Physics of Semiconductor: Lecture # Lec 7

Dr. Sudipta Som

Department of Physics

Shiv Nadar University Chennai

SHIV NADAR UNIVERSITY CHENNAI

What we have learnt earlier

Intrinsic semiconductor

$$n = \int_{E_C}^{\infty} Z(E) f(E) dE$$

$$Z(E)dE = \frac{4\pi}{h^3}\left(2m_e^*\right)^{3/2} E^{1/2} dE \quad \text{for } E > E_C.$$

$$f(E) = \frac{1}{1+\exp[(E-E_F)/kT]}$$

Electron density available for conduction, $n = N_c e^{-(E_c - E_F)/kT}$

$$dp = Z(E)[1 - f(E)]dE \qquad (30.31)$$

$$p = N_V e^{-(E_F - E_V)/kT}$$

FERMI LEVEL IN INTRINSIC SEMICONDUCTOR

$$E_F = \frac{E_C + E_V}{2} + \frac{1}{2} kT \ ln \frac{N_V}{N_C}$$

$$\therefore \quad E_F = \frac{E_C + E_V}{2} + \frac{3}{4} kT \ ln \left(\frac{m_h^*}{m_e^*}\right)$$

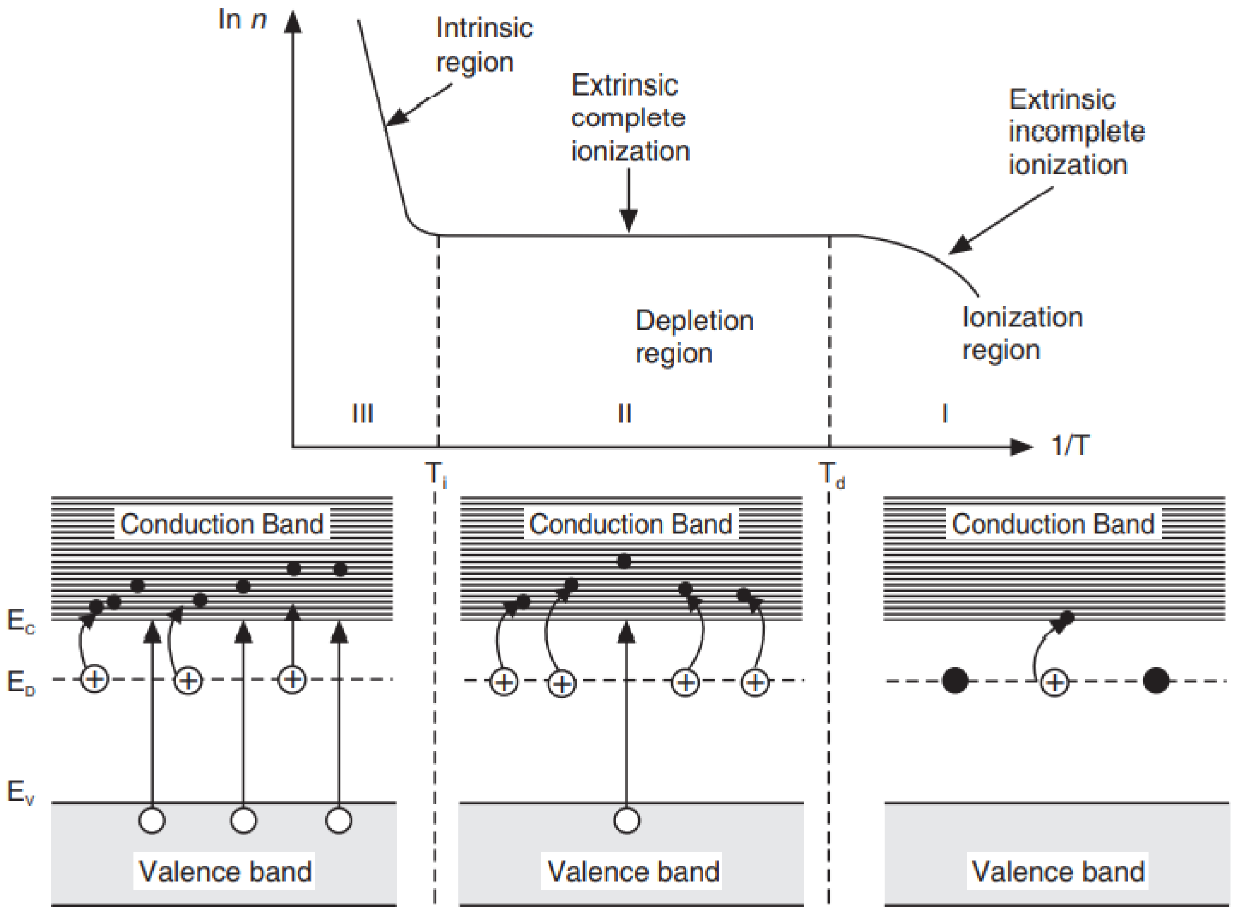
$$\sigma_i = A e^{-E_g/2kT}$$

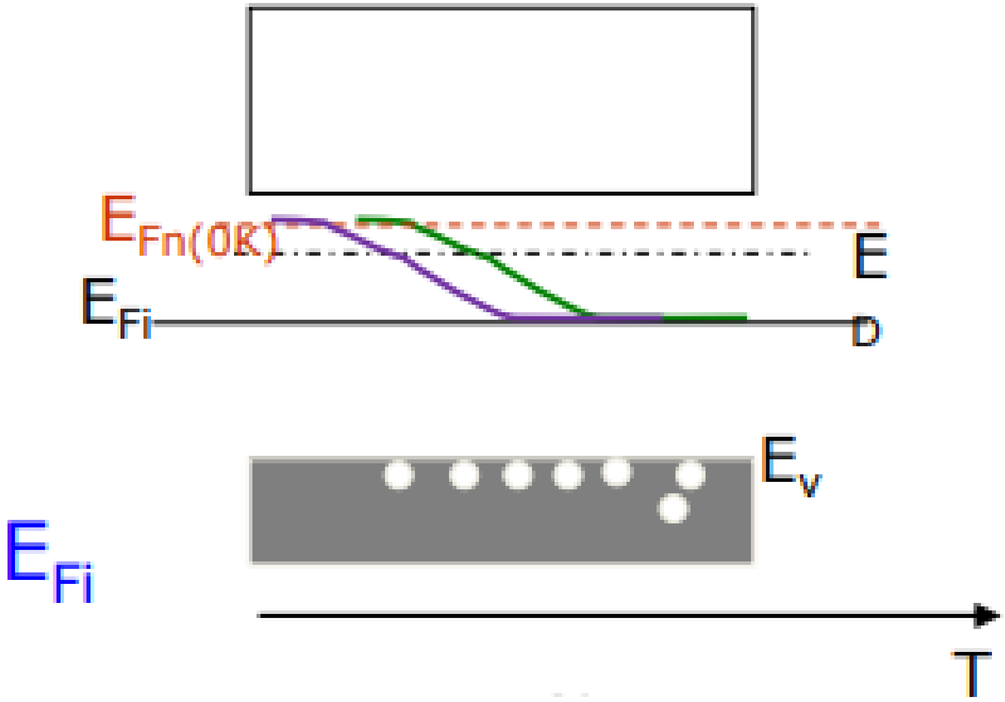
Electron density available for conduction, $n = N_c e^{-(E_c - E_F)/kT}$

Hole density available for conduction, $p = N_v e^{-(E_F - E_v)/kT}$

Extrinsic semiconductor

$$E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln\left[\frac{N_D}{N_c}\right]$$





Carrier Concentration in semiconductor

N -Type

$$\therefore E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln\left[\frac{N_D}{N_c}\right]$$

$$\boxed{E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln\left[\frac{N_D}{2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2}}\right]}$$

$$n_n = \sqrt{N_c N_D}\ exp\left(\frac{E_D - E_{cn}}{2k_B T}\right)$$

$$\boxed{n_n = (2N_D)^{1/2}\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/4} exp\left(\frac{E_D - E_{cn}}{2k_B T}\right)}$$

P -Type

$$\therefore E_{Fp} = \frac{E_A + E_{vp}}{2} - \frac{kT}{2} ln\left[\frac{N_A}{N_V}\right]$$

$$\boxed{E_{Fp} = \frac{E_A + E_{vp}}{2} - \frac{kT}{2} ln\left[\frac{N_A}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]}$$

$$p_p = \sqrt{N_v N_A}\ exp\left(\frac{E_{vp} - E_A}{2\ kT}\right)$$

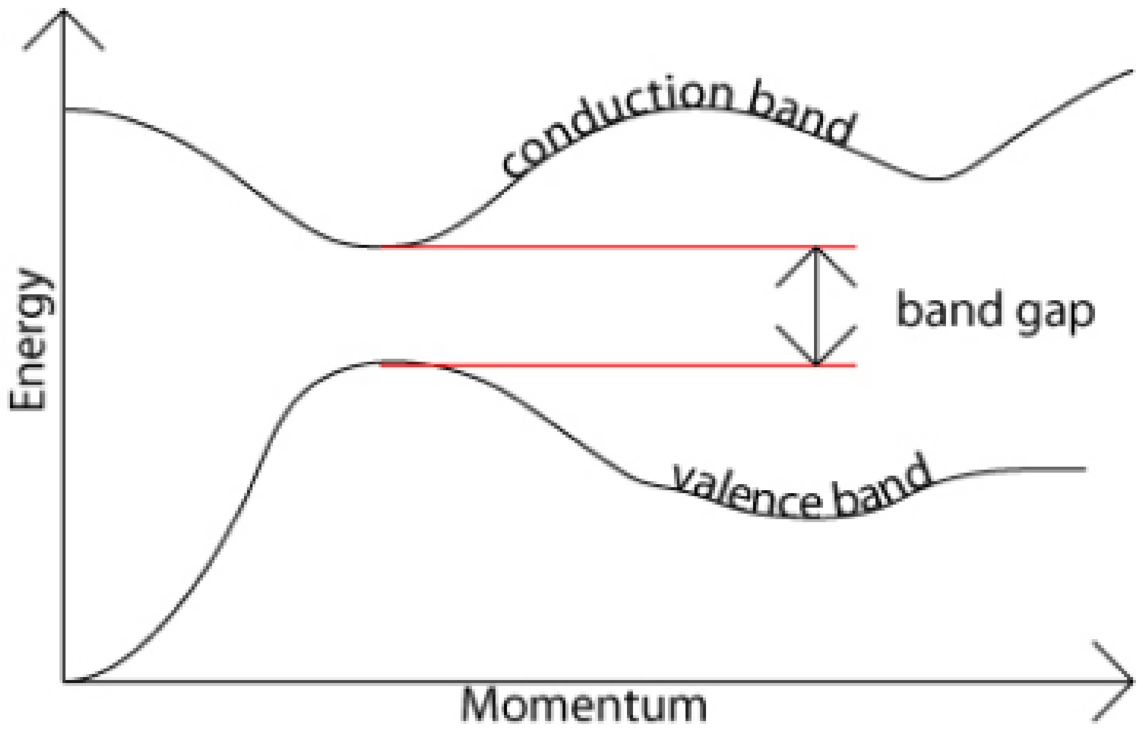
$$\boxed{p_p = (2N_A)^{1/2}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} exp\left(\frac{-(E_A - E_{vp})}{2\ kT}\right)}$$

Variation of semiconductor- types of band gap

What is bandgap??

The band gap represents the minimum energy difference between the top of the valence band and the bottom of the conduction band,

However, the top of the valence band and the bottom of the conduction band are not generally at the same value of the electron momentum.

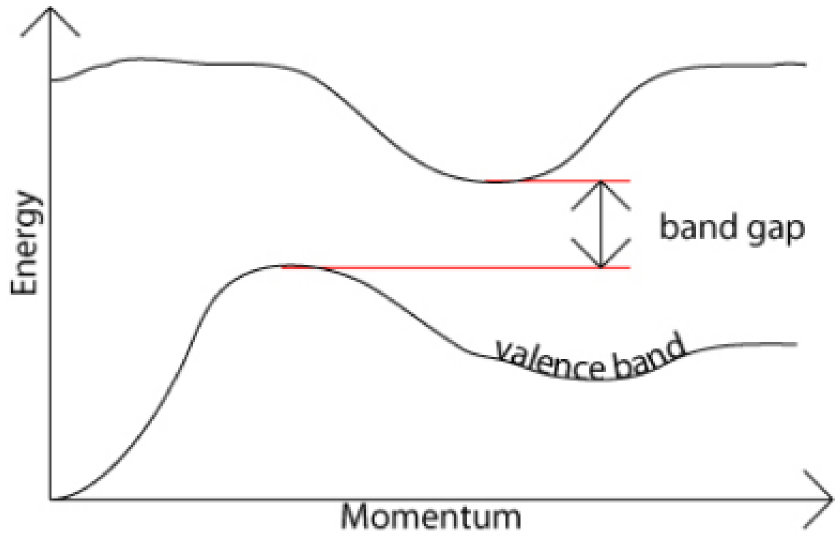


the top of the valence band and the bottom of the conduction band occur at the same value of momentum

direct band gap semiconductor

Variation of semiconductor- types of band gap

In an **indirect band gap semiconductor**, the maximum energy of the valence band occurs at a different value of momentum to the minimum in the conduction band energy:



The difference between the two is most important in optical devices. As has been mentioned in the section charge carriers in semiconductors, a photon can provide the energy to produce an electron-hole pair.

- ✓ photon of energy E_g, where E_g is the band gap energy, can produce an electron-hole pair in a direct band gap semiconductor quite easily, because the electron does not need to be given very much momentum.
- ✓ However, an electron must also undergo a significant change in its momentum for a photon of energy E to produce an electron-hole pair in an indirect band gap semiconductor.
- ✓ This is possible, but it requires such an electron to interact not only with the photon to gain energy, but also with a lattice vibration called a phonon in order to either gain or lose momentum.

Variation of semiconductor- types of band gap

- The indirect process proceeds at a much slower rate, as it requires three entities to intersect in order to proceed:
  - ✓ an electron,
  - ✓ a photon and
  - ✓ a phonon.

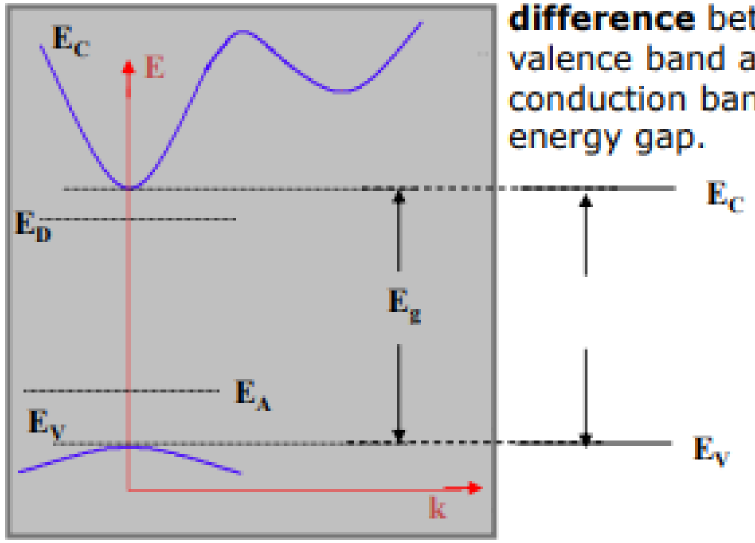
- The recombination process is much more efficient for a direct band gap semiconductor than for an indirect band gap semiconductor, where the process must be mediated by a phonon.

- As a result of such considerations, gallium arsenide and other direct band gap semiconductors are used to make optical devices such as LEDs and semiconductor lasers, whereas silicon, which is an indirect band gap semiconductor, is not.

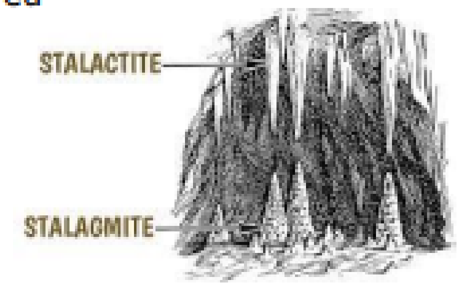
Variation of semiconductor

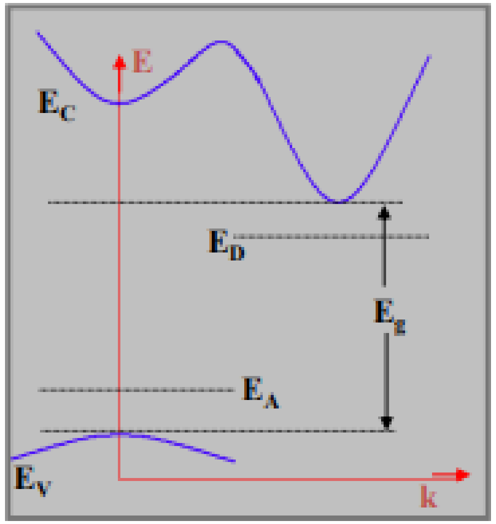
Classification of Semiconductor Materials

Energy (E) of an allowed electronic level α (Momentum)^2 i.e k^2



Minimum energy difference between valence band and conduction band is called energy gap.





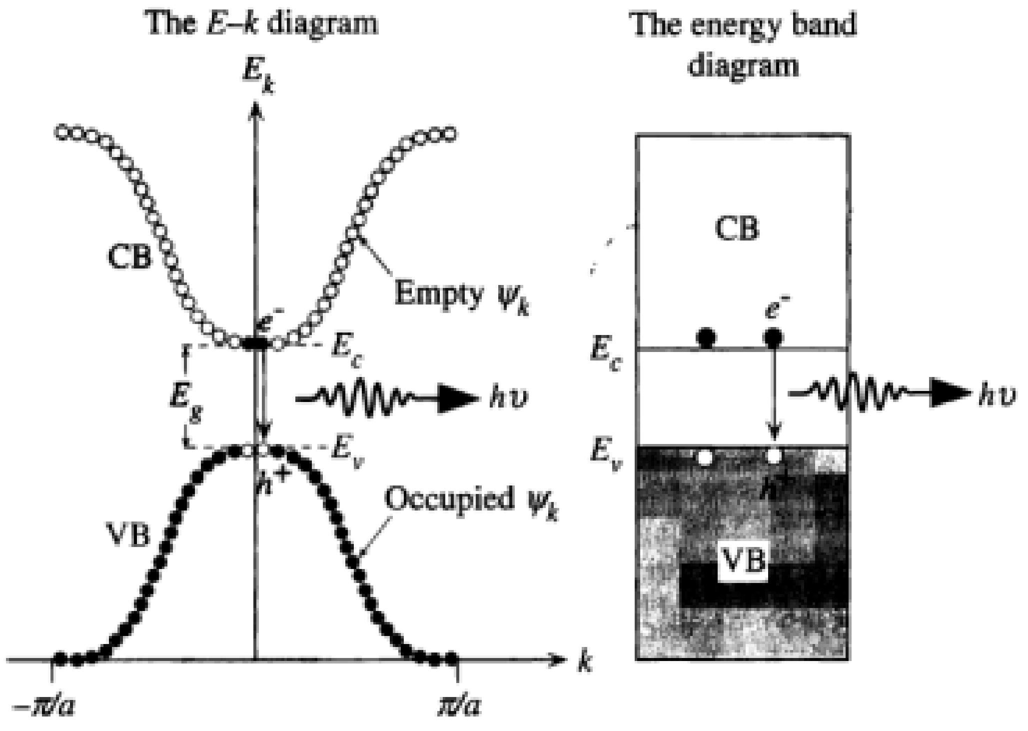
Direct Bandgap Semiconductors

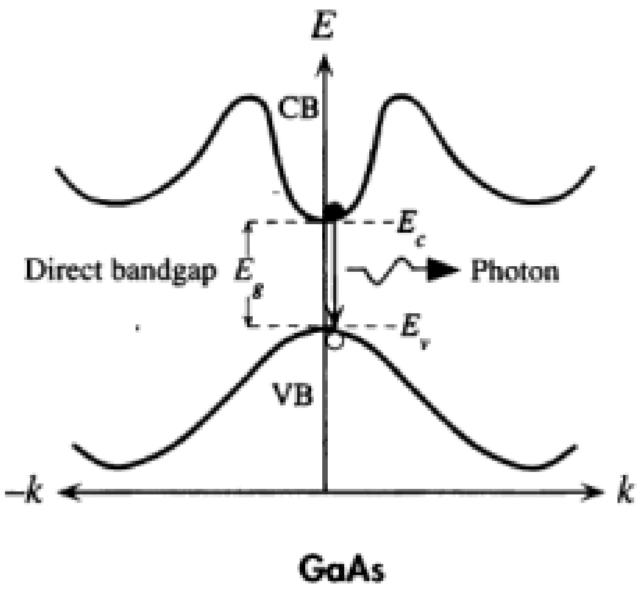
Minimum of conduction band and maximum of valence band occur at the same momentum value. So no momentum change is involved when electrons and holes combine. Results in
Maximum recombination efficiencies, fast (<1ns) decay time. Eg. GaAs, GaN (III-V compounds)

Indirect Bandgap Semiconductors
(Phonon assisted transitions)
Lower recombination efficiencies,
decay time higher by few orders
of magnitude Eg. Ge, Si, GaP, SiC

$$\lambda_{em} = \frac{hc}{E_g} = \frac{1240}{E_g(eV)} \text{ nm}$$

Variation of semiconductor





$$\lambda_{em} = \frac{hc}{E_g} = \frac{1240}{E_g(eV)} \text{ nm}$$

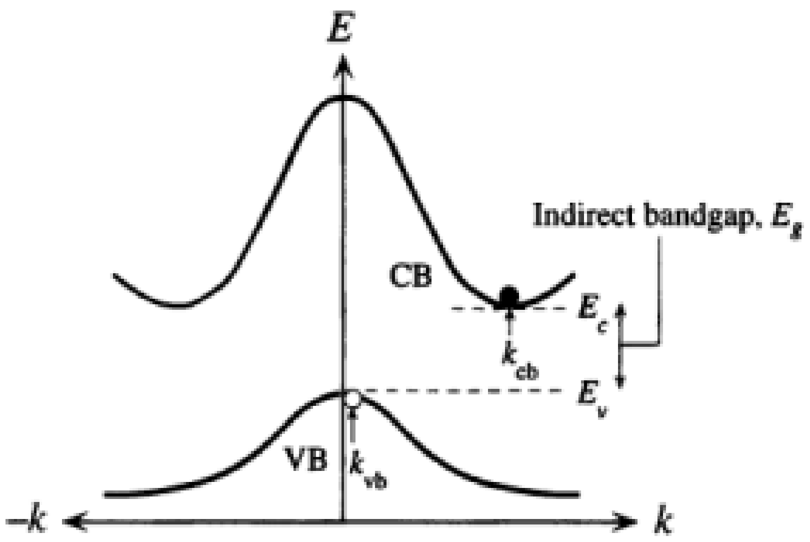
Direct Bandgap Semiconductors

Minimum of conduction band and maximum of valence band occur at the same momentum value. So no momentum change is involved when electrons and holes combine.

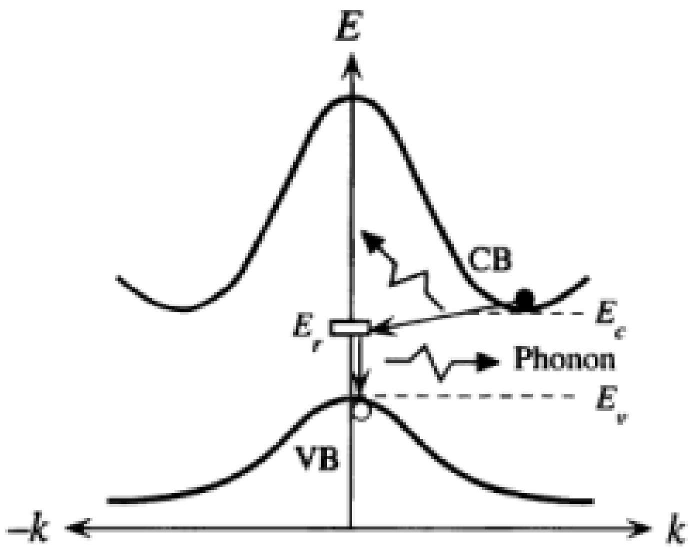
Maximum recombination efficiencies, fast (<1ns) decay time. Eg. GaAs, GaN (III-V compounds)

Conservation of momentum is necessary for efficient radiative recombination

Variation of semiconductor



Si



(c) Si with a recombination center

Indirect Bandgap Semiconductors

- An electron in conduction band cannot combine directly with hole in valence band as its momentum kcb is not equal to kvb and it will violate momentum conservation rule.
- Recombination can occur only through a recombination centre. Recombination center due to impurities are localized in space which means vacancy is localized leading to larger momentum spread of the vacancy/hole (as per uncertainty principle)

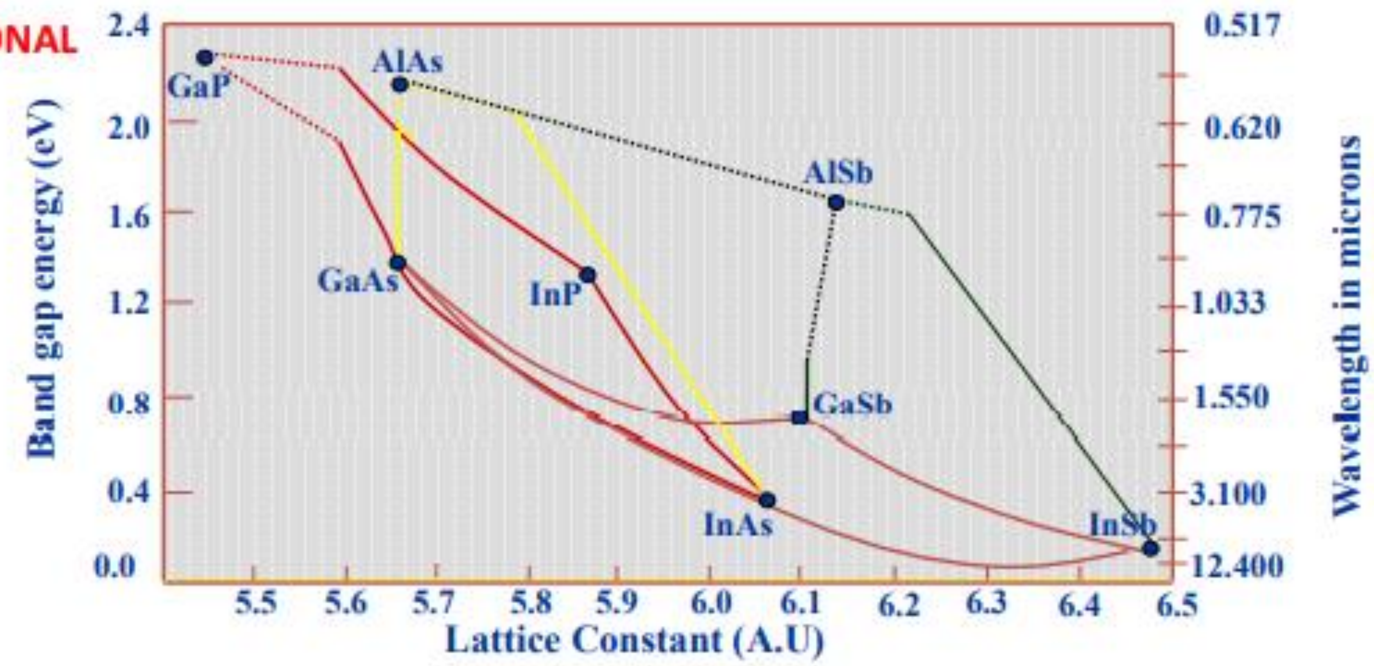
(Phonon assisted transitions)
Lower recombination efficiencies, decay time higher by few orders of magnitude Eg. Ge, Si, GaP, SiC

Bandgap of some Semiconductor Materials

Semiconductor	E_g(300K)	λ_C	Gap
InAs	0.3eV	4133nm	Direct
Ge/GaSb	0.7eV	1800nm	Indirect
Si	1.1eV	1100nm	Indirect
InP	1.3eV	950nm	Direct
GaAs	1.4eV	840nm	Direct
GaP	2.3eV	560nm	Indirect
SiC	2.8 eV	440nm	Indirect
GaN	3.5eV	350nm	Direct

Bandgap of some semiconductors

ADDITIONAL



Dotted lines represent indirect band gap compounds.

To cover the optical windows of interest in optical fiber networks, ternary compounds combining III-V elements as $IIIA_x\ IIIB_y\ VC$ or quarternary compounds combining III-V elements in in the form $IIIA_x\ IIIB_{(1-x)}\ VC_y\ VD_{(1-y)}$ are necessary.

Eg:

BINARY: $Ga_xAs_{(1-x)}$, $Ga_xP_{(1-x)}$

TERNARY: $Ga_1Al_xAs_{(1-x)}$, $In_1Ga_xP_{(1-x)}$

QUARTERNARY: $Ga_yAl_{(1-y)}In_xP_{(1-x)}$, $Ga_yAs_{(1-y)}In_xSb_{(1-x)}$

Effective mass

What is the implication of the slope of the E vs K curve and the inflections at the zone boundaries bands?

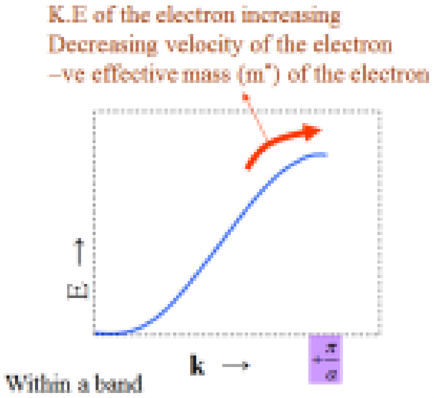
The curve represents change in kinetic energy or

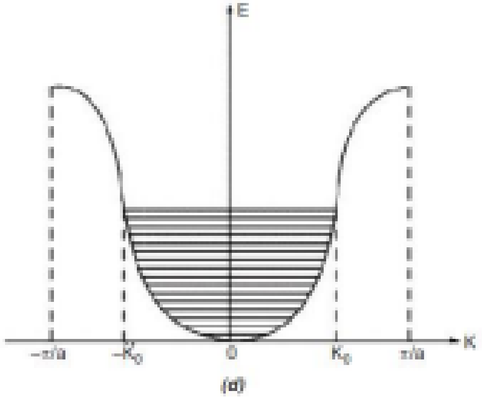
$E_{kinetic} = \frac{\hbar^2}{2m^*}\frac{k^2}{}$; differentiating once gives $dE/dk = \frac{2k\,\hbar^2}{2m^*} = \frac{k\,\hbar^2}{m^*}$

This is nearly zero at the start and end of the bands implying that electrons effective mass tends to infinity at the start and end of each band.

Second differential gives $d^2E/dk^2 = d/dk\left(\frac{k\,\hbar^2}{m^*}\right) = \left(\frac{\hbar^2}{m^*}\right)$ or $m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$

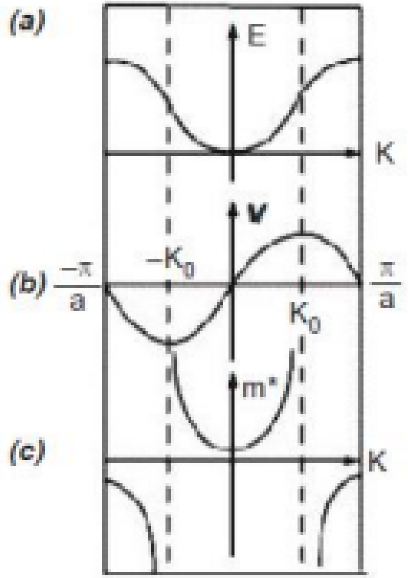
The E-k curve for each band thus indicates that the electrons in the states close to the band edges have higher effective mass and at the band edge they will be backscattered electrons giving rise to –ve effective mass values At intermediate energy levels in the middle of the bands, electrons behave as if they are comparatively lighter.





v=dE/dk

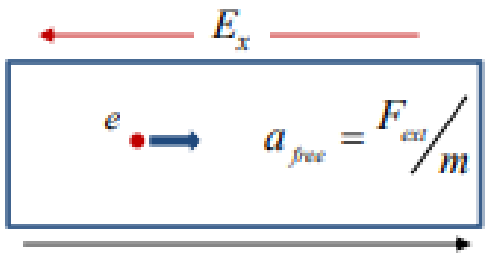
$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$



Effective mass

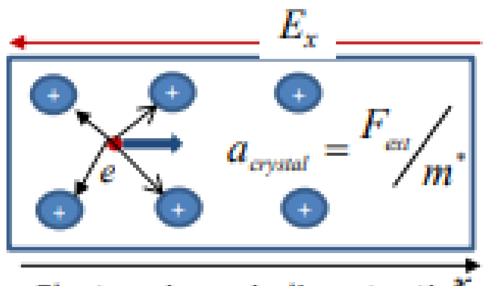
Effective mass of electron – Alternate derivation- ADDITIONAL

When an external field is applied , a free electron ($V_{internal}$=0) experiences an acceleration directly proportional to the external force F_{ext} i.e $F_{ext} = m\, a_{free}$



Free electron

When an external field is applied to an electron in periodic potential, ($V_{internal} \neq 0$) it experiences an acceleration directly proportional to the net force acting on it ie $F_R = F_{ext} + F_{int}$ i.e $F_r = m\, a_{crystal}$



Electron in periodic potential

Therefore the acceleration of electron $a_{crystal} = F_r / m$.

From the perspective of the external force, the electron in the periodic potential within the material now behaves as if it has different acceleration for the same applied field strength.

If we do not want to bring the internal force into picture , but want to express the acceleration of this electron also in terms of external force only., then for this electron which is also subjected to an internal force, we can incorporate the change as an effective mass of electron in the presence of internal periodic potential.

$a_{crystal} = F_{ext} / m^*$ inside the material .

List of formula

$$F(E) = \frac{1}{1+\exp\left(\frac{E-E_F}{k_B T}\right)}$$

$$g(E) = (4\pi/h^3)\ (2m_e^*)^{3/2}\sqrt{E}\,dE$$

$$n = \int_{E_1}^{E_2} g(E)F(E)\,dE$$

$$E_{F(0K)} = (h^2/2m^*_e)\ (3n_c/8\pi)^{2/3}$$

Average energy of electrons in metals at 0K.

$$= \frac{3}{5}E_{F(0K)}$$

$$n = N_c \times \exp\left(\frac{E_F - E_c}{k_B T}\right) \qquad p = N_v \exp\left(\frac{E_v - E_F}{kT}\right)$$

$$\boxed{E_F = \frac{(E_c + E_v)}{2} + \frac{3kT}{4}\ln\left(\frac{m_h^*}{m_e^*}\right)}$$

$$\boxed{E_F = \frac{E_g}{2} + \frac{3kT}{4}\ln\left(\frac{m_h^*}{m_e^*}\right)}$$

From top of valence band

$$\therefore\ n_i = \sqrt{np} = \sqrt{N_c N_v}\, e^{-\left(E_g/2kT\right)}$$

$$= 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_e^* m_h^*)^{3/4} e^{-\left(E_g/2kT\right)}$$

$$\sigma = (\mu_e + \mu_h)e \quad 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}(m_e^* m_h^*)^{3/4} e^{-\left(E_g/2kT\right)}$$

List of formula

N -Type

$$\therefore E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln\left[\frac{N_D}{N_c}\right]$$

$$\boxed{E_{Fn} = \frac{E_{cn} + E_D}{2} + \frac{k_B T}{2} ln\left[\frac{N_D}{2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/2}}\right]}$$

$$n_n = \sqrt{N_c N_D}\ exp\left(\frac{E_D - E_{cn}}{2k_B T}\right)$$

$$\boxed{n_n = (2N_D)^{1/2}\left(\frac{2\pi m_e^* k_B T}{h^2}\right)^{3/4} exp\left(\frac{E_D - E_{cn}}{2k_B T}\right)}$$

P -Type

$$\therefore E_{Fp} = \frac{E_A + E_{vp}}{2} - \frac{kT}{2} ln\left[\frac{N_A}{N_V}\right]$$

$$\boxed{E_{Fp} = \frac{E_A + E_{vp}}{2} - \frac{kT}{2} ln\left[\frac{N_A}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}\right]}$$

$$p_p = \sqrt{N_v N_A}\ exp\left(\frac{E_{vp} - E_A}{2\ kT}\right)$$

$$\boxed{p_p = (2N_A)^{1/2}\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/4} exp\left(\frac{-(E_A - E_{vp})}{2\ kT}\right)}$$

Hall Effect

When a metal or semiconductor carrying current is subjected to a magnetic field transverse to the current flow, a potential difference is produced in a direction normal to both the current and magnetic field. This phenomenon is called Hall effect and the voltage developed is called Hall voltage

It provided experimental proof that

- negatively charged carriers (electrons) are responsible for conductivity of monovalent metals
- there exists 2 types of charge carriers positive and negative in semiconductors

Hall effect helps in determining

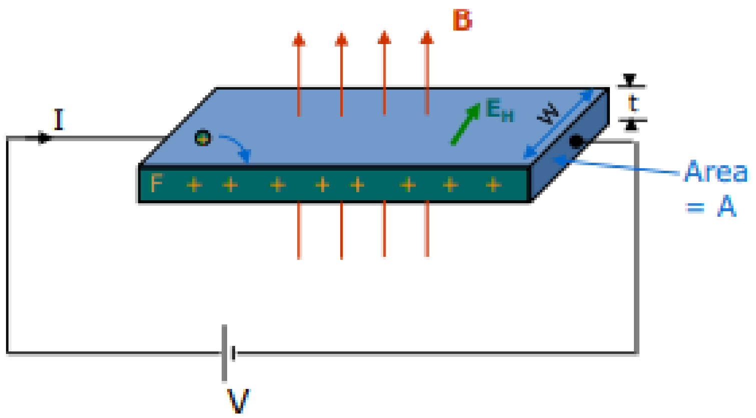
- sign of charge carriers
- concentration of charge carriers
- mobility of charge carriers.

Hall Effect

Consider a rectangular slab of P type material carrying current I

ADDITIONAL

- Holes flow parallel to face F of the crystal and this current $I = p\ e\ A\ v_d$



- Let a magnetic field B be applied normal to the surface as shown
- Holes now experience a deflection sideways due to Lorentz force, $F_L = e\,(v \times B) = Bev_d$ towards front face F and tend to pile up closer to this face.
- Therefore rear face, F' becomes relatively negative
- This results in a resultant electric field $\mathbf{E_H}$ from face F to its rear face F'
- This resultant electric field $\mathbf{E_H}$ prevents further build up of charges when force F_E due to E_H balances force F_L

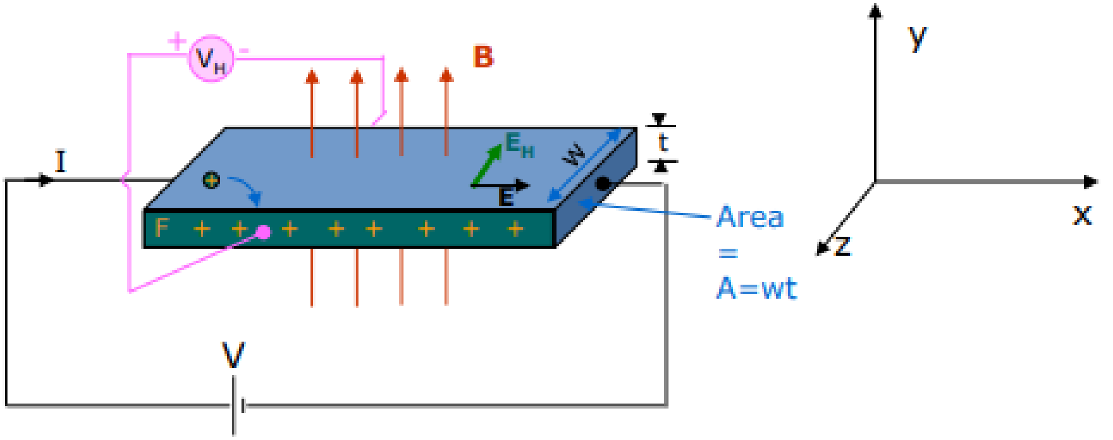
Hall Effect

$F_E = F_L$

$eE_H = B\,e\,v_d$ or $E_H = B\,v_d = B\,J/pe = B\,(I/Ape)$

$I = p\,e\,A\,v_d$
$J = I/A = p\,e\,v_d$

ADDITIONAL



$E_H = B\,J/pe = R_H BJ$

$\therefore$ Hall Coefficient , $R_H = E_H/BJ = 1/pe$

can be defined as Hall field generated per unit current density and per unit magnetic induction.

E_H, B and J can be measured experimentally, and therefore, R_H and hence carrier concentration , p, in this case, can be determined

How do you determine E_H experimentally?

$E_H = V_H/w$; $\therefore R_H = V_H/(w\,B\,J) = V_H / wB\,(I/A) = V_H wt / wBI = V_H t / BI$

Remember *t* is the dimension parallel to applied B direction

Hall Effect

ADDITIONAL

Determination of mobility

The net field on the semiconductor is the resultant of E_H and E acting at an angle θ to the X-axis

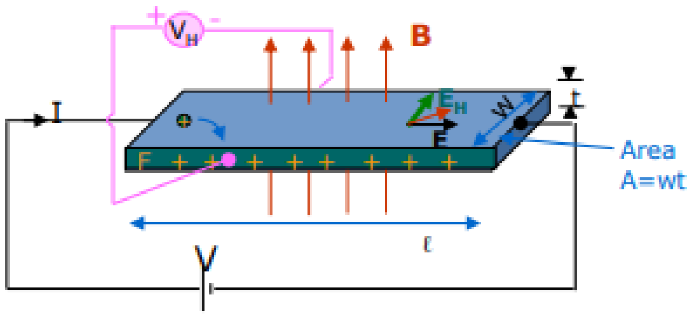
$$\tan\theta = E_H / E = V_H\,\ell / w\,V$$

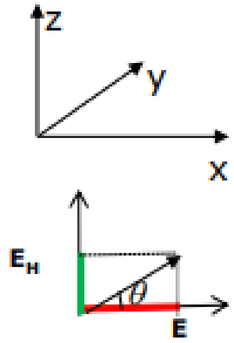
$$= R_H BJ / E$$

$$= R_H B\,\sigma E / E$$

$$= R_H B\,\sigma = R_H B\,pe\,\mu_h$$

$$\tan\theta = B\,pe\,\mu_h / pe = B\,\mu_h$$





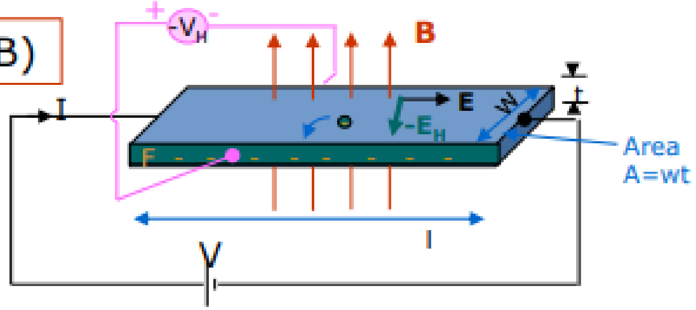
$$\boxed{\tan\theta = V_H\,\ell / w\,V = B\,\mu_h}$$

$$\boxed{\therefore \mu_h = R_H\sigma = \tan\theta / B = (V_H\,\ell)/(wVB)}$$

In N type semiconductors:

$$\boxed{\text{Hall Coefficient , } R_H = -E_H/BJ = -1/ne}$$

$$\boxed{\therefore \mu_e = R_H\sigma = (-V_H\,\ell)/(wVB)}$$



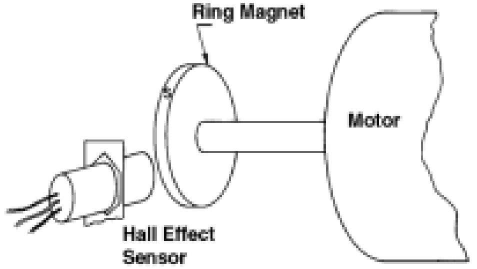
N-type semiconductor

Hall Effect

Current & magnetic sensor $E_H = R_H BJ$

ADDITIONAL

- the presence of a magnetic field can be detected by the appearance of Hall voltage across the hall effect sensor which is a conductor driven with appropriate voltage.

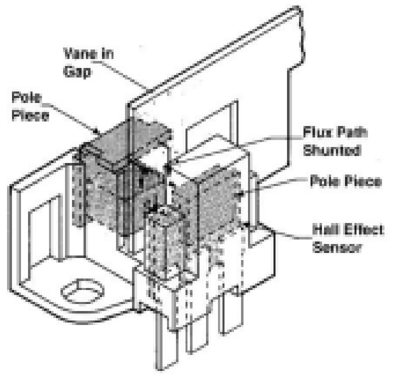


- if the magnetic field to be applied to the conductor is supplied by an electromagnet, then same device serves as a current sensor/electronic multiplier

Vane based position sensor

- If ferrous material inserted in gap, flux lines will get cut off, hence hall voltage will be zero.

- If vane is circular and rotated at fixed speed, this gives you a timing signal-timing for fuel injection and ignition.



Wattmeter

$$E_H = R_H BJ; \quad B \propto I_{coil}, \; J \propto V, \; \therefore E_H \text{ or } V_H \propto V\, I_{coil} \propto \text{power}$$

Applications of Hall effect

ADDITIONAL

Direction of Hall field (polarity of voltage) indicates type of charge carriers.

Measurement of Hall coefficient R_H leads to determination of carrier concentration .

If a semiconductor of known R_H is used, magnetic flux density B can be determined – Sensor application

If R_H and conductivity is measured for a given semiconductor, mobility of carriers can be determined.

If an intrinsic semiconductor is used in the Hall set-up given in earlier figure, what will be the polarity of the Hall voltage that you will observe, if a DVM is used?

$$R_H = \frac{-n\mu_e^2 + p\mu_h^2}{e(n\mu_e + p\mu_h)^2}$$