

2 conditions for orthogonal

- ↳ Natural
- ↳ Mutual exclusive terms

* K-MAP (Karnaugh Map) L-20

- Boolean variable → symbol which can take value 0 or 1
Ex: $n \leftrightarrow 0$ $n \leftrightarrow 1$
- Literal → use of variable or its complement in an expression

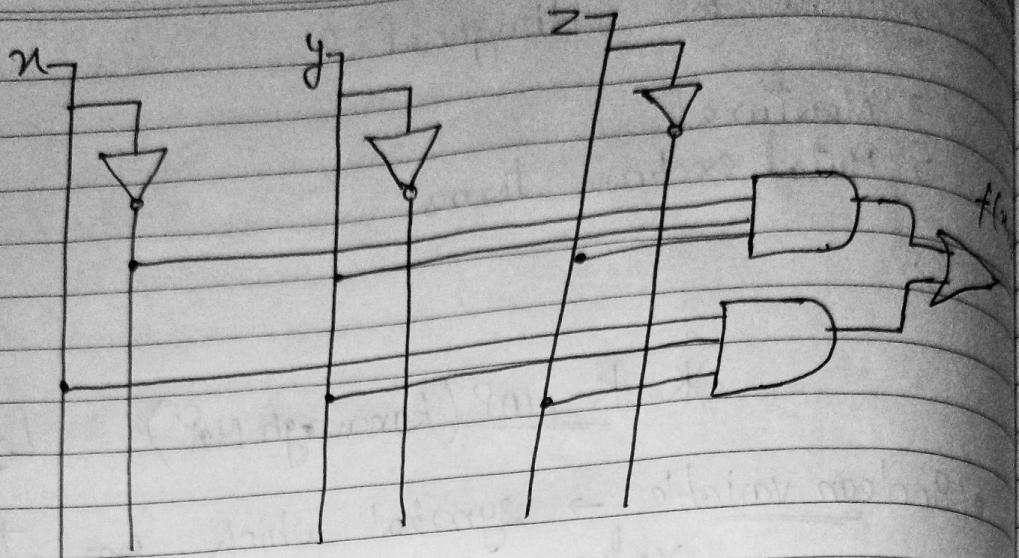
$$f(x, y) = \bar{y}x + \bar{x}\bar{y}$$

2 variables

4 Literals

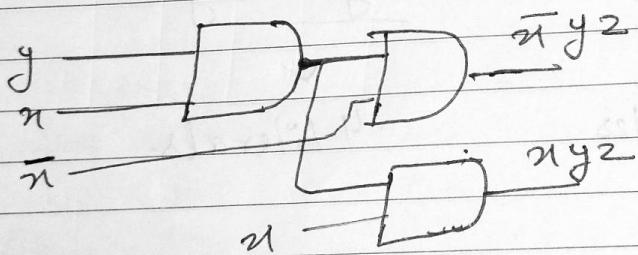
* Implement the following function

$$f(x, y, z) = \bar{x}yz + xy\bar{z}$$



2 AND gate (3 I/P)
 1 OR gate
 1 Not gate

Another way to implement



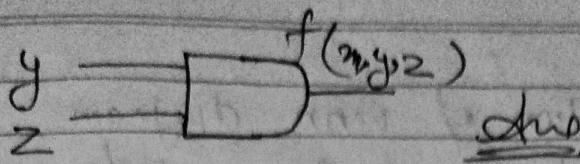
Minimization

$$f(x_1, y_1, z) = \bar{x}y_2 + xz$$

$y_2 (\bar{x} + x)$
 ↓ complement law

$$yz(1)$$

$$yz =$$



There are two way to minimize an expression

1) Boolean laws

2) ~~Karnaugh Map~~ Karnaugh Map

Summary :-

→ before implementing the function by logic gates we must first minimize it

→ for minimization we can apply boolean laws on identities. But sometime its difficult to minimize expression by laws like:

$$f(A, B, C) = AB + \bar{A}C + BC$$

↓ 1. BC (conclusion step)
identity law

$$AB + \bar{A}C + (A + \bar{A}) \cdot BC$$

$$AB + \bar{A}C + \underline{ABC} + \bar{A}BC$$

$$AB + \bar{A}C$$

Hence better option is Karnaugh Map.

K-Map

- Application of venn diagram
- based on gray code
- Minimum possible k map is for 1 var & max for n variable
- for "n" var boolean fn we've to to make 2^n minterms & for each minterm we have cell in K-Map
- Hence total no. of cells = 2^n (for n variables)

g 4 var k map has no. of cells $\Rightarrow 2^4 = 16$

g $f(A, B, C, D) = \Sigma m(0, 1, 13, 15)$

AB		00	01	11	10
CD	00	m ₀	m ₄	m ₁₂	m ₈
01	m ₁	m ₅	m ₉	m ₉	
11	m ₃	m ₇	m ₁₅	m ₁₁	
10	m ₂	m ₆	m ₁₄	m ₁₀	

AB		00	01	11	10
CD	00	m ₀	m ₁	m ₃	m ₂
01	m ₄	m ₅	m ₇	m ₆	
11	m ₁₂	m ₁₃	m ₁₅	m ₁₄	
10	m ₈	m ₉	m ₁₁	m ₁₃	

2 way to make K-Map

Size of rectangle

1
2
4
8
16
32

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

AB		00	01	11	10
CD	00	1			
01	1	1			
11					
10					

$\bar{A}\bar{B}\bar{C} + ABD$

$\left\{ \begin{array}{l} 2^n \times 2^m = 1 \times 2 \\ n=0, m=1 \end{array} \right.$

$$f(A_1, B_1, C_1, D) = \Sigma m(0, 7, 8, 15)$$

	AB	00	01	11	10
CD					
00	10	4	8	12	18
01	1	5	13	9	
11	2	7	11		11
10	3	6	14		10

$$BCD + \bar{B}\bar{C}\bar{D}$$

$$f(A_1, B_1, C_1, D) = \Sigma m(0, 2, 12, 14)$$

	AB	00	01	11	10
CD					
00	10	4	12	8	
01	1	5	13	9	
11	3	7	15	11	
10	12	6	14	10	

$$= \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{D}$$

$$f(A_1, B_1, C_1, D) = \Sigma m(3, 5, 6, 15)$$

	AB	00	01	11	10
CD					
00					
01			1		
11	1			1	
10		1			

$$\bar{A}BCD + ABCD + \bar{ABC}\bar{D} + \bar{A}\bar{B}CD$$

$$f(A_1, B_1, C_1, D) = \Sigma m(0, 1, 2, 3)$$

	AB	00	01	11	10
CD					
00		1			
01		1			
11		1			
10		1			

$$\bar{A}\bar{B}$$

Size of rectangle \rightarrow literal

$$1 \rightarrow 4$$

$$2 \rightarrow 3$$

$$4 \rightarrow 2$$

Q $f(A, B, C, D) = \Sigma m(5, 7, 13, 15)$

	AB 00	01	11	10
CD				
00				
01		1	1	
11		1	1	
10				

\overline{BD}
Ans

Q $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5)$

	AB 00	01	11	10
CD				
00	1	1		
01	1	1		
11	1			
10	1	.		

Q) $f(A, B, C, D) = \Sigma m(5, 7, 13, 15)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1	1		
01	11					
11	10					

\overline{BD}
Ans

Q) $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1	1		
01	11					
11	10					

$\overline{AB} + \overline{AC}$

Ans

Q) $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 10, 11)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1			
01	11					
11	10					

$\overline{AB} + \overline{BC}$

Ans

Q) $f(A, B, C, D) = \Sigma m(0, 2, 4, 6, 8, 10, 12, 14)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1	1	1	1
01	11					
11	10					

\overline{D}

Ans

$$f(A, B, C, D) = \Sigma m(0, 2, 5, 7, 13, 15, 8, 10)$$

		AB	00	01	11	10
		CD	00	1	.	1
		00	1	1	1	
		01	1	.	1	
		11	1	.	1	
		10	1	1	1	1

$$BD + \bar{B}\bar{D}$$

Ave

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)$$

		AB	00	01	11	10
		CD	00	1	1	1
		00	1	.	1	
		01	1	.	1	
		11	1	.	1	
		10	1	1	1	1

$$\bar{D} + \bar{B}$$

Ave

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5, 10, 11)$$

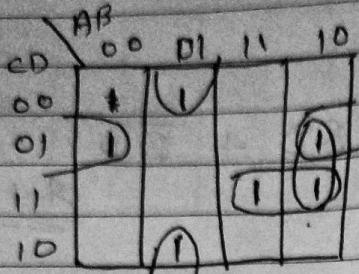
		AB	00	01	11	10
		CD	00	1	1	.
		00	1	1	1	
		01	1	.	1	
		11	1	.	1	
		10	1	1	1	1

$$\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{C}\bar{B} \times$$

$$\bar{A}\bar{C} + \bar{B}\bar{C}$$

Ave

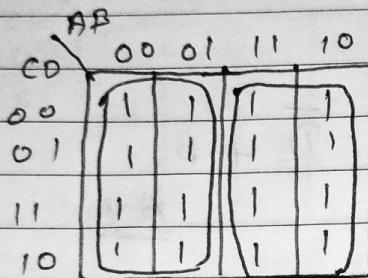
Ques $f(A, B, C, D) = \Sigma m(1, 4, 6, 9, 11, 15)$



$$\bar{B}\bar{C}D + A\bar{C}D + \bar{A}B\bar{D}$$

Ans

Ques $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, \dots, 15)$



1

Ans

if every cell is present then ans will always be "1" and literal count is "zero"

Proof!

~~Make 2 boxes of 8 - 8 cells~~

$$\bar{A} + A$$



"1" Ans

Size of box	→ Literal Count
1	→ 4
2	→ 3
4	→ 2
8	→ 1
16	→ 0

"n" Variable K-MAP

Rectangle — Literal count

1	—	n
2	—	$n-1$
4	—	$n-2$
8	—	$n-3$
16	—	$n-4$
32	—	$n-5$
2^n	—	$n-n$

* Cover

Consider 2 sets $f \& g$ we say f covers g if $g \subseteq f$

g	A	B	f_1	f_2
0	0	0	0	0
0	1	1	1	1
1	0	1	1	1
1	1	0	1	1

$$f_1 = \text{em}(1, 2)$$

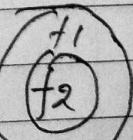
$$f_2 = \text{em}(1, 2, 3)$$

Since the element in f_1 is also present in f_2 , means f_2 covers

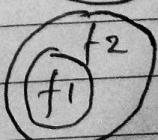
f_1

- a f_1 covers f_2
 b f_2 covers f_1 ✓
 c both
 d none

f_1 covers $f_2 \rightarrow$



f_2 covers $f_1 \rightarrow$



- In logic/B.A we say if $f_1 \rightarrow f_2$ is a tautology
- $f_1 \rightarrow f_2$ is a tautology
 f_1 implies f_2

A	B	f_1	f_2	$f_1 \rightarrow f_2$
0	0	0	0	1
0	1	1	1	1
1	0	1	1	1
1	1	0	1	1

implies Truth Table

P	Q	$P \rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

f_2 covers $f_1 \therefore f_1 \rightarrow f_2$ is tautology

* Implicant:

given 2 functions $f_1 \wedge f_2$ s.t $f_1 \leq f_2$
then f_1 is called implicant of
 f_2 i.e. $f_1 \rightarrow f_2$ is tautology

Simply we can say that every subset of
boolean f^n is an implicant

$$\text{Ex } f(A, B) = \Sigma(1, 2)$$

$$\begin{aligned} f_1(A, B) &= \emptyset \\ f_2(A, B) &= \Sigma(1) \\ f_3(A, B) &= \Sigma(2) \\ f_4(A, B) &= \Sigma(1, 2) \end{aligned} \quad \left[\begin{array}{l} \text{All function from} \\ f_1 \text{ to } f_4 \text{ are} \\ \text{implicants of } f(A, B) = \Sigma(1, 2) \end{array} \right]$$

If f^n has n minterms then how many
implicants?

2^n step

Q If f^n has ' n ' minterms then how many implicants functions can cover f ?

$$n \rightarrow 2^n = \text{minterms}$$

$$2^n - n = 2^{(2^n - n)}$$

Q $f(A, B, C) = AB + BC \rightarrow \underline{\text{SOP}}$

How many functions can cover f ?

$$f(A, B, C) = \Sigma m(6, 7, 3)$$

$$8 - 3 = 5 = 2^5 \rightarrow 32 \text{ Ans}$$

"3" are always present rest 5 having "2" possibilities
they can be or can be present

for $(6, 7, 3, 0)$ and so on...