

2 conditions for orthogonal

- ↳ Neutral
- ↳ Mutual exclusive terms

* K-MAP (Karnaugh Map) L-20

- Boolean variable → symbol which can take value 0 or 1
Ex: $n \leftrightarrow 0$ $n \leftrightarrow 1$
- Literal → use of variable or its complement in an expression

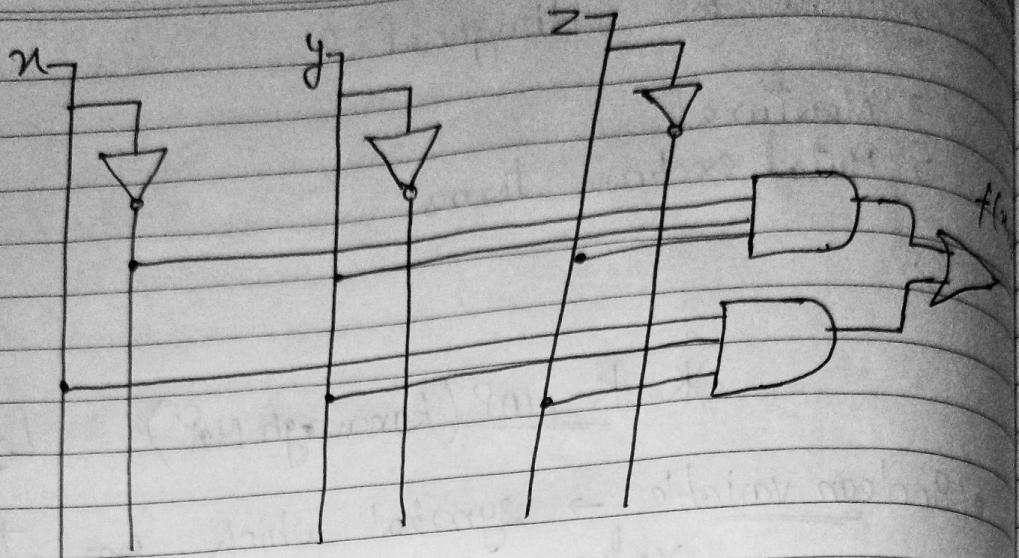
$$f(x, y) = \bar{y}x + \bar{x}\bar{y}$$

2 variables

4 Literals

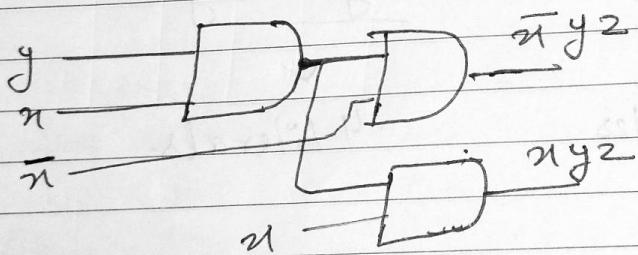
* Implement the following function

$$f(x, y, z) = \bar{x}yz + xy\bar{z}$$



2 AND gate (3 I/P)
1 OR gate
1 Not gate

* Another way to implement



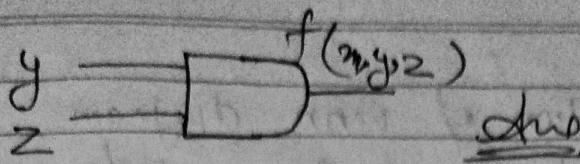
o Minimization

$$f(x_1, y_1, z) = \bar{x}y_2 + xz$$

$y_2 (\bar{x} + x)$
↓ complement law

$$yz(1)$$

$$yz =$$



There are two ways to minimize an expression

1) Boolean laws

2) ~~Karnaugh Map~~ Karnaugh Map

Summary :-

→ before implementing the function by logic gates we must first minimize it

→ for minimization we can apply boolean laws on identities. But sometime its difficult to minimize expression by laws like:

$$f(A, B, C) = AB + \bar{A}\bar{C} + BC$$

↓ 1. BC (convenient step)
identity law

$$AB + \bar{A}\bar{C} + (A + \bar{A}) \cdot BC$$

$$AB + \bar{A}\bar{C} + \underline{ABC} + \bar{ABC}$$

$$AB + \bar{A}\bar{C}$$

Hence better option is Karnaugh Map.

K-Map

- Application of venn diagram
- based on gray code
- Minimum possible k map is for 1 var & max for n variable
- for "n" var boolean fn we've to to make 2^n minterms & for each minterm we have cell in K-Map
- Hence total no. of cells = 2^n (for n variables)

g 4 var k map has no. of cells $\Rightarrow 2^4 = 16$

g $f(A, B, C, D) = \Sigma m(0, 1, 13, 15)$

AB		00	01	11	10
CD	00	m ₀	m ₄	m ₁₂	m ₈
01	m ₁	m ₅	m ₉	m ₉	
11	m ₃	m ₇	m ₁₅	m ₁₁	
10	m ₂	m ₆	m ₁₄	m ₁₀	

AB		00	01	11	10
CD	00	m ₀	m ₁	m ₃	m ₂
01	m ₄	m ₅	m ₇	m ₆	
11	m ₁₂	m ₁₃	m ₁₅	m ₁₄	
10	m ₈	m ₉	m ₁₁	m ₁₃	

2 way to make K-Map

Size of rectangle

1
2
4
8
16
32

0	4	12	8
1	5	13	9
3	7	15	11
2	6	14	10

AB		00	01	11	10
CD	00	1			
01	1	1			
11					
10					

$\bar{A}\bar{B}\bar{C} + ABD$

$\left\{ \begin{array}{l} 2^n \times 2^m = 1 \times 2 \\ n=0, m=1 \end{array} \right.$

$$f(A_1, B_1, C_1, D) = \Sigma m(0, 7, 8, 15)$$

	AB	00	01	11	10
CD					
00	10	4	8	12	18
01	1	5	13	9	
11	2	7	11		11
10	3	6	14		10

$$BCD + \bar{B}\bar{C}\bar{D}$$

$$f(A_1, B_1, C_1, D) = \Sigma m(0, 2, 12, 14)$$

	AB	00	01	11	10
CD					
00	10	4	12	8	
01	1	5	13	9	
11	3	7	15	11	
10	12	6	14	10	

$$= \bar{A}\bar{B}\bar{D} + A\bar{B}\bar{D}$$

$$f(A_1, B_1, C_1, D) = \Sigma m(3, 5, 6, 15)$$

	AB	00	01	11	10
CD					
00					
01			1		
11	1			1	
10		1			

$$\bar{A}BCD + ABCD + \bar{ABC}\bar{D} + \bar{A}\bar{B}CD$$

$$f(A_1, B_1, C_1, D) = \Sigma m(0, 1, 2, 3)$$

	AB	00	01	11	10
CD					
00		1			
01		1			
11		1			
10		1			

$$\bar{A}\bar{B}$$

Size of rectangle \rightarrow literal

$$1 \rightarrow 4$$

$$2 \rightarrow 3$$

$$4 \rightarrow 2$$

Q $f(A, B, C, D) = \Sigma m(5, 7, 13, 15)$

	AB 00	01	11	10
CD				
00				
01		1	1	
11		1	1	
10				

\overline{BD}
Ans

Q $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5)$

	AB 00	01	11	10
CD				
00	1	1		
01	1	1		
11	1			
10	1	.		

Q) $f(A, B, C, D) = \Sigma m(5, 7, 13, 15)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1	1		
01	11					
11	10					

$\bar{B}D$
Ans

Q) $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1	1		
01	11					
11	10					

$\bar{A}\bar{B} + \bar{A}\bar{C}$

Ans

Q) $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 10, 11)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1			
01	11					
11	10					

$\bar{A}\bar{B} + \bar{B}C$

Ans

Q) $f(A, B, C, D) = \Sigma m(0, 2, 4, 6, 8, 10, 12, 14)$

		AB	00	01	11	10
		CD	00	01	11	10
00	01		1	1	1	1
01	11					
11	10					

\bar{D}

Ans

$$f(A, B, C, D) = \Sigma m(0, 2, 5, 7, 13, 15, 8, 10)$$

		AB	00	01	11	10
		CD	00	1	.	1
		00	1	1	1	
		01	1	.	1	
		11	1	.	1	
		10	1	1	1	1

$$BD + \bar{B}\bar{D}$$

Ave

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 6, 8, 9, 10, 11, 12, 14)$$

		AB	00	01	11	10
		CD	00	1	1	1
		00	1	.	1	
		01	1	.	1	
		11	1	.	1	
		10	1	1	1	1

$$\bar{D} + \bar{B}$$

Ave

$$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 4, 5, 10, 11)$$

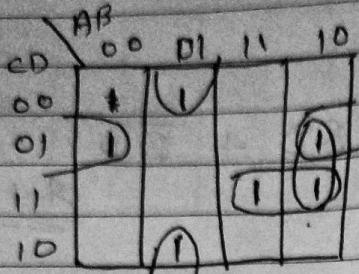
		AB	00	01	11	10
		CD	00	1	1	.
		00	1	1	1	
		01	1	.	1	
		11	1	.	1	
		10	1	1	1	1

$$\bar{A}\bar{B} + \bar{A}\bar{C} + \bar{C}\bar{B} \times$$

$$\bar{A}\bar{C} + \bar{B}\bar{C}$$

Ave

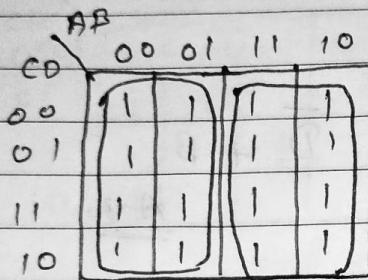
Ques $f(A, B, C, D) = \Sigma m(1, 4, 6, 9, 11, 15)$



$$\bar{B}\bar{C}D + A\bar{C}D + \bar{A}B\bar{D}$$

Ans

Ques $f(A, B, C, D) = \Sigma m(0, 1, 2, 3, \dots, 15)$



Ans

Ans

if every cell is present then ans will always be "1" and literal count is "zero"

Proof!

~~Make 2 boxes of 8 - 8 cells~~

$$\bar{A} + A$$



"1" Ans

Size of box \rightarrow Literal Count

1 \rightarrow 4

2 \rightarrow 3

4 \rightarrow 2

8 \rightarrow 1

16 \rightarrow 0

"n" Variable K-MAP

Rectangle — Literal count

1	—	n
2	—	$n-1$
4	—	$n-2$
8	—	$n-3$
16	—	$n-4$
32	—	$n-5$
2^n	—	$n-n$

* Cover

Consider 2 sets $f \& g$ we say f covers g if $g \subseteq f$

g	A	B	f_1	f_2
0	0	0	0	0
0	1	1	1	1
1	0	1	1	1
1	1	0	1	1

$$f_1 = \text{em}(1, 2)$$

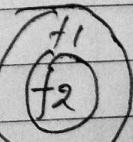
$$f_2 = \text{em}(1, 2, 3)$$

Since the element in f_1 is also present in f_2 , means f_2 covers

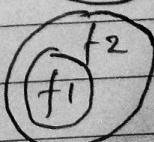
f_1

- a f_1 covers f_2
 b f_2 covers f_1 ✓
 c both
 d none

f_1 covers f_2 →



f_2 covers f_1 →



- In logic/B.A we say if $f_1 \rightarrow f_2$ is a tautology
- $f_1 \rightarrow f_2$ is a tautology
 f_1 implies f_2

A	B	f_1	f_2	$f_1 \rightarrow f_2$
0	0	0	0	1
0	1	1	1	1
1	0	1	1	1
1	1	0	1	1

implies Truth Table

P	Q	$P \rightarrow Q$
0	0	1
0	1	1
1	0	0
1	1	1

f_2 covers $f_1 \therefore f_1 \rightarrow f_2$ is tautology

* Implicant:

given 2 functions $f_1 \wedge f_2$ s.t $f_1 \leq f_2$
then f_1 is called implicant of
 f_2 i.e. $f_1 \rightarrow f_2$ is tautology

Simply we can say that every subset of
boolean f^n is an implicant

Ex $f(A, B) = \Sigma(1, 2)$

$$\begin{array}{l} f_1(A, B) = \emptyset \\ f_2(A, B) = \Sigma(1) \\ f_3(A, B) = \Sigma(2) \\ f_4(A, B) = \Sigma(1, 2) \end{array} \quad \left[\begin{array}{l} \text{All function from} \\ f_1 \text{ to } f_4 \text{ are} \\ \text{implicants of } f(A, B) = \Sigma(1, 2) \end{array} \right]$$

If f^n has n minterms then how many
implicants?

2^n step

Q If f^n has ' n ' minterms then how many implicants functions can cover f ?

$$n \rightarrow 2^n = \text{minterms}$$

$$2^n - n = 2^{(2^n - n)}$$

Q $f(A, B, C) = AB + BC \rightarrow \underline{\text{SOP}}$

How many functions can cover f ?

$$f(A, B, C) = \Sigma m(6, 7, 3)$$

$$8 - 3 = 5 = 2^5 \rightarrow 32 \text{ Ans}$$

"3" are always present rest 5 having "2" possibilities
they can be or can be present

for $(6, 7, 3, 0)$ and so on...

Q If f^n has 'n' minterms then how many implicants functions can cover f ?

$$n \rightarrow 2^n = \text{minterms}$$

$$2^n - n = 2^{(2^n - n)}$$

$$Q f(A, B, C) = AB + BC \rightarrow \underline{\text{SOP}}$$

How many functions can cover f ?

$$f(A, B, C) = \text{Em}(6, 7, 3)$$

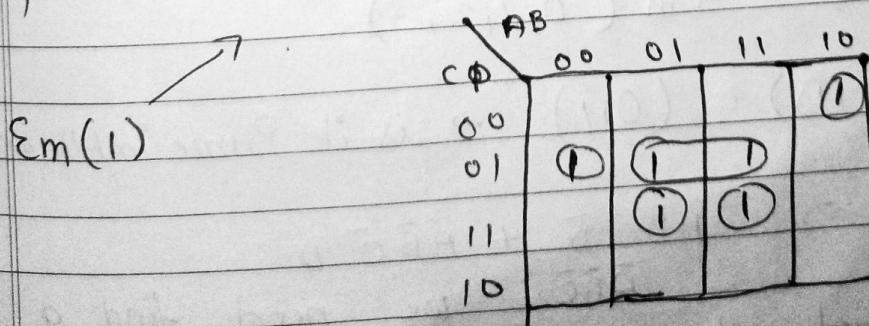
$$8 - 3 = 5 = 2^5 \rightarrow 32 \text{ Ans}$$

"3" are always present rest 5 having "2" possibilities
they can be or can be present

for $(6, 7, 3, 0)$ and so on...

Q How to visualize implicants in K-map?

$$f(A, B, C) = \text{Em}(1, 5, 7, 8, 13, 15)$$



$f \rightarrow f$

$$f(A, B, C, D) = \text{Em}(7, 8, 15)$$

- We have f^n '1's of 'n' variable & to minimize it we have designed K-Map
- This K-Map will contain 1's in those cells whose corresponding minterms are present in f^n
- Hence take any single 1 or group of 1's from the K-Map as it is called implicant of the f^n for which K-Map is designed

Defⁿ of Implicant:-

Any single 1 or group of 1's in K-Map is called implicant.

- * Prime implicant \rightarrow A prime implicant is an implicant that can not be covered by more reduced implicant

Implicant with less literal count than the given implicant

$$f(A, B, C, D) = \Sigma_m (0, 1, 2, 3)$$

$$f_1(A, B, C, D) = (0, 1) \rightarrow \text{is it Prime implicant?}$$

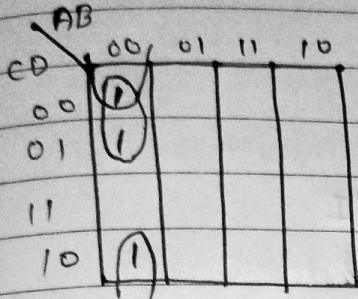
$$(0, 1) \Rightarrow \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D$$

$\bar{A}\bar{B}\bar{C}$ we need find a function which who is super set of (0, 1) and its literal count is less

than $\bar{A}\bar{B}\bar{C}$

$$(0,1,2) \Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{D} \quad \times$$

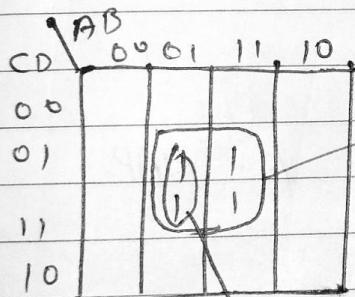
$$(0,1,3) \Rightarrow \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}D \quad \times$$



$$(0,1,2,3) \Rightarrow \bar{A}\bar{B} \quad \checkmark$$

Q How to visualize Prime implicant in k-MAP?

A Any single 1 or group of 1 that can't be included in bigger rectangle in k-MAP?

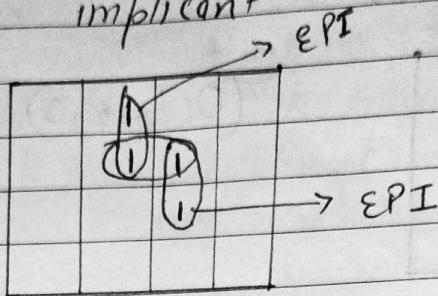


BD is a prime implicant

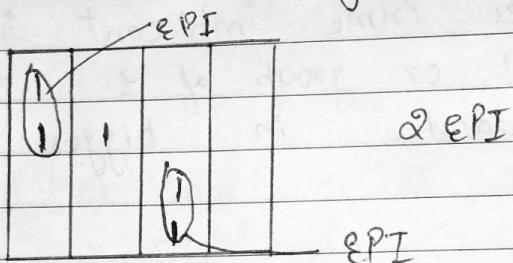
$\bar{A}\bar{B}D$ is not a prime implicant

* Essential Prime Implicant

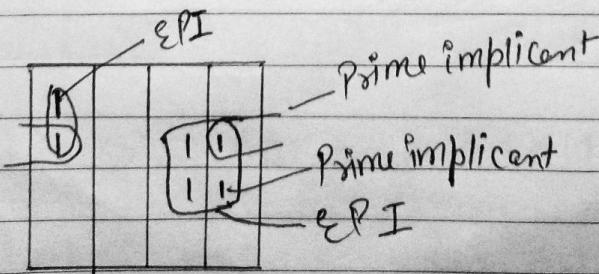
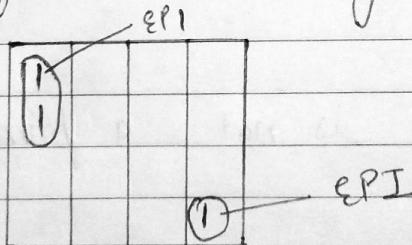
Those prime implicant which contain at least 1 minterm which is not included in any other implicant



- No. of EPI in given K-MAP ?



- No. of EPI in given K-MAP



1	1	1	
1	1	1	EPI
1	1	1	(2) <u>di</u>

1	1	1	1
1	1	1	EPI
1	1	1	(2) <u>di</u>

1	1	1	EPI
1	1	1	EPI
1	1	1	(3) <u>di</u>

1	1	1	1
1	1	1	1
1	1	1	1

0 EPI

Redundant Prime Implicant

A prime implicant in which each of its minterm is covered by some other EPI

1	1	1	
1	1	1	RPI
1	1	1	EPI

1	1	1	1
1	1	1	EPI
1	1	1	RPI

1	1	1	1
1	1	1	1
1	1	1	1

0 EPI
0 RPI

* Selective Prime implicant

- neither EPI nor RPI
- They ~~do~~ always occurs in pair

	AB	00	01	11	10
EPI	00	1			
SPI	01		1		
EPI	10				

② EPI
② SPI

$$M \cdot E = \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}C + \bar{A}\bar{C}D$$

or

$$\bar{A}\bar{B}\bar{C} + \bar{A}BC + A\bar{B}D$$

	AB	00	01	11	10
CD	00	1	1		
	01		1		
	11	1	1		
	10				

6 SPI

	AB	00	01	11	10
CD	00	1	1		
	01		1		
	11	1	1		
	10		1		

$$M \cdot E = \bar{A}\bar{C}\bar{D} + \bar{A}BD + \bar{A}\bar{B}C$$

or

$$\bar{A}\bar{B}\bar{D} + \bar{A}\bar{B}\bar{C} + \bar{A}CD$$

	AB	00	01	11	10
CD	00	1	1	1	
	01		1		
	11		1		
	10			1	

or

	AB	00	01	11	10
CD	00	1	1	1	
	01		1	1	
	11		1	1	
	10			1	1

$$M \cdot E = \bar{A}\bar{C}\bar{D} + B\bar{C}D + ACD + A\bar{B}\bar{D}$$

$$M \cdot E = \bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C} + ABD + A\bar{B}C$$

drw

* Practice Question EPI, RPI & SPI and minimized expression?

	00	01	11	10	EPI
CD	00	01	11	10	EPI
00	1	1	1	1	EPI
01	1	1	1	1	RPI
11	1	1	1	1	EPI

using PI chart

	3	4	5	7	9	13	14	15
P1	✓	✓						
P2				✓	✓			
P3							✓	✓
P4	✓							
P5		✓	✓			✓	✓	✓
P6								

$$M_OE = \bar{A}\bar{B}\bar{C} + A\bar{C}D + ABC \\ + \bar{A}CD$$

Ans

- EPI
- RPI
- SPI

$$P1 \dots P4 = 4 \text{ EPI}$$

$$P = 1 \text{ RPI}$$

	00	01	11	10
CD	00	01	11	10
00	1	1	1	1
01	1	1	1	1
11	1	1	1	1
10	1	1		

EPI

6 SPI

using PI chart

	0	2	3	4	5	7
P1	✓					
P2				✓		
P3					✓	
P4					✓	
P5		✓	✓			
P6	✓	✓				

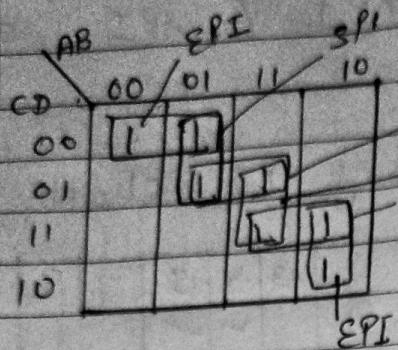
$$M_OE = \bar{A}\bar{C}D + \bar{A}BD \\ + \bar{A}\bar{B}C$$

or

$$\bar{A}\bar{B}\bar{D} + \bar{A}\bar{B}\bar{C} \\ + \bar{A}CD$$

Ans

$$P1 \dots P6 = 6 \text{ SPI}$$



SPI SPI SPI

EPI	0	4	5	10	11	13
P1	✓	✓				
P2				✓	✓	
P3					✓	
P4					✓	
P5				✓		
P6					✓	✓

2 EPI

4 SPI

$$\text{M.E} = \bar{A}\bar{C}D + B\bar{C}D \\ + ABD + A\bar{B}C$$

Single check vertically:

$\equiv 2 \text{ EPI}$

4 SPI

or

$$\bar{A}\bar{C}\bar{D} + B\bar{C}D$$

$$+ ACD + A\bar{B}C$$

or

$$\bar{A}\bar{C}D + \bar{A}B\bar{C} + ABD$$

$$+ A\bar{B}C$$

Anw

* Note \rightarrow Minimize expression for K-MAP contain all EPI

all EPI

No RPI all

Some but not SPI

Minimized expression

① $f(A, B, C, D) = \Sigma m(0, 1, 2, 5, 7, 8, 9, 10, 13, 15)$

	AB	00	01	11	10
CD	1				1
00	1				
01		1	1		D
11		1	1	*	
10	1				10

$$\bar{B}\bar{D} + BD + \bar{C}D$$

$$\text{or } \bar{B}\bar{D} + BD + \bar{B}\bar{C}$$

Ans

Ans

②

	AB	00	01	11	10
CD	1				
00	1	1			
01		1	1		
11					1
10	1				1

$$\bar{B}C + \bar{A}\bar{C}$$

Ans

③

	AB	00	01	11	10
CD	1				
00			1		
01	1				
11				1	
10		1		1	1

$$\bar{A}B\bar{D} + \bar{B}\bar{C}D + ACD$$

	AB	CD	00	01	11	10
00	1	1	1	1	1	1
01	1	1	1	1	1	1
11						
10						

$$\bar{A}D + \bar{A}\bar{B} + \bar{B}\bar{C} + \bar{B}D$$

Ans

	AB	CD	00	01	11	10
00	1	1	1	1	1	1
01						
11			1	1		
10						

$$\bar{C}\bar{D} + A\bar{C} + BCD$$

Ans

	AB	CD	00	01	11	10
00	1	1	1	1	1	1
01	1	1	1	1	1	1
11			1	1	1	1
10	1	1				

$$\bar{A}\bar{D} + \bar{A}\bar{C} + ACD + BC$$

Ans

	AB	CD	00	01	11	10
00	1	1	1	1	1	1
01						
11	1					
10	1			1	1	

$$\bar{A}\bar{C}\bar{D} + A\bar{B}\bar{C} + AC\bar{D} + \bar{A}\bar{B}C$$

Ans

	$\bar{A}B$	00	01	11	10
CD	00	1	1	1	
00	01	1	1	1	
01	11	1	1	1	
11	10	1	1	1	

$$A \oplus B \oplus C \oplus D$$

Ans

if we ~~ain't~~ able to make any cell then it would will be XOR or ~~XOR~~ XNOR

XOR \rightarrow gives ~~not~~ '1' if odd no. of literals are true.

XNOR \rightarrow gives '1' when even no. of literals are false.

Proof why it is XOR using boolean law

$$A \oplus B \oplus C \oplus D$$

$$\begin{aligned}
 & \cancel{1} \quad \cancel{3} \quad \cancel{4} \quad \cancel{7} \quad \cancel{13} \quad \cancel{14} \\
 & \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}C\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}\bar{C}D + ABC\bar{D} \\
 & + A\bar{B}\bar{C}\bar{D} + AB\bar{C}D
 \end{aligned}$$

* K-MAP representation in POS

① $f(A, B, C, D) = \text{PM}(1, 2, 3, 4, 5)$

- means given minterms are "absent"; rest everyone is "present"

AB		00	01	11	10
CD	00	0			
	01	0	0		
11	0				
10	0				

$$\textcircled{\times} (A + \bar{B} + C) \cdot (A + B + \bar{C})$$

$$\cdot (A + C + \bar{D})$$

or

$$(A + \bar{B} + C) + (A + B + \bar{C})$$

$$\cdot (A + B + \bar{D})$$

Ans

Note: 0 means as it is
= 1 means bar

② $f(A, B, C, D) = \text{PM}(0, 1, 2, 5, 7, 8, 9, 10, 14, 15)$

AB		00	01	11	10
CD	00	0		0	
	01	0	0	0	
11	0	0	0	0	
10	0		0	0	0

$$\textcircled{\times} (B + D) \cdot (B + C) \cdot (A + \bar{B} + \bar{D})$$

$$\cdot (\bar{A} + \bar{B} + \bar{C})$$

Ans

————— * ————— * —————

$f(A, B, C, D) = \Sigma m(0, 1, 2, 3, 8, 9, 10, 11, 13, 15)$
 minimized expression

- a) $(A + \bar{B}) \cdot (\bar{B} + D) \checkmark$
- b) $(A + B) \cdot (\bar{B} + D)$
- c) $(A + B) \cdot (\bar{B} + \bar{D})$
- d) $(A + \bar{B}) \cdot (B + D)$

Any is given Pos and
minterm is given in
SOP.
will write those terms
which are absent

~~$f(A, B, C, D) = \Sigma m(4, 5, 6, 7, 12, 14)$~~

		AB	00	01	11	10
		CD	00	0	0	
		01	0			
		11	0			
		10	0	0		

$$(\bar{B} + D) \cdot (A + \bar{B})$$

or if will be $\underline{\bar{B} + A \cdot D}$
in SOP Anse Ane

$f(W, x, y, z) = \Sigma m(0, 1, 2, 5, 8, 9, 10)$

- a) $(\bar{w} + \bar{x}) \cdot (\bar{y} + \bar{z}) \cdot (\bar{x} + \bar{z})$ in Pos = $\Sigma m(3, 4, 6, 7, 11, 12)$
- b) $(w + \bar{x}) \cdot (y + \bar{z}) \cdot (x + \bar{z})$
- c) $(w + x) \cdot (y + z) \cdot (x + z)$
- d) None $w \bar{w}$ ✓

		yz	00	01	11	10
		xy	00	0	0	
		01	0			
		11	0	0	0	0
		10	0	0		

$$(\bar{w} + \bar{x}) \cdot (\bar{y} + \bar{z}) \cdot (\bar{x} + z)$$

$f(A, B, C, D) = \text{XOR}(0, 1, 2, 3, \dots, 15)$

• $\begin{array}{c} AB \\ \diagdown \\ CD \end{array}$

	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	0
10	0	0	0	0

$\Rightarrow 0$
proof by makin 2
8-8 cells

$(A \cdot \bar{A}) = 0$

Literal Count 0 Ans

Similarly in SOP ans. will = 1
 $A + \bar{A} = 1$ Ans

• $f(A, B, C, D) = A\bar{B} + \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{C}D + A\bar{C}\bar{D}$

• $\begin{array}{c} AB \\ \diagdown \\ CD \end{array}$

	00	01	11	10
00	1		1	1
01	1		1	1
11	1			1
10	1			

$\bar{B} + A\bar{C} \rightarrow \text{SOP}$
or $(\bar{B} + A) \cdot (\bar{B} + \bar{C}) \rightarrow \text{POS}$