

L-15

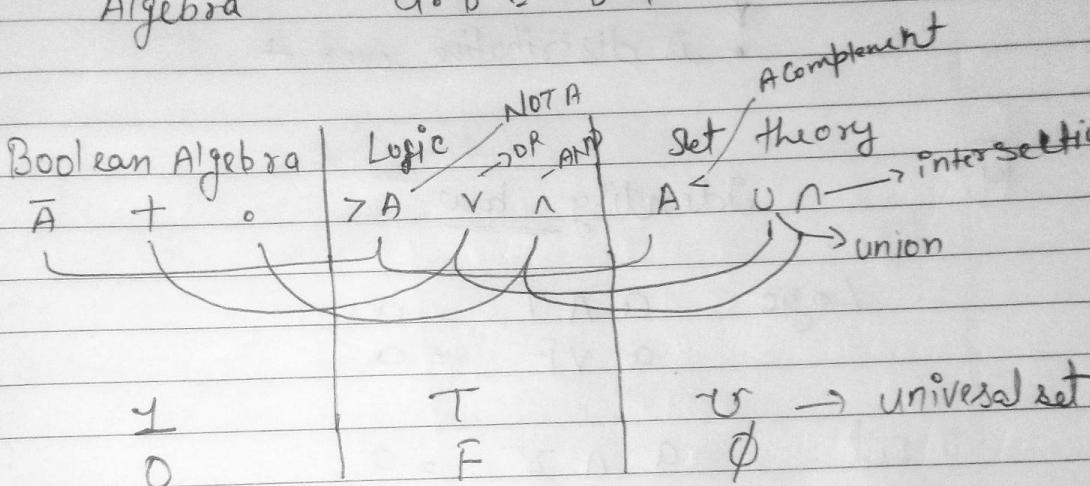
* Boolean laws

① Commutative law

Logic $a \vee b = b \vee a$
 $a \wedge b = b \wedge a$

set $a \cup b = b \cup a$
 theory $a \cap b = b \cap a$

Boolean
Algebra $a + b = b + a$
 $a \cdot b = b \cdot a$



② Associative law

Logic $= a \vee (b \vee c) = (a \vee b) \vee c$
 $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

Set theory $= a \cup (b \cup c) = (a \cup b) \cup c$
 $a \cap (b \cap c) = (a \cap b) \cap c$

Boolean
Algebra $= a + (b + c) = (a + b) + c$
 $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(3)

Distributive Law

Logic $\rightarrow a \cdot (b \vee c) = (a \cdot b) \vee (a \cdot c)$
 $a \vee (b \cdot c) = (a \vee b) \cdot (a \vee c)$

Set theory $\rightarrow a \cdot (b \cup c) = (a \cap b) \cup (a \cap c)$
 $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

Boolean Algebra \rightarrow

$a \cdot (b + c) = a \cdot b + a \cdot c$ $a + (b \cdot c) = (a + b) \cdot (a + c)$	\downarrow + is distributive over •
\downarrow • is distributive over +	

(4)

Identity law

Logic $a \wedge T = a$
 $a \vee F = a$

Set theory $a \cap \mathcal{U} = a$
 $a \cup \emptyset = a$

Boolean Algebra $a \cdot 1 = a$
 $a + 0 = a$

5Complement law

Logic

$$a \vee \neg a = T$$

$$a \wedge \neg a = F$$

set theory

$$a \cup a^c = U$$

$$a \cap a^c = \emptyset$$

Boolean

Boolean

$$a + \bar{a} = 1$$

Algebra

$$a \cdot \bar{a} = 0$$

6Idempotent law

Logic

$$a \vee a = a$$

$$a \wedge a = a$$

set

$$a \cup a = a$$

theory

$$a \cap a = a$$

Boolean

$$a + a = a$$

Algebra

$$a \cdot a = a$$

6double Complement

Logic

$$\neg(\neg a) = a$$

Boolean

$$\overline{\overline{A}} = A$$

Algebra

set

$$(a^c)^c = a$$

theory

(7)

DeMorgan's Law

Logic	$(a \vee b \vee c \dots) = \neg \neg a \vee \neg \neg b \vee \neg \neg c \dots$	boolean $a + b + c = 1$
	$\neg(a \wedge b \wedge c \dots) = \neg a \wedge \neg b \wedge \neg c \dots$	Algebra $\bar{a} \cdot \bar{b} \cdot \bar{c} = \bar{a+b+c}$

Set $(a \cup b \cup c \dots) = a^{\leftarrow} \cap b^{\leftarrow} \cap c^{\leftarrow}$
 $(a \cap b \cap c \dots) = a^{\leftarrow} \cup b^{\leftarrow} \cup c^{\leftarrow}$

Trick to remember \Rightarrow change the sign
 break the line

$$a + b \Leftrightarrow \bar{a} \cdot \bar{b}$$

(8)

Domination law

logic	$a \vee T = T$	boolean $a + 1 = 1$
	$a \wedge F = F$	Algebra $a \cdot 0 = 0$

Set $a \vee \bar{a} = T$
 theory $a \cap \emptyset = \emptyset$

(9)

Absorption law

Logic $a \vee (a \wedge b) = a$
 $a \vee (\neg a \wedge b) = a \vee b$
 $a \wedge (a \vee b) = a$

Set theory

$$a \cup (a \cap b) = a$$

$$a \cup (a \bar{\cup} b) = a \cup b$$

$$a \cap (a \cup b) = a$$

Boolean algebra

$$a + ab = a$$

$$a + \bar{a}b = a + b$$

$$a \cdot (a + b) = a$$

 \vdash \vdash * Proof

$$a + ab = a$$

$$a(\underline{1+b}) =$$

$$\underline{a \cdot 1}$$

$$\underline{\downarrow}$$

$$\underline{a}$$

domination law

Absorption law

• domination law proof

$$a+1 = 1 \quad ??$$

$$\begin{array}{cc|c} a & 1 & a+1 \\ \hline 0 & 1 & 1 \end{array} \rightarrow \text{proof}$$

$$a \cdot 0 = 0 \quad ??$$

$$\begin{array}{cc|c} a & 0 & a \cdot 0 \\ \hline 0 & 0 & 0 \\ 1 & 0 & 0 \end{array}$$

$$a + \bar{a} \cdot b = a + b$$

$$(a + \bar{a}) \cdot (a + b) \rightarrow$$

distributive

law applied

Absorption law

Complement

$$\text{law} \rightarrow 1 \cdot (a + b)$$

$$\downarrow \rightarrow \text{identity}$$

$$a + b \quad \underline{\text{law}}$$

• Identity law

$$\begin{array}{c}
 a \cdot 1 = a \\
 \hline
 1 \cdot 1 = 1 \\
 \text{Same}
 \end{array}$$

$$a + 0 = a$$

$$\begin{array}{c}
 a + 0 = a \\
 \hline
 1 + 0 = 1 \\
 \text{Same}
 \end{array}$$

• $a \cdot (a+b) = a$

\downarrow distributive law

$$a \cdot a + a \cdot b$$

\downarrow idempotent law

$$\underline{a + ab = a}$$

~~Absorb~~

Absorption law

Q

$$P + Q + \bar{P} \rightarrow \text{minimize this}$$

$$P + (Q + \bar{P}) \rightarrow \text{commutative law}$$

$$\downarrow \quad \downarrow \rightarrow \text{commutative law}$$

$$(P + \bar{P}) + Q$$

$$\downarrow \rightarrow \text{complement law}$$

$$1 + Q$$

$$\downarrow \rightarrow \text{domination law}$$

$$9 \quad P \vee P' \wedge Q \rightarrow \text{logic style question}$$

$$\downarrow \quad P + \bar{P} + Q$$

~~changed
syntax~~ to boolean Algebra

$$\downarrow \quad 1 + Q = 1 \Rightarrow T \text{ is true in logic}$$

Tautology \Rightarrow means always true

$$9 \quad P \wedge (Q \wedge P')$$

$$P \cdot (Q \cdot \bar{P})$$

$$\downarrow \quad \rightarrow \text{commutative law}$$

$$P \cdot (\bar{P} \cdot Q)$$

$$\downarrow \quad \rightarrow \text{Associative law}$$

$$(P \cdot \bar{P}) \cdot Q$$

$$\downarrow \quad \rightarrow \text{Complement law}$$

$$\underline{0 \cdot Q} \Rightarrow \text{Domination law}$$

$$\downarrow$$

$$0 \Rightarrow F \Rightarrow \text{Contradiction}$$

$$10 \quad (P \vee Q) \wedge (P \vee Q')$$

$$(P + Q) \cdot (P + \bar{Q})$$

\downarrow distributive law

$$P + Q \cdot \bar{Q}$$

\downarrow \rightarrow Complement law

$$P + 0$$

\downarrow \rightarrow Identity law

$$P$$

$$\text{f_w} \quad PQR + (PQR + PR) \rightarrow \text{associative law}$$

\downarrow Commutative law

$$PQR + (PQR + \bar{P}\bar{Q}R) \rightarrow \text{associative law}$$

$$(PQR + PQR) + \bar{P}\bar{Q}R \rightarrow \text{idempotent law}$$

$$PQR + \bar{P}\bar{Q}R \rightarrow \text{idempotent law}$$

Ans

$$\text{f_w} \quad PnQnPRnS'$$

\downarrow

$$P \cdot Q \cdot R \cdot S'$$

$$\underline{P \cdot P \cdot Q \cdot R \cdot S'}$$

$$\underline{P \cdot Q \cdot R \cdot S'} \rightarrow \underline{PnQnRnS'}$$

$$\text{f_w} \quad PnQ \cup (PnQ \cap (RnSNT))$$

$$P \cdot Q + (P \cdot Q \cdot (RST))$$

$$PQ + PQ R ST$$

$$PQ (I + RST)$$

$$PQ \cdot I \Rightarrow PQ \quad \underline{\text{Ans}}$$

Ques. $T \vee P \wedge Q \vee P' \wedge Q' \wedge R \vee P' \wedge Q' \wedge S'$

$$1 + \bar{P}Q + \bar{P} \cdot Q + \bar{P} \cdot \bar{Q} \cdot \bar{R} + \bar{P} \cdot Q \cdot \bar{S}$$

\downarrow 1 + something is 1

$$1 \Rightarrow T$$

How

$$1 + \bar{P} + PQR + ST$$

\downarrow

$$(1 + \bar{P}) + PQR + ST$$

\downarrow

$$(1 + PQR) + ST$$

$$1 + ST$$

\downarrow

$$1$$

solved using domination law

Ques.

$$A \cup (B - C) = (A \cup B) - (A \cap C)$$

$$\frac{A \cap B \Rightarrow A \cap B^c}{\downarrow 1} \quad \frac{\downarrow 2, 3 \cap (1 \cup 2)}{\downarrow 1}$$

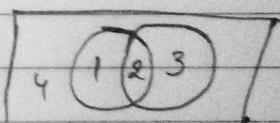
$$A \cup (B \cap C^c) = (A \cup B) \cap (A \cap C)^c$$

$$A + B\bar{C} = (A + B) \cdot (\bar{A} + \bar{C})$$

$$\bar{A}\bar{C} + \bar{A}B + B\bar{C}$$

$$A + B\bar{C} = \bar{A} \cdot B\bar{C}$$

Not equal



\cup is distributive over set diff or not?

No

• $A \cap (B - C) = (A \cap B) - (A \cap C)$

\cap is distributive of set difference
or not?
intersection

$$A \cdot B \cdot \bar{C} = \cancel{AB} (A \cap B) \cap (A \cap C)^c$$

$$\begin{array}{l} A \cdot B + \bar{A} \cdot C \\ AB(\bar{A} + \bar{C}) \\ A\bar{B} \end{array}$$

distributive

• $(A \cap B) \vee (A \cap B')$

$$\begin{array}{l} AB + A\bar{B} \\ A(B + \bar{B}) \\ A \cdot 1 \Rightarrow A \end{array}$$

↓ D.L

• $(A \vee B) \cap (A \vee B')$

$$\begin{array}{l} (A + B) \cdot (A + \bar{B}) \\ A + B + \bar{B} \\ A + 0 = A \end{array}$$

↓ D.L
↓ CL
↓ g.L

$$A \vee (A' \cap B) \vee (A \cap B')$$

$$A + \bar{A}B + A\bar{B}$$

$$A(1 + \bar{B}) + \bar{A}B$$

↓ Domination law

$$A \cdot 1 + \bar{A}B$$

$$\begin{array}{l} A + \bar{A}B \\ AB \end{array}$$

↓ Identity law
↓ Absorption law

$$\bullet \quad \overline{A \cdot B} \cdot \overline{A} + \overline{A \cdot B} \cdot \overline{B}$$

$$\overline{\overline{A \cdot B} \cdot A} + \overline{\overline{A \cdot B} \cdot B}$$

$$(A\bar{B} \cdot A) + (\bar{A}\bar{B} \cdot B)$$

$$(\bar{A} + \bar{B}) \cdot A + [(\bar{A} + \bar{B}) \cdot B]$$

$$A\bar{B} + \bar{A}B$$

$$A \oplus B$$

↓
XOR

$$\bullet \quad A \otimes B = AB + A\bar{B} + \bar{A}B$$

↓ DL
A \cdot (B + \bar{B}) + \bar{A}B

↓ CL

$$A \cdot 1 + \bar{A}B$$

↓ g

$$A + \bar{A}B$$

↓ A
A + B

or
function

$$\bullet \quad (AVC) \cap (A'VB)$$

$$(A+C) \cdot (\bar{A}+B)$$

Simpl

$$AB + \bar{A}C + BC, \text{ by step}$$

$$AB + AC + (A + \bar{A}) \cdot BC$$

$$AB + \bar{A}C + ABC + \bar{A}BC$$

$$\rightarrow AB + \bar{A}C$$

A	B	OP
0	0	0
0	1	1
1	0	1
1	1	1

$$\bullet \quad B \cap (AVB)$$

$$B \cdot (A+B)$$

$$\bullet \quad A - (A - B)$$

$$A - (A \cap B^c)$$

$$A \cap (A \cap B^c)^c$$

$$\cancel{A \cdot B} \quad A \cdot \overline{\overline{A} \cdot B}$$

$$A \cdot (\bar{A} \cdot B)$$

$$A \cdot B \Rightarrow A \cap B \quad \underline{\text{Ans}}$$

$$AB + B \cdot B$$

↓ idempotent law

$$AB + B$$

$$B(A + B(1+A))$$

$$B \quad \underline{\text{Ans}}$$

• gate question 2008

$$(P \wedge Q \wedge R) \vee (P^c \wedge Q \wedge R) \vee Q^c \vee R^c$$

$$PQR + \bar{P}QR + \bar{Q} + \bar{R}$$

$$QR(P + \bar{P}) + \bar{Q} + \bar{R}$$

$$\underbrace{QR}_{\text{grouped}} + \bar{Q} + \bar{R}$$

$$\bar{Q} + R + \bar{R}$$

$$\bar{Q} + 1 = 1 \Rightarrow 1 \rightarrow 25 \quad \underline{\text{Ans}}$$

$$A + \bar{A}B$$

$$AB$$

$$\bar{A} + A \cdot B$$

$$(\bar{A} + A) \cdot (\bar{A} + B)$$

$$1 + B$$

• gate 2004

$$\bar{x}\bar{y} + xy + \bar{x}\bar{y}$$

$$\begin{matrix} \bar{x}\bar{y} + y \cdot (x + \bar{x}) \\ \bar{x}\bar{y} + y \end{matrix}$$

$$\begin{matrix} y + \bar{y}\bar{x} \\ y + \bar{x} \end{matrix} \quad \underline{\text{Ans}}$$

• gate 2008

$$(P + \bar{Q}) \cdot \underbrace{(P\bar{Q} + PR)}_{(Q + P \cdot \bar{P} \cdot R)} \cdot (\bar{P}\bar{R} + \bar{Q})$$

$$(Q + P \cdot \bar{P} \cdot R) \cdot (P\bar{Q} + PR)$$

$$Q \cdot (P\bar{Q} + PR)$$

$$P\bar{S} + P\bar{S}R$$

$$P\bar{S}(1+R)$$

$$P\bar{S} \quad \underline{\text{Ans}}$$

• Jethi 2014

$$P\bar{S} + \bar{P}S\bar{R} + \bar{P}S\bar{R}\bar{S}$$

$$\bar{S}(P + \bar{P}R) + \bar{P}S\bar{R}\bar{S}$$

$$\bar{S}(P + R) + \bar{P}S\bar{R}\bar{S} \quad \downarrow \text{absorption law}$$

$$P\bar{S} + S\bar{R} + \bar{P}S\bar{R}\bar{S}$$

$$P\bar{S} + \bar{S}(R + \bar{R}\bar{P}\bar{S})$$

$$P\bar{S} + \bar{S}(R + \bar{P}\bar{S})$$

$$\underline{P\bar{S} + \bar{S}R + \bar{P}S\bar{S}}$$

$$\cancel{\bar{S}(P + R)} -$$

$$\bar{S}(P + \bar{P}S) + \bar{S}R$$

$$\bar{S}(P + S) + \bar{S}R$$

$$P\bar{S} + \bar{S}S + \bar{S}R$$

Ans