2 Conditions for orthogonal

↳ Neutural
↳ Mutal exclusive terms

* K-MAP (Konaugh Map) L-20

- Boolean variable → symbol which can take value 0 or 1

Ex $x \to 0, 1$

- Literal → use of variable or its complement in an expression

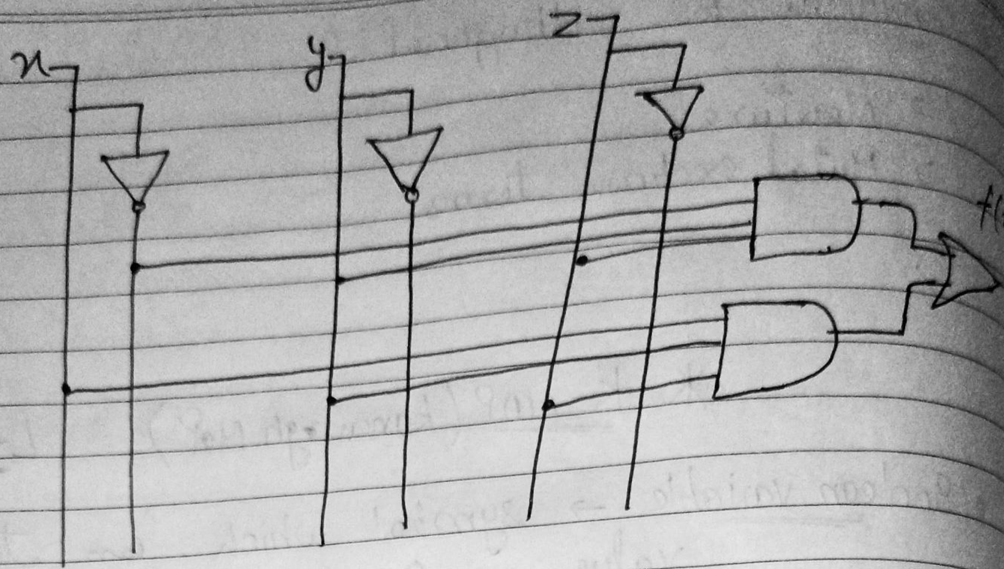
$$f(x, y) = \bar{x}y + x\bar{y}$$

$f(x,y)$ ↓ 2 variables

$\bar{x}y + x\bar{y}$ ↓ 4 Literals

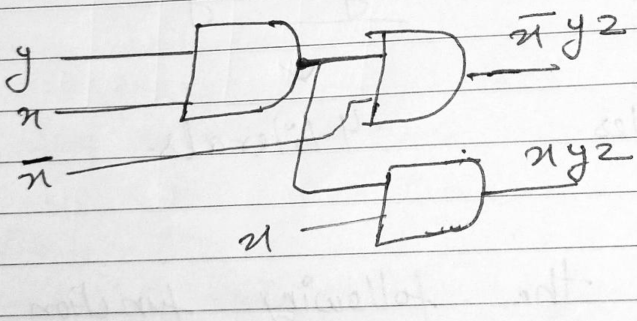
* Implement the following function

$$f(x, y, z) = \bar{x}yz + xyz$$



2 AND gate (3 I/P)
1 OR gate
1 Not gate

Another way to implement:-



- Minimization

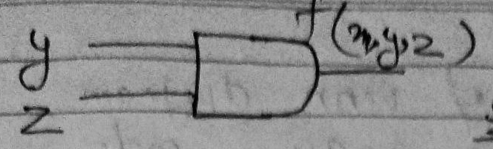
$$f(x,y,z) = \bar{x}yz + xyz$$

$$yz(x+\bar{x})$$

$\downarrow$ complement law

$$yz(1)$$

$$yz$$



Ans

There are two way to minimize an expression

1) Boolean laws   2) ~~kara~~ Karnaugh Map

Summary :-

→ before implementing the function by logic gates we must first minimize it

→ for minimization we can apply boolean laws on identities. But some time its difficult to minimize expression by laws like:

$f(A, B, C) = AB + \bar{A}C + BC$

$\downarrow$ $1 \cdot BC$ (crucial step) identity law

$AB + \bar{A}C + (A+\bar{A}) \cdot BC$

$AB + \bar{A}C + ABC + \bar{A}BC$

$AB + \bar{A}C$

Hence better option is Karnaugh Map.

K-Map

→ Application of venn diagram
→ based on gray code
→ Minimum possible k map is for 1 var & max for n variable
→ for "n" var boolean f^n we've to make 2^n minterms & for each minterm we have cell in K-Map

Hence total no. of cells = 2^n (for n variable)

Q 4 var k map has no. of cells ⇒ $2^4 = 16$

Q $f(A,B,C,D) = \Sigma m(0,1,13,15)$

CD \ AB	00	01	11	10
00	m0	m4	m12	m8
01	m1	m5	m13	m9
11	m3	m7	m15	m11
10	m2	m6	m14	m10

AB \ CD	00	01	11	10
00	m0	m1	m3	m2
01	m4	m5	m7	m6
11	m12	m13	m15	m14
10	m8	m9	m11	m13

2 way to make ~~ma~~ K-Map

Size of rectangle
1
2
4
8
16
32

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

CD \ AB	00	01	11	10
00	1			
01	1		1	
11			1	
10				

$\bar{A}\bar{B}\bar{C} + ABD$

$2^n \times 2^m = 1 \times 2$
$n = 0, \; m = 1$

Q $f(A, B, C, D) = \Sigma m(0, 7, 8, 15)$

CD \ AB	00	01	11	10
00	1_0	4	12	1_8
01	1	5	13	9
11	2	1_7	1_{15}	11
10	3	6	14	10

$BCD + \bar{B}\bar{C}\bar{D}$

Q $f(A, B, C, D) = \Sigma m(0, 2, 12, 14)$

CD \ AB	00	01	11	10
00	1_0	4	1_{12}	8
01	1	5	13	9
11	3	7	15	11
10	1_2	6	1_{14}	10

$= \bar{A}\bar{B}\bar{D} + AB\bar{D}$

Q $f(A, B, C, D) = \Sigma m(3, 5, 6, 15)$

CD \ AB	00	01	11	10
00				
01		1		
11	1		1	
10		1		

$\bar{A}BCD + ABCD + \bar{A}BC\bar{D} + \bar{A}\bar{B}CD$

Q $f(A, B, C, D) = \Sigma m(0, 1, 2, 3)$

CD \ AB	00	01	11	10
00	1			
01	1			
11	1			
10	1			

$\bar{A}\bar{B}$

Size of rectang → literal

1 → 4

2 → 3

4 → 2

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Q $f(A,B,C,D) = \Sigma m(5,7,13,15)$

CD \ AB	00	01	11	10
00				
01		1	1	
11		1	1	
10				

BD

Ans

Q $f(A,B,C,D) = \Sigma m(0,1,2,3,4,5)$

CD \ AB	00	01	11	10
00	1	1		
01	1	1		
11	1			
10	1			

Q $f(A, B, C, D) = \Sigma m(5, 7, 13, 15)$

CD \ AB	00	01	11	10
00				
01		1	1	
11		1	1	
10				

BD

Ans

Q $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5)$

CD \ AB	00	01	11	10
00	1	1		
01	1	1		
11	1			
10	1			

$\bar{A}\bar{B} + \bar{A}\bar{C}$

Ans

Q $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 10, 11)$

CD \ AB	00	01	11	10
00	1			
01	1			
11	1			1
10	1			1

$\bar{A}\bar{B} + \bar{B}C$

Ans

Q $f(A, B, C, D) = \Sigma m(0, 2, 4, 6, 8, 10, 12, 14)$

CD \ AB	00	01	11	10
00	1	1	1	1
01				
11				
10	1	1	1	1

$\bar{D}$

Ans

Que $f(A,B,C,D) = \Sigma m(0,2,5,7,13,15,8,10)$

CD \ AB	00	01	11	10
00	1			1
01		1	1	
11		1	1	
10	1			1

$BD + \bar{B}\bar{D}$

Ans

Que $f(A,B,C,D) = \Sigma m(0,1,2,3,4,6,8,9,10,11,12,14)$

CD \ AB	00	01	11	10
00	1	1	1	1
01	1			1
11	1			1
10	1	1	1	1

$\bar{D} + \bar{B}$

Ans

Que $f(A,B,C,D) = \Sigma m(0,1,2,3,4,5,10,11)$

CD \ AB	00	01	11	10
00	1	1		
01	1	1		
11	1			1
10	1			1

$\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{C}\bar{B}$ ✗

$\bar{A}\bar{C} + \bar{B}C$

Ans

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Que $f(A,B,C,D) = \Sigma m(1,4,6,9,11,15)$

CD \ AB	00	01	11	10
00		1		
01	1			1
11			1	1
10		1		

$\bar{B}\bar{C}D + ACD + \bar{A}B\bar{D}$

Ans

Que $f(A,B,C,D) = \Sigma m(0,1,2,3 \ldots 15)$

CD \ AB	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1	1	1

1 Ans

if every cell is present then ans will always be "1" and literal count is "zero"

Proof :

Make 2 boxes of 8 – 8 cells

$\bar{A} + A$

↓

"1" Ans

Size of box	→	literal Count
1	→	4
2	→	3
4	→	2
8	→	1
16	→	0

"n" Variable K-MAP

Rectangle		Literal count
1	—	n
2	—	$n-1$
4	—	$n-2$
8	—	$n-3$
16	—	$n-4$
32	—	$n-5$
2^x	—	$n-x$

* Cover

Consider 2 sets f & g we say f covers g if $g \subseteq f$

q

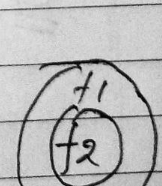
A	B	f1	f2
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	1

$f1 = \Sigma m(1, 2)$
$f2 = \Sigma m(1, 2, 3)$

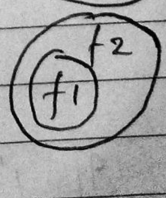
Since the element in f_1 is also present in f_2, means f_2 covers f_1

a. f_1 covers f_2
b. f_2 covers f_1 ✓
c. both
d. none

f_1 cover f_2 →



f_2 cover f_1 →



- In logic/B.A we say f covers g ~~ment~~ means $g \Rightarrow f$ is a tautology

$g \Rightarrow f$
g implies in f

implies Truth table

P	Q	$P \to Q$
0	0	1
0	1	1
1	0	0
1	1	1

A	B	f_1	f_2	$f_1 \to f_2$
0	0	0	0	1
0	1	1	1	1
1	0		1	1
1	1	0	1	1

f_2 covers f_1 $\therefore$ $f_1 \to f_2$ is tautology

* Implicant:
given 2 functions f_1 & f_2 s.t $f_1 \subseteq f_2$ then f_1 is called implicant of f_2 i.e $f_1 \to f_2$ is tautology

Simply we can say that every subset of boolean f^n is an implicant

Ex $f(A,B) = \Sigma(1,2)$

$f_1(A,B) = \phi$
$f_2(A,B) = \Sigma(1)$
$f_3(A,B) = \Sigma(2)$
$f_4(A,B) \quad \Sigma(1,2)$

All function from f_1 to f_4 are implicant of $f(A,B) = \Sigma m(1,2)$

If f^n 'f' has n minterms then how many implicant?
2^n Ans

Q If f^n 'f' has 'x' minterms of n var then how many ~~implicants~~ functions can cover f?

$n \rightarrow 2^n$ = minterms

$2^n - x = 2^{(2^n - x)}$

Q $f(A,B,C) = AB + BC \rightarrow$ SOP

How many functions can cover f?

$f(A,B,C) = \Sigma m(6,7,3)$

$8 - 3 = 5 = 2^5 \rightarrow 32$ Ans

"3" are always present rest 5 having "2" possibilite

they can be or can be present

for $(6,7,3,0)$ and so on...

Q If f^n 'f' of n var has 'x' minterms then how many ~~implicants~~ functions can cover f?

$n \rightarrow 2^n$ = minterms

$2^n - x = 2^{(2^n - x)}$

Q $f(A,B,C) = AB + BC \rightarrow$ SOP

How many functions can cover f?

$f(A,B,C) = \Sigma m(6,7,3)$

$8 - 3 = 5 = 2^5 \Rightarrow 32$ Ans

"3" are always present rest 5 having "2" possibilites they can be or can be present

for $(6,7,3,0)$ and so on...

Q How to visualize implicants in K-map?

$f(A,B,C) = \Sigma m(1,5,7,8,13,15)$

$\Sigma m(1)$

CD \ AB	00	01	11	10
00				1
01	1	1	1	
11		1	1	
10				

$f_1 \rightarrow f$

$f_1(A,B,C,D) = \Sigma m(7,8,15)$

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- We have f^n 'f' of 'n' variable & to minimize it we have designed K-Map
- This K-Map will contain 1s in those cells whose corresponding minterms are present in f^n
- Hence take any single 1 or group of 1s from the the K-Map & it is called implicant of the f^n for which K-MAp is designed

Def^n of Implicant:-
Any single 1 or group of 1s in K-Map is called implicant.

* Prime implicant → A prime implicant is an implicant that can not be covered by more reduced implicant
↓
Implicant with less literal count than the given implicant

$f(A,B,C,D) = \Sigma m(0,1,2,3)$

$f_1(A,B,C,D) = (0,1)$ → is it Prime implicant?

$(0,1) \Rightarrow \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D$
$\bar{A}\bar{B}\bar{C}$
we need find a function ~~which~~ who is super set of (0,1) and its literal count is less

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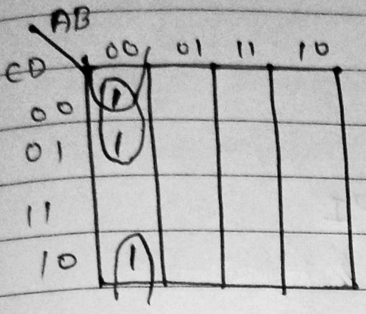
than $\bar{A}\bar{B}\bar{C}$

$(0,1,2) \Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D}$ ✗

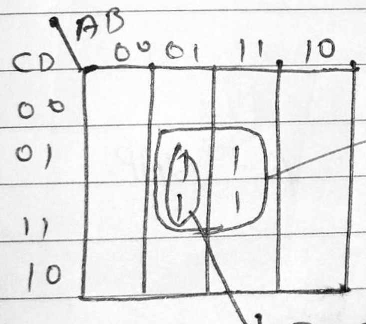
$(0,1,3) \Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}D$ ✗



$(0,1,2,3) \Rightarrow \bar{A}\bar{B}$ ✓

Q How to visulize Prime implicant in K-MAP?

A Any single 1 or group of 1 that can't be inculded in bigger rectangle in K-MAP?

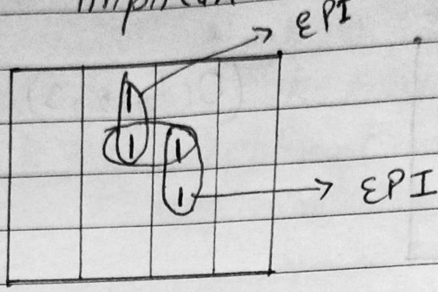


BD is a prime implicant

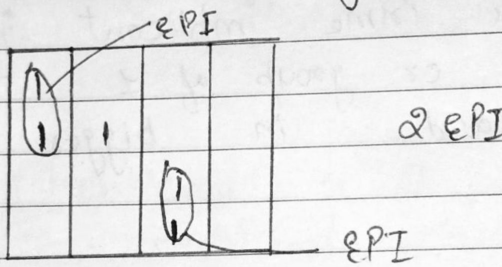
$\bar{A}BD$ is not a prime implicant

* Essential Prime Implicant

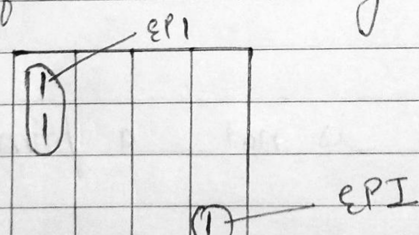
Those prime implicant which contain at least 1 minterm which is not included in any other implicant

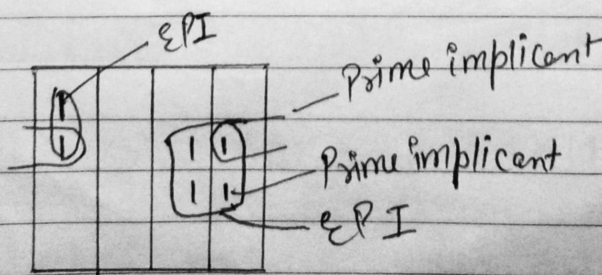


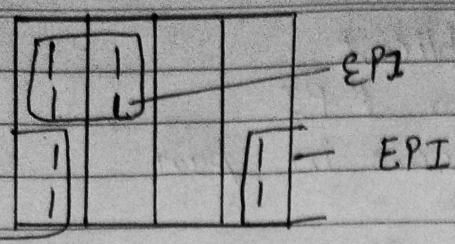
- No. of EPI in given K-MAP ?



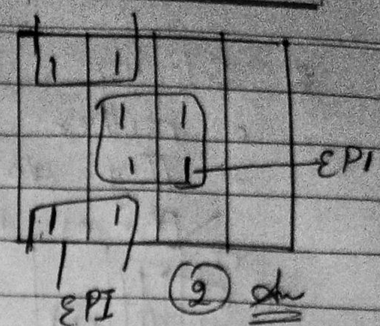
- No of EPI in given K-MAP



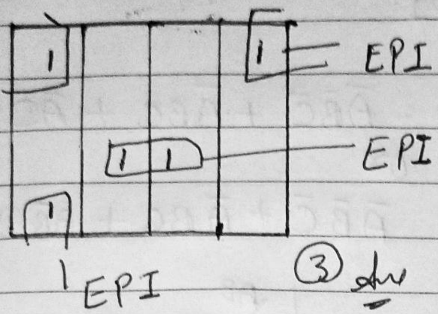




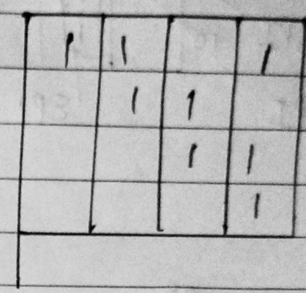
② Ans



② Ans



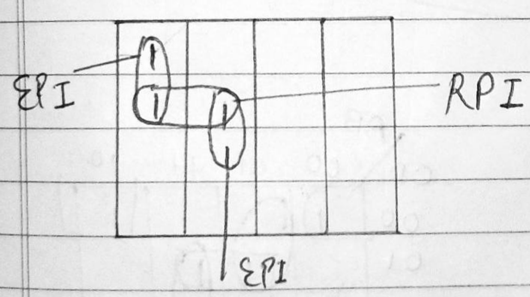
③ Ans

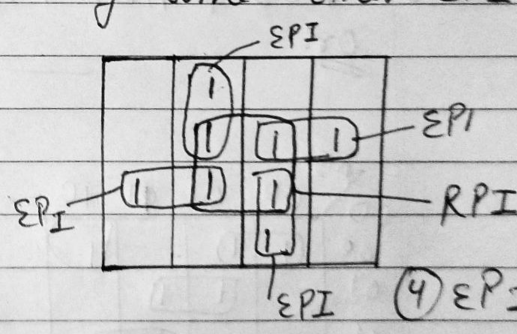


0 EPI

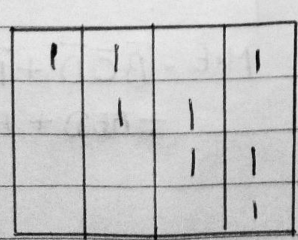
* Redundant Prime Implicant

A prime implecant in which ~~which~~ each of its minterm is coverd by some other EPI





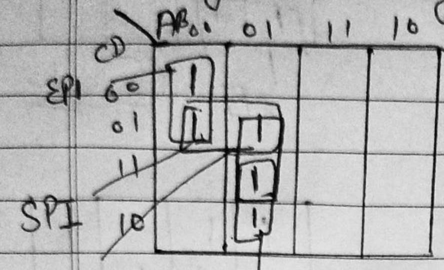
④ EPI



0 EPI
0 RPI

* Selective Prime implicant

- neither EPI nor RPI
- They ~~do~~ alway accurs in pair



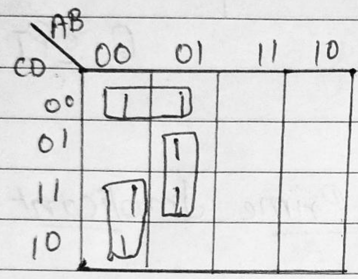
(2) EPI

(2) SPI

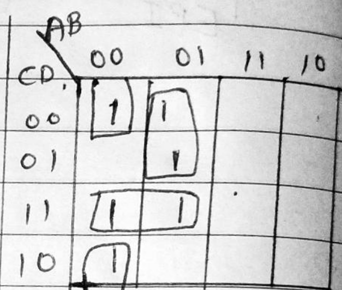
M.E = $\bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{C}D$

or

$\bar{A}\bar{B}\bar{C} + \bar{A}BC + \bar{A}\bar{B}D$



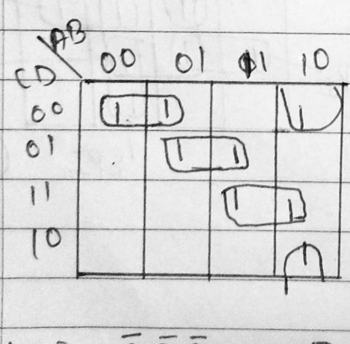
6 SPI



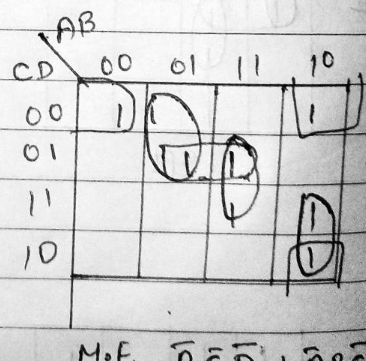
M.E = $\bar{A}\bar{C}\bar{D} + \bar{A}BD + \bar{A}\bar{B}C$

Or

$\bar{A}\bar{B}\bar{D} + \bar{A}B\bar{C} + \bar{A}CD$



or



M.E = $\bar{A}\bar{C}\bar{D} + B\bar{C}D + ACD + A\bar{B}\bar{D}$

M.E = $\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C} + ABD + A\bar{B}C$