L-21

* Boolean function

↳ Completely specified f^n | Incompletely specified f^n

only 2 possibility for each minterm i.e present or absent

3 possibility for each minterm i.e Present, absent or don't care

⇓

we really don't care about them i.e they may be present or absent it hardly matters

Don't care minterm

→ It is a combination of variable whose logical value is not specified

→ In K-MAP it is represented by "x" → 0, 1

Q = $f(A,B,C,D) = \Sigma m(0,1,2,3) + d(4)$

CD \ AB	00	01	11	10
00	1	X		
01	1			
11	1			
10	1			

$\bar{A}\bar{B}$ Ans

$X = 0$

Q $f(A,B,C,D) = \Sigma m(0,1,2) + d(3)$

CD \ AB	00	01	11	10
00	1			
01	1			
11	X			
10	1			

$\bar{A}\bar{B}$ Ans

$X = 1$

- $f(A,B,C,D) = \Sigma m(5,6,7,8,9) + d(10,11,12,13,14,15)$

CD \ AB	00	01	11	10
00			X	1
01		1	X	1
11		1	X	X
10		1	X	X

$A + BD + BC$

Ans

-

CD \ AB	00	01	11	10
00			1	
01	X	X	1	X
11		1	1	
10		1	1	

$AB + BC$

Ans

-

CD \ AB	00	01	11	10
00		1	1	
01	X			1
11	X			1
10		1	1	X

$B\bar{D} + \bar{B}D$

Ans

-

CD \ AB	00	01	11	10
00	1	X	X	1
01	X			1
11				
10	1			X

$\bar{B}\bar{C} + \bar{B}\bar{D}$

Ans

- Minimum literal Count

CD \ AB	00	01	11	10
00	X	1		1
01		1	X	
11	1	X	X	
10	X			X

2+3+3=8

According to SOP LC is 8

CD \ AB	00	01	11	10
00	X		0	
01	0		X	0
11		X	X	0
10	X	0	0	X

3+3+2+2 = 10

According to POS LC is 10

Minimum 8 Ans

CD \ AB	00	01	11	10
00	X	1	1	X
01			1	X
11			1	
10			1	

CD \ AB	00	01	11	10
00	X			X
01	0	0		X
11	0	0		0
10	0	0		0

2+2 = 4

SOP

1+2+2= 5

POS

Min 4 Ans

CD \ AB	00	01	11	10
00	1	1		1
11	X			
10	X			
	1	1		X

$2+2=4$

SOP

CD \ AB	00	01	11	10
00			0	
01	X	0	0	0
11	X	0	0	0
10			0	X

$1+2=3$

POS

3 Min Ans

0

xy \ wz	00	01	11	10
00	0	X	0	0
01	0	X	1	1
11	1	1	1	1
10	0	X	0	0

$y(w+x)$

which of the foll. is not eq. minimized expression

a) $(x+w)\cdot y$

b) $xy + wy$

c) $(w+x)(\bar{w}+y)(\bar{x}+y)$ ✓ Ans

d none

——— * ——— * ———

Why incompletely specifid f^n exists ??

OR

what is the need of incompletely specified f^n?

~~BD~~ BCD

0000

⋮

1001

1010 ⎤
⋮ ⎥ invalid don't care term "X"
1111 ⎦ BCD

- find minimized expression for a function involving BCD digits "f^n is on" w.r.t odd BCD digits

$$f(A,B,C,D) = \Sigma m(1,3,5,7,9) + \cancel{d(11,13,15)}$$
$$+ d(10,11,12,13,14,15)$$

CD \ AB	00	01	11	10
00			X	
01	1	1	X	1
11	1	1	X	X
10			X	X

D Ans

Since we have 6 don't care minterm so it is an incompletely specified f^n & represent $2^6 = 64$ completely specified f^n & out of those 64 functions we have choosen only one f^n having minimum literal count i.e

$$\Sigma m(1,3,5,7,9,11,13,15)$$

Q $f(A,B,C,D) = \Sigma m(1,3,7,11,15) + d(0,2,5)$

CD \ AB	00	01	11	10
00	X			
01	1	X		
11	1	1	1	1
10	X			

CD \ AB	00	01	11	10
00	X			
01	1	X		
11	1	1	1	1
10	X			

$CD + \bar{A}D$

$CD + \bar{A}\bar{B}$

$\Sigma(1,3,5,7,11,15)$

$\Sigma(0,1,2,3,7,11,15)$

* both are Correct

Q If f^n Completely specified then its minimized SOP & POS form are equivalent

CD \ AB	00	01	11	10
00	1	1	1	1
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

CD \ AB	00	01	11	10
00	1	1	1	1
01	0	0	1	0
11	0	0	1	0
10	0	0	1	0

$AB + \bar{C}\bar{D}$

$(A+C)\cdot(A+\bar{D})$
$\cdot(B+\bar{D})\cdot(B+\bar{C})$

same

~~$A(\bar{C}\bar{D}) \cdot (B \cdot$~~

$(A + \bar{C}\bar{D}) \cdot (B + \bar{C}\bar{D})$

$AB + \bar{C}\bar{D}$

Ans

- In incompletely specified then its minimized SOP & POS may and may not be equivalent algebrically ∵ they may be representing diff completely specified function

Ex

CD \ AB	00	01	11	10
00	X	1	1	X
01	0	0	1	X
11	0	0	1	0
10	0	0	1	0

CD \ AB	00	01	11	10
00	X	1	1	X
01	0	0	1	X
11	0	0	1	0
10	0	0	1	0

$AB + \bar{C}\bar{D}$

$B \cdot (A + \bar{C}) \cdot (A + \bar{D})$

$B \cdot (A + \bar{C}\bar{D})$

$AB + A\bar{C}\bar{D}$

Ans

not same

CD \ AB	00	01	11	10
00	0	0	0	X
01	1	1	1	X
11	1	X	X	X
10	1	X	0	0

CD \ AB	00	01	11	10
00	0	0	0	X
01	1	1	1	X
11	1	X	X	X
10	1	X	0	0

$D + \bar{A}C$

$(C + D) \cdot (\bar{A} + D)$

same

$D + \bar{A}C$

Same, bcuz we used diff don't care term in both of the tables

* 3 Variable K-MAP

no. of cells $2^3 = 8$ cells

BC \ A	0	1
00	0	4
01	1	5
11	3	7
10	2	6

- $f(A,B,C) = \Sigma m(2,3)$

BC \ A	0	1
00	0	4
01	1	5
11	3	7
10	2	6

$\bar{A}B$ Ans

- $\Sigma m(2,3,6,7)$

BC \ A	0	1
00		
01		
11	1	1
10	1	1

B Ans

- $f(A,B,C) = \Sigma m(0,1,4,5)$

BC \ A	0	1
00	1	1
01	1	1
11		
10		

$\bar{B}$ Ans

- $\Sigma m(0,1,2,3 \ldots 7)$

BC \ A	0	1
00	1	1
01	1	1
11	1	1
10	1	1

1 Ans

- $f(A,B,C) = \Pi M(0,1,2,3)$

BC \ A	0	1
00	0	
01	0	
11	0	
10	0	

A Ans

cell size	literal count
	3
1	2
2	1
4	0
8	

$f(A,B,C) = \bar{A}B + BC + AC$

BC \ A	0	1
00	1	
01	1	1
11	1	1
10		

• $f(A,B,C) = \bar{A}B + BC + AC$

BC \ A	0	1
00		
01		1
11	1	1
10	1	

$\cancel{C + \bar{A}}$

using boolean law

$\bar{A}B + AC + (A + \bar{A})BC$

$\bar{A}B + AC + ABC + \bar{A}BC$

$\bar{A}B + AC$

Ans

MoE $AC + \bar{A}B$

Ans

dual $\bar{A}B + BC + AC$

$(\bar{A} + B) \cdot (B + C) \cdot (A + C) = (\bar{A} + B) + (A \cdot C)$

Proof

BC \ A	0	1
00	1	1
01		1
11		
10	1	

$(\bar{A} + B) \cdot (A + C)$

Ans

* 5 variable K-MAP

no. of cells $2^5 = 32$

DE \ BC	00	01	11	10
00	0	4	12	8
01	1	5	13	9
11	3	7	15	11
10	2	6	14	10

A = 0

DE \ BC	00	01	11	10
00	16	20	28	24
01	17	21	29	25
11	19	23	31	27
10	18	22	30	26

A = 1

$f(A,B,C,D,E) = \Sigma m(4,5,6,7) = \bar{A}\bar{B}C$ Ans

$f(A,B,C,D,E) = \Sigma m(4,5,6,7,12,13,14,15)$
$= \bar{A}C$ Ans

$f(A,B,C,D,E) = \Sigma m(4,5,6,7,20,21,22,23)$

$\bar{A}\bar{B}C + A\bar{B}C$
$\bar{B}C(A+\bar{A}) = \bar{B}C$ Ans

$f(A,B,C,D,E) = \Sigma m(9,25) = B\bar{C}\bar{D}E$

$f(A,B,C,D,E) = \Sigma m(0,2,8,10,16,18,24,26)$

$\bar{A}\bar{C}\bar{E} + A\bar{C}\bar{E}$
$\bar{C}\bar{E}(A+\bar{A}) = \bar{C}\bar{E}$ Ans

Q The boolean expression

$$\bar{A}BE + BCDE + B\bar{C}\bar{D}E + \bar{B}\bar{C}D\bar{E}$$

Can be simplified to

$$BE + \bar{B}D\bar{E}$$

if the don't care minterms are

$$ABCDE + A\bar{B}CD\bar{E}$$

a) $AB\bar{C}DE + A\bar{B}CD\bar{E} + ABC\bar{D}E$

b) $ABCDE + A\bar{B}CD\bar{E} + ABCD\bar{E}$

c) none ✓ Ans

d)

DE \ BC	00	01	11	10
00				
01			1	1
11			1	1
10	1	X		

A=0

DE \ BC	00	01	11	10
00				
01			X	1
11			1	X
10	1	X		

A=1

$BE + \bar{B}\bar{C}D\bar{E}$, 6, 22, 29, 27

4 don't care terms

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* 1 var KMAP

$2^1 = 2$ cells

	0	1
A	1	

cell size	L.C
1	1
2	0

$f(A) = \Sigma m(0) = \bar{A}$ Ans

$f(A) = \Sigma m(0, 1) = (A + \bar{A}) = 1$ Ans

$f(A) = \Sigma m(1) = A$ Ans

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* 2 var KMAP
$2^2 = 4$ cells

B \ A	0	1
0	0	2
1	1	3

or

A \ B	0	1
0	0	1
1	2	3

Cell size	LC
1	2
2	1
4	0

$f(A,B) = \Sigma m(0,1) = \bar{A}$

$f(A,B) = \Sigma m(1) = \bar{A}B$

* 6 variable K-MAP → 2+4 $f(A,B,C,D,E,F)$

EF \ CD	00	01	11	10
00				
01			1	1
11			1	1
10				

A=0, B=0

8cell

EF \ CD	00	01	11	10
00				
01			1	1
11			1	1
10				

A=1, B=0

EF \ CD	00	01	11	10
00				
01				
11				
10				

A=0, B=1

8cell

EF \ CD	00	01	11	10
00				
01			1	1
11			1	1
10				

A=1, B=1

$\bar{B}CF + ACF$

Ans

Sumarry of L C

6 var cell size	LC	n var cell size	LC
1	6	1	n
2	5	2	$n-1$
4	4	4	$n-2$
8	3	8	$n-3$
16	2	16	$n-4$
32	1	2^x	$n-x$
64	0		

Practice Question

① which minterm must be added to make foll f^n self dual?

$f(A,B,C,D) = \bar{A}BC + (AC + B)D$

① Neutral - 8
② MC ×

CD \ AB	00	01	11	10
00				
01		1	1	
11		1	1	1
10		1		

5, 6, 7, 11, 13, 15

(0, 15), (1, 14), (2, 13), (3, 12)
(4, 11), (5, 10), (6, 9), (7, 8)

④ [(1, 3)
(1, 12)
(14, 3)
(14, 12)

Q2 Consider the following function of four variable.
$f(A,B,C,D) = \Sigma m(0,2,5,7,8,13,15)$
the function is:
1) Independent of one variable
2) " " two "
3) " " three "
4) dependent on all the variable ✓ Ans

Sol

CD \ AB	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			

$BD + \bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{D}$ Ans

Q3 $f_1(A,B,C,D) = \Sigma m(1,2,5,7,9,13,15)$
$f_2(A,B,C,D) = \Sigma m(2,5,7,10,11,14)$

what minterms are in

a $\bar{f_1} = \Sigma m(0,3,4,6,8,10,11,12,14) = \pi M$
b $\bar{f_2} = \Sigma m(0,1,3,4,6,8,9,12,13)$
AND c $f_1 \cdot f_2 = \Sigma m(2,5,7)$ AND fun (intersection)
OR d $f_1 + f_2 = \Sigma m(1,2,5,7,9,10,11,13,14,15)$ OR (union)
XOR e $f_1 \oplus f_2 = \Sigma m(1,9,10,11,13,14,15)$ exactly 1
XNOR f $f_1 \odot f_2 = \Sigma m(0,2,3,4,5,6,7,8,12,)$ Same
NAND g $\overline{f_1 \cdot f_2} = \Sigma m(0,1,3,4,6,8,9,10,11,12,13,14,15)$
NOR h $\overline{f_1 + f_2} = \Sigma m(0,3,4,6,8,12)$

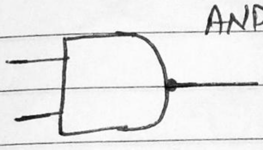
Q How many function does $f1 \cdot f2$ & $f1 + f2$ represent

$f_1(a,b,c) = \Sigma m\,(0,2,4) + d\,(3,5,7)$
$f_2(a,b,c) = \Sigma m\,(2,3) + d\,(1,6,7)$

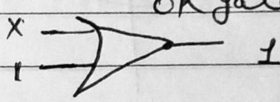
Suppose : $f(a,b) = \Sigma m(1) + d\,(0,2)$
$f(a,b) = \Sigma m\,(2) + d\,(0,1,3)$

a	b	f_1	f_2	$f_1 \cdot f_2$	$f_1 + f_2$
0	0	X	X	X	X
0	1	1	X	X	1
1	0	X	1	X	1
1	1	0	X	0	X

AND gate



OR gate



$f_1(a,b,c) = \Sigma m\,(0,2,4) + d\,(3,5,7)$
$f_2(a,b,c) = \Sigma m(2,3) + d\,(1,6,7)$

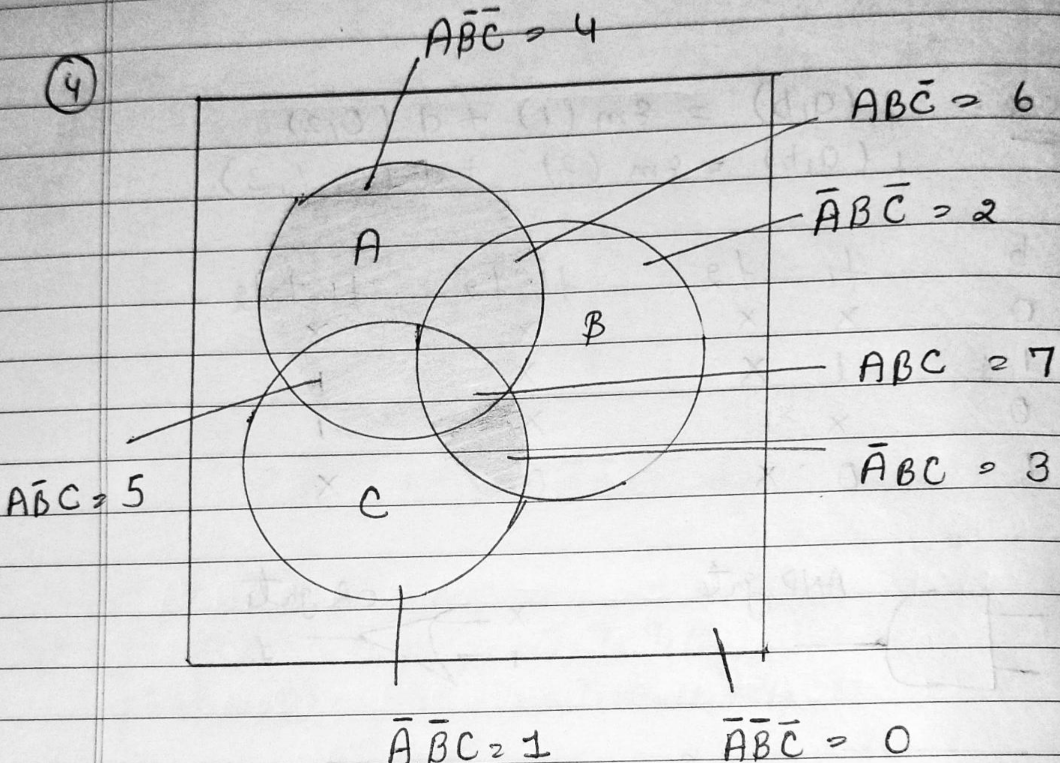
a	b	c	f_1	f_2	$f_1 \cdot f_2$	$f_1 + f_2$
0	0	0	1	0	0	1
0	0	1	0	X	0	X
0	1	0	1	1	1	1
0	1	1	X	1	X	1
1	0	0	1	0	0	1
1	0	1	X	0	0	X
1	1	0	0	X	0	X
1	1	1	X	X	X	X

$f1 \cdot f2 = \Sigma m(2) + d(3,7)$

$f1 + f2 = \Sigma m(0,2,3,4) + d(1,5,6,7)$

Ans

(4)



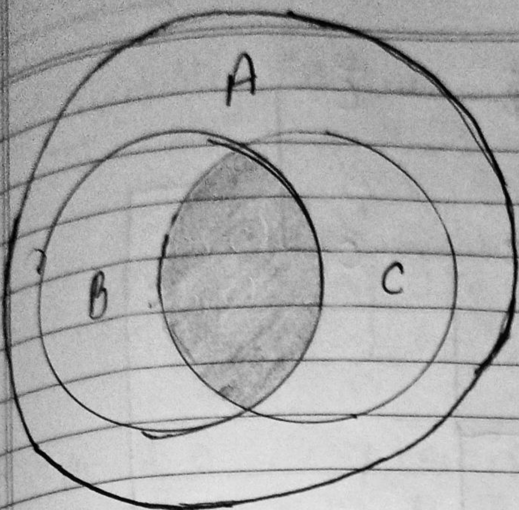
$f(A,B,C) = \Sigma m(4, 6, 3, 5, 7)$

BC \ A	0	1
00		1
01		1
11	1	1
10		1

$A + BC$

Ans

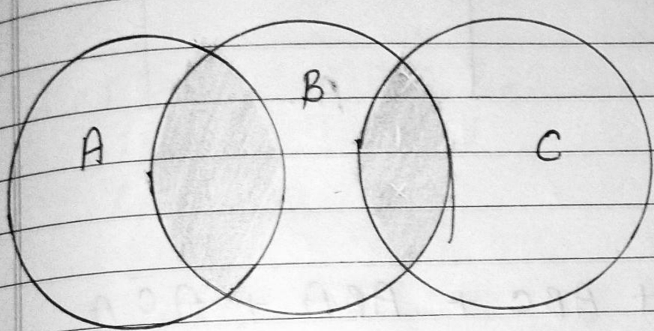
5



ABC

Ans

6

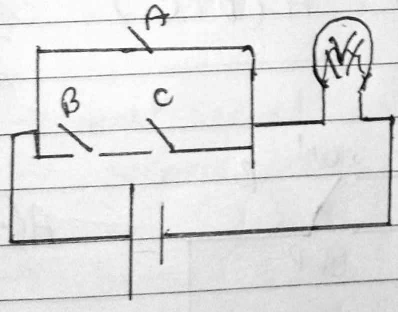


$AB + BC$

$A \cap B \cup B \cap B$ or

Ans

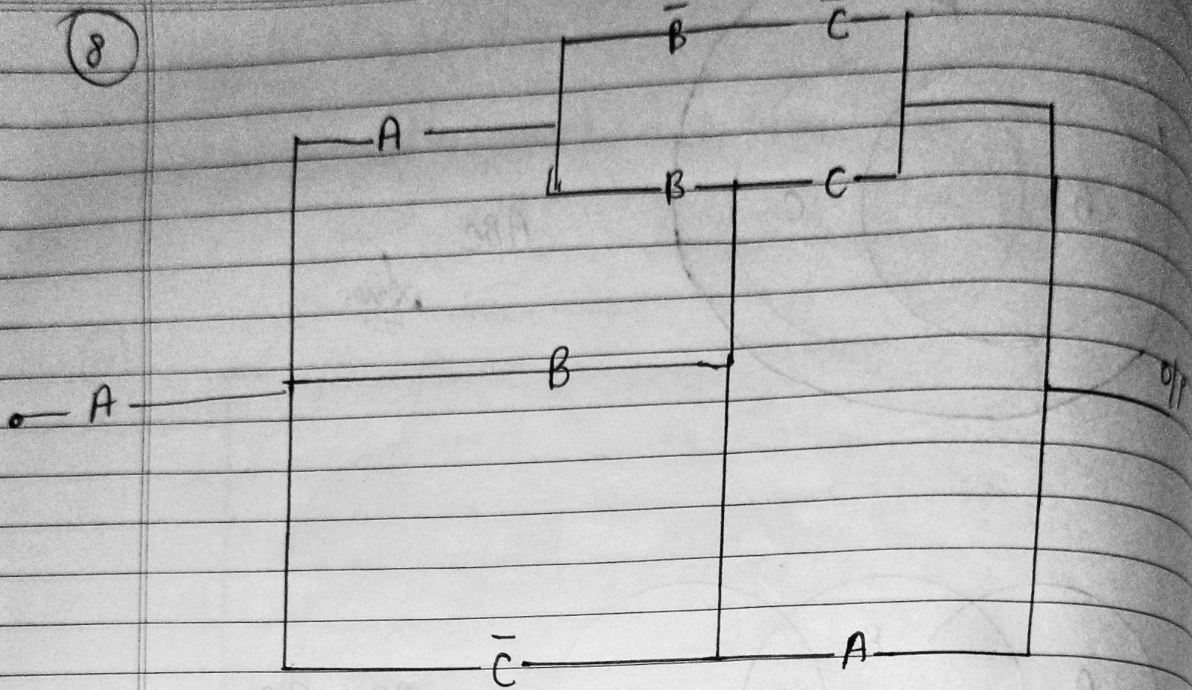
7



$A + BC$

Ans

8



$$A\cdot A\bar{B}\bar{C} + A\circ ABC + ABC + ABA + A\bar{C}A$$

$$A\bar{B}\bar{C} + ABC + ABC + AB + A\bar{C}A$$

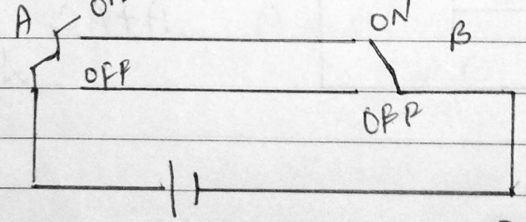
$$A\bar{B}\bar{C} + AB + A\bar{C}$$

$$A\bar{C} + AB \Rightarrow A(B+\bar{C})$$ Aw

x

x

9



$A \odot B$

XNOR

Aw

0 0 = 1
1 1 = 1 Aw
0 1 = 0
1 0 = 0