

$$A + \bar{A} = 1 \rightarrow \text{Tautology}$$

$$A \cdot \bar{A} = 0 \rightarrow \text{Tautology}$$

Expression is ~~not~~ tautology iff its dual is tautology

* SOP and POS

L-18

We have two way to represent function SOP and POS.

* Function

- SOP (Sum of Products)
- POS (Product of sum)

• SOP (disjunctive normal form) → Principal

Standard SOP ← $f(A, B) = A\bar{B} + \bar{A}B$ → P DNF

Canonical SOP

$f(A, B, C) = A\bar{B} + B\bar{C} + AC$ } SOP ✓

$f(A, B, C, D) = AB + CD$ } SSOP X

POS (Conjunctive normal form)

$f(A, B) = (A+B) \cdot (\bar{A} + \bar{B}) \rightarrow$ PCNF

$f(A, B, C) = (\bar{A} + B + C) \cdot (A + \bar{B}) \rightarrow$ POS ✓

$f(A, B, C, D) = (A + B) \cdot (\bar{C} D) \rightarrow$ SPOS ✗

SPOS
non-minimal
POS

* SSOP \rightarrow Standard Sum of Product

A	B	
0	0	$\bar{A}\bar{B}$ ← minterms
0	1	$\bar{A}B$
1	0	$A\bar{B}$
1	1	AB

If each term in function is minterm/boolean term then we call it SSOP

$f(A, B, C) = AB\bar{C} + \bar{A}BC$

$A\bar{B}C + \bar{A}BC + AB\bar{C} + ABC$

$\rightarrow A\bar{B} + \bar{A}B \rightarrow$ Not SSOP

Every var must be present.

SSOP \Rightarrow SSOP

$$\bullet f(A, B, C) = A\bar{B} + \bar{A}B$$

add missing variables

$$A\bar{B} \cdot [C + \bar{C}] + \bar{A}B[C + \bar{C}]$$

$$A\bar{B}C + A\bar{B}\bar{C} + \bar{A}BC + \bar{A}B\bar{C} \quad \underline{\text{Ans}}$$

$$\bullet f(A, B, C) \Rightarrow AB + BC$$

$$AB[C + \bar{C}] + (A + \bar{A})BC$$

$$ABC + A\bar{B}C + \underline{ABC} + \bar{A}BC$$

$$ABC + A\bar{B}C + \bar{A}BC \quad \underline{\text{Ans}}$$

$$\bullet f(A, B, C, D) = AB + \bar{C}D$$

$$AB(C + \bar{C})(D + \bar{D}) + (A + \bar{A})(B + \bar{B})\bar{C}D$$

$$\downarrow$$

$$ABCD + ABC\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}D$$

$$ABCD + A\bar{B}CD + \bar{A}BCD + \bar{A}\bar{B}CD \quad \underline{\text{Ans}}$$

$$\bullet f(A, B, C) = \bar{A}B + AC$$

$$\downarrow \quad \downarrow$$

$$\bar{A}BC + \bar{A}B\bar{C} + ABC + A\bar{B}C$$

Ans

SSOP \rightarrow New representation

A B

0 0

$\bar{A}\bar{B} \Rightarrow m_0$

0 1

$\bar{A}B \Rightarrow m_1$

1 0

$A\bar{B} \Rightarrow m_2$

1 1

$AB \Rightarrow m_3$

$$f(A, B) = m_0 + m_2$$

$$f(A, B) = AB + \bar{A}\bar{B}$$

\downarrow

$$f(A, B) = \sum m(0, 3) \quad \sum m(3, 0)$$

$$f(A, B, C) = A\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + ABC$$

\downarrow

$$f(A, B, C) = \sum m(5, 2, 3, 7)$$

$$f(A, B) = AB + BC$$

\downarrow

$$f(A, B) = \sum m(7, 6, 3)$$

$$f(A, B, C) = CB + \bar{C}A$$

$$\sum m(7, 3, 6, 4)$$

* Product of sum

POS \rightarrow SPOS

$$\begin{aligned}
 \bullet f(A, B, C) &= (A+B) \cdot (\bar{A}+\bar{B}) \\
 &= (A+B+C \cdot \bar{C}) \cdot (\bar{A}+\bar{B}+C \cdot \bar{C}) \\
 &= (A+B+C)(A+B+\bar{C}) \cdot (\bar{A}\bar{B}C)(\bar{A}\bar{B}+\bar{C}) \quad \underline{\underline{Ans}} \\
 &= \pi M(0, 1, 6, 7)
 \end{aligned}$$

$$\begin{aligned}
 \bullet f(A, B, C, D) &= (\bar{A}+B) \cdot (C+\bar{D}) \\
 &\quad \downarrow \\
 &= (\bar{A}+B+\bar{C}+\bar{D}) \cdot (\bar{A}+B+\bar{C}+D) \cdot (\bar{A}+B+C+\bar{D}) \\
 &\quad (\bar{A}+B+C+D) \\
 &= (\bar{A}+\bar{B}+C+\bar{D}) \cdot (\bar{A}+B+C+\bar{D}) \cdot (A+\bar{B}+C+\bar{D}) \cdot (A+B+C+\bar{D}) \quad \underline{\underline{Ans}} \\
 &= \pi M(4, 7, 3)
 \end{aligned}$$

A	B	r				
0	0	$\bar{A}\bar{B}$	m_0	$\bar{\bar{A}}\bar{\bar{B}}$	\Rightarrow	AB M_0
0	1	$\bar{A}B$	m_1	$\bar{\bar{A}}\bar{\bar{B}}$	\Rightarrow	$A\bar{B}$ M_1
1	0	$A\bar{B}$	m_2	$\bar{\bar{A}}\bar{\bar{B}}$	\Rightarrow	$\bar{A}B$ M_2
1	1	AB	m_3	$\bar{\bar{A}}\bar{\bar{B}}$	\Rightarrow	$\bar{A}\bar{B}$ M_3

minterms

maxterm

Standard pos \rightarrow every term must be maxterm.

$$f(A, B) \rightarrow (\bar{A} + B) \cdot (B + \bar{C})$$

$$\Rightarrow \pi M(4, 5, 2, 3)$$

$$\Rightarrow \pi M(4, 5, 1) \text{ dup}$$

$$f(A, B, C, D) = (\bar{A} + C) \cdot (B + \bar{C})$$

$$\pi M(8, 11, 12, 13, 1, 3, 9)$$

A	B	$f(A, B)$
0	0	1
0	1	0
1	0	0
1	1	1

Minterm ON SOP
 $f(A, B) = \bar{A}\bar{B} + AB$

$$f(A, B) = (A + \bar{B}) \cdot (\bar{A} + B)$$

\Downarrow

Maxterm OFF POS