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Source: *The Journal of Finance*, Mar., 1990, Vol. 45, No. 1 (Mar., 1990), pp. 221-229

Published by: Wiley for the American Finance Association

Stable URL: https://www.jstor.org/stable/2328817

THE JOURNAL OF FINANCE • VOL. XLV, NO. 1 • MARCH 1990

Heteroskedasticity in Stock Return Data: Volume versus GARCH Effects

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ABSTRACT

This paper provides empirical support for the notion that Autoregressive Conditional Heteroskedasticity (ARCH) in daily stock return data reflects time dependence in the process generating information flow to the market. Daily trading volume, used as a proxy for information arrival time, is shown to have significant explanatory power regarding the variance of daily returns, which is an implication of the assumption that daily returns are subordinated to intraday equilibrium returns. Furthermore, ARCH effects tend to disappear when volume is included in the variance equation.

THE AUTOREGRESSIVE CONDITIONAL HETEROSKEDASTICITY (ARCH) process of Engle (1982) has been shown to provide a good fit for many financial return time series.[1] ARCH imposes an autoregressive structure on conditional variance, allowing volatility shocks to persist over time. This persistence captures the propensity of returns of like magnitude to cluster in time and can explain the well documented nonnormality and nonstability of empirical asset return distributions. (See especially Fama (1965).)

An appealing explanation for the presence of ARCH is based upon the hypothesis that daily returns are generated by a mixture of distributions, in which the rate of daily information arrival is the stochastic mixing variable. As suggested by Diebold (1986), Gallant, Hsieh, and Tauchen (1988), and Stock (1987, 1988), ARCH might capture the time series properties (e.g., serial correlation) of this mixing variable. However, this linkage has not been broadly documented with the data.

The objective of this study is to examine the validity of this explanation for daily stock returns. The empirical strategy exploits the implication of the mixture model that the variance of daily price increments is heteroskedastic—specifically, positively related to the rate of daily information arrival. Using daily trading volume as a proxy for the mixing variable, we show that, for a sample of 20 common stocks, ARCH effects vanish when volume is included as an explanatory variable in the conditional variance equation.

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[1] See, e.g., Bollerslev (1987), Lamoureux and Lastrapes (1988), Baillie and Bollerslev (1989), and Lastrapes (1989).

The paper is organized as follows. Section I provides theoretical motivation for the empirical analysis. ARCH in daily returns is shown to follow from serial correlation in the mixing variable—the number of intraday price changes—and testable hypotheses are noted. Section II describes the data and discusses the empirical results. Concluding remarks are contained in Section III.

I. The Heteroskedastic Mixture Model and ARCH

The Generalized ARCH (GARCH) model of Bollerslev (1986) restricts the conditional variance of a time series to depend upon past squared residuals of the process. Such a model for daily stock returns is given below:

$$r_t = \mu_{t-1} + \varepsilon_t, \tag{1}$$

$$\varepsilon_t | (\varepsilon_{t-1}, \varepsilon_{t-2}, \cdots) \sim N(0, h_t), \tag{2}$$

$$h_t = \alpha_0 + \alpha_1(L)\, \varepsilon^2_{t-1} + \alpha_2(L)\, h_{t-1}, \tag{3}$$

where r_t represents the rate of return (cum-dividend), μ_{t-1} is the mean r_t conditional on past information, L is the lag operator, and $\alpha_0 > 0$. If the parameters of the lag polynomials $\alpha_1(L)$ and $\alpha_2(L)$ are positive, then shocks to volatility persist over time. The degree of persistence is determined by the magnitude of these parameters.

To motivate the empirical tests of this paper, let δ_{it} denote the ith intraday equilibrium price increment in day t, which implies

$$\varepsilon_t = \textstyle\sum_{i=1}^{n_t} \delta_{it}. \tag{4}$$

The random variable n_t is the mixing variable, representing the stochastic rate at which information flows into the market. Note that ε_t is drawn from a mixture of distributions, where the variance of each distribution depends upon information arrival time. Equation (4) implies that daily returns are generated by a subordinated stochastic process, in which ε_t is subordinate to δ_i and n_t is the directing process. (See Mandelbrot and Taylor (1967), Clark (1973), Westerfield (1977), and Harris (1987).)

If δ_i is i.i.d. with mean zero and variance σ^2, and n_t is sufficiently large, then $\varepsilon_t | n_t \sim N(0, \sigma^2 n_t)$. The normal law follows from the Central Limit Theorem (CLT). As Osborne (1959) notes, variation in n_t over time will lead to rejection of normality in the unconditional distribution even if the CLT applies.

GARCH may be explained as a manifestation of time dependence in the rate of evolution of intraday equilibrium returns. To make the argument precise, assume that the daily number of information arrivals is serially correlated, which can be expressed as follows:

$$n_t = k + b(L) n_{t-1} + u_t, \tag{5}$$

where k is a constant, $b(L)$ is a lag polynomial of order q, and u_t is white noise. Innovations to the mixing variable persist according to the autoregressive structure of $b(L)$. Define $\Omega_t = E(\varepsilon_t^2 | n_t)$. As noted above, if the mixture model is valid, then $\Omega_t = \sigma^2 n_t$. Substituting the moving average representation of (5) into this expression for variance yields

$$\Omega_t = \sigma^2 k + b(L)\Omega_{t-1} + \sigma^2 u_t. \tag{6}$$

Equation (6) captures the type of persistence in conditional variance that can be picked up by estimating a GARCH model. In particular, innovations to the information process lead to momentum in the squared residuals of daily returns.

The focus of our empirical tests is on the variance of returns conditional on knowledge of the mixing variable.[2] Because n_t is generally not observed, a proxy is required. We choose daily trading volume as a measure of the amount of daily information that flows into the market. Tauchen and Pitts (1983) model volume and price change as being a joint (random) function of information flow. If this specification is correct, our estimation is subject to an unquantified specification bias. Nevertheless, using volume as the mixing variable is consistent with the sequential information models of Copeland (1976) and others and the mixture of Epps and Epps (1976). In general, despite the imprecise role of volume in financial research (Ross (1987)), volume is likely to contain information about the disequilibrium dynamics of asset markets.

The model to be estimated for each stock in the sample is given by equation (1) and the following generalized variance specification:

$$\varepsilon_t | (V_t, \varepsilon_{t-1}, \varepsilon_{t-2}, \cdots) \sim N(0, h_t), \tag{2'}$$

$$h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1} + \alpha_2 h_{t-1} + \alpha_3 V_t. \tag{3'}$$

Under the assumption that volume (V_t) is the mixing variable, volume is weakly exogenous in the sense of Engle, Hendry, and Richard (1983). We restrict our attention to a GARCH (1,1) specification since it has been shown to be a parsimonious representation of conditional variance that adequately fits many economic time series (e.g., Bollerslev (1987)). A succinct measure of the persistence of variance as measured by GARCH is the sum $(\alpha_1 + \alpha_2)$: as this sum approaches unity, the greater is the persistence of shocks to volatility.

The mixture model of the previous section predicts that $\alpha_3 > 0$. Furthermore, in the presence of volume with $\alpha_3 > 0$, α_1 and α_2 will be small and statistically insignificant if daily volume is serially correlated. In particular, the persistence of variance as measured by $(\alpha_1 + \alpha_2)$ should become negligible if accounting for the uneven flow of information explains the presence of GARCH in the data.

II. Data and Empirical Results

The data set comprises daily return and volume data for 20 actively traded stocks. Actively traded stocks are most likely to have a sufficiently large number of information arrivals per day to satisfy the conditions for the CLT. Our sample is chosen from a population of stocks for which options trade on the CBOE. The rationale for using this population is that in general only stocks that are actively traded have options listed on an exchange. The use of stocks with listed options

[2] Inferences regarding the unconditional distribution can be made by assuming a specific stochastic process for the mixing variable and the intraday price increments. See especially Mandelbrot and Taylor (1967) and Mandelbrot (1973).

mitigates the potential asymmetry between positive and negative returns (a problem noted by Morgan (1976)). Finally, stocks with splits during the sample period are dropped to eliminate potential bias from split effects on volume.[3]

Daily stock returns are obtained from the 1986 version of the CRSP data base and, as such, are based upon the last daily transaction price of the security. Daily transactions volume (number of shares traded during the day) for each stock is taken from Standard and Poor's Daily Stock Price Records. Table I lists the sample stocks, the range of the data used to generate the reported results, and the number of observations. Table I also documents the nonnormality of the unconditional distribution of daily returns, and the time dependence of daily volume. The table reports the Kiefer-Salmon (1983) (KS) skewness test statistic (S), the KS kurtosis statistic (K), the KS joint statistic for normality ($S + K$), and the p-value (marginal significance level) of the Shapiro-Wilk (SW) statistic, as modified for large samples by Royston (1982). S and K are distributed as $\chi^2(1)$, while the sum is $\chi^2(2)$, under normality. At reasonable significance levels, the null hypothesis of normality is generally rejected. The Box-Ljung Q-statistic, constructed for maximum lag of 20, tests for serial correlation in the daily volume series. The Q-statistics are generally large and statistically significant at low significance levels.

Table II reports the estimated coefficient estimates and asymptotic t-statistics of the model (1), (2′), and (3′) under the restriction that $\alpha_3 = 0$ and $\mu_{t-1} \equiv 0$. (Allowing for first-order serial correlation in returns had no effect on any of the results.) The parameters are estimated jointly using numerical techniques to maximize the likelihood function. The parameter space of the variance equation is constrained to be nonnegative. Experimentation with various starting values for the parameters shows that global maxima are obtained in all models estimated. The table provides strong evidence that daily stock returns can be characterized by the GARCH model when volume is excluded from the variance equation. Coupled with the noted serial correlation in volume, this evidence provides prima facie support for the hypothesis that ARCH reflects an uneven but persistent flow of information to stock markets.

The results from the unrestricted variance model for $\mu_{t-1} \equiv 0$ are reported in Table III. The coefficient on volume, α_3, is significantly positive for each company, as predicted. This inference is consistent with the results of Clark (1973) and Westerfield (1977). The final four columns show the goodness-of-fit statistics for the adjusted residuals ($\varepsilon_t h_t^{-1/2}$) from the variance model with volume only. All statistics suggest that volume explains much of the nonnormality of the unconditional distributions. This result is consistent with the applicability of the CLT to daily stock returns, given adjustment for an uneven rate of information flow. Also, according to the SW statistic, the model is generally well specified, so that the test statistics are valid.

[3] Another potential problem in relating volume to volatility is tax-induced dividend trading. The original sample included a company (Celanese) that paid a $1 quarterly, regular dividend, with stock price around $50 a share. Volume was very high around the ex-day, which was unrelated to information flows (Lakonishok and Vermaelen (1986)). For this reason, Celanese was dropped. The final sample includes low-yield companies only.

Table I

Empirical Properties of Daily Returns and Volume for the 20 Stocks in the Sample

The estimation period and number of observations used in estimation (T) are reported for each company. $S(K)$ is the Kiefer-Salmon (1983) statistic testing the null hypothesis of normality against the alternative of skewness (excess kurtosis). $S + K$ is the joint Kiefer-Salmon (1983) statistic for normality; the alternative hypothesis is skewness and/or excess kurtosis. $SW(p)$ is the p-value of the modified Shapiro-Wilk statistic. $Q(20)$ is the Box-Ljung Q-statistic for autocorrelations up to 20 lags.

Company	T	Period	S[a]	K[a]	$S+K$[b]	$SW(p)$	$Q(20)$[c]
1 Amdahl	245	7/14/81–6/30/82	3.30	31.77	35.07	0.67	100.53
2 Diebold	399	6/7/82–12/30/83	19.14	286.95	306.09	0.00	377.92
3 Englehard	370	7/19/82–12/30/83	2.44	68.10	70.54	0.00	146.97
4 Paine-Webber	272	12/6/82–12/30/83	0.05	8.73	8.78	0.93	116.42
5 Sabine	374	7/13/82–12/30/83	24.97	89.77	114.74	0.00	1362.11
6 Loral	372	7/14/82–12/30/83	9.24	10.86	20.10	0.72	125.00
7 Cont. Ill.	338	9/1/82–12/30/83	7.50	69.78	77.28	0.00	434.93
8 1st Chicago	273	9/1/83–9/28/84	1.07	7.02	8.09	0.47	22.10
9 Medtronic	429	4/22/82–12/30/83	1.31	81.15	82.46	0.00	148.97
10 NBI	271	9/6/83–9/28/84	1.83	28.67	30.50	0.06	83.95
11 Northrup	433	4/19/82–12/30/83	10.30	157.99	168.29	0.00	569.76
12 Rolm	432	4/20/82–12/30/83	184.14	1620.21	1804.35	0.00	84.24
13 Viacom	441	4/6/82–12/30/83	30.35	35.35	65.70	0.00	109.71
14 Winnebago	269	9/8/83–9/28/84	92.70	518.25	610.95	0.00	49.23
15 Alex & Alex	333	3/7/84–6/28/85	0.18	21.88	22.06	0.31	107.69
16 Brist. Meyers	393	6/12/80/12/31/81	5.40	13.52	18.92	0.95	63.09
17 Corning Glass	389	6/18/80–12/31/81	9.86	53.01	62.87	0.00	67.41
18 Harris Corp.	342	8/25/80–12/31/81	0.10	0.99	1.09	0.04	23.71
19 Litton	392	6/13/80–12/31/81	3.33	1.36	4.69	0.40	139.79
20 Owens-Ill.	387	6/20/80–12/31/81	16.06	42.65	58.71	0.00	103.87
Mean			21.16	157.40	178.56	0.23	211.87
Median			6.45	39.00	60.79	0.00	108.70

[a] Under the null, distributed as χ^2 (1). Five percent critical value is 3.87.

[b] Under the null, distributed as χ^2 (2). Five percent critical value is 5.99.

[c] Under the null of no serial correlation, distributed as χ^2 (20). Five percent critical value is 31.41.

Table II
Maximum Likelihood Estimates of GARCH (1,1) Model without Volume

$\varepsilon_t | (\varepsilon_{t-1}, \cdots) \sim N(0, h_t)$, $h_t = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \alpha_2 h_{t-1}$. ε_t is the daily return. Goodness-of-fit tests on $\varepsilon_t h_t^{-1/2}$ (not reported) suggest that conditional returns are not normal for most of the 20 companies, so that estimated standard errors may be biased. Asymptotic t-statistics appear in parentheses.

Co.	α_1	α_2	$\alpha_1 + \alpha_2$
1	0.134*	0.768*	0.902
	(2.07)	(7.28)	
2	0.030*	0.937*	0.967
	(2.37)	(34.59)	
3	0.069	0.710*	0.779
	(1.78)	(4.74)	
4	0.005	0.992*	0.997
	(0.53)	(35.99)	
5	0.216*	0.446*	0.662
	(4.34)	(3.70)	
6	0.035	0.901*	0.936
	(1.29)	(8.03)	
7	0.036*	0.953*	0.989
	(3.41)	(68.05)	
8	0.026	0.952*	0.978
	(1.11)	(19.95)	
9	0.248*	0.000	0.248
	(4.02)	(0.00)	
10	0.341*	0.000	0.341
	(3.71)	(0.00)	
11	0.041*	0.953*	0.994
	(3.16)	(55.83)	
12	0.011	0.795	0.806
	(0.53)	(1.51)	
13	0.063*	0.836*	0.899
	(2.57)	(9.54)	
14	0.285*	0.524*	0.809
	(3.87)	(4.22)	
15	0.167*	0.270	0.437
	(2.26)	(1.10)	
16	0.073*	0.723*	0.796
	(2.04)	(3.40)	
17	0.007	0.000	0.007
	(0.24)	(0.00)	
18	0.095	0.378	0.473
	(1.44)	(0.74)	
19	0.025	0.699	0.724
	(0.52)	(1.22)	
20	0.123*	0.692*	0.815
	(2.11)	(5.40)	
Mean	0.102	0.626	0.728
Median	0.066	0.716	0.807

* Statistically significant at 5% assuming that returns are conditionally normally distributed.

Table III

Maximum Likelihood Estimates of GARCH (1,1) Model with Volume

$\varepsilon_t|(V_t,\varepsilon_{t-1},\cdots) \sim N(0,h_t)$, $h_t = \alpha_0 + \alpha_1\varepsilon_{t-1}^2 + \alpha_2 h_{t-1} + \alpha_3 V_t$, where ε_t is daily return and V_t is daily volume. $S(K)$ is the Kiefer-Salmon (1983) statistic testing the null hypothesis of normality against the alternative of skewness (excess kurtosis). $S + K$ is the joint Kiefer-Salmon (1983) statistic for normality; the alternative hypothesis is skewness and/or excess kurtosis. $SW(p)$ is the p-value of the modified Shapiro-Wilk statistic. These goodness-of-fit statistics are constructed for $\varepsilon_t h_t^{-1/2}$, where $h_t = \alpha_0 + \alpha_3 V_t$ (i.e., α_1 and α_2 are restricted to be zero). Asymptotic t-statistics appear in parentheses. In those cases in which normality is rejected, standard errors may be biased.

Co.	α_1	α_2	α_3	$\alpha_1 + \alpha_2$	S	K	$S + K$	$SW(p)$
1	0.026	0.168	1.560*	0.194	0.04	0.57	0.61	0.59
	(0.32)	(0.95)	(3.40)					
2	0.000	0.108	0.908*	0.108	5.45	22.06	27.51	0.82
	(0.00)	(1.82)	(7.07)					
3	0.000	0.036	0.906*	0.036	7.12	16.79	23.91	0.46
	(0.00)	(0.52)	(8.59)					
4	0.000	0.000	0.650*	0.000	0.34	0.02	0.36	0.91
	(0.00)	(0.00)	(5.07)					
5	0.207*	0.000	0.350*	0.207	10.10	45.48	55.58	0.00
	(3.70)	(0.00)	(3.27)					
6	0.042	0.000	0.885*	0.042	0.03	0.10	0.13	0.10
	(0.76)	(0.00)	(4.58)					
7	0.082	0.000	0.227*	0.082	2.10	6.56	8.66	0.09
	(1.47)	(0.00)	(8.48)					
8	0.048	0.000	0.353*	0.048	0.25	0.01	0.26	0.33
	(0.74)	(0.00)	(5.25)					
9	0.022	0.000	0.755*	0.022	3.79	8.50	12.29	0.60
	(0.42)	(0.00)	(6.76)					
10	0.086	0.000	1.425*	0.086	1.98	1.66	3.64	0.29
	(1.26)	(0.00)	(5.37)					
11	0.108*	0.000	0.671*	0.108	0.01	10.58	10.59	0.50
	(2.06)	(0.00)	(7.81)					
12	0.065	0.000	0.552*	0.065	14.54	82.19	96.73	0.03
	(0.96)	(0.00)	(7.75)					
13	0.111*	0.000	0.921*	0.111	24.77	23.63	49.40	0.01
	(3.02)	(0.00)	(4.86)					
14	0.190*	0.000	1.004*	0.190	18.42	3.99	22.41	0.00
	(2.35)	(0.00)	(5.08)					
15	0.096	0.000	0.196*	0.096	0.03	7.88	7.91	0.98
	(1.09)	(0.00)	(3.64)					
16	0.000	0.000	0.151*	0.000	0.04	1.85	1.89	0.98
	(0.00)	(0.00)	(4.10)					
17	0.009	0.000	0.686*	0.009	6.04	18.35	24.39	0.01
	(0.26)	(0.00)	(4.51)					
18	0.032	0.000	0.510*	0.032	0.56	0.97	1.53	0.22
	(0.71)	(0.00)	(3.46)					
19	0.000	0.000	0.370*	0.000	0.44	0.04	0.48	0.49
	(0.00)	(0.00)	(4.08)					
20	0.025	0.000	0.349*	0.025	0.17	3.39	3.56	0.77
	(0.40)	(0.00)	(6.32)					
Mean	0.057	0.016	0.671	0.073	4.86	12.73	17.59	0.41
Median	0.037	0.000	0.668	0.057	1.27	5.28	8.29	0.40

* Statistically significant at 5% assuming conditional normality.

Our primary hypothesis is given strong support by the results from the unrestricted model: α_1 and α_2 generally become small and statistically insignificant when α_3 is unconstrained; ARCH effects remain for only four companies. However, for these stocks, as well as all the others, the persistence in volatility as measured by $(\alpha_1 + \alpha_2)$ is much smaller when α_3 is unconstrained than when α_3 is restricted to be zero. These results are highly suggestive that lagged squared residuals contribute little if any additional information about the variance of the stock return process after accounting for the rate of information flow, as measured by contemporaneous volume.[4]

III. Conclusions

This paper provides empirical support for the hypothesis that ARCH is a manifestation of the daily time dependence in the rate of information arrival to the market for individual stocks. Thus, this form of heteroskedasticity is an artifact of the arbitrary, albeit natural, choice of observation frequency. While this conclusion is strictly valid only for our sample of actively traded stocks, it is plausible to surmise that similar results would be found for other asset return series that can be explained by ARCH (e.g., foreign exchange rates), where in many instances more appropriate measures of information arrival time are not available. The results properly motivate the use of ARCH models to study the behavior of asset prices.

[4] To resolve the possible simultaneity bias, we considered lagged volume and fitted values from univariate autoregressions on volume as instruments for the mixing variable. These were found to be poor instruments for contemporaneous volume and therefore had little explanatory power in the variance equation. A detailed analysis of this issue is left for future research. It should be noted that, if volume is not exogenous, any study that regresses return volatility on volume (see Karpoff (1987)) is subject to this simultaneity bias.

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