: Jianbo Shi

Bundle adjustment



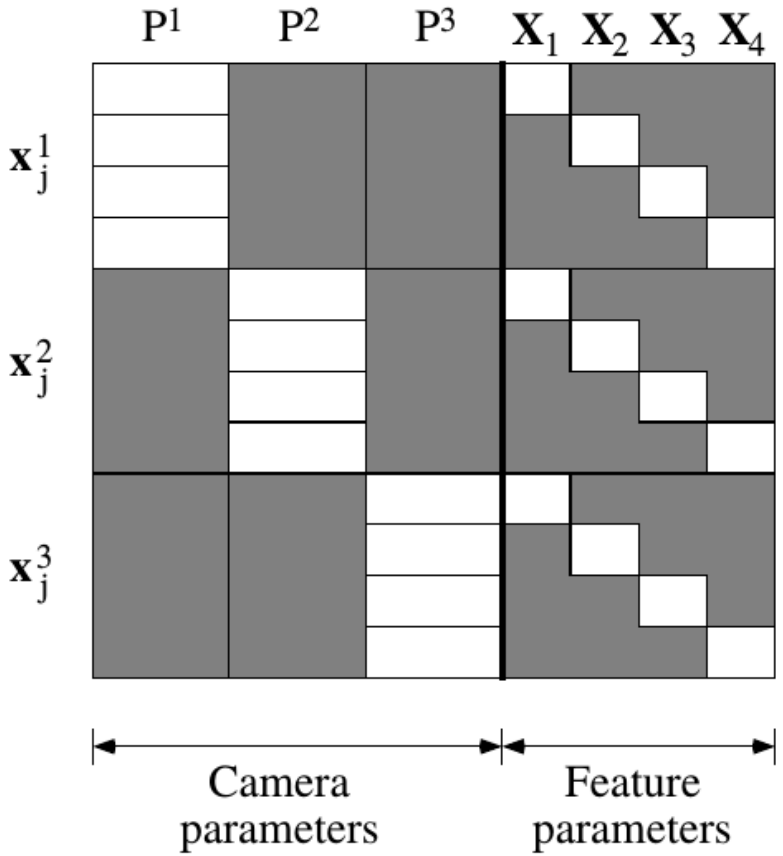
Courtesy: Jianbo Shi

Bundle Adjustment

- Bundle here refers to the bundles of light rays leaving each3D feature and converging on the camera centre, which are then adjusted optimally with respect to both feature and camera parameters.
- It is typically solved using the Levenberg Marquardt solver. Refer to - Bundle Adjustment by Triggs et al.- for an extensive survey on different approaches to solve the problem.

Bundle Adjustment

Structure of the Jacobian for a typical BA problem,



Quiz

https://bit.ly/2Wm8K8M