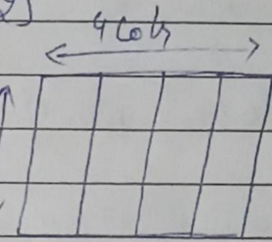
Lecture 7 - 2D Array

- Collection of 1D Array

In C language

* type name[size1][size2]

int A[3][4]



A[0][0] A[0][1] - - - -

A[2][0] A[2][1] --- A[2][3]

LB for row number } ⇒ 0
LB " Col " }

* In data structure

name[LB_i : UB_i] [LB_j : UB_j]

↓ row ↓ Columns

Ex. A[2 : 5] [-3 : 0] ~~A[2 4][-3]~~

↓ rows from 2 to 5

↓ cols from -3 to 0

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$A[2][-3]$ $A[2][-2]$ $A[2][-1]$ $A[2][0]$
$A[3][-3]$ $A_{3\,-2}$ $A_{3\,-1}$ $A_{3\,0}$
$A_{4\,-3}$ $A_{4\,-2}$ $A_{4\,-1}$ $A_{4\,0}$
$A_{5\,-3}$ $A_{4\,-2}$ $A_{5\,-1}$ $A_{5\,0}$

~~Note~~ No. of rows $(m) = UB_i - LB_i + 1$
No. of Columns $(n) = UB_j - \cancel{UB_j}\,LB_j + 1$

Total no. of elements in array $= m * n$

2D Array : Storage | each element 2 location

~~A_{00}~~ ~~A_{04}~~ ~~A_{02}~~ ~~A_{04}~~
~~A_{10}~~

- Row major order
- Column major order

| row 1 | | row 2 |

A_{00} A_{01} A_{02} A_{03} A_{10} A_{11} A_{12} A_{13}
↓
200 204 206 208 210 212 214 216
↓
base address

* How to find Location : RMO

$$Loc(A[i][j]) = Base + w * \underbrace{\text{relative index}}_{\text{no. of element before}}$$

Base ~~*~~ w * [No. of elements from LB_i to $i-1$ rows] ~~No. of elements~~ " LB_j to $j-1$ col. with ith row

$$Base + w[(i - LB_i)n + (j - LB_j)]$$

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* Column Major Order

$$\overbrace{A_{00}\quad A_{10}\quad A_{20}}^{\text{1st col}}\quad \overbrace{A_{10}\quad A_{11}\quad A_{21}}^{\text{2nd col}}$$

Location in : CMO

$Loc(A[i][j])$ Base $+ w *$ [~~No. of elements in column from~~ LB_j to $j-1$

+

No. of elements from row LB_i to $i-1$ within col j

$$Base + w * [(j - LB_j)\, m + (i - LB_i)]$$

Quiz: Consider a 2d array $A[-3 \ldots 7][6 \ldots 12]$. The starting address of array in memory is 1000. Each element occupies 4 memory location

(1) what is the address of element $A[0][9]$ in row major order?

$$LB_i = -3 \quad , \quad LB_j = 6$$

$m = 7 - (-3) + 1 = 11$ rows

$n = 12 - 6 + 1 = 7$ Cols

$$Loc(A[0][9]) = 1000 + 4[(0-(-3) * 7 + (9-6)]$$

$$1000 + 4 * 24 = 1096 \text{ Ans}$$

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Q what is the address of element A[5][7] in row major order

Base add   w $i - LBi * n + j - LBj$

$1000 + 4 \times [(5-(-3)) * 7 + (7-6))]$

$1000 + 4 \times 57 = 1228$ Ans

Quiz Consider a 2d array A[2..12][-6..5]. The starting address of array in memory is 1000. Each element occupies 4 memory location

o what is the address of element A[3][2] in col major order

$LBi = 2, \quad LBj = -6$

$m = 12 - 2 + 1 = 11$ — row

$n = 5 - (-6) + 1 = 12$ — cols

LOC(A[3][2]) = $1000 + 4[2-(-6)12 + 3(-2)]$

$1000 + 356 = 1356$

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Q What is the address of element A[6][5] in Col ~~row~~ major order

$1000 + 4\{(6-6)\ 11 +$

$j - LBj \qquad i - LBi$

$1000 + 4[5-(-6)\ 11 + (6-2)]$

$1000 + 500 = 1500$

Lecture 8 - Sparse Metrix

* Lower Tringular Matrix

every element out side thir tringle is 0
nxn, Square matrix

stored element at every level

1	A_{11}	A_{12}	A_{13}	A_{14}
2	A_{21}	A_{22}	A_{23}	A_{24}
3	A_{31}	A_{32}	A_{33}	A_{34}
4	A_{41}	A_{42}	A_{43}	A_{44}

hence we will not store these element which are "0"'s

if $(i<j)$
  $A_{ij} = 0$
else
  $A_{ij} \neq 0$ (store these only)

Stored element $= 1 + 2 + 3 + \ldots + n$

$$= \frac{n(n+1)}{2}$$

~~Meaning~~ Means the we'll have 10 element stored in memory of 4×4 matrix $= \frac{4(4+1)}{2} = 10$

* How this metrix will stored in the memory

- RMO – Row major order

A_{11}	A_{12}	A_{13}	A_{14}
A_{21}	A_{22}	A_{23}	A_{24}
A_{31}	A_{32}	A_{33}	A_{34}
A_{41}	A_{42}	A_{43}	A_{44}

Relative Index

R_1 0	1	R_2 2	3	R_3 4	5	6	7 R_4	8	9
A_{11}	A_{21}	A_{22}	A_{31}	A_{32}	A_{33}	A_{41}	A_{42}	A_{43}	A_{44}
200	202	204	206	208	210	212	214	216	218

base address

* Finding location in : RMO

$$Loc(A_{ij}) = Base + w * [\text{no. of elements from 1 to } i-1 \text{ rows} + (j - LB_j)]$$

No. of elements in Ist row $= 1$

— " — 2nd row $= 2$

⋮

— " — $(i-1)^{th}$ row $= i-1$

No. of elements from row 1 to $i-1 = 1+2+3 \cdots i-1$

$$= \frac{(i-1)\,i}{2}$$

$$= \text{Base} + w * \left[\frac{i(i-1)}{2} + j - \overset{1}{\cancel{LBj}} \right]$$

for every row we are are have $LBj = 1$

Example

$$LOC(A_{43}) = \text{Base} + w * \left[\frac{4 \times 3}{2} + (3-1) \right]$$

$i = 4,\ j = 3$

$$\text{Base} + w * [\ 8\ ]$$

* Lower Tringular Matrix : CMO

A11	A12	A13	A14
A21	A22	A23	A24
A31	A32	A33	A34
A41	A42	A43	A44

Col 1				Col 2			Col 3		Col 4
0	1	2	3	4	5	6	7	8	9
A11	A21	A31	A41	A22	A32	A42	A33	A43	A44

$$LOC(A_{ij}) = \text{Base} + w * \left[\text{No. of elements in } \cancel{\text{cols}} \text{ cols from } 1 \text{ to } j-1 + i - LBi \right]$$

$$Base + w * \left[n(j-1) - \frac{(j-1)(j-2)}{2} + (i-j) \right]$$

No. of element is 1 col $= n$

—— " —— " 2 col $= n-1$

—— " —— " 3 col $= n-2$

$\vdots$

—— " —— j^{th} col $= n-(j-1)$

—— " —— $(j-1)^{th}$ col $= n-(j-2)$

Total elements from col 1 to $j-1$ =

$$(n-0)+(n-1)+ \dots + n-(j-2)$$

$$n(j-1) - [0+1+ \dots j-2]$$

$\cancel{n(}$

$$n(j-1) - \frac{(j-1)(j-2)}{2}$$

ex

$$Loc(A_{3,3}) = Base + w * \left[4\times2 - \frac{2\times1}{2} + (3-3) \right]$$

$i=3,\ j=3$

$$Base + w \times 7$$

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* Upper Triangular Matrix

A_{11}	A_{12}	A_{13}	A_{14}
A_{21}	A_{22}	A_{23}	A_{24}
A_{31}	A_{32}	A_{33}	A_{34}
A_{41}	A_{42}	A_{43}	A_{44}

if $(i > j)$
$A_{ij} = 0$
else
$A_{ij} \neq 0$ (these only Stored)

No. of element stored $= 1+2+ \ldots + n$

$$\frac{n(n+1)}{2}$$

- Row major order

0	1	2	3	4	5	6	7	8	9
A_{11}	A_{12}	A_{13}	A_{14}	A_{22}	A_{23}	A_{24}	A_{33}	A_{34}	A_{44}

Loc (A_{ij}) = Base + w × [no. of elements from row 1 to $i-1$
+
$j - LB_j$]

$$= \text{Base} + w \times \left[n(i-1) - \frac{(i-1)(i-2)}{2} + j - i \right]$$

No of elements in 1^{st} row $= n$
—— " —— " 2nd row $= n-1$
—— " —— " 3rd row $= n-2$
|
|
—— " —— $(i-1)^{th}$ row $= n-(i-2)$

No. of elements from row 1 to $i-1$ $= (n+0) + (n-1) \ldots n-(i$

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$$n(i-1) - \frac{(i-1)(i-2)}{2}$$

* CMO

$$Loc(A_{ij}) = Base + w * [\text{no. of elements from col 1 to } j - 1 + i - LB_i$$

$$Base + w * \left[\frac{j(j-1)}{2} - i - 1\right]$$

Quiz (1) Consider a 2d array A of size 8×8 the starting address of array in memory is 1000. Each element occupies 4 memory location

(1) what is the address of element A[4][2] in Row major for lower tringular metrix

Ans

$$Base + w * \frac{i(i-1)}{2} + j - LB_j$$

$$LOC[A_{4,2}] = 1000 + 4\left[\frac{4 \times 3}{2} + (2-1)\right]$$

$$1000 + 4 + 7$$

$$= \cancel{102}\ 1028$$

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(2) what is the address of element A[2][6] in row major order for upper Tringula matrix

$Loc(A_{2,6}) = Base + w \times \left[8 \times 1 - \frac{1 \times 0}{2} + 6 - 2\right]$

$i = 2,\ j = 6$

$1000 + 4 \times [8 + 4]$

$1000 + 48 = 1048$ Ans