Week 1 - 8

50 - 60 %

Week 5 Normalisation

Week 5
→ functional dependency & Axioms
→ closure
→ C.K & S.K
→ Canonical cover
→ Lossless decomposition
→ Dependency Preservation

Week 7 Lec Assignment
Week 8 :- Assignment

Quiz 2 Revision Session 1

Week 5 & 6

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1NF
2NF
3NF
BCNF

MVD & 4NF

Functional Dependencies

- Let R be a relation schema
  $\alpha \subseteq R$ and $\beta \subseteq R$
- The functional dependency or FD
  $\alpha \rightarrow \beta$
  holds on R if and only if for any legal relations $r(R)$, whenever any two tuples t_1 and t_2 of r agree on the attributes α they also agree on the attributes β. That is,

$$t_1[\alpha] = t_2[\alpha] \Rightarrow t_1[\beta] = t_2[\beta]$$

$\alpha \longrightarrow \beta$

$A \rightarrow B$ $\cancel{\checkmark}$

$B \longrightarrow A$

- Example: Consider $r(A, B)$ with the following instance of r.

A	B
1	4
1	5
3	7

- On this instance, $A \rightarrow B$ does **NOT** hold, but $B \rightarrow A$ does hold. So we cannot have tuples like (2, 4), or (3, 5), or (4, 7) added to the current instance.

8) Consider relational instance given in Figure 1.

α

V	W	X	Y	Z
b	9	8	7	5
9	b	8	7	1
b	9	8	2	2
b	9	8	3	6

$b \rightarrow 98 \; \beta$

b

$9 \rightarrow b \; 8$

Figure 1: Relational instance

Which of the following functional dependencies does not hold on the given table?

- $\overset{\alpha}{V} \rightarrow WX \rightarrow \beta$ ✓
- $YZ \rightarrow X$ ✓
- $X \rightarrow YZ$ ✓
- $WY \rightarrow Z$

Functional Dependencies : Armstrong's Axioms: Derived Rules

$\alpha \to \beta\gamma$

$\gamma\alpha \to \beta\gamma$

$\gamma\beta \to \delta$

- Additional Derived Rules:
  - **Union**: if $\alpha \to \beta$ holds and $\alpha \to \gamma$ holds, then $\alpha \to \beta\gamma$ holds
  - **Decomposition**: if $\alpha \to \beta\gamma$ holds, then $\alpha \to \beta$ holds and $\alpha \to \gamma$ holds
  - **Pseudotransitivity**: if $\alpha \to \beta$ holds and $\gamma\beta \to \delta$ holds, then $\alpha\gamma \to \delta$ holds
- The above rules can be inferred from basic Armstrong's axioms (and hence are not included in the basic set). They can be proven independently too
  - **Reflexivity**: if $\beta \subseteq \alpha$, then $\alpha \to \beta$
  - **Augmentation**: if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$
  - **Transitivity**: if $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$

$\boxed{\gamma\alpha \to \delta}$

12) Pseudo-transitivity is a derived Armstrong's axiom. From among the options, select those basic Armstrong's axiom(s) which are used to derive the pseudo-transitivity axiom. ***2 points***

- [ ] Composition Axiom
- [ ] Decomposition Axiom
- [ ] Transitivity Axiom
- [ ] Augmentation Axiom

$A \to BCD$

$A \to B$

$A \to C$

$A \to D$

$A \to BCGI$

Functional Dependencies : Closure of Attribute Sets: Example

- $R = (A, B, C, G, H, I)$
- $F = \{A \rightarrow B,\ A \rightarrow C,\ CG \rightarrow H,\ CG \rightarrow I,\ B \rightarrow H\}$
- $(AG)^+$
  1. result = AG
  2. result = ABCG ($A \rightarrow C$ and $A \rightarrow B$)
  3. result = ABCGH ($CG \rightarrow H$ and $CG \subseteq AGBC$)
  4. result = ABCGHI ($CG \rightarrow I$ and $CG \subseteq AGBCH$)
- Is AG a candidate key?
  1. Is AG a super key?
     1. Does $AG \rightarrow R$? == Is $(AG)^+ \supseteq R$
  2. Is any subset of AG a superkey?
     1. Does $A \rightarrow R$? == Is $(A)^+ \supseteq R$
     2. Does $G \rightarrow R$? == Is $(G)^+ \supseteq R$

A   A, B, C

$(AG) = \{AGBCHI\} = R$

AG must be part C.K

$(\text{Key})^+ = \{R\}$

AG   AGB, AGC

$(A)^+ = \{ABCH\}$

$(G)^+ = \{G\} \neq R$

$A^+ = R$

BC

BD

A is C.K

→ Every C.K is S.K

15. Consider a relation $\boldsymbol{R}(P, Q, R, S, T, U)$ with the following functional dependencies:

$$\mathcal{F} = \{PQ \rightarrow R, S \rightarrow P, RS \rightarrow T, PR \rightarrow UT, TS \rightarrow Q\}$$

Which among the following option(s) is/are the super key(s)? [MSQ: 1 Point]

Answer: Option A & Option D

- ○ PQS $\quad (PQS)^+ = \{PQSRTU\} = R$
- ○ PS $\quad (PS)^+ = \{PS\}$
- ○ QRU
- ○ QS

$(QRU)^+ = \{QRU\}$

$(QS)^+ = \{QSPR\} = R$

$(SQ)^+ =$

$(SP)^+$

$(SQ)^+ = \{SQPR$

SR

ST

SU

$(key)^+ = R$

S must be part of C.K

$(S)^+ = \{SP\}$

7) Let $R(V, W, X, Y, Z)$ be a given relation with the following functional dependencies:
$\mathcal{F} = \{V \to W, WX \to Z, YZ \to V\}$
Then, which among the following is/are the candidate key(s)?

- [ ] XY
- [ ] WXY
- [ ] VXY
- [ ] VW

XYVZV
XYVW

(X,Y) must be part of C.K

$(XY)^+ = \{XY\}$
$(XYV)^+ = \{XYVWZ\} = R$
$(XYW)^+ = \{XYWZV\} = R$
$(XYZ)^+ = \{XYZVW\} = R$

Extraneous Attributes

- Consider a set F of FDs and the FD $\alpha \rightarrow \beta$ in F.
  - Attribute A is **extraneous** in α if $A \in \alpha$ and F logically implies $(F - \{\alpha \rightarrow \beta\}) \cup \{(\alpha - A) \rightarrow \beta\}$.
  - Attribute A is **extraneous** in β if $A \in \beta$ and the set of FDs $(F - \{\alpha \rightarrow \beta\}) \cup \{\alpha \rightarrow (\beta - A)\}$ logically implies F.
- *Note:* Implication in the opposite direction is trivial in each of the cases above, since a "stronger" functional dependency always implies a weaker one
- Example: Given $F = \{A \rightarrow C, AB \rightarrow C\}$ $\rightarrow \{A \rightarrow C, A \rightarrow C\}$ $BA \rightarrow C$
  - B is extraneous in $AB \rightarrow C$ because $\{A \rightarrow C, AB \rightarrow C\}$ logically implies $A \rightarrow C$ (that is, the result of dropping B from $AB \rightarrow C$). $B - C$
  - $A^+ = AC$ in $\{A \rightarrow C, AB \rightarrow C\}$
- Example: Given $F = \{A \rightarrow C, AB \rightarrow CD\}$ $\Rightarrow \{A \rightarrow C, AB \rightarrow D\} \Rightarrow (AB)^+ = \{ABC$
  - C is extraneous in $AB \rightarrow CD$ since $AB \rightarrow C$ can be inferred even after deleting C
  - $AB^+ = ABCD$ in $\{A \rightarrow C, AB \rightarrow D\}$ $(AB \rightarrow D)$

$\rightarrow$ Check for 'D' = $A \rightarrow C$, $AB \rightarrow C$

Canonical Cover

- A **Canonical Cover** for F is a set of dependencies F_c such that ALL the following properties are satisfied:
  - $F^+ = F_c^+$. Or,
    - F logically implies all dependencies in F_c
    - F_c logically implies all dependencies in F
  - No functional dependency in F_c contains an extraneous attribute
  - Each left side of functional dependency in F_c is unique. That is, there are no two dependencies $\alpha_1 \rightarrow \beta_1$ and $\alpha_2 \rightarrow \beta_2$ in such that $\alpha_1 \rightarrow \alpha_2$
- Intuitively, a **Canonical cover** of F is a **minimal** set of FDs
  - Equivalent to F
  - Having no redundant FDs
  - No redundant parts of FDs
- **Minimal / Irreducible Set of Functional Dependencies**

Ra

F_c^+ G

F = G

F covers G

⟹ G covers F

Fc

Fc: Step 1: Find out extraneous

Step 2: Remove Reducdnt FDs

$\{A \rightarrow B$

$A \rightarrow CE\}$

$\{A \rightarrow BCE\}$

mod 2

11) Consider a relation $\mathbf{R}(I, J, K, L, M, N)$ with the following set of functional dependencies.

$KN \to J$

(1) Find out Extran

$\mathcal{F} = \{IK \to N, L \to MN, JK \to L, KN \to JL, IKL \to J, KM \to IN\}$

$KM \to N$

From among the following options, choose all the sets of functional dependencies which serve as canonical covers of $\mathcal{F}$ on $\mathbf{R}$.

take closure of $(IK)^+$ based Fc

- [ ] $\mathcal{F}_C = \{L \to MN, JK \to L, KM \to I, KM \to N\}$
- [ ] $\mathcal{F}_C = \{L \to MN, JK \to L, KM \to I, IK \to N, KN \to J\}$
- [ ] $\mathcal{F}_C = \{L \to MN, JK \to L, KM \to IJ, IK \to J\}$
- [ ] $\mathcal{F}_C = \{L \to MN, JK \to L, KM \to I, IK \to J, KN \to L\}$

$(IK)^+ =$

$(KN)^+ = \{KN\}$

$KN \to L$

$(KN)^+ = \{KNJL$

$(KM)^+ = \{KMIN\}$

$\Rightarrow$ Both F & Fc must be equiv

$(KM)^+ = \{KMINJL$

$(IK)^+ = \{IKNJL$

$KM \to N$

$\Rightarrow$ Equivalence

$F^+ = Fc^+$

Lossless Join Decomposition

- For the case of $R = (R_1, R_2)$, we require that for all possible relations r on schema R

$$r = \pi_{R_1}(r) \bowtie \pi_{R_2}(r)$$

$R_2 \bowtie R_1$

- A decomposition of R into R_1 and R_2 is lossless join if at least one of the following dependencies is in F^+:
  - $R_1 \cap R_2 \rightarrow R_1$
  - $R_1 \cap R_2 \rightarrow R_2$
- The above functional dependencies are a sufficient condition for lossless join decomposition; the dependencies are a necessary condition only if all constraints are functional dependencies

To Identify whether a decomposition is lossy or lossless, it must satisfy the following conditions:

- $R_1 \cup R_2 = R$
- $R_1 \cap R_2 \neq \phi$ and
- $R_1 \cap R_2 \rightarrow R_1$ or $R_1 \cap R_2 \rightarrow R_2$

all the condition must be satisfied

4) Consider a relation $\mathbf{R}(A, B, C, D, E, F, G, H, I, J)$ having functional dependencies as follows $\mathcal{F} = \{AB \to C, B \to F, D \to IJ, A \to DE, F \to GH\}$. ***4 points***

Then, which among the following is a lossless decomposition of $\mathbf{R}$

1

- R1 (A, B, C, I, J), R2 (D, E, F) and R3 (G, H, I)
- R1(A, B, C), R2 (D, E, F), R3(G, H) and R4 (D, I, J) lossy
- R1 (A, B, C), R2 (A, D, E), R3(B, F) R4 (F, G, H) and R5 (D, I, J)
- R1 (A, B, C, D), R2 (D, E), R3 (B, F), R4 (F, G, H) and R5 (D, I, J)

$R_1 \cup R_2 \cup R_3 \ldots \cup R_n = R$

$R_1 \cup R_2 \cup$

$R_3 \cup R_4 = \{R_3$

$R_2 \cap R_4 = (D)^+ = \{DIJ\}$

So $R_{24} = R_2 \cup R_4 = (DEFIJ)$

Option C $R_1 \cup R_2 \cup R_3 \cup R_4 \cup R_5 = \{R\}$

$R_1 \cap R_2 = \{A\}^+ = \{ADE\} = R_2$

So $R_{12} = R_1 \cup R_2 = (ABCDE)$,

$R_3 \cap R_4 = (F)^+ = \{FGH\} = R_4$

So $R_{34} = R_3 \cup R_4 = \{BFGH\}$

$R_{12} \cap R_5 = (D)^+ = \{DIJ\} = R_5$

So $R_{125} = R_{12} \cup R_5 = \{ABCDEIJ\}$

$R_{34} \cap R_{125} = \{B\}^+ = \{BFGH\} = R_{34}$

So $R_{34} \cup R_{125} = R_{12345} = R$

Dependency Preservation

$F = \{A \to BC, C \to E, B \to C,$ R_1

- Let F_i be the set of dependencies F^+ that include only attributes in R_i
  - A decomposition is **dependency preserving**, if

$$(F_1 \cup F_2 \cup \cdots \cup F_n)^+ = F^+$$

  - If it is not, then checking updates for violation of functional dependencies may require computing joins, which is expensive

Let R be the original relational schema having FD set F. Let R_1 and R_2 having FD set F_1 and F_2 respectively, are the decomposed sub-relations of R. The decomposition of R is said to be preserving if

- $F_1 \cup F_2 \equiv F$ {Decomposition Preserving Dependency}
- If $F_1 \cup F_2 \subset F$ {Decomposition NOT Preserving Dependency} and
- $F_1 \cup F_2 \supset F$ {this is not possible}

9) In the relational schema given below, all attributes take atomic values only.

$\mathbf{R}(bike\_name, bike\_model, bike\_engine\_no)$

Suppose $\mathbf{R}$ satisfies the following functional dependencies:

$F =$ { $bike\_name \rightarrow bike\_model$

$bike\_model \rightarrow bike\_engine\_no$

$bike\_engine\_no \rightarrow bike\_name$

$\boxed{be \rightarrow bn}$

If $\mathbf{R}$ is decomposed into:

$\mathbf{R1}(bike\_name, bike\_model)$ and

$\mathbf{R2}(bike\_model, bike\_engine\_no)$,

then the decomposition is:

- a lossless decomposition as well as a dependency preserving one
- not a lossless decomposition but dependency preserving
- a lossless decomposition but not dependency preserving
- neither a lossless decomposition nor dependency preserving

$F_1 = \{ bn \rightarrow bm$, $bm \rightarrow bn \}$

$F_2 = \{ bm \rightarrow bE$, $bE \rightarrow bm \}$

$R_1 \cup R_2 = R \}$

$R_1 \cap R_2 = (bike\_model)^+ = (bm, bE, bn)$

$= \{ bm, bE$ R_2

$R_1 \cap R_2 \rightarrow R_1$

or

R_2

$(be)^+ = \{ be, bm$

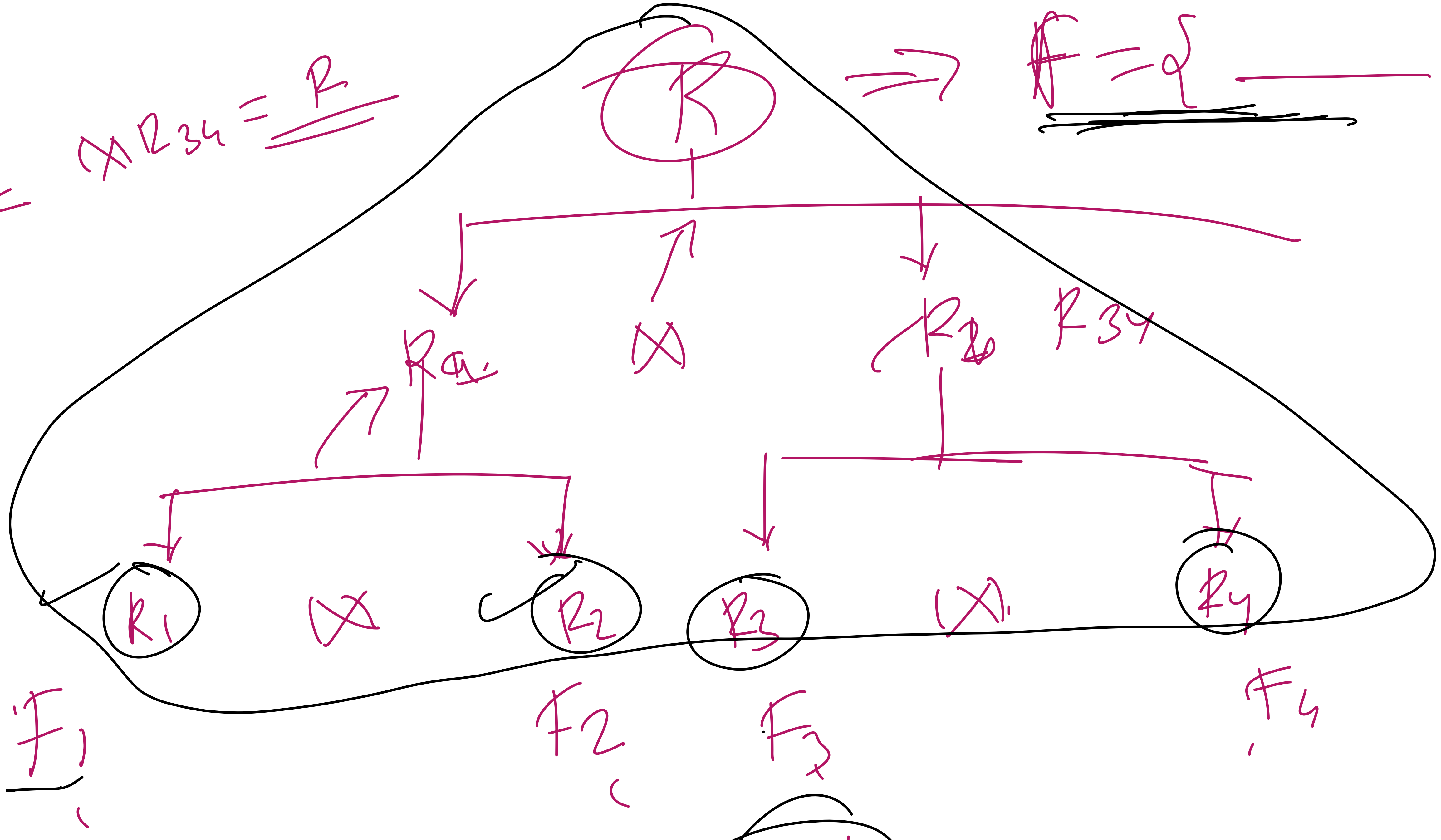
$F_1 \cup F_2 = \{ bn \rightarrow bm, bm \rightarrow bn, bm \rightarrow bE, be \rightarrow bm \}$

$be = \{ be, bm, bn \}$

$\boxed{be \rightarrow bn}$

$R_{12} \bowtie R_{34} = R$

$F = \{ \quad \}$



$F_1 \cup F_2 \cup F_3 \cup F_4 = F^+$

case 2 C.K :- A & B, & C

S.K based of A = 2^{n-m} = 2^{4-1} = 2^3 = 8

S.K based on B = 8

AB = 2^{n-m} = 2^{4-2} = 4

$(A \cap B) = n(A) + n(B) - n(A \cup B)$

= 8 + 8 = 4

= 12

AB

$2^{n-m} = 2^{4-2} = 4$

A, B, C & D

AB

ABC

ABC

2^{n-m} → C.K

2^{n-m}

= $2^n - 1$

$2^4 - 1$

= 15

$R(\underline{A\ B\ C\ D})$. $\underline{n}$, $\underline{m}$

$F = \{A \to B,\ B \to C,\ C \to D,\ D \to A\}$

(1) C.K :- $(A)^+ = \{R\}$ & B

$\to$ no. of S.K $= 2^{n-m} = 2^{4-1} = 2^3 = 8$

$n \to$ total no. of attribut in R

$m \to$ no. of attribute present in C.K

$\boxed{\text{no. of S.K} = 2^{n-m}}$ =

A

AB

AC

AD

ACD

ABC

ABD

ABCD

8

15. In the relational schema given below, the domains of all its attributes are atomic only.

R$(employee\_num, employee\_name, department\_num, department\_name)$

Suppose **R** satisfies the following functional dependencies:

$$\{employee\_num \rightarrow employee\_name,$$
$$department\_num \rightarrow department\_name,$$
$$employee\_num \rightarrow department\_name\}$$

If **R** is decomposed into:
R1$(employee\_num, employee\_name)$ and
R2$(department\_num, department\_name)$,
then, which among the following statement(s) is/are correct? [MSQ: 4 points]
Answer: Option A & Option B

- ◯ **R1** and **R2** are in BCNF, but decomposition is not a dependency preserving one.
- ◯ **R1** and **R2** are in BCNF, but decomposition is a lossy.
- ◯ **R1** and **R2** are in BCNF, but decomposition is a lossless decomposition as well as dependency preserving.
- ◯ **R1** and **R2** are not in BCNF, and decomposition is neither a lossless nor dependency preserving.

3. Consider the relational schema $\mathbf{R}(A, B, C, X, U)$ with the following functional dependencies (assume that all the attributes have atomic values).

$\mathcal{F} = \{U \rightarrow B,$
$XA \rightarrow C,$
$XA \rightarrow U,$
$B \rightarrow A,$
$XA \rightarrow A$
$\}$

Check if the relation schema $\mathbf{R}$ is in third normal form or not. If not, which of the following functional dependency can be removed to make the relation in third normal form?

[MCQ: 3 points]

Answer: Option D

- ○ U → B
- ○ B → A
- ○ XA → U
- ○ R is in third normal form.

5. Consider a relation **CustomerLogs**($Name, Items, Restaurant, Date$) with the following data values.

[MCQ: 4 points]

Name	Items	Restaurant	Date
Zury	Coffee	Your's cafe	19-10-21
Zury	Tea	Our's cafe	21-10-21
Zury	Tea	C	E
Zury	A	B	D

If multivalued dependency ($Name \twoheadrightarrow \{Items, Date\}$) exists in the above **CustomerLogs** relation, then what are the values of A, B, C, D, E?

Answer: Option C

- ○ A = Tea, B = Your's cafe, C = Our's cafe, D = 21-10-21, E = 19-10-21
- ○ A = Coffee, B = Your's cafe, C = Our's cafe, D = 21-10-21, E = 19-10-21
- ○ A = Coffee, B = Our's cafe, C = Your's cafe, D = 19-10-21 , E = 21-10-21
- ○ A = Tea, B = Our's cafe, C = Your's cafe, D = 19-10-21, E = 21-10-21