Statistical Modeling and Inference – Problem Set #4

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Solution to proposed exercises.

Exercise 1

We need to show that $y(\mu + \sigma)$ is a point less than one standard deviation away from the mean of the marginal distribution of t, that is:

$$y(\mu + \sigma) \leq \bar{t} + \sqrt{\mathbb{V}[t]}$$

Given that $x \sim \mathcal{N}(\mu, \sigma^2)$ and $\varepsilon \sim \mathcal{N}(0, \tau^2)$ are assumed to be uncorrelated:

$$\begin{aligned} \mathbb{E}[t] &= \mathbb{E}[x + \varepsilon] = \mathbb{E}[x] + \mathbb{E}[\varepsilon] = \mu = \bar{t} \\ \mathbb{V}[t] &= \mathbb{V}[x + \varepsilon] = \mathbb{V}[x] + \mathbb{V}[\varepsilon] = \sigma^2 + \tau^2 \end{aligned}$$

We see that $\mu = \bar{t}$ because given the distribution of its components t is a normally (and thus symmetrically) distributed around its mean, and has the same expected value as x. On the other hand:

$$y(\mu + \sigma) = \mathbb{E}[t | x = \mu + \sigma] = \mu + \sigma$$

And so we would need to show that:

$$\begin{aligned} y(\mu + \sigma) &\leq \mu + \sqrt{\mathbb{V}[t]} \\ \mu + \sigma &\leq \mu + \sqrt{\sigma^2 + \tau^2} \\ \sigma^2 &\leq \sigma^2 + \tau^2 \\ \tau^2 &\geq 0 \end{aligned}$$

We know that $\tau^2 \geq 0$ is indeed non-negative, thus proved.

Exercise 2

Part (a)

Answer.

Part (b)

Answer.

Part (c)

Answer.

Part (d)

Answer.

Exercise 3

Part (a)

Answer.

Part (b)

Answer.

Part (c)

Answer.

Exercise 4

Answer.