

Physics of Semiconductor: Lecture

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P1. Estimate the fraction of electrons in conduction band at room temperature in Ge with $E_g = 0.72$ eV and in diamond with $E_g = 5.6$ eV

$$f(E) = \frac{1}{1 + \exp[(E - E_F) / kT]}$$

$$f(E_C) = \frac{1}{1 + \exp[(E_C - E_F) / kT]}$$

$$f(E_C) = \frac{n}{N}$$

$$\frac{n}{N} = \frac{1}{1 + \exp[(E_C - E_F) / kT]}$$

$$(E_C - E_F) = E_g / 2$$

$$\frac{n}{N} = \frac{1}{1 + \exp[E_g / 2kT]}$$

$$E_g > 2kT,$$

$$\frac{n}{N} = \frac{1}{\exp[E_g / 2kT]} = e^{-E_g / 2kT}$$

(i) Fraction of electrons in conduction band in Ge,

$$\frac{n}{N} = \exp\left[-\frac{0.72 \text{ eV}}{2 \times 0.026 \text{ eV}}\right] = \exp(-13.846) = \mathbf{9.7 \times 10^{-7}}.$$

(ii) Fraction of electrons in conduction band in diamond,

$$\begin{aligned} \frac{n}{N} &= \exp\left[-\frac{5.6 \text{ eV}}{2 \times 0.026 \text{ eV}}\right] \\ &= \exp(-107.692) = \mathbf{1.7 \times 10^{-47}}. \end{aligned}$$

P2. Estimate the fraction of electrons in the conduction band at 300°K of (i) Germanium ($E_g = 0.72$ eV) (ii) Silicon ($E_g = 1.1$ eV) and (iii) Diamond ($E_g = 5.6$ eV).
What is the significance of these results?

Solution.

$$(i) \text{ Germanium: } f(E_C) = e^{-E_g/2kT} = e^{\frac{0.72\text{eV}}{2 \times 0.026\text{eV}}} = e^{-13.85} = 9.66 \times 10^{-7}$$

$$(ii) \text{ Silicon: } f(E_C) = e^{-E_g/2kT} = e^{\frac{1.1\text{eV}}{2 \times 0.026\text{eV}}} = 6.5 \times 10^{-10}$$

$$(iii) \text{ Diamond: } f(E_C) = e^{-E_g/2kT} = e^{\frac{5.6\text{eV}}{2 \times 0.026\text{eV}}} = e^{-107.7} = 1.7 \times 10^{-47}$$

The above results show that the larger the band gap the smaller the electrons that can go into the conduction band, at a given temperature.

P3. Assuming that the number of electrons near the top of the valence band available for thermal excitation is $5 \times 10^{25}/\text{m}^3$ and the intrinsic carrier density is $2.5 \times 10^{19}/\text{m}^3$, calculate the energy gap of germanium at room temperature

Solution: Fraction of electrons in conduction band $\frac{n}{N} = \exp\left[-\frac{E_g}{2kT}\right]$

$$\begin{aligned}\therefore E_g &= -2kT \ln\left(\frac{n}{N}\right) \\ &= -2(8.61 \times 10^{-5} \text{ eV/K})(300 \text{ K}) \ln\left[\frac{2.5 \times 10^{19} / \text{m}^3}{5 \times 10^{25} / \text{m}^3}\right] \\ &= 0.052 \times 14.509 \text{ eV} = \mathbf{0.75 \text{ eV}}.\end{aligned}$$

P4. What is the probability of an electron being thermally promoted to the conduction band in diamond at 27° C, if the bandgap is 5.6 eV wide?

Solution. The probability that an electron being thermally promoted to the conduction band is given by

$$\begin{aligned} f(E_C) &= \frac{1}{1 + \exp(E_g/2kT)} = \frac{1}{1 + \exp\left[\frac{5.6\text{ eV}}{2(0.026)}\text{ eV}\right]} \\ &= \frac{1}{1 + e^{107.69}} = 1.7 \times 10^{47} \end{aligned}$$

P5. Determine the position of Fermi level in silicon semiconductor at 300 K. Given that the band gap is 1.12 eV, and $m_e^* = 0.12 m$ and $m_h^* = 0.28m$.

$$\begin{aligned} E_F &= \frac{E_g}{2} + \frac{3kT}{4} \ln \left(\frac{m_h^*}{m_e^*} \right) = \frac{1.12 \text{ eV}}{2} + \frac{3 \times 8.61 \times 10^{-5} \text{ eV/K} \times 300 \text{ K}}{4} \ln \left(\frac{0.28m}{0.12m} \right) \\ &= 0.56 \text{ eV} + (0.0194) \ln 2.333 \text{ eV} \\ &= 0.56 \text{ eV} + (0.0194) (0.8473) \text{ eV} \\ &= 0.56 \text{ eV} + 0.016 \text{ eV} = \mathbf{0.576 \text{ eV}.} \end{aligned}$$

P6. If the effective mass of an electron is equal to twice the effective mass of hole, determine the position of the Fermi level in an intrinsic semiconductor from the center of forbidden gap at room temperature.

$$E_F = \frac{E_g}{2} + \frac{3}{4}kT \ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$\frac{E_g}{2} - \frac{3}{4}kT \ln 2 = \frac{E_g}{2} - \frac{3}{4}(0.026 \text{ eV})(0.69)$$

$$E_F = \frac{E_g}{2} - 0.0135 \text{ eV}$$

P7. The resistivity of an intrinsic semiconductor is $4.5 \, \Omega \cdot \text{m}$ at 20°C and $2.0 \, \Omega \cdot \text{m}$ at 32°C . Find the energy gap.

$$\sigma = \sigma_o e^{-E_g/2kT}$$

$$\rho_i = \rho_o e^{E_g/2kT}$$

$$\frac{R_i A}{L} = \rho_o e^{E_g/2kT}$$

$$R_i = C e^{E_g/2kT}$$

$$E_g = 2k \times \text{slope}$$

$$\begin{aligned} E_g &= 2k \left(\frac{dy}{dx} \right) = 2k \left(\frac{\log \rho_2 - \log \rho_1}{(1/T_1 - 1/T_2)} \right) \\ &= 2 \times 8.61 \times 10^{-5} \text{ eV/K} \left(\frac{\log 2 - \log 4.5}{(1/293) - (1/305)} \right) \text{ K} \\ &= 2 \times 8.61 \times 10^{-5} \text{ eV} \left(\frac{0.6532 - 0.3010}{0.134 \times 10^{-3}} \right) \\ &= 2 \times 8.61 \times 10^{-5} \text{ eV} \times 2.63 \times 10^3 = \mathbf{0.45 \text{ eV}} \end{aligned}$$

P8. In a doped semiconductor, there are 4.52×10^{24} holes and 1.25×10^{14} electrons per cubic metre. What will be the carrier density in undoped specimen? Electron and hole mobilities are $0.38 \text{ m}^2/\text{V.s}$ and $0.18 \text{ m}^2/\text{V.s}$ respectively. Calculate the conductivity of intrinsic and the doped semiconductors.

Intrinsic carrier density, $n_i = \sqrt{p_p n_p}$

$$n_i = \sqrt{(4.52 \times 10^{24} / \text{m}^3)(1.25 \times 10^{14} / \text{m}^3)}$$
$$= (5.65 \times 10^{38} / \text{m}^6)^{1/2} = 2.38 \times 10^{19} / \text{m}^3$$

Conductivity of the doped semiconductor

$$\sigma_p = ep_p \mu_h$$

$$= (1.602 \times 10^{-19} \text{ C})(4.52 \times 10^{24} / \text{m}^3)(0.18) \frac{\text{m}^2}{\text{V.s}}$$
$$= 1.30 \times 10^5 \frac{\text{C}}{\text{m.V.s}} = \mathbf{130 \text{ kS/m}}$$

Conductivity of the intrinsic semiconductor

$$\sigma_i = en_i (\mu_e + \mu_h)$$
$$= (1.602 \times 10^{-19} \text{ C})(2.38 \times 10^{19} / \text{m}^3)(0.38 + 0.18) \frac{\text{m}^2}{\text{V.s}}$$
$$= 2.14 \frac{\text{C}}{\text{m.V.s}} = \mathbf{2.14 \text{ S/m}}$$

Example 30.13: *Silicon has a conductivity of only $5 \times 10^{-4} \Omega^{-1}\text{m}^{-1}$ in its pure form. An engineer wanted it to have conductivity of $200 \Omega^{-1}\text{m}^{-1}$ and doped it with aluminium to produce p-type semiconductor. Calculate the impurity concentration. Assume $\mu_h = 0.05 \text{ m}^2/\text{V.s}$.*

Solution.

$$\sigma = pe\mu_h = N_A e\mu_h.$$

\therefore

$$\begin{aligned} N_A &= \frac{\sigma}{e\mu_h} = \frac{200 \Omega^{-1}\text{m}^{-1}}{1.602 \times 10^{-19} \text{ C} \times 0.05 \text{ m}^2/\text{V.s}} \\ &= 2.5 \times 10^{22} \text{ atoms/m}^3. \end{aligned}$$

P10. A sample of intrinsic germanium at room temperature has a carrier concentration of $2.4 \times 10^{19} / \text{m}^3$. It is doped with antimony at a rate of one antimony atom per million atoms of germanium. If the concentration of the germanium atoms is $4 \times 10^{28} / \text{m}^3$, determine the hole concentration.

$$N_D = \frac{4 \times 10^{28} \text{ atoms/m}^3}{10^6 \text{ atoms/m}^3} = 4 \times 10^{22} \text{ donors/m}^3$$

$$n_n = N_D = 4 \times 10^{22} \text{ electrons/m}^3$$

$$p_n = \frac{n_i^2}{n_n} = \frac{(2.4 \times 10^{19} \text{ carriers/m}^3)^2}{(4 \times 10^{22} \text{ electrons/m}^3)}$$

$$p_n = 1.4 \times 10^{16} \text{ holes/m}^3.$$

Write the expressions for electron and hole concentrations in an intrinsic semiconductors and hence derive the expression:

$$E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ln \left(\frac{m_h^*}{m_e^*} \right)$$

for Fermi level in the intrinsic semiconductor. Assume the symbols to have their usual meanings.

Describe a method of determining the band gap of a semiconductor. How does electrical conductivity vary with temperature for an intrinsic semiconductor? (Anna Univ., 2004)

Intrinsic silicon with a bandgap of 1.1 eV is doped with boron. If ionization energy of boron is 0.05 eV in silicon, calculate the change in Fermi energy due to doping at 0K .

Draw the variation of natural logarithm of electron concentration (n_n) in a N type semiconductor with reciprocal of temperature (T) marking out three distinct regions and the slopes in each of these regions.

$$F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{k_B T}\right)}$$

$$g(E) = (4\pi/h^3) (2m_e^*)^{3/2} \sqrt{E} dE$$

$$n = \int_{E_1}^{E_2} g(E) F(E) dE$$

$$E_{F(0K)} = (h^2/2m_e^*) (3n/8\pi)^{2/3}$$

Average energy of electrons in metals at 0K.

$$= \frac{3}{5} E_{F(0K)}$$

$$n = N_c \times \exp\left(\frac{E_F - E_c}{k_B T}\right) \quad p = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$E_F = \frac{(E_c + E_v)}{2} + \frac{3kT}{4} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

$$E_F = \frac{E_g}{2} + \frac{3kT}{4} \ln\left(\frac{m_h^*}{m_e^*}\right)$$

From top of valence band

$$\begin{aligned} \therefore n_i &= \sqrt{np} = \sqrt{N_c N_v} e^{-\left(E_g/2kT\right)} \\ &= 2 \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-\left(E_g/2kT\right)} \end{aligned}$$

$$\sigma = (\mu_e + \mu_h) e \quad 2 \left(\frac{2\pi kT}{h^2}\right)^{3/2} (m_e^* m_h^*)^{3/4} e^{-\left(E_g/2kT\right)}$$

N -Type

$$\therefore E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} \ln \left[\frac{N_D}{N_c} \right]$$

$$E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} \ln \left[\frac{N_D}{2 \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/2}} \right]$$

$$n_n = \sqrt{N_c N_D} \exp \left(\frac{E_D - E_{cn}}{2 k_B T} \right)$$

$$n_n = (2N_D)^{1/2} \left(\frac{2\pi m_e^* k_B T}{h^2} \right)^{3/4} \exp \left(\frac{E_D - E_{cn}}{2 k_B T} \right)$$

P -Type

$$\therefore E_{Fp} = \frac{E_A + E_{vp}}{2} - \frac{k_B T}{2} \ln \left[\frac{N_A}{N_v} \right]$$

$$E_{Fp} = \frac{E_A + E_{vp}}{2} - \frac{k_B T}{2} \ln \left[\frac{N_A}{2 \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/2}} \right]$$

$$p_p = \sqrt{N_v N_A} \exp \left(\frac{E_{vp} - E_A}{2 k_B T} \right)$$

$$p_p = (2N_A)^{1/2} \left(\frac{2\pi m_h^* k_B T}{h^2} \right)^{3/4} \exp \left(\frac{-(E_A - E_{vp})}{2 k_B T} \right)$$