Physics of Semiconductor: Lecture # Lec 5

Dr. Sudipta Som

Department of Physics

Shiv Nadar University Chennai

SHIV NADAR UNIVERSITY CHENNAI

What we have learnt earlier

Intrinsic semiconductor- Calculation of electron density

$$n = \int_{E_C}^{\infty} Z(E) f(E) dE$$

$$Z(E)dE = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} E^{1/2} dE \quad \text{for } E > E_C.$$

$$f(E) = \frac{1}{1 + \exp\left[\left(E - E_F\right)/kT\right]}$$

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} \int_{E_C}^{\infty} \left(E - E_C\right)^{1/2} e^{-(E-E_F)/kT} dE$$

$$n = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} e^{(E_F-E_C)/kT} \int_{E_C}^{\infty} \left(E - E_C\right)^{1/2} e^{-(E-E_C)/kT} dE$$

$$n = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} e^{-(E_C-E_F)/kT}$$

The above equation is the expression for the **electron concentration** in the conduction band of an intrinsic semiconductor.

Designating
$$N_C = 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2} \qquad (30.29)$$

$$n = N_C e^{-(E_C - E_F)/kT} \qquad (30.30)$$

N_C is a temperature dependent material constant known as the **effective density of states** in the conduction band. In silicon at 300 K, $N_C = 2.8 \times 10^{25}/\text{m}^3$.

Electron density available for conduction, $\boxed{n = N_c e^{-(Ec- EF)/kT}}$

Intrinsic semiconductor- Calculation of hole density

$$dp = Z(E)[1 - f(E)]dE \qquad (30.31)$$

$$[1 - f(E)] = 1 - \frac{1}{1+e^{(E-E_F)/kT}} = \frac{1}{1+e^{(E_F-E)/kT}} \approx e^{-(E_F-E)/kT} \qquad (30.32)$$

$$Z(E)dE = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} E^{1/2}dE \qquad (30.33)$$

$$Z(E)dE = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2}\left(E_V - E\right)^{1/2} dE \qquad (30.34)$$

$$dp = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2}\left(E_V - E\right)^{1/2} e^{-(E_F-E)/kT} dE \qquad (30.35)$$

$$p = \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2}\int_{-\infty}^{E_V}\left(E_V - E\right)^{1/2} e^{-(E_F-E)/kT} dE \qquad (30.36)$$

$$= \frac{4\pi}{h^3}\left(2m_h^*\right)^{3/2} e^{-(E_F-E_V)/kT}\int_{-\infty}^{E_V}\left(E_V - E\right)^{1/2} e^{-(E_V-E)/kT} dE \qquad (30.37)$$

$$N_V = 2\left[\frac{2\pi\, m_h^* kT}{h^2}\right]^{3/2}$$

$$p = N_V e^{-(E_F - E_V)/kT}$$

Intrinsic semiconductor- carrier concentration

INTRINSIC CARRIER CONCENTRATION

$$n = p = n_i$$

$$n^2{}_i = np \tag{30.42}$$

$$= (N_C e^{-(E_C - E_F)/kT})(N_V e^{-(E_F - E_V)/kT})$$

$$= (N_C N_V)^{-(E_C - E_V)/kT} \tag{30.43}$$

$\therefore$ $$n_i^2 = (N_C N_V) e^{-E_g/kT}$$

$$= 4\left[\frac{2\pi\, kT}{h^2}\right]^3 \left(m_e^* m_h^*\right)^{3/2} e^{-E_g/kT}$$

$\therefore$ $$n_i = 2\left[\frac{2\pi kT}{h^2}\right]^{3/2} \left(m_e^* m_h^*\right)^{3/4} e^{-E_g/2kT} \tag{30.45}$$

This is the expression for intrinsic carrier concentration.

Fermi level in Intrinsic semiconductor

FERMI LEVEL IN INTRINSIC SEMICONDUCTOR

$$n = N_C e^{-(E_C - E_F)/kT} \qquad p = N_V e^{-(E_F - E_V)/kT}$$

$$N_C e^{-(E_C - E_F)/kT} = N_V e^{-(E_F - E_V)/kT}$$

$$E_F = \frac{E_C + E_V}{2} + \frac{1}{2} kT \ ln \frac{N_V}{N_C}$$

But $N_C - 2\left[\frac{2\pi m_e^* kT}{h^2}\right]^{3/2}$ and $N_V - 2\left[\frac{2\pi m_h^* kT}{h^2}\right]^{3/2}$

$$\therefore \quad E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ ln\left(\frac{m_h^*}{m_e^*}\right)$$

If the effective mass of a free electron is assumed to be equal to the effective mass of a hole, i.e.,

$$m_h^* = m_e^*$$

$$ln\left(\frac{m_h^*}{m_e^*}\right) = 0$$

$$\therefore \quad E_F = \frac{E_C + E_V}{2}$$

To make the meaning of the above equation more explicit, we write

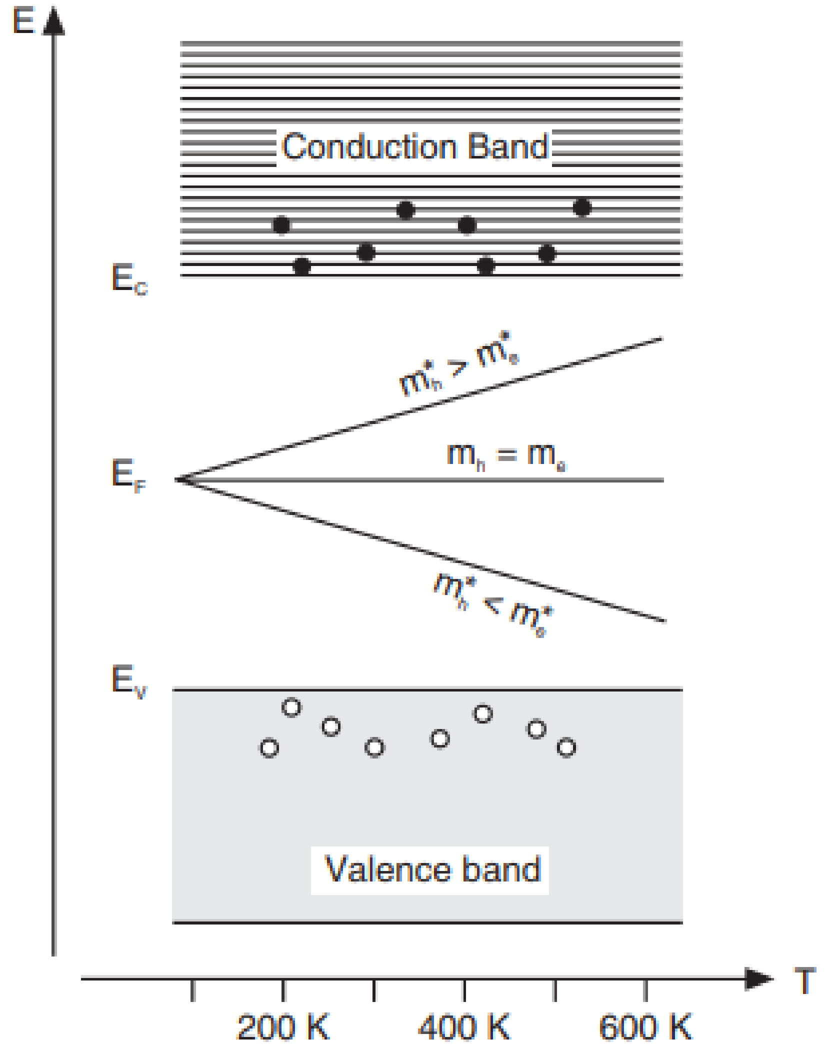
$$E_F = \frac{E_C - E_V}{2} + E_V$$

$$E_F = \frac{E_g}{2} + E_V$$

If we denote the top of the valence band E_V as zero level, $E_V = 0$

$$E_F = \frac{E_g}{2}$$

FERMI LEVEL IN INTRINSIC SEMICONDUCTOR



Electrical conductivity

$$\sigma_i = ne\mu_e + pe\mu_h$$

$$n_i = 2\left[\frac{2\pi kT}{h^2}\right]^{3/2}(m_e^* m_h^*)^{3/4} e^{-E_g/2kT}$$

$$= n_i e(\mu_e + \mu_h) \quad as\ n_i = n = p$$

$$= (\mu_e + \mu_h)e \quad 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_e^* m_h^*)^{3/4} e^{-\left(E_g/2kT\right)}$$

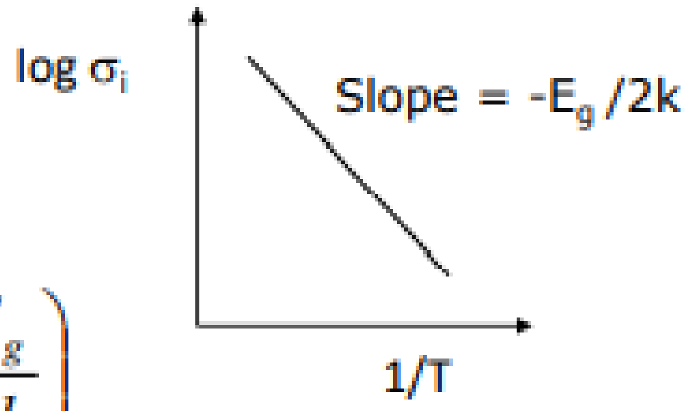
μ_e and μ_h are determined by scattering from lattice vibrations (phonons) and has a dependence of $T^{-3/2}$, therefore temperature dependence of conductivity is determined by the exponential term only and all the rest of the terms can be accounted for as a constant A. Expression for conductivity can be then written as

$$\sigma_i = Ae^{-E_g/2kT}$$

Electrical conductivity

$$\sigma_i = Ae^{-E_g/2kT}$$

Taking log on either side $\quad \log \sigma_i = \log A - \dfrac{E_g}{2kT}$



$$slope\ \ of\ \ (\log\sigma)\, vs\, (1/T)\, curve = \frac{\partial(\log\sigma)}{\partial(1/T)} = 0 - \left(\frac{E_g}{2k}\right) = -\left(\frac{E_g}{2k}\right)$$

$I/A = J = \sigma E = \sigma V/d$

Do an experiment with a semiconductor keeping E constant (ensure that only thermally generated carriers are contributing to the current), and then heat the device .Measure change in I with temperature, you will essentially be tracking variation of σ with temperature.

Determination of band gap

Experimental determination of bandgap from R vs T variation

Resistivity , $\rho_{intrinsic} = 1/\sigma_{intrinsic} = R_i a/L$, where L-length, a-Area of semiconductor

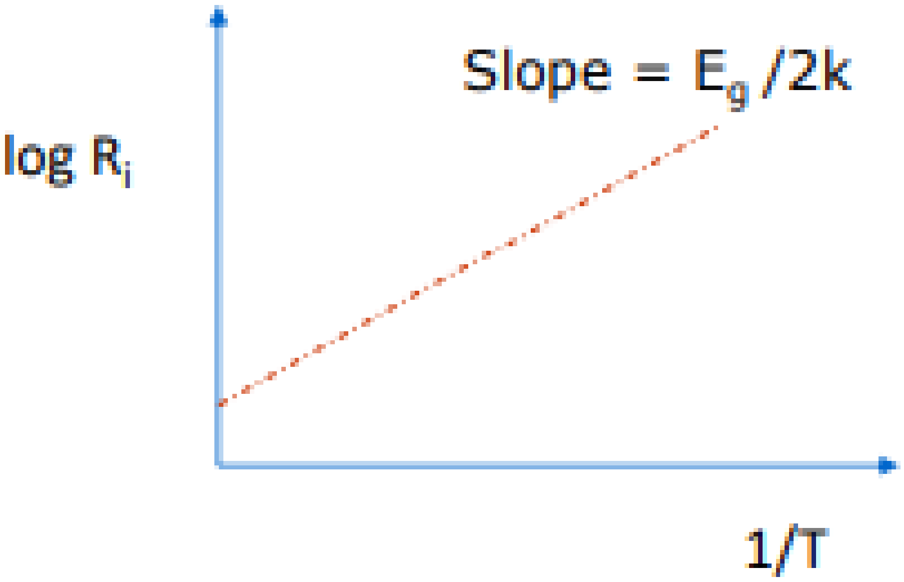
Therefore, resistance of intrinsic semiconductor

$$R_i = L/a\sigma_{intrinsic}$$

$$= \underbrace{\frac{L}{aA}}_{C} e^{E_g/2kT}$$

$$\log R_i = \log C + \frac{E_g}{2kT}$$

$$\sigma_i = Ae^{-E_g/2kT}$$



- Resistance R_i of a given slab of intrinsic material is measured as a function of temperature
- A graph is plotted between log of resistance in Y axis and 1/T in the X- axis.
- E_g is determined as (2k x slope of line)

Limitation of intrinsic semiconductor

30.14 LIMITATIONS OF INTRINSIC SEMICONDUCTOR

Intrinsic semiconductors are not useful for device manufacture because of low conductivity and the strong dependence of conductivity on temperature. If we take a crystal of pure silicon or germanium and connect it in the circuit, we will find that the current in the circuit will gradually increase as the temperature of the crystal is increased. We would also expect the current to increase if the voltage is increased and it does. But *increasing the voltage increases the current only proportionally, obeying Ohm's law, while increasing the temperature increases the current at an exponential rate*. Thus the temperature over which we have no control exerts more influence upon the current than the voltage, which we customarily do control.

We summarize the limitations as follows:

- Conductivity is low. Germanium has a conductivity of 1.67 S/m, which is nearly 10^7 times smaller than that of copper.
- Conductivity is a function of temperature and increases exponentially as the temperature increases.
- Conductivity cannot be controlled from outside.

Types of semiconductors (i) Intrinsic (Elemental & Compound)
(ii) Extrinsic

Intrinsic semiconductors are pure semiconductors characterized by completely filled valence shell at 0K ,with conductivity at higher temperatures due to thermally or optically generated electron-hole pairs .($n=p=n_i$)

Conductivity in semiconductors is due to both holes (electrons in valence band) and electrons (in conduction band)

Electron density available for conduction, $\boxed{n = N_c e^{-(Ec-EF)/kT}}$

Hole density available for conduction, $\boxed{p = N_v e^{-(EF-Ev)/kT}}$

Fermi level and its position as a function of temperature

$$\boxed{E_F = \frac{(E_c + E_v)}{2} + \frac{3kT}{4}\ln\left(\frac{m_h^*}{m_e^*}\right)}$$

OR

$$\boxed{E_F - \frac{E_g}{2} + \frac{3kT}{4}\ln\left(\frac{m_h^*}{m_e^*}\right)}$$

From top of valence band

Intrinsic carrier concentration and conductivity

$$\boxed{\therefore\ n_i = \sqrt{np} = \sqrt{N_c N_v}\, e^{-\left(E_g/2kT\right)} = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(m_e^* m_h^*\right)^{3/4} e^{-\left(E_g/2kT\right)}}$$

$$\sigma = (\mu_e + \mu_h)e \quad 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_e^* m_h^*)^{3/4} e^{-\left(E_g/2kT\right)}$$

Bandgap measurement

$$slope\ \ of\ \ (\log\sigma)\,vs\,(1/T)\,curve = \frac{\partial(\log\sigma)}{\partial(1/T)} = -\left(\frac{E_g}{2k}\right)$$

Bandgap determined from experiment measuring R vs T. as (2k x slope of plot between log R & 1/T)

For Silicon with a bandgap of 1.12eV, determine position of Fermi level at 300K if $m^*_e = 0.12m_o$ and $m^*_h = 0.28m_o$

For an intrinsic semiconductor with Eg =0.7eV and carrier concentration of $33.49 \times 10^{18} / m^3$, calculate conductivity at 300K assuming $m^*_e = m^*_h = m_o$; $\mu_e = 0.39\ m^2/Vs$, $\mu_h = 0.19\ m^2/Vs$

Find the resistance of an intrinsic Ge slab 1cm long, 1mm wide and 1mm thick if the intrinsic carrier concentration is $2.5 \times 10^{19} /m^3$, $\mu_e = 0.39\ m^2/Vs$, $\mu_h = 0.19\ m^2/Vs$ at 300K

If the effective mass of holes in an intrinsic material is 4 times that of the electron, find the temperature at which the Fermi level is shifted by 10% from the middle of the forbidden gap of energy 1eV.

For Silicon with a bandgap of 1.12eV, determine position of Fermi level at 300K if $m^*_e = 0.12m_o$ and $m^*_h = 0.28m_o$

Given $E_g = 1.12eV$, $T = 300K$, $m^*_e = 0.12m_o$, $m^*_h = 0.28m_o$, position of Fermi level ?

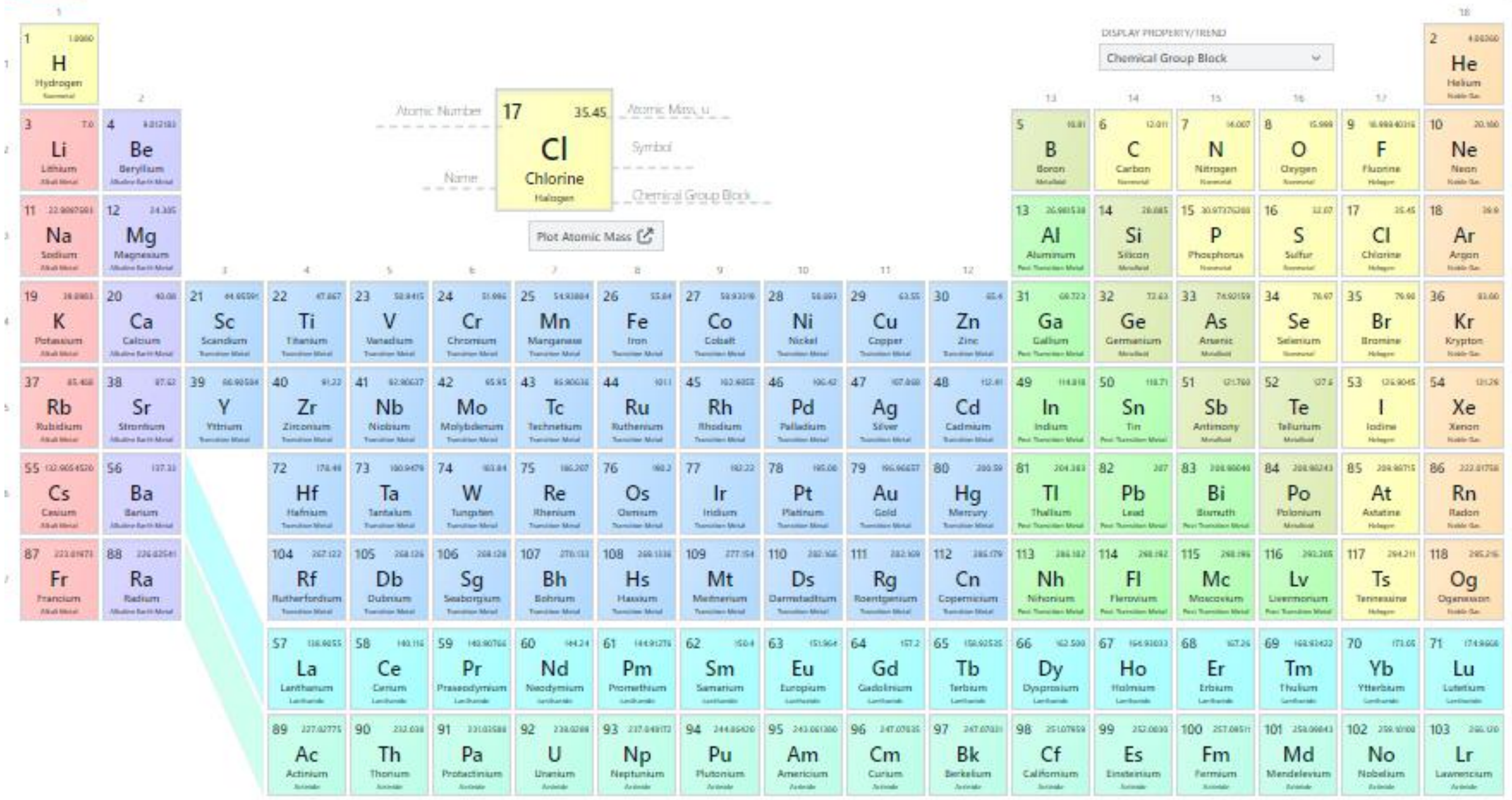
$$\boxed{E_F = \frac{E_g}{2} + \frac{3kT}{4}\ln\left(\frac{m_h^*}{m_e^*}\right)}$$ From top of valence band

$$E_F = \frac{1.12}{2} + \frac{3\,x\,1.38x10^{-23}\,x\,300}{4*1.6\,x\,10^{-19}}\ln\left(\frac{0.28}{0.12}\right) \quad eV \quad \textit{from top of valence band}$$

$$= 0.56 \quad + \quad 0.0164 = 0.5764\ eV \qquad \textit{from top of valence band}$$

Extrinsic semiconductor

- A judicious introduction of impurity atoms in an otherwise perfect semiconductor crystal produces useful modifications of its electrical conductivity.
- It makes the current more voltage dependent than temperature dependent.
- An intentional introduction of controlled amount of impurity into an intrinsic semiconductor is called doping.
- The impurity added is called a dopant.
- A semiconductor doped with impurity atoms is called an extrinsic semiconductor.
- The impurity-produced electrons are not temperature-dependent but are voltage-dependent and they will be under our control.
- Typical doping levels range from 10^{20} to 10^{27} impurity atoms/m^3.
- Pentavalent elements from Group V or trivalent elements from Group III are used as dopants.


DISPLAY PROPERTY/TREND
Chemical Group Block
Atomic Number
17 35.45 Cl Chlorine Halogen
Atomic Mass, u
Symbol
Name
Chemical Group Block
Plot Atomic Mass
1 H Hydrogen
2 He Helium
3 Li Lithium
4 Be Beryllium
5 B Boron
6 C Carbon
7 N Nitrogen
8 O Oxygen
9 F Fluorine
10 Ne Neon
11 Na Sodium
12 Mg Magnesium
13 Al Aluminum
14 Si Silicon
15 P Phosphorus
16 S Sulfur
17 Cl Chlorine
18 Ar Argon
19 K Potassium
20 Ca Calcium
21 Sc Scandium
22 Ti Titanium
23 V Vanadium
24 Cr Chromium
25 Mn Manganese
26 Fe Iron
27 Co Cobalt
28 Ni Nickel
29 Cu Copper
30 Zn Zinc
31 Ga Gallium
32 Ge Germanium
33 As Arsenic
34 Se Selenium
35 Br Bromine
36 Kr Krypton
37 Rb Rubidium
38 Sr Strontium
39 Y Yttrium
40 Zr Zirconium
41 Nb Niobium
42 Mo Molybdenum
43 Tc Technetium
44 Ru Ruthenium
45 Rh Rhodium
46 Pd Palladium
47 Ag Silver
48 Cd Cadmium
49 In Indium
50 Sn Tin
51 Sb Antimony
52 Te Tellurium
53 I Iodine
54 Xe Xenon
55 Cs Cesium
56 Ba Barium
72 Hf Hafnium
73 Ta Tantalum
74 W Tungsten
75 Re Rhenium
76 Os Osmium
77 Ir Iridium
78 Pt Platinum
79 Au Gold
80 Hg Mercury
81 Tl Thallium
82 Pb Lead
83 Bi Bismuth
84 Po Polonium
85 At Astatine
86 Rn Radon
87 Fr Francium
88 Ra Radium
104 Rf Rutherfordium
105 Db Dubnium
106 Sg Seaborgium
107 Bh Bohrium
108 Hs Hassium
109 Mt Meitnerium
110 Ds Darmstadtium
111 Rg Roentgenium
112 Cn Copernicium
113 Nh Nihonium
114 Fl Flerovium
115 Mc Moscovium
116 Lv Livermorium
117 Ts Tennessine
118 Og Oganesson
57 La Lanthanum
58 Ce Cerium
59 Pr Praseodymium
60 Nd Neodymium
61 Pm Promethium
62 Sm Samarium
63 Eu Europium
64 Gd Gadolinium
65 Tb Terbium
66 Dy Dysprosium
67 Ho Holmium
68 Er Erbium
69 Tm Thulium
70 Yb Ytterbium
71 Lu Lutetium
89 Ac Actinium
90 Th Thorium
91 Pa Protactinium
92 U Uranium
93 Np Neptunium
94 Pu Plutonium
95 Am Americium
96 Cm Curium
97 Bk Berkelium
98 Cf Californium
99 Es Einsteinium
100 Fm Fermium
101 Md Mendelevium
102 No Nobelium
103 Lr Lawrencium

Extrinsic semiconductor

- **The atoms belonging to these two groups are nearly of the same size as silicon or germanium atoms and easily substitute themselves in place of some of the host atoms in the semiconductor crystal.**
- **Thus, they are substitutional impurities and do not cause any distortion in the original crystal structure.**
- **Depending on the two different types of doping, two types of extrinsic semiconductors are possible.**
- **They are n-type and p-type semiconductors.**

Common Dopant Elements for Silicon and Germanium

n-type	p-type
Phosphorous	Aluminium
Arsenic	Boron
Antimony	Gallium
	Indium

Advantages of Extrinsic Semiconductors

- Conductivity is high.
- Conductivity can be tailored to the desired value through the control of doping concentration.
- Conductivity is not a function of temperature.

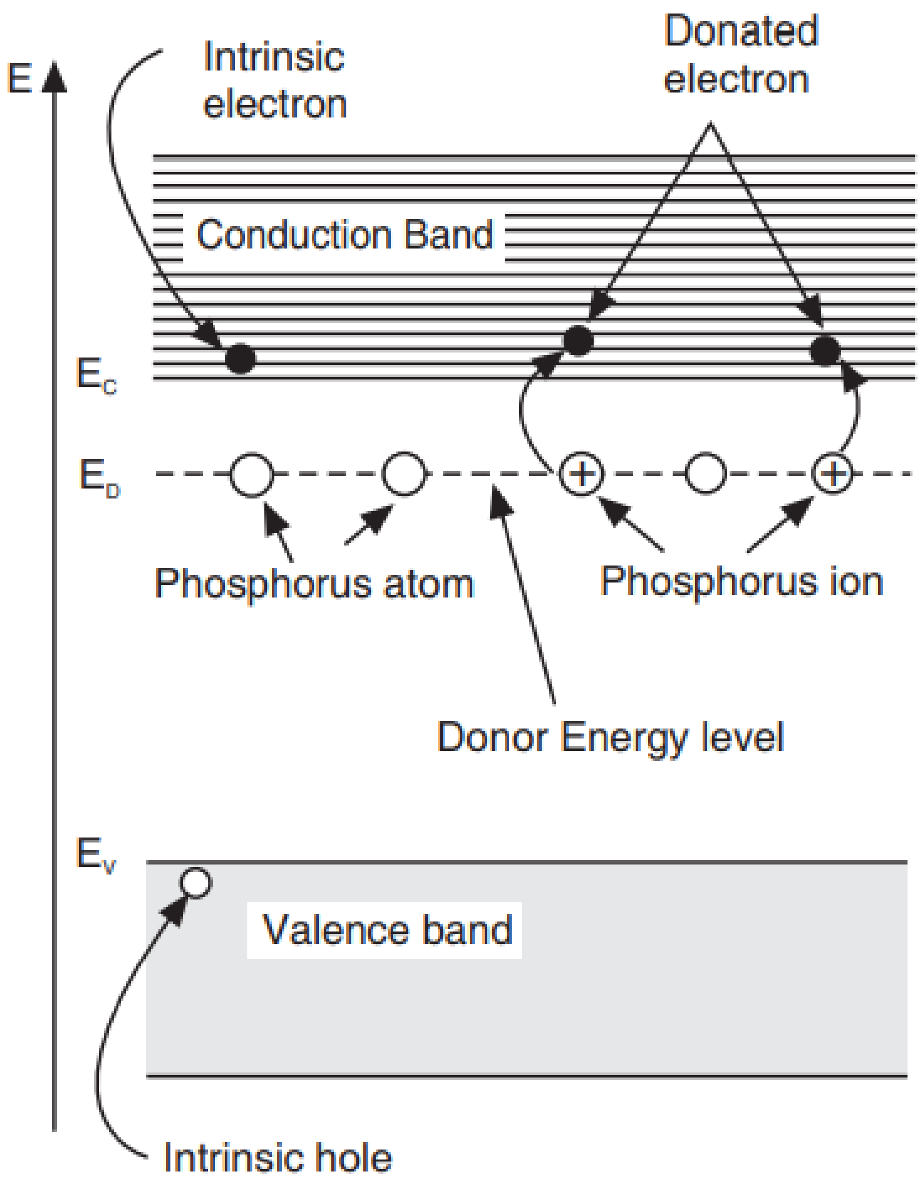
n- type semiconductor

- An n-type semiconductor is produced when a pure semiconductor is doped with a pentavalent impurity such as phosphorous.
- A phosphorous atom has five valence electrons.
- Out of the five electrons, only four participate in bonding with four host silicon atoms while the fifth electron remains loosely bound.
- The host silicon lattice is a dielectric medium having a dielectric constant of 12.
- As a result, the Coulomb force between the phosphorous nucleus and the fifth electron is smaller than that it would be in free space.
- Therefore, the ionization energy of the fifth electron is very small.
- It is found to be 0.045 eV.
- The ionization energy is so small that the thermal energy can easily liberate the fifth electron from the nucleus.

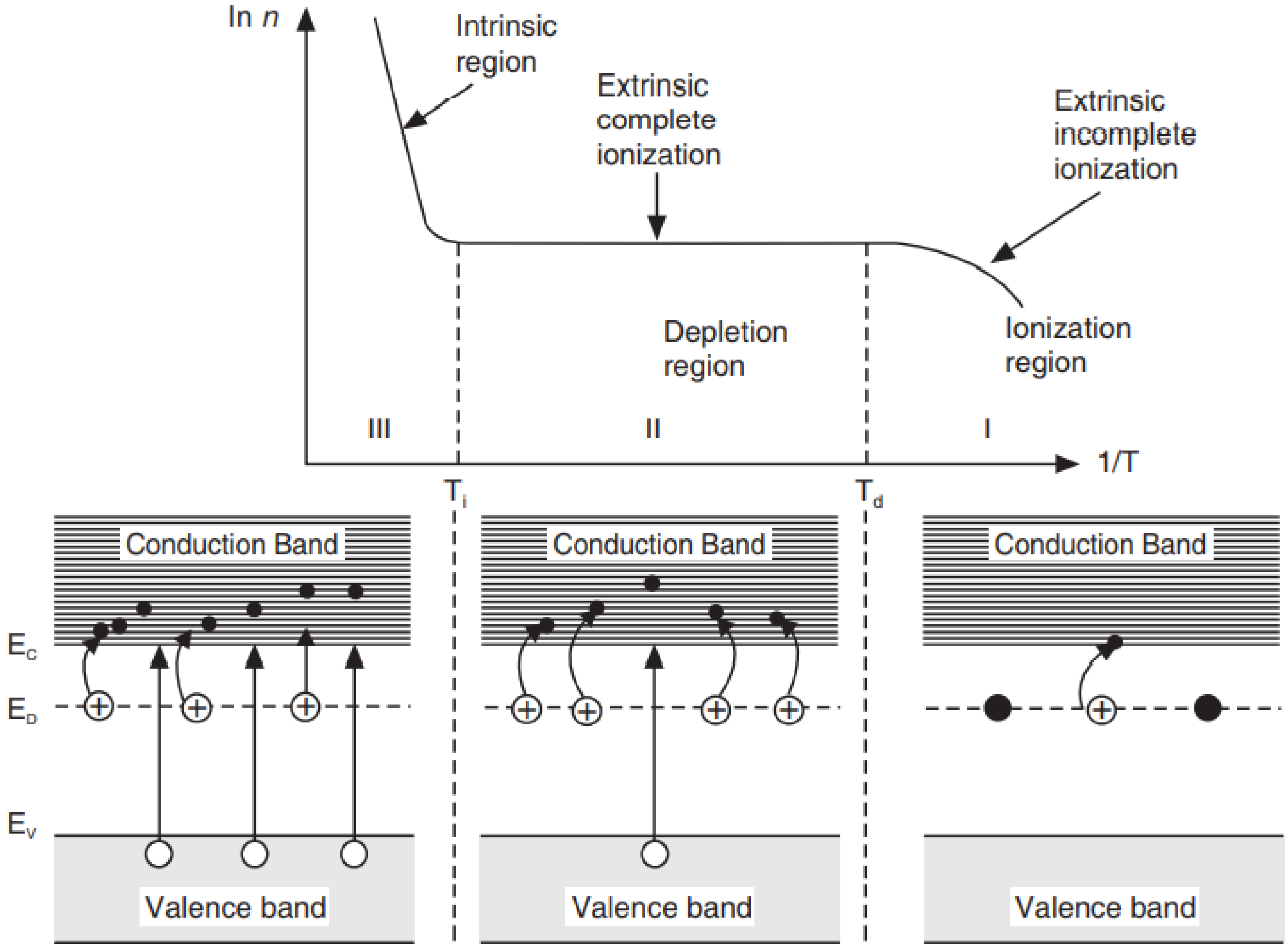
n- type semiconductor

- It means that the energy levels corresponding to phosphorous atoms are nearer to the bottom edge of the conduction band.
- At normal temperatures, the fifth electron becomes free to move about in the crystal and acts as a charge carrier.
- That is, the electron jumps into the conduction band leaving behind the positive phosphorous ion that is fixed in the crystal lattice.
- As the phosphorous atom is donating an electron for the purpose of electrical conduction, it is called a donor atom.
- If the donor atom density is low, the donor atoms are distantly spaced from one another, approximately, by 100 atom spacings.
- In such a situation, the donor atoms cannot interact with each other, and their energy levels are discrete levels, E_D.
- They are called donor levels and represent the ground state of the fifth electron of impurity atom.
- As even small amount of thermal energy can readily liberate the fifth electron from the atom and send it into the conduction band, the donor levels are expected to be located very near to the bottom edge of the conduction band.

n- type semiconductor



Temperature Variation of Carrier Concentration



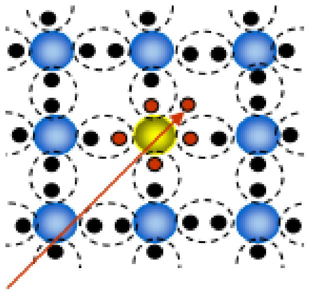
1. **@ 0K**
2. **Then raise temperature**
3. **@ 100 K all donor atoms are ionized**
4. **@ high temperature**

In n-type material the electrons outnumber the holes and constitute the majority carriers (in region II). Holes are minority carriers. The number of carriers is independent of temperature in the depletion region. The current in this type of crystal is mainly due to the negatively charged electrons and hence the material is called n-type semiconductor

Carrier Concentration in n-type semiconductor

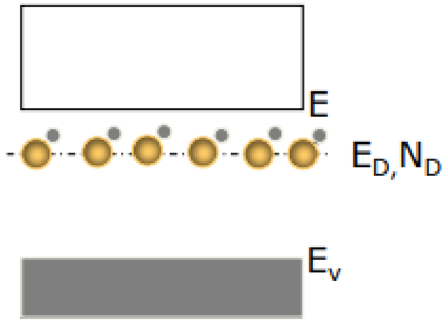
- In a N type semiconductor pentavelent impurity replaces the covalently bonded host semiconductor atoms.
- Electron of the pentavalent impurity atom that is not involved in any covalent bond is loosely bound to its nucleus with small binding energies (~0.045eV) .
- This is so small that the electron, acquires this small amount of energy, and becomes "free" leaving the impurity +vely charged) without generation of holes in the valence band
- The pentavalent impurity which donates electrons to the conduction band structure is called a donor.
- The ionization energy of the electron is 1/25th of energy gap and since the donated electron occupies the conduction band it implies energy level of donor atoms are close to E_c and lesser than E_c by 0.045eV

Let the no. density of impurity atoms be N_D



an electron for conductivity

and is hence called N-type semiconductor



N-type Semiconductor at 0K

Carrier Concentration in n-type Semiconductor at Low Temperatures: (In the Ionization Region)

N_D= concentration of donors in the material

@ 0K, the donor atoms are not ionized----- are at the level E_D which is very near to E_C

When temperature is raised above 0 K, the donor atoms get ionized and free electrons appear in the CB.

With increase in temperature more, more donor atoms get ionized and electron concentration in the conduction band increases

n= electron concentration in the conduction band

$$n = N_D^+$$

$$n = N_D - N_D^0$$

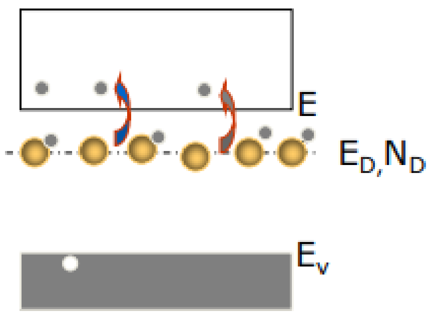
where N_D^+ is the number of donor atoms that are ionized and N_D^0 is the number of atoms left unionized at the energy level E_D.

The concentration of ionized donors $N_D^+ = \left(N_D - N_D^0\right) = N_D[1 - f(E_D)]$

Therefore, at low temperatures, no. of carriers for conduction in a N type material , n_n = no. density of electrons in conduction band
= no. of ionized impurity atoms (N_D^+)

N_D^+ = no. density of impurity atoms in the donor energy level x probability that they are not occupied by an electron

= N_Dx [1-F(E_D)]



N-type Semiconductor

$$(1 - F(E_D)) = 1 - \frac{1}{1+exp\left(\frac{E_D - E_{Fn}}{k_B T}\right)} = \frac{1 + exp\left(\frac{E_D - E_{Fn}}{k_B T}\right) - 1}{1 + exp\left(\frac{E_D - E_{Fn}}{k_B T}\right)}$$

$$= \frac{1}{1 + exp\left(\frac{E_{Fn} - E_D}{k_B T}\right)}$$

$$exp\left(\frac{E_{Fn} - E_D}{k_B T}\right) \gg 1$$

$$Therefore, (1 - F(E_D)) \approx \frac{1}{exp\left(\frac{E_{Fn} - E_D}{k_B T}\right)} \approx exp\left(\frac{E_D - E_{Fn}}{k_B T}\right)$$

$$(n_n) = (N_D^+) = N_D(1 - F(E_D)) = N_D exp\left(\frac{E_D - E_{Fn}}{k_B T}\right)$$

$$n_n = N_c \times \exp\left(\frac{E_{Fn} - E_{cn}}{k_B T}\right) = (N_D^+) = N_D exp\left(\frac{E_D - E_{Fn}}{k_B T}\right)$$

$$\exp\left(\frac{E_{Fn} - E_{cn} - E_D + E_{Fn}}{k_B T}\right) = \frac{N_D}{N_c} \qquad \exp\left(\frac{2E_{Fn} - (E_{cn} + E_D)}{k_B T}\right) = \frac{N_D}{N_c}$$

Taking natural logarithms on both sides

$$\frac{2E_{Fn} - (E_{cn} + E_D)}{k_B T} = ln\left[\frac{N_D}{N_c}\right] \qquad 2E_{Fn} - (E_{cn} + E_D) = k_B T \;\; ln\left[\frac{N_D}{N_c}\right] \qquad \therefore E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln\left[\frac{N_D}{N_c}\right]$$

$$\boxed{E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln\left[\frac{N_D}{2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2}}\right]}$$

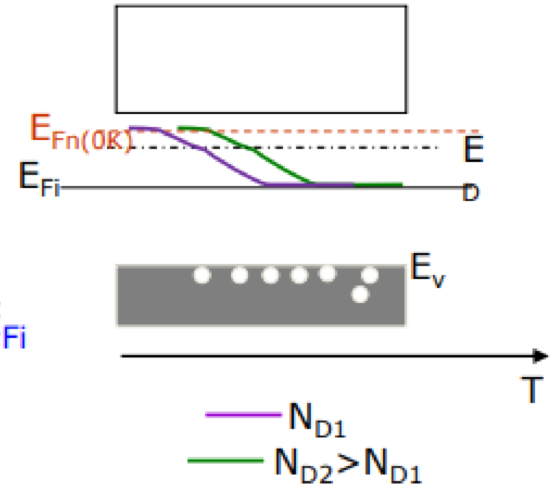
i.e Fermi level of N-type semiconductor lies midway between bottom of conduction band E_{cn} and the donor level , E_D at 0K and falls with temperature as $N_D/N_c < 1$ or ln (N_D/N_c) is –ve.

$$E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln \left[\frac{N_D}{N_c}\right]$$

- At 0K, Fermi level of N-type semiconductor lies midway between bottom of conduction band E_c and the donor level , E_D

- As temperature increases E_{Fn} falls as $N_D/N_c < 1$ or ln (N_D/N_c) is –ve.

- Once all the donor atoms are ionized, the Fermi level falls and becomes equal to the center of the bandgap i.e becomes equal to the Fermi level of intrinsic semiconductor, E_{Fi}



- For semiconductors doped with higher concentration of impurities, the Fermi level drops more slowly and attains E_{Fi} at relatively higher temperatures (green curve)