

* Boolean laws

L-15

① Commutative law

Logic $a \vee b = b \vee a$
 $a \wedge b = b \wedge a$

set theory $a \cup b = b \cup a$
 $a \cap b = b \cap a$

Boolean Algebra $a + b = b + a$ \rightarrow (OR)
 $a \cdot b = b \cdot a$

Boolean Algebra	Logic	Set theory
\bar{A} $+$ \cdot	$\neg A$ \vee \wedge	A^c \cup \cap
$\neg A$ \rightarrow NOT A	$\neg A$ \rightarrow NOT A	A^c \rightarrow A Complement
\vee \rightarrow OR	\vee \rightarrow OR	\cup \rightarrow union
\wedge \rightarrow AND	\wedge \rightarrow AND	\cap \rightarrow intersection
1	T	U \rightarrow universal set
0	F	\emptyset

② Associative law

Logic = $a \vee (b \vee c) = (a \vee b) \vee c$
 $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

Set theory = $a \cup (b \cup c) = (a \cup b) \cup c$
 $a \cap (b \cap c) = (a \cap b) \cap c$

Boolean Algebra = $a + (b + c) = (a + b) + c$
 $a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(3)

Distributive law

Logic $\rightarrow a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$
 $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

Set theory $\rightarrow a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$
 $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$

Boolean
Algebra \rightarrow

$$a \cdot (b + c) = a \cdot b + a \cdot c$$

$$a + (b \cdot c) = (a + b) \cdot (a + c)$$

\cdot is distributive over $+$

\rightarrow $+$ is distributive over \cdot

(4)

Identity law

Logic $a \cap T = a$
 $a \cup F = a$

Set theory $a \cap U = a$
 $a \cup \emptyset = a$

Boolean Algebra $a \cdot 1 = a$
 $a + 0 = a$

5Complement law

Logic

$$a \vee \neg a = T$$

$$a \wedge \neg a = F$$

Set theory

$$a \cup a^c = U$$

$$a \cap a^c = \emptyset$$

Boolean

Boolean

Algebra

$$a + \bar{a} = 1$$

$$a \cdot \bar{a} = 0$$

6Idempotent law

Logic

$$a \vee a = a$$

$$a \wedge a = a$$

Set

theory

$$a \cup a = a$$

$$a \cap a = a$$

Boolean

Algebra

$$a + a = a$$

$$a \cdot a = a$$

6Double Complement

Logic

$$\neg(\neg a) = a$$

Boolean

Algebra

$$\overline{\overline{a}} = a$$

Set

theory

$$(a^c)^c = a$$

⑦

Demorgan's Law

Logic $(a \vee b \vee c \dots) = > a > b > c \dots$
 $(a \wedge b \wedge c \dots) = > a^v > b^v > c^v \dots$

boolean $a + b + c \dots$
 Algebra $a \cdot b \cdot c \dots$

set $(a \cup b \cup c \dots) = a^c \cap b^c \cap c^c \dots$
 $(a \cap b \cap c \dots)^c = a^c \cup b^c \cup c^c \dots$

Trick to remember \Rightarrow change the sign
 break the line

$$\overline{a + b} \Leftrightarrow \overline{a} \cdot \overline{b}$$

⑧

Domination law

logic $a \vee T = T$
 $a \wedge F = F$

boolean $a + 1 = 1$
 Algebra $a \cdot 0 = 0$

Set theory $a \cup U = U$
 $a \cap \emptyset = \emptyset$

⑨

Absorption law