L-15

* Boolean laws

① Commutative law

Logic $a \vee b = b \vee a$
$a \wedge b = b \wedge a$

Set theory $a \cup b = b \cup a$
$a \cap b = b \cap a$

Boolean Algebra $a + b = b + a$ → OR
$a \cdot b = b \cdot a$

A Complement

NOT A

Boolean Algebra	Logic (→OR, AND)	Set theory
$\bar{A}$ $+$ $\circ$	$\neg A$ $\vee$ $\wedge$	A' $\cup$ $\cap$ → intersection, → union
1	T	U → universal set
0	F	ϕ

② Associative law

Logic = $a \vee (b \vee c) = (a \vee b) \vee c$
$a \wedge (b \wedge c) = (a \wedge b) \wedge c$

Set theory = $a \cup (b \cup c) = (a \cup b) \cup c$
$a \cap (b \cap c) = (a \cap b) \cap c$

Boolean Algebra = $a + (b + c) = (a + b) + c$
$a \cdot (b \cdot c) = (a \cdot b) \cdot c$

(3) Distributive law

Logic - $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee b)$

Set theory - $a \cap (b \cup c) = (a \cap b) \cup (a \cap c)$

$a \cup (b \cap c) = (a \cup b) \cap (a \cup b)$

Boolean Algebra - $a \cdot (b + c) = a \cdot b + a \cdot c$

$a + (b \cdot c) = (a + b) \cdot (a + c)$

$+$ is distributive ~~ov~~ over $\cdot$

$\cdot$ is distributive over $+$

(4) Identity law

Logic

$a \wedge T = a$

$a \vee F = a$

Set theory

$a \cap U = a$

$a \cup \emptyset = a$

Boolean Algebra

$a \cdot 1 = a$

$a + 0 = a$

5 Complement law

Logic $a \vee \neg a = T$

$a \wedge \neg a = F$

Set theory $a \cup a^c = U$

$a \cap a^c = \phi$

~~Boolean~~

Boolean Algebra $a + \bar{a} = 1$

$a \cdot \bar{a} = 0$

6 Idempotent law

Logic $a \vee a = a$

$a \wedge a = a$

Set theory $a \cup a = a$

$a \cap a = a$

Boolean Algebra $a + a = a$

$a \cdot a = a$

6 double Complement

Logic $\neg(\neg a) = a$

Boolean Algebra $\bar{\bar{A}} = a$

Set Theory $(a^c)^c = a$

✓The Good

(7) Demorgan's law

Logic $\neg(a \vee b \vee c \cdots) = \neg a \wedge \neg b \wedge \neg c \cdots$

$\neg(a \wedge b \wedge c \cdots) = \neg a \vee \neg b \vee \neg c \cdots$

Set $(a \cup b \cup c \cdots)^c = a^c \cap b^c \cap c^c$

$(a \cap b \cap c \cdots)^c = a^c \cup b^c \cup c^c \cdots$

boolean Algebra $\overline{a+b+c} \cdots = \bar{a} \cdot \bar{b} \cdot \bar{c} \cdots$

$\overline{a \cdot b \cdot c} \cdots = \bar{a} + \bar{b} + \bar{c} \cdots$

Trick to ~~rember~~ remember $\Rightarrow$ change the sign
break the line

$$\overline{a+b} \Leftrightarrow \bar{a} \cdot \bar{b}$$

(8) Domination law

logic	$a \vee T = T$	boolean Algebra	$a + 1 = 1$
	$a \wedge F = F$		$a \cdot 0 = 0$
Set theory	$a \cup U = U$		
	$a \cap \phi = \phi$		

(9) Absorbtion law

Logic $a \vee (a \wedge b) = a$

$a \vee (\neg a \wedge b) = a \vee b$

$a \wedge (a \vee b) = a$

Set theory

$a \cup (a \cap b) = a$
$a \cup (a^c \cap b) = a \cup b$
$a \cap (a \cup b) = a$

Boolean algebra

$a + ab = a$
$a + \bar{a}b = a + b$
$a \cdot (a + b) = a$

* Proof

$a + ab = a$ — Absorbtion law

$a(\underline{1+b}) =$ → domination law

$a \cdot 1$ → identity law

a

• domination law proof

$a + 1 = 1$??

a	1	a+1
0	1	1
1	1	1

⇒ Proof

$a \cdot 0 = 0$?

a	0	a·0
0	0	0
1	0	0

• $a + \bar{a} \cdot b = a + b$ — Absorbtion law

$(a + \bar{a}) \cdot (a + b)$ → ~~dis~~ distributive law applied

Complement law → $1 \cdot (a + b)$

→ Identity law

$a + b$

- **Identity law**

a	1	$a \cdot 1$
0	1	0
1	1	1

same

$a \cdot 1 = a$

$a + 0 = 0$

a	0	$a+0$
0	0	0
1	0	1

same

$a + 0 = a$

- $a \cdot (a+b) = 0$

$\downarrow$ distributive law

$a \cdot a + a \cdot b$

$\downarrow$ Idempotent law

$a + ab = a$

~~Absorb~~ Absorbtion law

Q $P + Q + \overline{P}$ Minimize this

$P + (Q + \overline{P})$ → ~~Commutative law~~

$\downarrow \quad \downarrow$ → Commutative law

$(P + \overline{P}) + Q$

$\downarrow$ → Complement law

$1 + Q$

$\downarrow$ → domination law

1

Q $P \vee P' \wedge Q$ → logic style question

↓ ~~Bonged~~ changed to boolean Algebra

$P + \bar{P} + Q$

↓

$1 + Q = 1$ ⇒ T Aw in logic

↓

Tautology ⇒ means always true

Q $P \wedge (Q \wedge P')$

$P \cdot (Q \cdot \bar{P})$

↓ → Commutative law

$P \cdot (\bar{P} \cdot Q)$

↓ → Associative law

$(P \cdot \bar{P}) \cdot Q$

↓ → Compliment law

$0 \cdot Q$ → domination law

↓

0 ⇒ F ⇒ Contradiction

Que $(P \vee Q) \wedge (P \vee Q')$

$(P + Q) \cdot (P + \bar{Q})$

↓ distributive law

$P + Q \cdot \bar{Q}$ → compliment law

↓

$P + 0$

↓ → Identity law

P

✓The Good Pap

Qus $PQR + (\bar{P}\bar{Q}R + PQR) \rightarrow$ Associative law

$\downarrow$ Commutative law

$PQR + (PQR + \bar{P}\bar{Q}R)$

$\downarrow \Rightarrow$ Associative law

$(PQR + PQR) + \bar{P}\bar{Q}R$

$\downarrow$ Idempotent law

$PQR + \bar{P}\bar{Q}R$

Ans

Qus $P \wedge Q \wedge P \wedge R \wedge S'$

$\downarrow$

$P \cdot Q \cdot P \cdot R \cdot \bar{S}$

$P \cdot P \cdot Q \cdot R \cdot \bar{S}$

$P \cdot Q \cdot R \cdot \bar{S} \Rightarrow P \wedge Q \wedge R \wedge \neg S$

Qus $P \wedge Q \vee (P \wedge Q \wedge (R \wedge S \wedge T))$

$P \cdot Q + (P \cdot Q \cdot (RST))$

$PQ + PQRST$

$PQ(1 + RST)$

$PQ \cdot 1 \Rightarrow PQ$ Ans

Que: $T \vee P\wedge Q \vee P'\wedge Q \vee P'\wedge Q'\wedge R' \vee P'\wedge Q\wedge S'$

$$1 + PQ + \bar{P}\cdot Q + \bar{P}\cdot\bar{Q}\cdot\bar{R} + \bar{P}\cdot Q\cdot\bar{S}$$

↓ 1+ somthing is 1

$\underline{1} \Rightarrow T$

How

$$1+\bar{P} + PQR + ST$$
↓
$$(1+\bar{P}) + PQR + ST$$
↓
$$(1+PQR) + ST$$
$$1+ST$$
↓
$$\underline{1}$$

solved using domination law

Que: $A\cup(B-C) = (A\cup B)-(A\cup C)$

$A\cup(B\cap C^c) = (A\cup B)\cap(A\cup C)^c$

$A + B\bar{C} = (A+B)\cdot(\overline{A+C})$

$A + B\bar{C} = (A+B)\cdot(\bar{A}\cdot\bar{C})$

$\cancel{A\bar{C} + \bar{A}B + B\bar{C}}$

$A + B\bar{C} = \bar{A}\cdot B\bar{C}$ Not equel

$A-B \Rightarrow A\cap B^c$

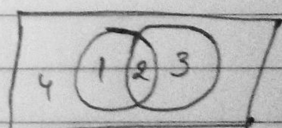
↓

1

$\{1,2\}\cap\{1\ldots$

↓

1



U is distributive over set diff or not?

No