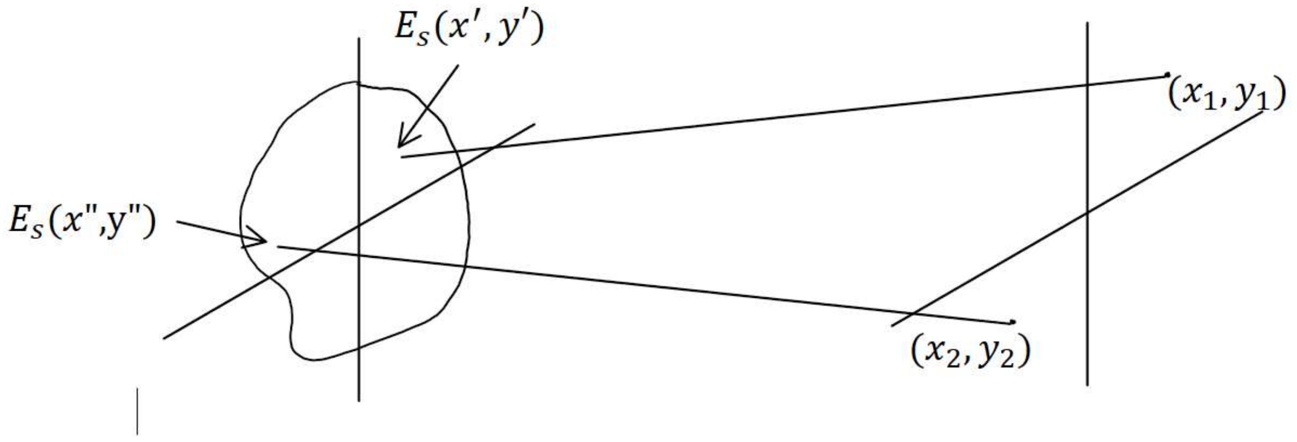
Lab 5

1. Abstract

This study explores the simulation of partial coherence based on the van Cittert–Zernike theorem. By employing multiple realizations of a random phase mask and an aperture of varying sizes, we analyze the impact of aperture dimensions on coherence.

2. Van-Cittert Zernike Theorem



The field at at far field distance is given by:

$$E_{out} = \int\int dx'dy' E_s(x',y')e^{-\iota 2\pi(xx'+yy')/\lambda z}$$

The correlation at two points in the far field:

$$< E_{out}(x_1,y_1)E_{out}(x_2,y_2) >= \int\int\int\int dx'dy'dx" \ dy" < E_s^*(x',y')E_s(x",y") > e^{\iota 2\pi[(x'x_1+y'y_1)-(x"x_2+y"y_2)]/\lambda z}$$

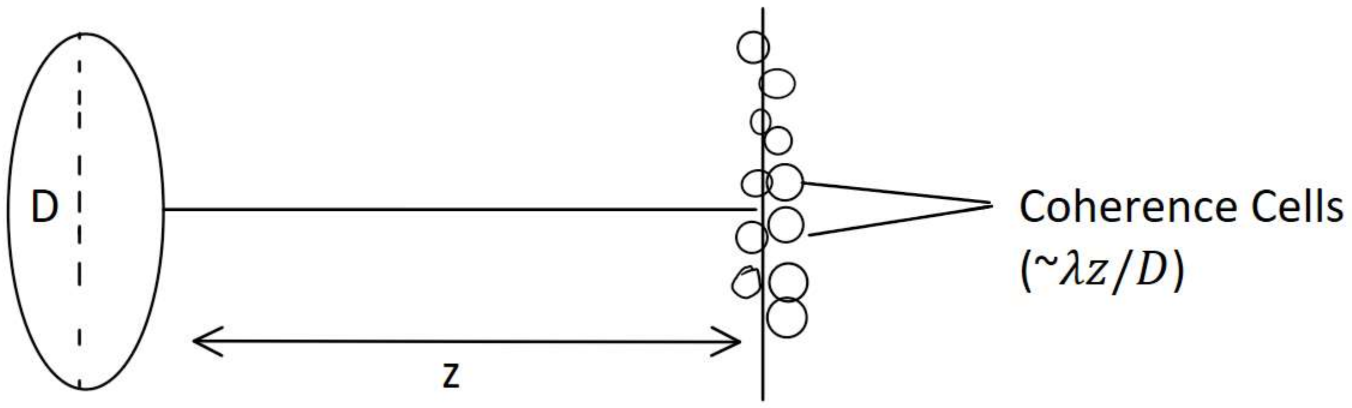
for incoherent source,

$$< E_s^*(x',y')E_s(x",y") >= I_s(x',y')\delta(x'-x",y'-y")$$

Therefore,

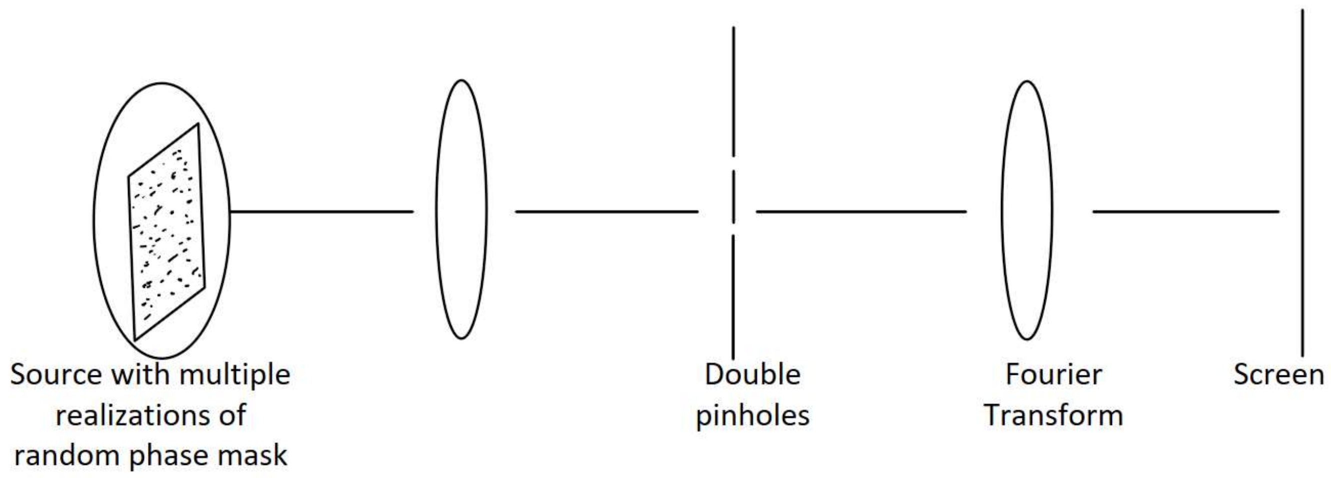
$$\Gamma(x_1,y_1:x_2,y_2) = \int\int dx'dy'I_s(x',y')e^(-\iota 2\pi[x'(x_2-x_1)+y'(y_2-y_1)]/\lambda z)$$

This result is known as Van-Cittert and Zernike Theorem. The correlation between two points due to an incoherent source will be like the diffraction pattern of the aperture.



If the aperture is circular, the size over which the two points are coherent, i.e. the size of the coherence cell will be given by $\lambda z/D$.

3. Parital Coherence Simulation

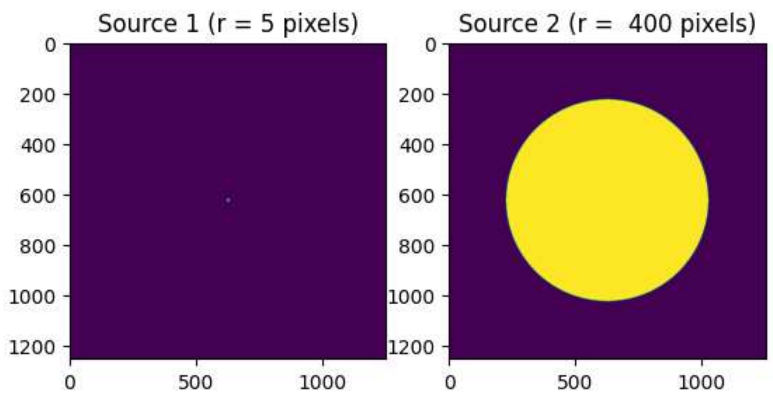



```python
import numpy as np
import matplotlib.pyplot as plt
from tqdm.notebook import tqdm
```

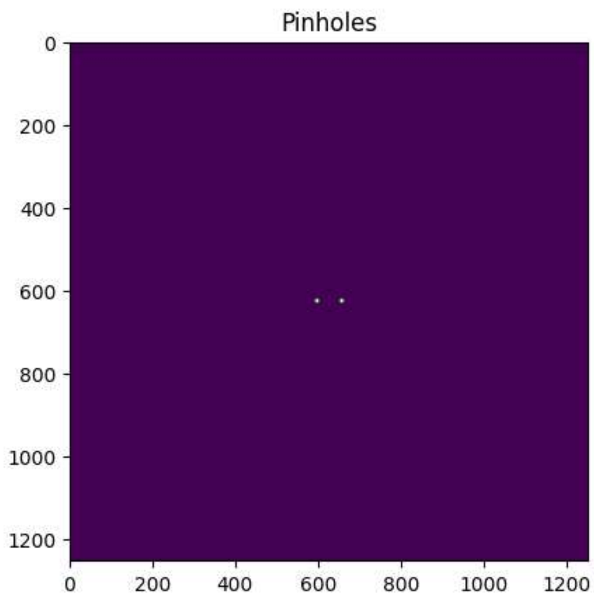
```python
# Helper functions
def ft2(x):
  return np.fft.fftshift(np.fft.fft2(np.fft.ifftshift(x))) # Fourier transform

def ift2(x):
  return np.fft.fftshift(np.fft.ifft2(np.fft.ifftshift(x))) # Inverse fourier transform
```

```python
# Constants
N = 1254  # For a screen size of 6.27 mm * 6.27 mm
p = 5e-6  # Pixel Size
wavelength = 0.6328e-6 # Wavelength of He-Ne laser = 0.6328 micron
k = 2*np.pi/wavelength # Wave vector
D = 2e-3  # Diameter of the aperture is 2 mm

# Coordinates
x0 = np.linspace(-N/2, N/2, N, endpoint = True)
x, y = np.meshgrid(x0*p, x0*p)

f0 = x0/N
fx, fy = np.meshgrid(f0/p, f0/p)

# Source 1 of radius 5 pixels
radius1 = 5 * p
circ1 = np.zeros((N,N))
circ1[(x**2 + y**2) <= radius1**2] = 1
print(f"Diameter = {2*radius1} m.")

# Source 2 of radius 400 pixels
radius2 = 400*p
circ2 = np.zeros((N, N))
circ2[(x**2 + y**2) <= radius2**2] = 1
print(f"Diameter = {2*radius2} m.")

plt.figure()

plt.subplot(1,2,1)
plt.imshow(circ1)
plt.title("Source 1 (r = 5 pixels)")

plt.subplot(1,2,2)
plt.imshow(circ2)
plt.title("Source 2 (r =  400 pixels)")
plt.show()
```

```
Diameter = 5e-05 m.
Diameter = 0.004 m.
```




```python
# Pinholes in fourier plane
pinhole = np.zeros((N, N))
x1 = 30*p # Pinhole separation
radius = 5*p # Pinhole radius
pinhole[(x-x1)**2 + (y)**2 <= radius**2] = 1
pinhole[(x+x1)**2 + (y)**2 <= radius**2] = 1

plt.figure()
plt.imshow(pinhole)
plt.title("Pinholes")
plt.show()
```

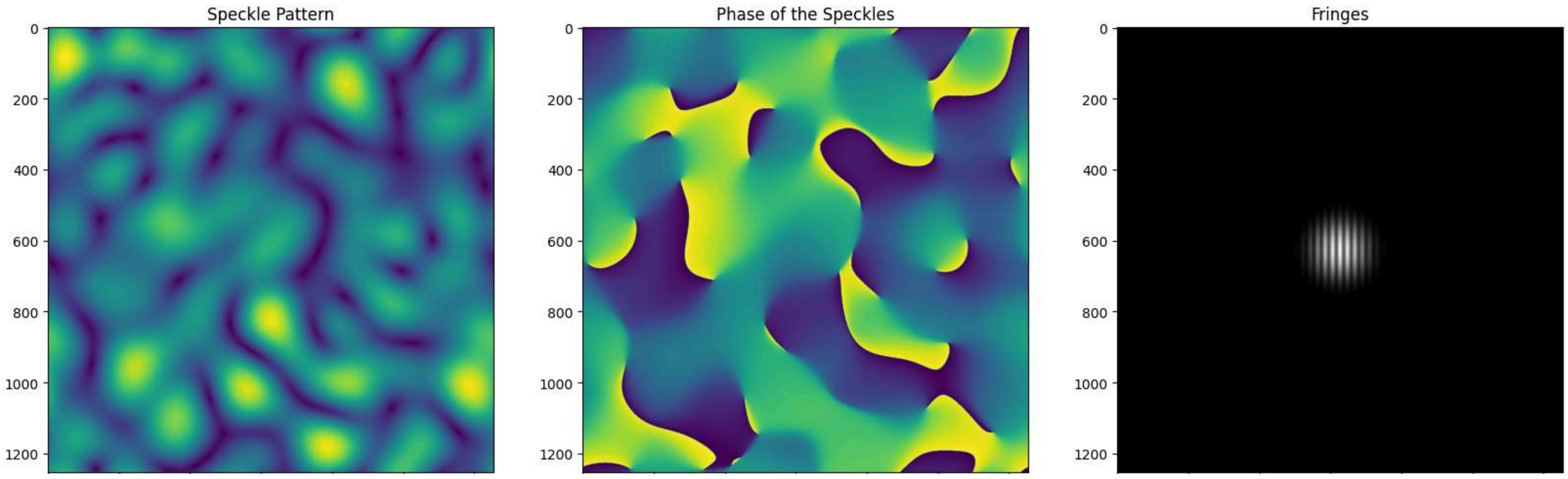



```python
#Initialize intensity with zeros
intensity1 = np.zeros((N, N))

# Generate multiple realizations of random phase mask and observe the averaged intensity
for i in tqdm(range(100)):
  mask = np.exp(1j * 2*np.pi*np.random.rand(N, N)) # random phase
  aperture = circ1 * mask # Source 1 with random phase
  partial = ft2(aperture) # field at fourier plane
  after_pinhole = partial * pinhole # field through two pinholes
  final_field = ft2(after_pinhole) # interference field
  intens = np.abs(final_field)**2 # intensity
  intens = intens/intens.max() # normalized intensity
  intensity1 += intens # add intensity to previous intensities
```

```
100%                                        100/100 [00:37<00:00,    3.19it/s]
```

```python
# Display results
plt.figure(figsize = (20, 10))

plt.subplot(1,3,1)
```

```
plt.imshow(np.abs(partial))
plt.title("Speckle Pattern")

plt.subplot(1,3,2)
plt.imshow(np.angle(partial))
plt.title("Phase of the Speckles")

plt.subplot(1,3,3)
plt.imshow(intensity1/100, cmap = "gray") # Displays average intensity
plt.title("Fringes")

plt.show()
```

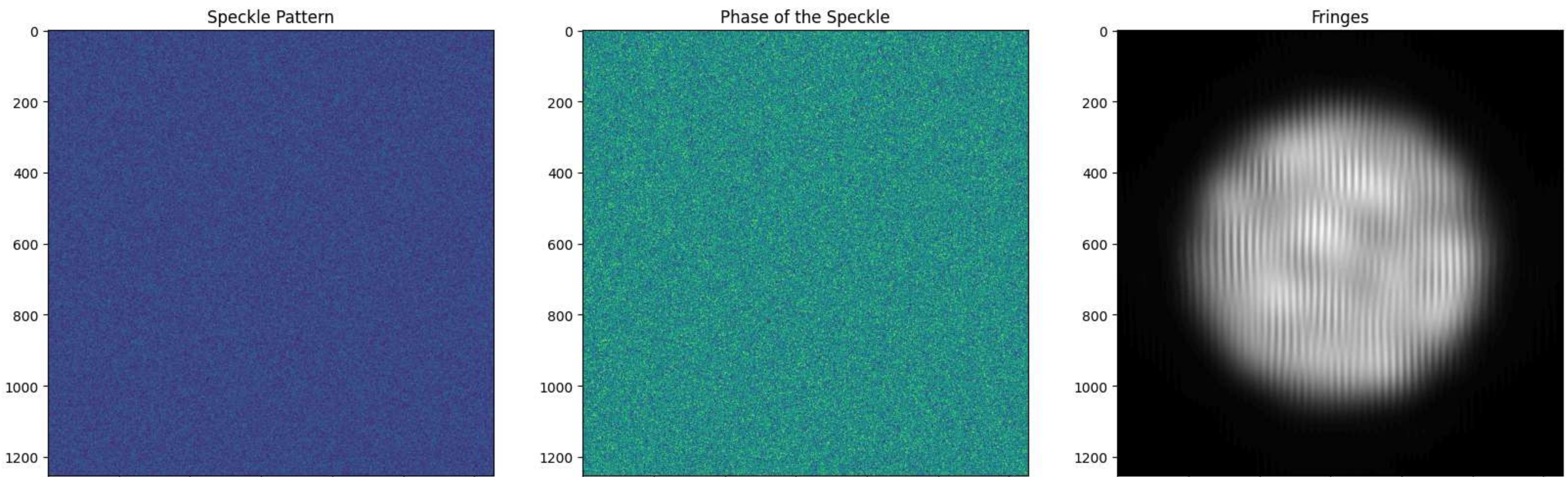


The fringes have well contrast. The size of the coherence cells is large, because the aperture size is very small, and therefore the field on the two pinholes is coherent and the resultant fringes have good contrast.

```
# Initialize intensity with zeros
intensity2 = np.zeros((N, N))

# Generate multiple realizations of random phase mask and observe the averaged intensity
for i in tqdm(range(100)):
  mask = np.exp(1j * 2*np.pi*np.random.rand(N, N))
  aperture = circ2 * mask # Source 2 with random phase
  partial = ft2(aperture)
  after_pinhole = partial * pinhole
  final_field = ft2(after_pinhole)
  intens = np.abs(final_field)**2
  intens = intens/intens.max()
  intensity2 += intens
```

```
100%                                   100/100 [00:38<00:00,   3.27it/s]
```

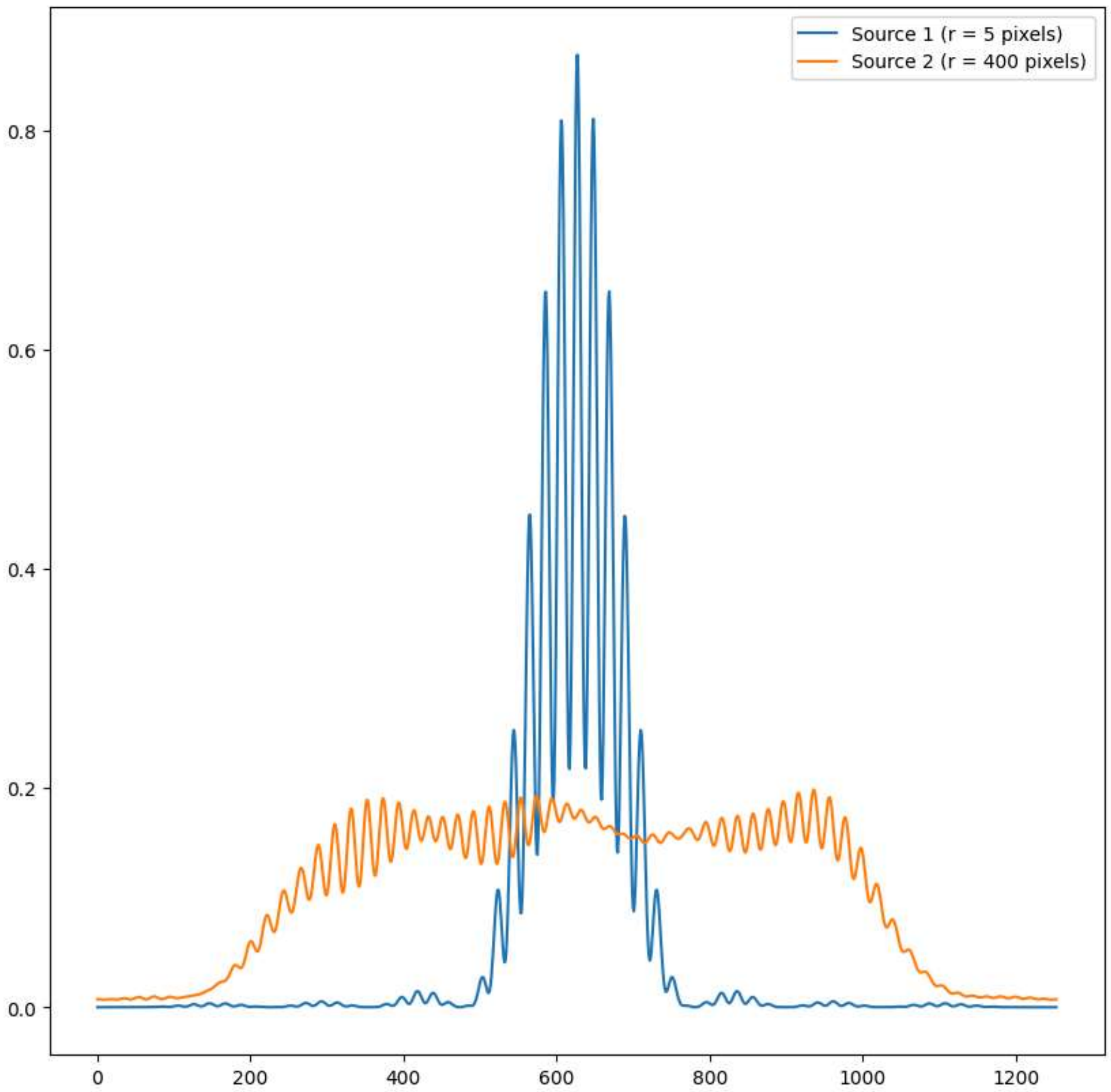
```
# Display results

plt.figure(figsize = (20, 10))
plt.subplot(1,3,1)
plt.imshow(np.abs(partial))
plt.title("Speckle Pattern")

plt.subplot(1,3,2)
plt.imshow(np.angle(partial))
plt.title("Phase of the Speckle")

plt.subplot(1,3,3)
plt.imshow(intensity2/100, cmap = "gray") # Displays average intensity
plt.title("Fringes")

plt.show()
```



The source size is now very large and therefore the size of the coherence cells is very small. As a result, the field at the two pinholes is incoherent and we get interference fringes with low contrast.

```
# The contrast of the fringes
first = intensity1/100 # Source 1
second = intensity2/100 # Source 2

profile1 = first[N//2, :] # Intensity profile for Source 1
profile2 = second[N//2, :] # Intensity profile for Source 2

plt.figure(figsize = (10,10))
plt.plot(profile1, label =  "Source 1 (r = 5 pixels)")
plt.plot(profile2, label = "Source 2 (r = 400 pixels)")
```

```
plt.legend()
plt.show()
```



From the plot we can observe that the contrast for Source 2 is greatly reduced.

4. Conclusion

The simulation highlights the crucial relationship between aperture size and fringe contrast in partially coherent systems. As the aperture size increases, the contrast of the interference fringes decreases, consistent with the theoretical predictions of the van Cittert-Zernike theorem. These results validate the simulation approach and emphasize the importance of aperture size in optical coherence experiments.