

Physics of Semiconductor: Lecture # Lec 4

Dr. Sudipta Som

Department of Physics

Shiv Nadar University Chennai

Intrinsic semiconductor- Calculation of electron density

$$n = \int_{E_C}^{\infty} Z(E) f(E) dE$$

The above equation is the expression for the **electron concentration** in the conduction band of an intrinsic semiconductor.

Designating

$$N_C = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2} \quad (30.29)$$

$$Z(E) dE = \frac{4\pi}{h^3} (2m_e^*)^{3/2} E^{1/2} dE \quad \text{for } E > E_C.$$

$$f(E) = \frac{1}{1 + \exp[(E - E_F) / kT]}$$

$$n = N_C e^{-(E_C - E_F)/kT} \quad (30.30)$$

N_C is a temperature dependent material constant known as the **effective density of states** in the conduction band. In silicon at 300 K, $N_C = 2.8 \times 10^{25}/\text{m}^3$.

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} \int_{E_C}^{\infty} (E - E_C)^{1/2} e^{-(E - E_F)/kT} dE$$

$$n = \frac{4\pi}{h^3} (2m_e^*)^{3/2} e^{(E_F - E_C)/kT} \int_{E_C}^{\infty} (E - E_C)^{1/2} e^{-(E - E_C)/kT} dE$$

$$n = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2} e^{-(E_C - E_F)/kT}$$

Electron density available for conduction, $n = N_C e^{-(E_C - E_F)/kT}$

30.8.2 Calculation of Hole Density

Let dp be the number of holes whose energy lies in the energy interval E and $E + dE$ in the valence band. Then,

$$dp = Z(E)[1 - f(E)]dE \quad (30.31)$$

We can write

$$[1 - f(E)] = 1 - \frac{1}{1 + e^{(E - E_F)/kT}} = \frac{1}{1 + e^{(E_F - E)/kT}} \approx e^{-(E_F - E)/kT} \quad (30.32)$$

The density of states in the valence band is given by

$$Z(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} E^{1/2} dE \quad (30.33)$$

Intrinsic semiconductor- Calculation of hole density

The top edge of the valence band E_V corresponds to the potential energy of a hole at rest. Therefore, $(E_V - E)$ will be the kinetic energy of the hole at lower energy levels. Hence, equ. (30.23) is to be modified as follows.

$$Z(E)dE = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{1/2} dE \quad (30.34)$$

The number of holes in the energy interval E and $E + dE$ is

$$dp = \frac{4\pi}{h^3} (2m_h^*)^{3/2} (E_V - E)^{1/2} e^{-(E_F - E)/kT} dE \quad (30.35)$$

$$p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_F - E)/kT} dE \quad (30.36)$$

$$= \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{-(E_F - E_V)/kT} \int_{-\infty}^{E_V} (E_V - E)^{1/2} e^{-(E_V - E)/kT} dE \quad (30.37)$$

Intrinsic semiconductor- Calculation of hole density

The integral in equ. (30.37) is of the standard form which has a solution of the following form.

$$\int_0^{\infty} x^{1/2} e^{-ax} dx = \frac{\sqrt{\pi}}{2a\sqrt{a}}$$

where $a = 1/kT$ and $x = (E_V - E)$.

$$\therefore p = \frac{4\pi}{h^3} (2m_h^*)^{3/2} e^{-(E_F - E_V)/kT} \left[\frac{\sqrt{\pi}}{2} (kT)^{3/2} \right] \quad (30.38)$$

Rearranging the terms we get

$$\text{or} \quad p = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2} e^{-(E_F - E_V)/kT} \quad (30.39)$$

The above equation is the **expression for the hole concentration** in the valence band of an intrinsic semiconductor.

$$\text{Denoting} \quad N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2} \quad (30.40)$$

$$\therefore p = N_V e^{-(E_F - E_V)/kT} \quad (30.41)$$

N_V is called the **effective density of states** in the valence band. For silicon at 300 K, $N_V = 10^{25}/\text{m}^3$. It is seen that $N_C/N_V = 2.8$. To a first approximation, N_C is taken to be equal to N_V .

30.9 INTRINSIC CARRIER CONCENTRATION

A single event of bond breaking in a pure semiconductor leads to generation of an *electron-hole* pair. At any temperature T , the number of electrons generated will be equal to the number of holes generated. As the two charge carrier concentrations are equal, they are denoted by a common symbol n_i , which is called *intrinsic density* or *intrinsic concentration*. Thus,

$$n = p = n_i$$

We can write that

$$n_i^2 = np \quad (30.42)$$

$$\begin{aligned} &= (N_C e^{-(E_C - E_F)/kT}) (N_V e^{-(E_F - E_V)/kT}) \\ &= (N_C N_V) e^{-(E_C - E_V)/kT} \end{aligned} \quad (30.43)$$

The term $(E_C - E_V)$ stands for the difference in energy between the top level of valence band and the bottom level of conduction band. Thus, it represents the separation between the valence and conduction bands, which is the band gap E_g .

$$(E_C - E_V) = E_g \quad (30.44)$$

$$\therefore n_i^2 = (N_C N_V) e^{-E_g/kT}$$

Intrinsic semiconductor- carrier concentration

Substituting the values of N_C and N_V into the above equation, we obtain

$$= 4 \left[\frac{2\pi kT}{h^2} \right]^3 (m_e^* m_h^*)^{3/2} e^{-E_g/kT}$$

$$\therefore n_i = 2 \left[\frac{2\pi kT}{h^2} \right]^{3/2} (m_e^* m_h^*)^{3/4} e^{-E_g/2kT} \quad (30.45)$$

This is the expression for intrinsic carrier concentration.

30.9.1 Variation of Intrinsic Carrier Concentration with Temperature

Equ. (30.45) may be written as

$$n_i = 2 \left[\frac{2\pi k}{h^2} \right]^{3/2} (m_e^* m_h^*)^{3/4} T^{3/2} e^{-E_g/2kT} \quad (30.46)$$

The above relation shows that the free charge carrier concentration varies with temperature.

Equ. (30.46) may be approximated to

$$n_i = 10^{21.7} T^{3/2} \times 10^{-2500 E_g/T} \quad (30.46a)$$

The following important points may be inferred from the relation (30.46):

- *The intrinsic concentration is independent of Fermi level.*
- *The intrinsic concentration has an exponential dependence on the band gap value E_g .*
- *It strongly depends on the temperature.*
- *The factor 2 in the exponent indicates that two charge carriers are produced for one covalent bond broken.*

The experimental value of n_i in silicon at room temperature is 1.5×10^{16} carriers/m³ and in germanium 2.5×10^{19} carriers/m³.

Intrinsic semiconductor- carrier concentration

Example 30.1: *The forbidden gap in pure silicon is 1.1 eV. Compare the number of conduction electrons at temperatures 37°C and 27°C.*

Solution: Let n_1 be the number of conduction electrons at 27°C and n_2 at 37°C. Using equn. (30.46 a), we get

and

$$\begin{aligned}n_1 &= 10^{21.7} (300 \text{ K})^{3/2} (10^{-2500 \times 1.1\text{eV}/300 \text{ K}}) \\n_2 &= 10^{21.7} (300 \text{ K})^{3/2} (10^{-2500 \times 1.1\text{eV}/310 \text{ K}}) \\ \frac{n_2}{n_1} &= \frac{(310\text{K})^{3/2} \times 10^{-8.87}}{(300\text{K})^{3/2} \times 10^{-9.2}} = \mathbf{2.96}\end{aligned}$$

Thus the number of electrons in the conduction band increased nearly three-fold as the temperature of the material increased from 27°C to 37°C.

Intrinsic semiconductor- carrier concentration

Example. 30.2: Compute the concentration of intrinsic charge carriers in a germanium crystal at 300 K. Given that $E_g = 0.72 \text{ eV}$ and assume $m_e^* = m_e$.

Solution: Intrinsic charge carrier concentration, $n_i = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2} \exp\left(-\frac{E_g}{2kT}\right)$

$$2 \left[\frac{2\pi m_e^* k}{h^2} \right]^{3/2} = 2 \left[\frac{2 \times 3.143 \times 9.11 \times 10^{-31} \text{ kg} \times 1.38 \times 10^{-23} \text{ J/K}}{(6.626 \times 10^{-34} \text{ J.s})^2} \right]^{3/2}$$

$$= 4.83 \times 10^{21}, T^{3/2} = 300^{3/2} = 5196$$

and

$$\exp\left(-\frac{E_g}{2kT}\right) = \exp\left(-\frac{0.72 \text{ eV}}{2 \times 8.61 \times 10^{-5} \text{ eV/K} \times 300 \text{ K}}\right)$$

$$= \exp(-13.846) = 9.7 \times 10^{-7}$$

\therefore

$$n_i = 4.83 \times 10^{21} \times 5196 \times 9.7 \times 10^{-7} = \mathbf{3.4 \times 10^{19}/m^3}.$$

Intrinsic semiconductor- carrier concentration

30.10 THE FRACTION OF ELECTRONS IN THE CONDUCTION BAND

The Fermi-Dirac probability function gives the fractional occupancy of the energy states.

Thus, $f(E) = \frac{1}{1 + \exp[(E - E_F)/kT]}$ gives the probability that an electron occupies the energy state E . The probability that an electron occupies the energy state E_C may then be found from

$$f(E_C) = \frac{1}{1 + \exp[(E_C - E_F)/kT]} \quad (30.47)$$

But, according to the definition of probability, $f(E_C) = \frac{n}{N}$

where n is the number of electrons excited to conduction band levels and N is the total number of electrons available in the valence band initially.

$$\therefore \frac{n}{N} = \frac{1}{1 + \exp[(E_C - E_F)/kT]} \quad (30.48)$$

As $(E_C - E_F) = E_g/2$, we write

$$\frac{n}{N} = \frac{1}{1 + \exp[E_g/2kT]}$$

Since $E_g > 2kT$, the factor unity may be neglected in comparison to the exponential term.

$$\therefore \frac{n}{N} = \frac{1}{\exp[E_g/2kT]} = e^{-E_g/2kT} \quad (30.49)$$

The above equation gives the fraction of electrons in the conduction band.

Intrinsic semiconductor- carrier concentration

Example 30.3. What is the probability of an electron being thermally promoted to the conduction band in diamond at 27° C, if the bandgap is 5.6 eV wide?

Solution. The probability that an electron being thermally promoted to the conduction band is given by

$$f(E_C) = \frac{1}{1 + \exp\left(\frac{E_g}{2kT}\right)} = \frac{1}{1 + \exp\left[\frac{5.6 \text{ eV}}{2(0.026) \text{ eV}}\right]}$$

$$= \frac{1}{1 + e^{107.69}} = 1.7 \times 10^{-47}$$

Example 30.5: Assuming that the number of electrons near the top of the valence band available for thermal excitation is $5 \times 10^{25}/\text{m}^3$ and the intrinsic carrier density is $2.5 \times 10^{19}/\text{m}^3$, calculate the energy gap of germanium at room temperature.

Solution: Fraction of electrons in conduction band $\frac{n}{N} = \exp\left[-\frac{E_g}{2kT}\right]$

$$\therefore E_g = -2kT \ln\left(\frac{n}{N}\right)$$

$$= -2(8.61 \times 10^{-5} \text{ eV/K})(300 \text{ K}) \ln\left[\frac{2.5 \times 10^{19}/\text{m}^3}{5 \times 10^{25}/\text{m}^3}\right]$$

$$= 0.052 \times 14.509 \text{ eV} = 0.75 \text{ eV}.$$

Example 30.6. Estimate the fraction of electrons in conduction band at room temperature in Ge with $E_g = 0.72 \text{ eV}$ and in diamond with $E_g = 5.6 \text{ eV}$.

Solution. Fraction of electrons in conduction band $\frac{n}{N} = \exp\left[-\frac{E_g}{2kT}\right]$

(i) Fraction of electrons in conduction band in Ge,

$$\frac{n}{N} = \exp\left[-\frac{0.72 \text{ eV}}{2 \times 0.026 \text{ eV}}\right] = \exp(-13.846) = 9.7 \times 10^{-7}.$$

(ii) Fraction of electrons in conduction band in diamond,

$$\frac{n}{N} = \exp\left[-\frac{5.6 \text{ eV}}{2 \times 0.026 \text{ eV}}\right]$$

$$= \exp(-107.692) = 1.7 \times 10^{-47}.$$

Fermi level in Intrinsic semiconductor

30.11 FERMİ LEVEL IN INTRINSIC SEMICONDUCTOR

In a pure semiconductor, the electrons in the conduction band cluster very close to the bottom edge of the band, and we assume that electrons are located right at the bottom edge of the conduction band, as shown in Fig. 30.9. Similarly, we assume that the holes are at the top edge of the valence band. The electron concentration in the conduction band is given by

$$n = N_C e^{-(E_C - E_F)/kT}$$

The hole concentration in the valence band is given by

$$p = N_V e^{-(E_F - E_V)/kT}$$

In an intrinsic semiconductor, the electron and hole concentrations are equal. Thus, $n = p$

$$N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT} \quad (30.50)$$

Taking logarithm on both sides, we get

$$-\frac{(E_C - E_F)}{kT} = \ln \frac{N_V}{N_C} - \frac{(E_F - E_V)}{kT}$$

$$-E_C + E_F = kT \ln \frac{N_V}{N_C} - E_F + E_V$$

$$2E_F = (E_C + E_V) + kT \ln \frac{N_V}{N_C}$$

$$E_F = \frac{E_C + E_V}{2} + \frac{1}{2} kT \ln \frac{N_V}{N_C} \quad (30.51)$$

$$\text{But } N_C = 2 \left[\frac{2\pi m_e^* kT}{h^2} \right]^{3/2} \text{ and } N_V = 2 \left[\frac{2\pi m_h^* kT}{h^2} \right]^{3/2}$$

$$\therefore \frac{N_V}{N_C} = \left(\frac{m_h^*}{m_e^*} \right)^{3/2}$$

$$\therefore \ln \left(\frac{N_V}{N_C} \right) = \frac{3}{2} \ln \left(\frac{m_h^*}{m_e^*} \right)$$

$$\therefore E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln \left(\frac{m_h^*}{m_e^*} \right)$$

$$E_F = \frac{E_g}{2}$$

If the effective mass of a free electron is assumed to be equal to the effective mass of a hole, i.e.,

$$m_h^* = m_e^* \\ \ln \left(\frac{m_h^*}{m_e^*} \right) = 0$$

\therefore

$$E_F = \frac{E_C + E_V}{2}$$

To make the meaning of the above equation more explicit, we write

$$E_F = \frac{E_C - E_V}{2} + E_V$$

$$E_F = \frac{E_g}{2} + E_V$$

If we denote the top of the valence band E_V as zero level, $E_V = 0$.

The above result shows that in an intrinsic semiconductor the Fermi level lies in the middle of the forbidden gap.

30.11.1 Variation of Fermi Level with Temperature in an Intrinsic Semiconductor

With an increase in temperature, the Fermi level gets displaced upward to the bottom edge of the conduction band if $m_h^* > m_e^*$ or downward to the top edge of the valence band if $m_h^* < m_e^*$, as indicated in Fig. 30.10.

In most of the materials, the shift of Fermi level on account of $m_h^* \neq m_e^*$ is insignificant. The Fermi level in an intrinsic semiconductor may be considered as independent of temperature and as staying in the middle of the band gap.

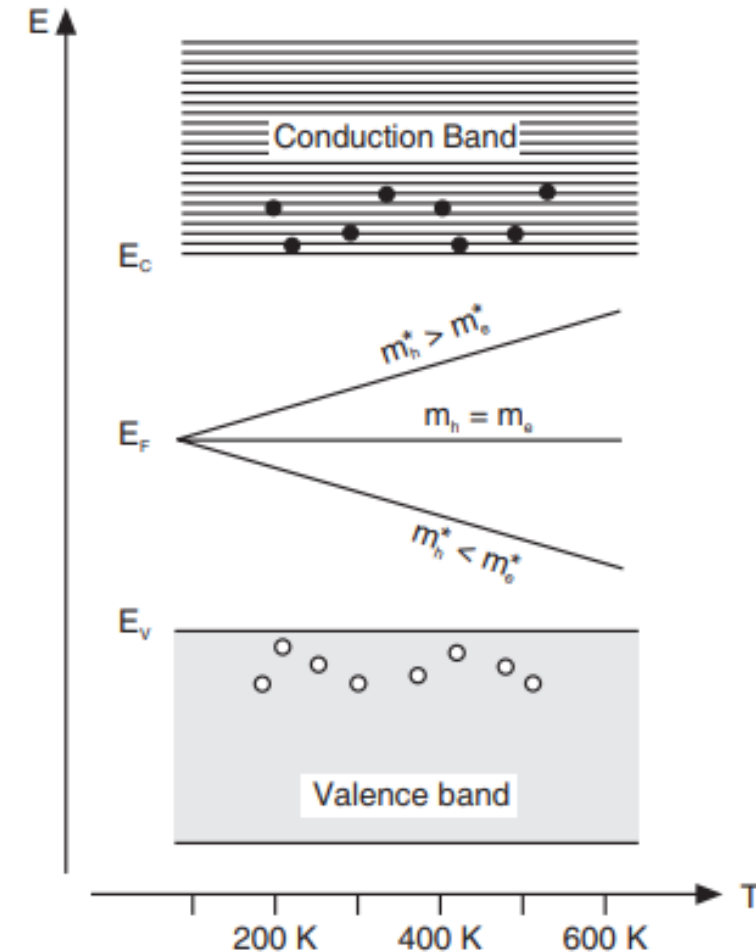


Fig. 30.10: Variation of Fermi level with temperature in an intrinsic semiconductor

Electrical conductivity

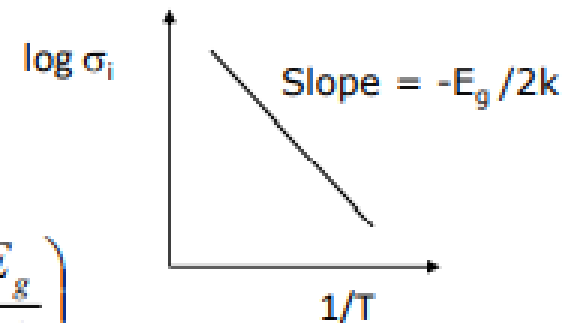
$$\begin{aligned}
 \sigma_i &= ne\mu_e + pe\mu_h \\
 &= n_i e(\mu_e + \mu_h) \quad \text{as } n_i = n = p \quad \text{Substituting for } n_i \\
 &= (\mu_e + \mu_h)e \quad 2\left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-\left(E_g/2kT\right)}
 \end{aligned}$$

μ_e and μ_h are determined by scattering from lattice vibrations (phonons) and has a dependence of $T^{-3/2}$, therefore temperature dependence of conductivity is determined by the exponential term only and all the rest of the terms can be accounted for as a constant A. Expression for conductivity can be then written as

$$\sigma_i = Ae^{-E_g/2kT}$$

Taking log on either side $\log \sigma_i = \log A - \frac{E_g}{2kT}$

$$\text{slope of } (\log \sigma) \text{ vs } (1/T) \text{ curve} = \frac{\partial(\log \sigma)}{\partial(1/T)} = 0 - \left(\frac{E_g}{2k}\right) = -\left(\frac{E_g}{2k}\right)$$



$$I/A = J = \sigma E = \sigma V/d$$

Do an experiment with a semiconductor keeping E constant (ensure that only thermally generated carriers are contributing to the current), and then heat the device. Measure change in I with temperature, you will essentially be tracking variation of σ with temperature.

Experimental determination of bandgap from R vs T variation

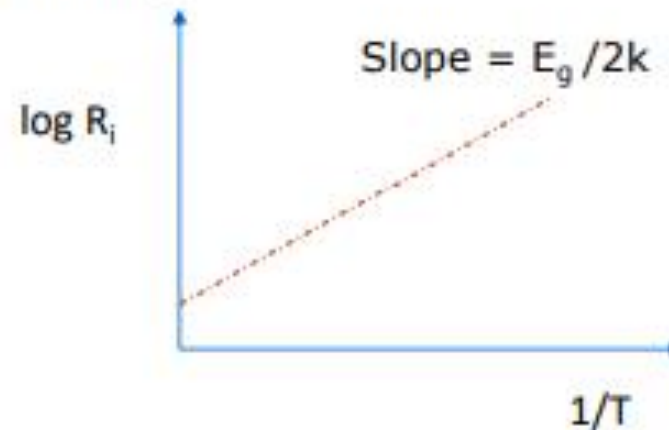
Resistivity, $\rho_{\text{intrinsic}} = 1/\sigma_{\text{intrinsic}} = R_i a/L$, where L-length, a-Area of semiconductor

Therefore, resistance of intrinsic semiconductor,

$$R_i = L/a\sigma_{\text{intrinsic}}$$
$$= \frac{L}{aA} e^{E_g/2kT}$$

$\boxed{\frac{L}{aA}} = C$

$$\log R_i = \log C + \frac{E_g}{2kT}$$



- ▶ Resistance R_i of a given slab of intrinsic material is measured as a function of temperature
- ▶ A graph is plotted between log of resistance in Y axis and $1/T$ in the X- axis.
- ▶ E_g is determined as $(2k \times \text{slope of line})$

30.14 LIMITATIONS OF INTRINSIC SEMICONDUCTOR

Intrinsic semiconductors are not useful for device manufacture because of low conductivity and the strong dependence of conductivity on temperature. If we take a crystal of pure silicon or germanium and connect it in the circuit, we will find that the current in the circuit will gradually increase as the temperature of the crystal is increased. We would also expect the current to increase if the voltage is increased and it does. But *increasing the voltage increases the current only proportionally, obeying Ohm's law, while increasing the temperature increases the current at an exponential rate*. Thus the temperature over which we have no control exerts more influence upon the current than the voltage, which we customarily do control.

We summarize the limitations as follows:

- Conductivity is low. Germanium has a conductivity of 1.67 S/m, which is nearly 10^7 times smaller than that of copper.
- Conductivity is a function of temperature and increases exponentially as the temperature increases.
- Conductivity cannot be controlled from outside.

- Types of semiconductors
- (i) Intrinsic (Elemental & Compound)
 - (ii) Extrinsic

Intrinsic semiconductors are pure semiconductors characterized by completely filled valence shell at 0K ,with conductivity at higher temperatures due to thermally or optically generated electron-hole pairs .($n=p=n_i$)

Conductivity in semiconductors is due to both holes (electrons in valence band) and electrons (in conduction band)

Electron density available for conduction, $n = N_c e^{-(E_c - E_F)/kT}$

Hole density available for conduction, $p = N_v e^{-(E_F - E_v)/kT}$

Fermi level and its position as a function of temperature

$$E_F = \frac{(E_c + E_v)}{2} + \frac{3kT}{4} \ln \left(\frac{m_h^*}{m_e^*} \right)$$

OR

$$E_F = \frac{E_g}{2} + \frac{3kT}{4} \ln \left(\frac{m_h^*}{m_e^*} \right)$$

From top of valence band

Intrinsic carrier concentration and conductivity

$$\therefore n_i = \sqrt{np} = \sqrt{N_c N_v} e^{-\left(E_g / 2kT\right)} = 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-\left(E_g / 2kT\right)}$$

$$\sigma = (\mu_e + \mu_h) e \cdot 2 \left(\frac{2\pi kT}{h^2} \right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-\left(E_g / 2kT\right)}$$

Bandgap measurement

$$\text{slope of } (\log \sigma) \text{ vs } (1/T) \text{ curve} = \frac{\partial(\log \sigma)}{\partial(1/T)} = -\left(\frac{E_g}{2k} \right)$$

Bandgap determined from experiment measuring R vs T. as (2k x slope of plot between log R & 1/T)

For Silicon with a bandgap of 1.12eV, determine position of Fermi level at 300K if $m_e^* = 0.12m_0$ and $m_h^* = 0.28m_0$

For an intrinsic semiconductor with $E_g = 0.7\text{eV}$ and carrier concentration of $33.49 \times 10^{18} / \text{m}^3$, calculate conductivity at 300K assuming $m_e^* = m_h^* = m_0$; $\mu_e = 0.39 \text{ m}^2 / \text{Vs}$, $\mu_h = 0.19 \text{ m}^2 / \text{Vs}$

Find the resistance of an intrinsic Ge slab 1cm long, 1mm wide and 1mm thick if the intrinsic carrier concentration is $2.5 \times 10^{19} / \text{m}^3$, $\mu_e = 0.39 \text{ m}^2 / \text{Vs}$, $\mu_h = 0.19 \text{ m}^2 / \text{Vs}$ at 300K

If the effective mass of holes in an intrinsic material is 4 times that of the electron, find the temperature at which the Fermi level is shifted by 10% from the middle of the forbidden gap of energy 1eV.

For Silicon with a bandgap of 1.12eV, determine position of Fermi level at 300K if $m_e^* = 0.12m_0$ and $m_h^* = 0.28m_0$

Given $E_g = 1.12\text{eV}$, $T = 300\text{K}$, $m_e^* = 0.12m_0$, $m_h^* = 0.28m_0$, position of Fermi level ?

$$E_F = \frac{E_g}{2} + \frac{3kT}{4} \ln \left(\frac{m_h^*}{m_e^*} \right) \quad \text{From top of valence band}$$

$$\begin{aligned} E_F &= \frac{1.12}{2} + \frac{3 \times 1.38 \times 10^{-23} \times 300}{4 \times 1.6 \times 10^{-19}} \ln \left(\frac{0.28}{0.12} \right) \text{ eV} \quad \text{from top of valence band} \\ &= 0.56 + 0.0164 = 0.5764 \text{ eV} \quad \text{from top of valence band} \end{aligned}$$