Shiv Nadar University, Chennai

School of Engineering

Department of Computer Science

CS1802 -- Programming in Python Lab   Class: 2024-2028 B. Tech CSE (Cyber)

Date: 26/03/2025   Continuous Lab Evaluation – 8 (10 Marks)

Statement: Consider a system of linear equations with 3 and 4 variables as given in the *sample.txt* file.

- As you know, any system of linear equations can be expressed as Ax = B, Where A is the coefficient matrix and B is the constant vector and x is the unknown variables.

For these equations to be solved, consistency must be checked for three possible solutions.

1. Unique Solution.
2. No Solution.
3. Infinite Solution.

Task: Develop a python program that can check the consistency condition for the equations in the text file based on **Rank Method using Determinants.**

Summary of the Rank Method:

- Rank of coefficient matrix = Rank of augmented matrix = Number of variables: Unique solution.
- Rank of coefficient matrix = Rank of augmented matrix < Number of variables: Infinitely many solutions.
- Rank of coefficient matrix < Rank of augmented matrix: No solution (inconsistent system).

Start with the file reading commands as below

```
file_pointer = open('sample1.txt')
array = [[x for x in line.split()] for line in file_pointer]
print(array[0][0][0]);
```

Steps in the Rank Method using Determinants:

1. **Write the system as an augmented matrix**: Convert the system of linear equations into the augmented matrix [A|B], where A is the coefficient matrix and B is the column matrix of constants.
2. **Find the determinant of the coefficient matrix**: For a system with n variables and n equations, calculate the determinant of the **coefficient matrix** A:

$$\det(A)$$

   - If $\mathbf{det(A) \neq 0}$, then the rank of the matrix is **n** (full rank), and the system has a **unique solution**.
   - If $\mathbf{det(A) = 0}$, then the system has **no unique solution**, and we need to investigate further using submatrices.
3. **Use submatrices to find the rank**: If $\mathbf{det(A) = 0}$, the system may have infinitely many solutions or no solution. To find the rank, we calculate the determinant of smaller submatrices (minor determinants).
   - Calculate the determinant of any $(n-1) \times (n-1)$ submatrix of A. If this determinant is **non-zero**, then the rank of the matrix is $\mathbf{n-1}$.
   - If the determinant of any $(n-1) \times (n-1)$ submatrix is also **zero**, then the rank of the matrix is less than n-1. Continue to calculate the determinant of smaller submatrices until you determine the rank.
4. **Check the rank of the augmented matrix**:
   - Calculate the determinant of the augmented matrix [A|B].
   - If $\mathbf{det([A|B]) \neq 0}$, the system is **inconsistent** and has **no solution**.
   - If $\mathbf{det([A|B]) = 0}$, the system is **consistent** and has **either a unique solution or infinitely many solutions** depending on the rank.