DEEN DAYAL UPADHYAYA COLLEGE
UNIVERSITY OF DELHI

Practical File For The Paper COMPLEX ANALYSIS

In[ ]:=

B.Sc (H) Mathematics
Session-2023

In[ ]:=

Name - Nitin Ahlawat

Roll No. - 20HMT3224

Semester - VI th

Session - 2023

Paper - Complex Analysis

Submitted to - Dr. Sanjay Kumar Pant
Mr. Ravi Kumar Meena
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S.NO.	PRACTICAL	DATE
1.	Practical No. 01	03 Jan 23
2.	Practical No. 02	10 Jan 23
3.	Practical No. 03	17 Jan 23
4.	Practical No. 04	24 Jan 23
5 .	Practical No. 05	24 Jan 23
6.	Practical No. 06	24 Jan 23
7.	Practical No. 07	31 Jan 23
8.	Practical No. 08	07 Feb 23
9.	Practical No. 09	14 Feb 23
10.	Practical No. 10	21 Feb 23
11.	Practical No. 11	28 Feb 23
12.	Practical No. 12	14 Mar 23
13.	Practical No. 13	21 Mar 23
14.	Practical No. 14	28 Mar 23

PRACTICAL NO. - 01

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 03 Jan 2023

Question 1: Write the following in the form a+ib and find real,imaginary, argument,absolute,polar form and conjugate. Also give graphical representation.

(1) $z1 = 2 + i - \frac{1}{3-i}$

In[ ]:= `z1 = 2 + i - 1/(3 - i)`

$$z1 = 2 + i - \frac{1}{3 - i}$$

Out[ ]=

$$\frac{17}{10} + \frac{9\,i}{10}$$

In[ ]:= `Re[z1]`

Out[ ]=

$$\frac{17}{10}$$

In[ ]:= `Im[z1]`

Out[ ]=

$$\frac{9}{10}$$

In[ ]:= `Abs[z1]`

Out[ ]=

$$\sqrt{\frac{37}{10}}$$

In[ ]:= `Arg[z1]`

Out[ ]=

$$\text{ArcTan}\left[\frac{9}{17}\right]$$

In[ ]:= `r = Abs[z1]`

Out[ ]=

$$\sqrt{\frac{37}{10}}$$

In[ ]:= `θ = Arg[z1]`

Out[ ]=

$$\text{ArcTan}\left[\frac{9}{17}\right]$$

In[ ]:=
```
PolarForm = r * e^(i*θ)
```

Out[ ]=

$$\sqrt{\frac{37}{10}}\, e^{i\,\text{ArcTan}\left[\frac{9}{17}\right]}$$

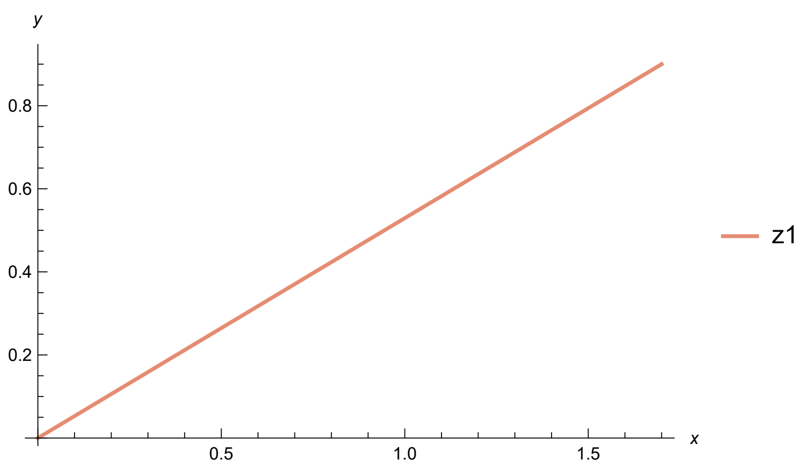
In[ ]:=
```
ComplexExpand[PolarForm]
```

Out[ ]=

$$\frac{17}{10}+\frac{9\,i}{10}$$

In[ ]:=
```
ListLinePlot[{{0, 0}, {Re[z1], Im[z1]}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
  PlotStyle → {Blend[{Green, Pink}, 0.9], Thick}, PlotLegends → {"z1"}]
```

Out[ ]=



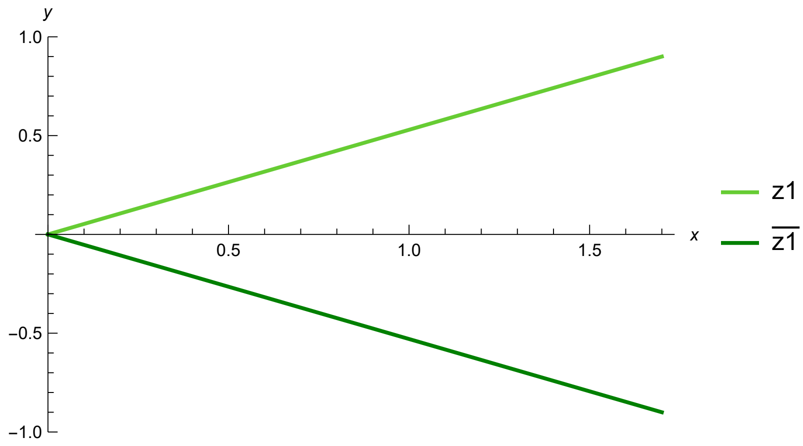
In[ ]:=
```
z1̄ = Conjugate[z1]
```

Out[ ]=

$$\frac{17}{10}-\frac{9\,i}{10}$$

In[ ]:=
```
ListLinePlot[{{{0, 0}, {Re[z1], Im[z1]}}, {{0, 0}, {Re[z̄1], Im[z̄1]}}},
 AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Green, Pink}, 0.4], Thick}, {Blend[{Green, Black}, 0.5], Thick}},
 PlotLegends → {"z1", "z̄1"}]
```

Out[ ]=



(2) $z2 = \frac{5i}{2+3i}$

In[ ]:= $z2 = \frac{5 i}{2 + 3 i}$

Out[ ]= $\frac{15}{13} + \frac{10 i}{13}$

In[ ]:= `Re[z2]`

Out[ ]= $\frac{15}{13}$

In[ ]:= `Im[z2]`

Out[ ]= $\frac{10}{13}$

In[ ]:= `Abs[z2]`

Out[ ]= $\frac{5}{\sqrt{13}}$

In[ ]:= `Arg[z2]`

Out[ ]= $\text{ArcTan}\left[\frac{2}{3}\right]$

In[ ]:=
```
r = Abs[z2]
```
Out[ ]=

$$\frac{5}{\sqrt{13}}$$

In[ ]:=
```
θ = Arg[z2]
```
Out[ ]=

$$\text{ArcTan}\left[\frac{2}{3}\right]$$

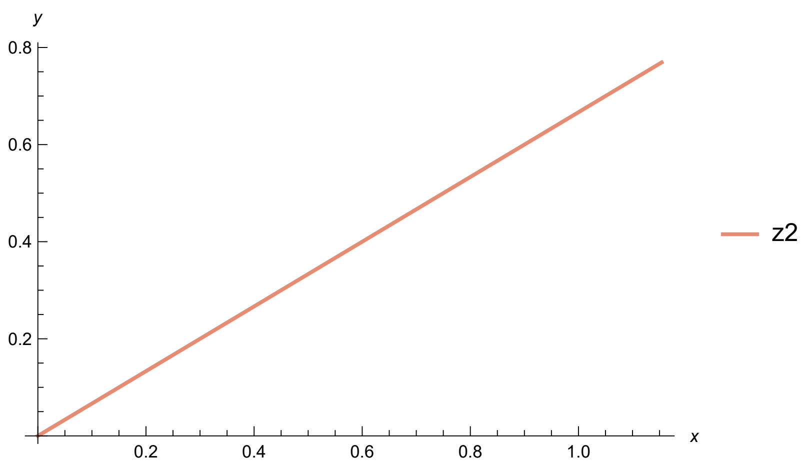
In[ ]:=
```
PolarForm = r * ⅇ^(ⅈ*θ)
```
Out[ ]=

$$\frac{5\, e^{i\, \text{ArcTan}\left[\frac{2}{3}\right]}}{\sqrt{13}}$$

In[ ]:=
```
ComplexExpand[PolarForm]
```
Out[ ]=

$$\frac{15}{13} + \frac{10\, i}{13}$$

In[ ]:=
```
ListLinePlot[{{0, 0}, {Re[z2], Im[z2]}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {Blend[{Green, Pink}, 0.9], Thick}, PlotLegends → {"z2"}]
```
Out[ ]=

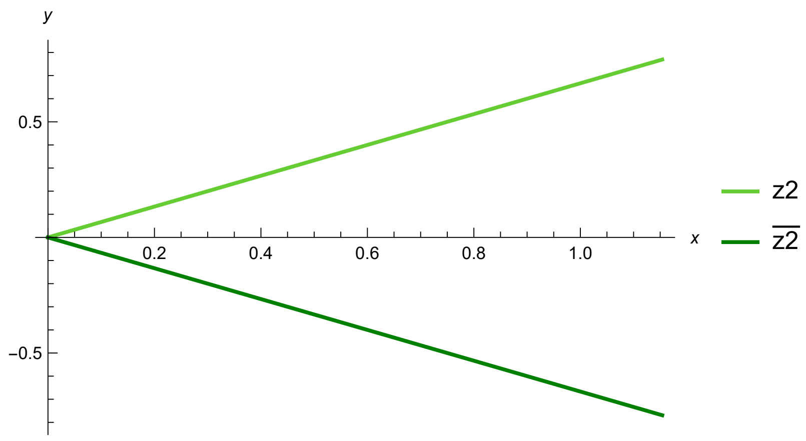


In[ ]:=
```
z2̄ = Conjugate[z2]
```
Out[ ]=

$$\frac{15}{13} - \frac{10\, i}{13}$$

In[ ]:=
```
ListLinePlot[{{{0, 0}, {Re[z2], Im[z2]}}, {{0, 0}, {Re[z̄2], Im[z̄2]}}},
 AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Green, Pink}, 0.4], Thick}, {Blend[{Green, Black}, 0.5], Thick}},
 PlotLegends → {"z2", "z̄2"}]
```

Out[ ]=



(3) $z3 = \frac{1+i}{2-5i}$

In[ ]:= $z3 = \frac{1+i}{2-5i}$

Out[ ]= $-\frac{3}{29} + \frac{7 i}{29}$

In[ ]:= `Re[z3]`

Out[ ]= $-\frac{3}{29}$

In[ ]:= `Im[z3]`

Out[ ]= $\frac{7}{29}$

In[ ]:= `Abs[z3]`

Out[ ]= $\sqrt{\frac{2}{29}}$

In[ ]:= `Arg[z3]`

Out[ ]= $\pi - \text{ArcTan}\left[\frac{7}{3}\right]$

In[ ]:= `r = Abs[z3]`

Out[ ]=

$$\sqrt{\frac{2}{29}}$$

In[ ]:= `θ = Arg[z3]`

Out[ ]=

$$\pi - \text{ArcTan}\left[\frac{7}{3}\right]$$

In[ ]:= `PolarForm = r * ⅇ^(ⅈ*θ)`

Out[ ]=

$$\sqrt{\frac{2}{29}}\, e^{i\left(\pi - \text{ArcTan}\left[\frac{7}{3}\right]\right)}$$

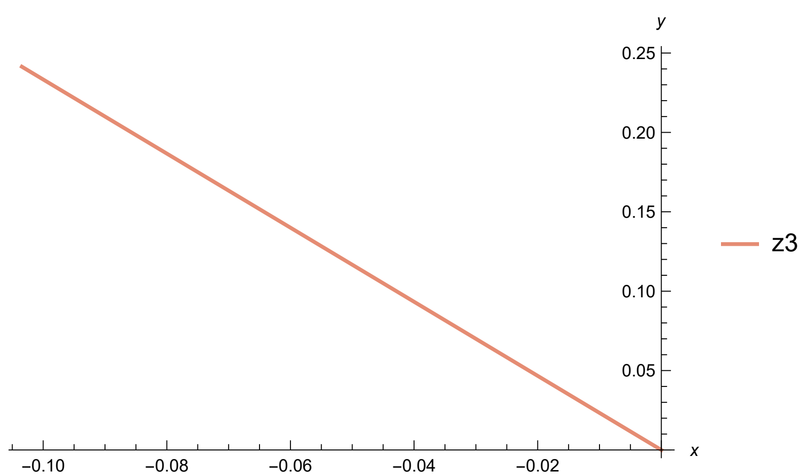
In[ ]:= `ComplexExpand[PolarForm]`

Out[ ]=

$$-\frac{3}{29} + \frac{7\,i}{29}$$

In[ ]:=
```
ListLinePlot[{{0, 0}, {Re[z3], Im[z3]}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {Blend[{Green, Pink}, 0.9], Thick}, PlotLegends → {"z3"}]
```

Out[ ]=



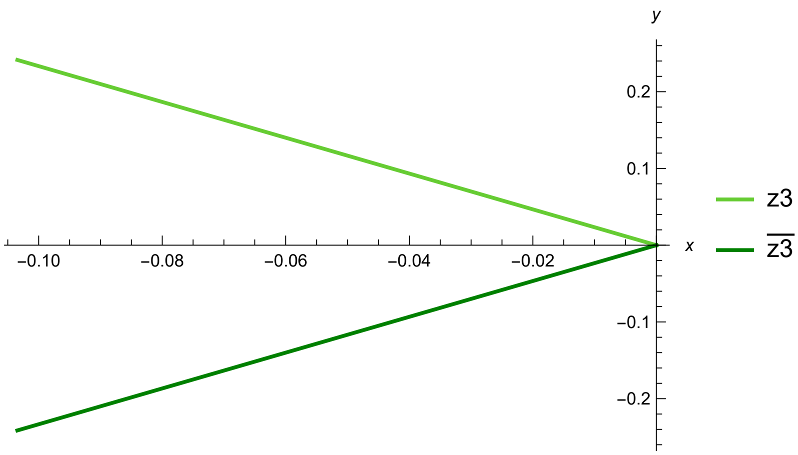
In[ ]:= `z3̅ = Conjugate[z3]`

Out[ ]=

$$-\frac{3}{29} - \frac{7\,i}{29}$$

In[ ]:=
```
ListLinePlot[{{{0, 0}, {Re[z3], Im[z3]}}, {{0, 0}, {Re[z̄3], Im[z̄3]}}},
 AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Green, Pink}, 0.4], Thick}, {Blend[{Green, Black}, 0.5], Thick}},
 PlotLegends → {"z3", "z̄3"}]
```

Out[ ]=



(4) z4 = $\frac{(1+3i)(2+i)}{3-i}$

In[ ]:= $z4 = \frac{(1 + 3 i)\ (2 + i)}{3 - i}$

Out[ ]= $-1 + 2 i$

In[ ]:= `Re[z4]`

Out[ ]= -1

In[ ]:= `Im[z4]`

Out[ ]= 2

In[ ]:= `Abs[z4]`

Out[ ]= $\sqrt{5}$

In[ ]:= `Arg[z4]`

Out[ ]= π − ArcTan[2]

In[ ]:= `r = Abs[z4]`

Out[ ]= $\sqrt{5}$

In[ ]:=
```
θ = Arg[z4]
```
Out[ ]=

$\pi - \text{ArcTan}[2]$

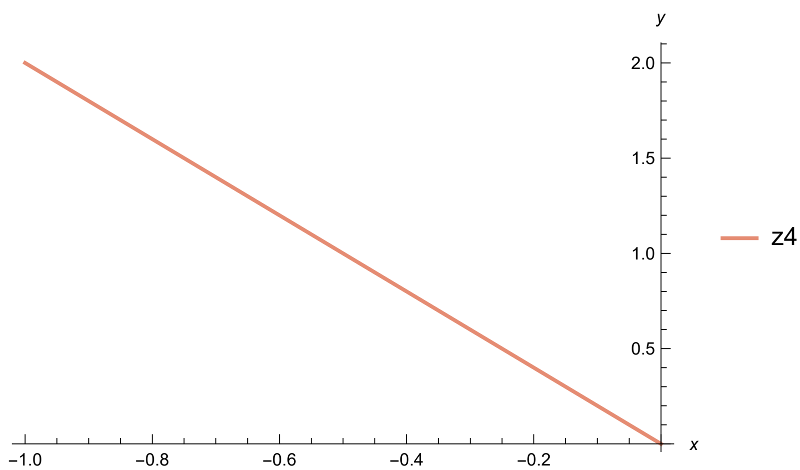
In[ ]:=
```
PolarForm = r * ⅇ^(ⅈ*θ)
```
Out[ ]=

$\sqrt{5}\, e^{i\,(\pi - \text{ArcTan}[2])}$

In[ ]:=
```
ComplexExpand[PolarForm]
```
Out[ ]=

$-1 + 2\,i$

In[ ]:=
```
ListLinePlot[{{0, 0}, {Re[z4], Im[z4]}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {Blend[{Green, Pink}, 0.9], Thick}, PlotLegends → {"z4"}]
```
Out[ ]=

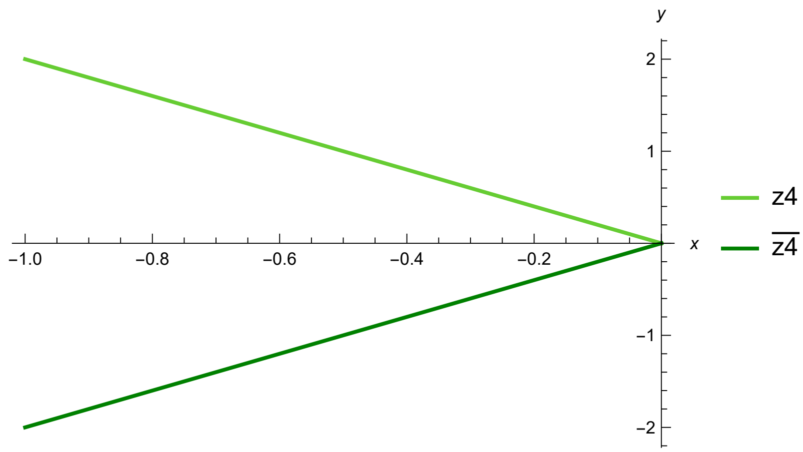


In[ ]:=
```
z4̄ = Conjugate[z4]
```
Out[ ]=

$-1 - 2\,i$

In[ ]:=
```
ListLinePlot[{{{0, 0}, {Re[z4], Im[z4]}}, {{0, 0}, {Re[z̅4], Im[z̅4]}}},
 AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Green, Pink}, 0.4], Thick}, {Blend[{Green, Black}, 0.5], Thick}},
 PlotLegends → {"z4", "z̅4"}]
```

Out[ ]=



(5) $z5 = \frac{(1.4+2 i)(0.3+1.2 i)}{6.5-3.9 i}$

In[ ]:= $z5 = \frac{(1.4 + 2\,i)\,(0.3 + 1.2\,i)}{6.5 - 3.9\,i}$

Out[ ]=

-0.378733 + 0.123529 i

In[ ]:= `Re[z5]`

Out[ ]=

-0.378733

In[ ]:= `Im[z5]`

Out[ ]=

0.123529

In[ ]:= `Abs[z5]`

Out[ ]=

0.398369

In[ ]:= `Arg[z5]`

Out[ ]=

2.82631

In[ ]:= `r = Abs[z5]`

Out[ ]=

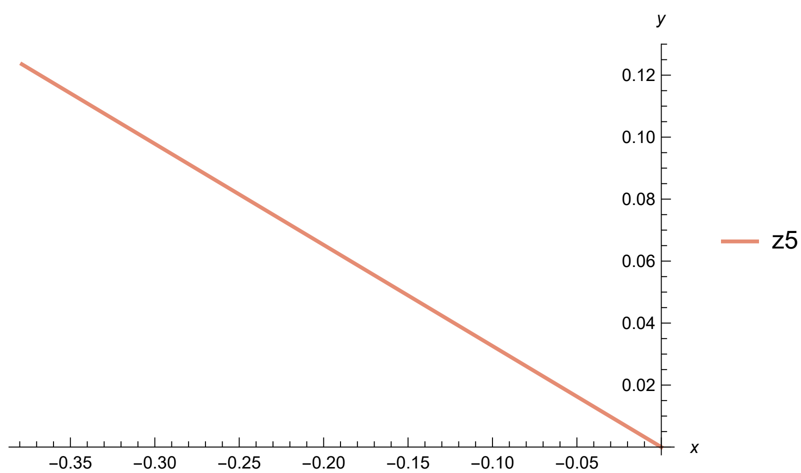
0.398369

In[ ]:=
```
θ = Arg[z5]
```
Out[ ]=
```
2.82631
```

In[ ]:=
```
PolarForm = r * ⅇ^(ⅈ*θ)
```
Out[ ]=
```
-0.378733 + 0.123529 ⅈ
```

In[ ]:=
```
ComplexExpand[PolarForm]
```
Out[ ]=
```
-0.378733 + 0.123529 ⅈ
```

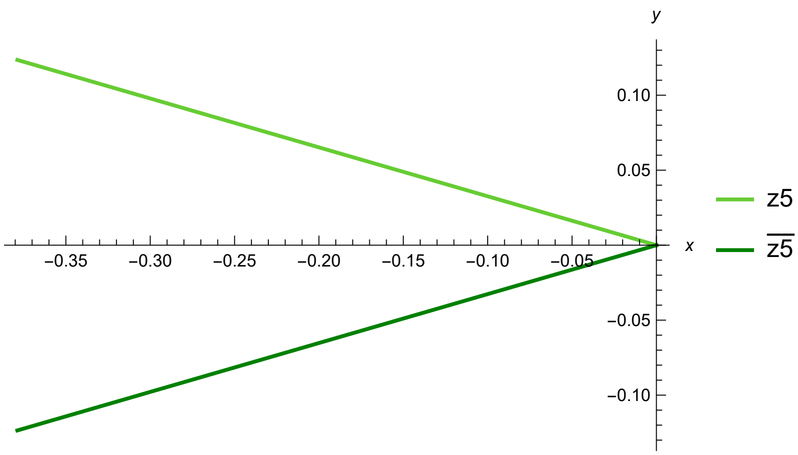
In[ ]:=
```
ListLinePlot[{{0, 0}, {Re[z5], Im[z5]}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {Blend[{Green, Pink}, 0.9], Thick}, PlotLegends → {"z5"}]
```
Out[ ]=



In[ ]:=
```
z5‾ = Conjugate[z5]
```
Out[ ]=
```
-0.378733 - 0.123529 ⅈ
```

In[ ]:=
```
ListLinePlot[{{{0, 0}, {Re[z5], Im[z5]}}, {{0, 0}, {Re[z̅5̅], Im[z̅5̅]}}},
 AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Green, Pink}, 0.4], Thick}, {Blend[{Green, Black}, 0.5], Thick}},
 PlotLegends → {"z5", "z̅5̅"}]
```

Out[ ]=



Question 2

In[ ]:=
```
ClearAll;
```

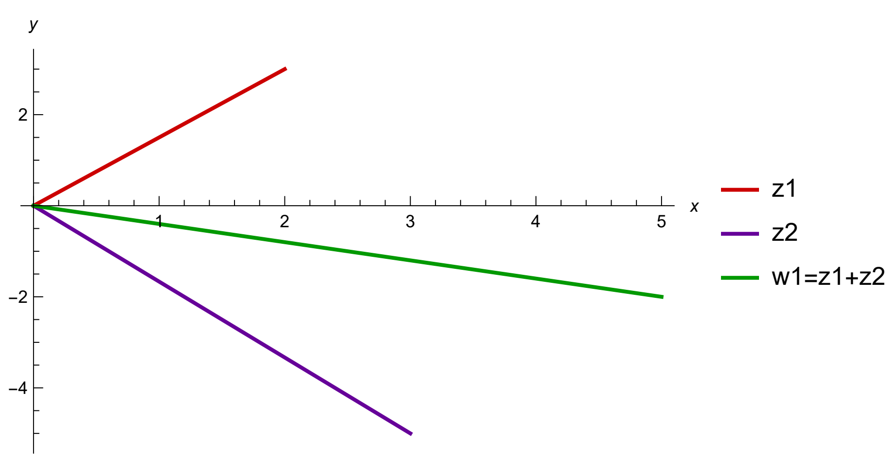
In[ ]:=
```
z1 = 2 + 3 i;
z2 = 3 - 5 i;
```

In[ ]:=
```
w1 = z1 + z2
ListLinePlot[{{{0, 0}, {Re[z1], Im[z1]}}, {{0, 0}, {Re[z2], Im[z2]}},
   {{0, 0}, {Re[w1], Im[w1]}}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Red, Black}, 0.2], Thick}, {Blend[{Purple, Blue}, 0.2], Thick},
    {Blend[{Green, Black}, 0.4], Thick}}, PlotLegends → {"z1", "z2", "w1=z1+z2"}]
```

Out[ ]=

5 − 2 i

Out[ ]=

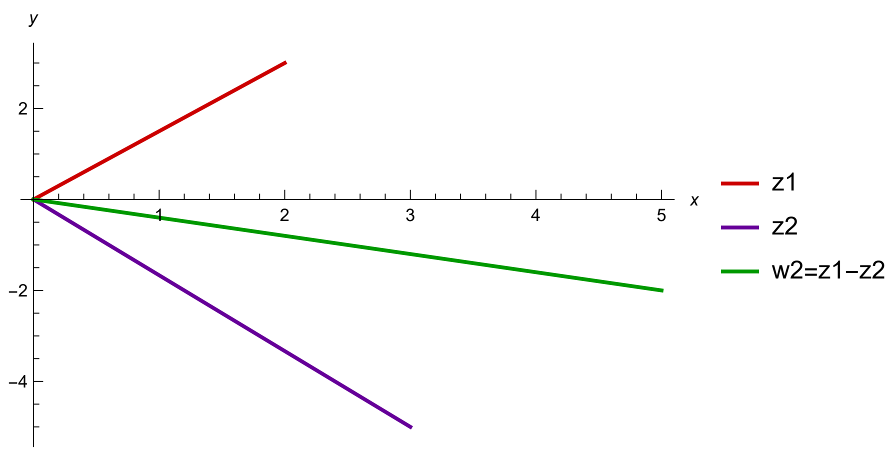


In[ ]:=
```
w2 = z1 - z2
ListLinePlot[{{{0, 0}, {Re[z1], Im[z1]}}, {{0, 0}, {Re[z2], Im[z2]}},
  {{0, 0}, {Re[w1], Im[w1]}}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Red, Black}, 0.2], Thick}, {Blend[{Purple, Blue}, 0.2], Thick},
   {Blend[{Green, Black}, 0.4], Thick}}, PlotLegends → {"z1", "z2", "w2=z1-z2"}]
```

Out[ ]=

$-1+8\,i$

Out[ ]=

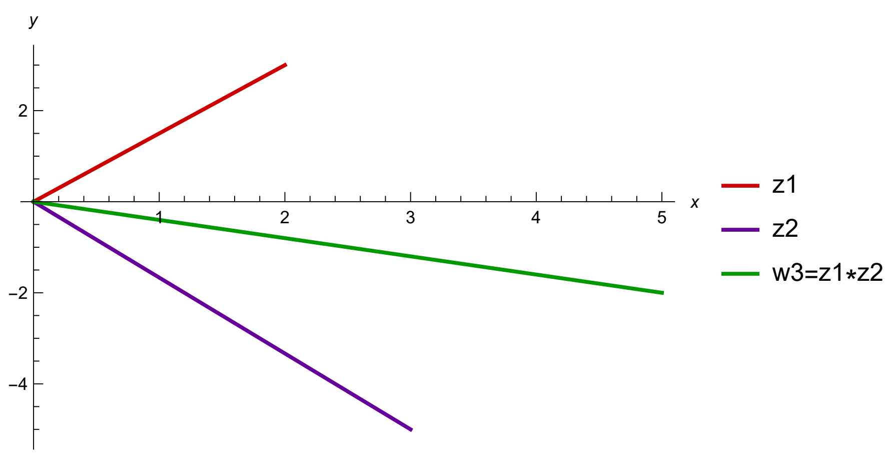


In[ ]:=
```
w3 = z1 * z2
ListLinePlot[{{{0, 0}, {Re[z1], Im[z1]}}, {{0, 0}, {Re[z2], Im[z2]}},
  {{0, 0}, {Re[w1], Im[w1]}}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
 PlotStyle → {{Blend[{Red, Black}, 0.2], Thick}, {Blend[{Purple, Blue}, 0.2], Thick},
   {Blend[{Green, Black}, 0.4], Thick}}, PlotLegends → {"z1", "z2", "w3=z1*z2"}]
```

Out[ ]=

$21-i$

Out[ ]=

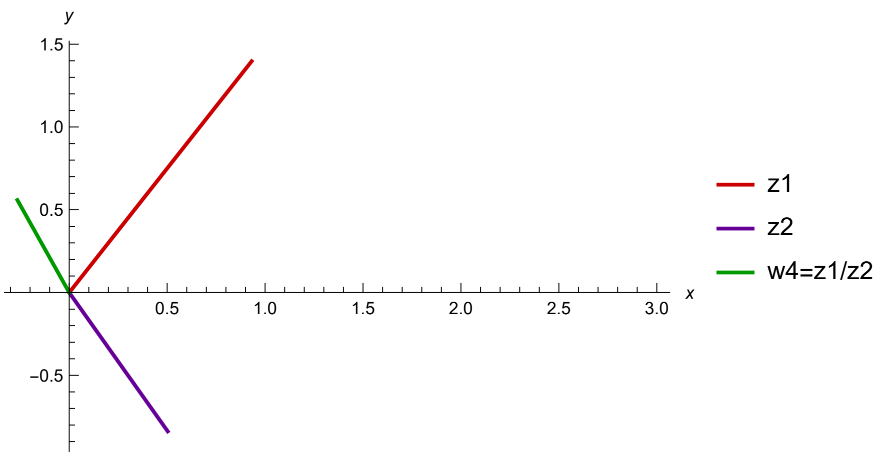


In[ ]:=
```
w4 = z1/z2
ListLinePlot[{{{0, 0}, {Re[z1], Im[z1]}}, {{0, 0}, {Re[z2], Im[z2]}},
   {{0, 0}, {Re[w4], Im[w4]}}}, AxesLabel → {x, y}, AxesOrigin → {0, 0},
  PlotStyle → {{Blend[{Red, Black}, 0.2], Thick}, {Blend[{Purple, Blue}, 0.2], Thick},
    {Blend[{Green, Black}, 0.4], Thick}}, PlotLegends → {"z1", "z2", "w4=z1/z2"}]
```

Out[ ]=

$$-\frac{9}{34}+\frac{19\,i}{34}$$

Out[ ]=



CONCLUSION- Addition and Subtraction of complex number is based on the parallelogram law of vector addition. In multiplication of complex numbers, their arguments get added

Question3: Make a Geometric plot to show nth roots unity are equally spaced points that lie on the unit circle C1(0)= {z: z =1} and form the vertices of a regular polygon with n sides, for n=4,5,6,7,8.

In[ ]:=
```
ClearAll;
```

In[ ]:=
```
Sol1 = Solve[z^4 == 1]
```

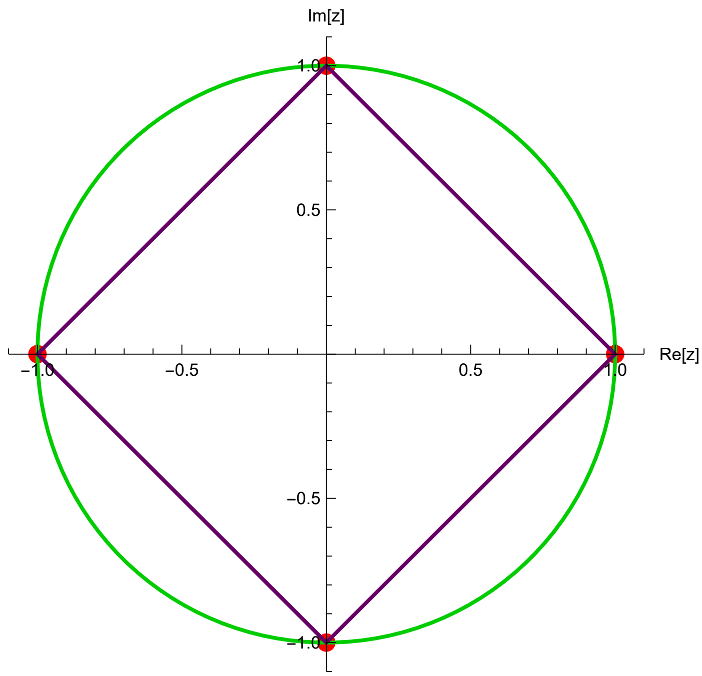
Out[ ]=
```
{{z → -1}, {z → -i}, {z → i}, {z → 1}}
```

In[ ]:=
```
Sol2 = ComplexExpand[z /. Sol1]
```

Out[ ]=
```
{-1, -i, i, 1}
```

In[ ]:=
```
Show[ComplexListPlot[{Sol2}, PlotStyle → {Red, Thick}, AxesLabel → {"Re[z]", "Im[z]"},
  PlotMarkers → {Automatic, 10}, PlotRange → {{-1.1, 1.1}, {-1.1, 1.1}}],
 Graphics[{Thick, Blend[{Purple, Black}, 0.2], {Blend[{Green, Black}, 0.2],
   Circle[{0, 0}, 1]}, Line[{{0, 1}, {-1, 0}, {0, -1}, {1, 0}, {0, 1}}]}]]
```

Out[ ]=



PRACTICAL NO. - 02

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 10-Jan-2023

Find all the solutions of the equation $z^3 = 8i$ and represent these geometrically.

In[ ]:=
```
ClearAll
```
Out[ ]=
```
ClearAll
```

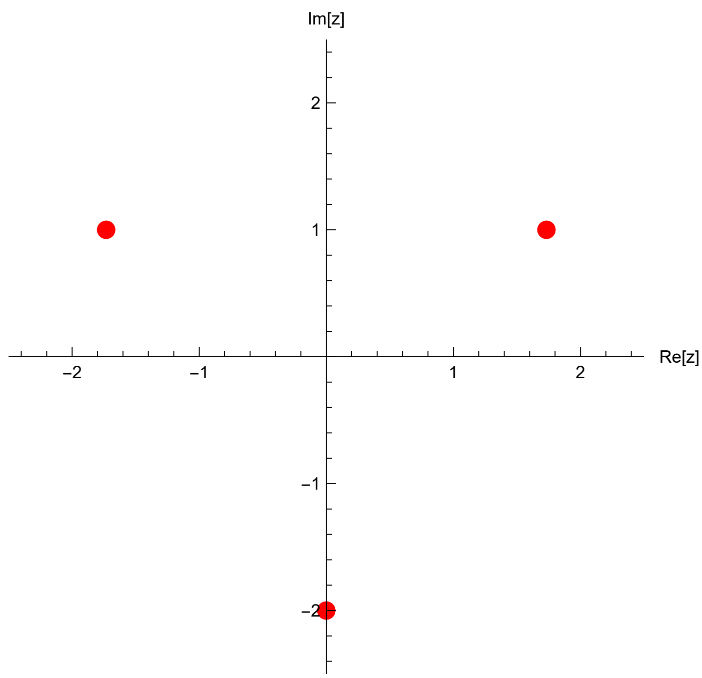
In[ ]:=
```
sol = Solve[z^3 - 8 i == 0]
```
Out[ ]=

$\{\{z \to -2 i\}, \{z \to 2 (-1)^{1/6}\}, \{z \to 2 (-1)^{5/6}\}\}$

In[ ]:=
```
pts = ComplexExpand[z /. sol]
```
Out[ ]=

$\{-2 i, i + \sqrt{3}, i - \sqrt{3}\}$

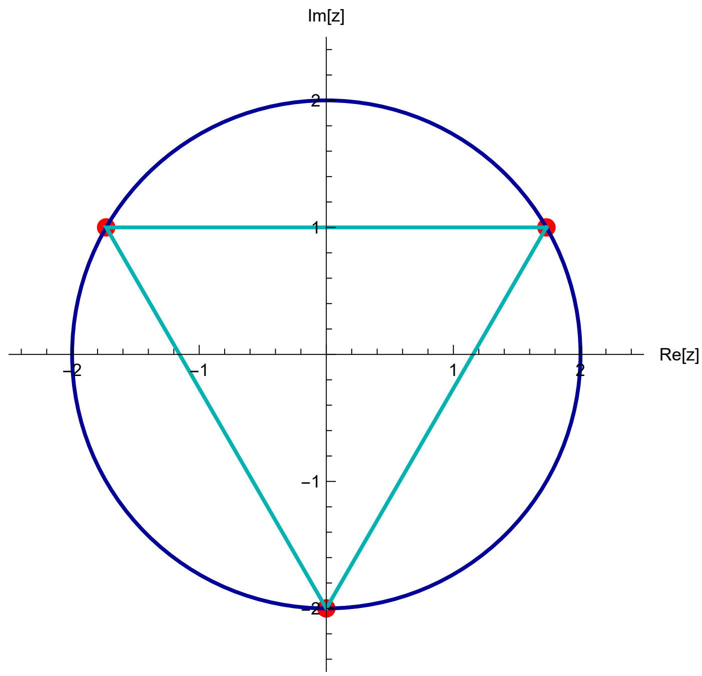
In[ ]:=
```
ComplexListPlot[{pts}, PlotStyle → {Red, Thick}, AxesLabel → {"Re[z]", "Im[z]"},
 PlotMarkers → {Automatic, 10}, PlotRange → {{-2.5, 2.5}, {-2.5, 2.5}}]
```
Out[ ]=



In[ ]:=

```
Show[ComplexListPlot[{pts}, PlotStyle → {Red, Thick}, AxesLabel → {"Re[z]", "Im[z]"},
  PlotMarkers → {Automatic, 10}, PlotRange → {{-2.5, 2.5}, {-2.5, 2.5}}],
 Graphics[{Thick, Blend[{Cyan, Black}, 0.3], {Blend[{Blue, Black}, 0.4], Circle[{0, 0}, 2]},
   Line[{{Sqrt[3], 1}, {-Sqrt[3], 1}, {0, -2}, {Sqrt[3], 1}}]}]]
```

Out[ ]=



PRACTICAL NO. - 03

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 17 Jan 2023

Write parametric equations and make a parametric plot for an ellipse centered at the origin with horizontal major axis of 4 units and vertical axis of 2 units. Show the effect of rotation of this ellipse by an angle of 30 degree and shifting of the centre from (0,0) to (2,1) by making a major plot.

Given, length of the horizontal major axis = 4 units,
length of the vertical minor axis = 2 units,
Centre is (0, 0)

Therefore, equation of the ellipse is given by $\frac{x^2}{4} + \frac{y^2}{2} = 1$.

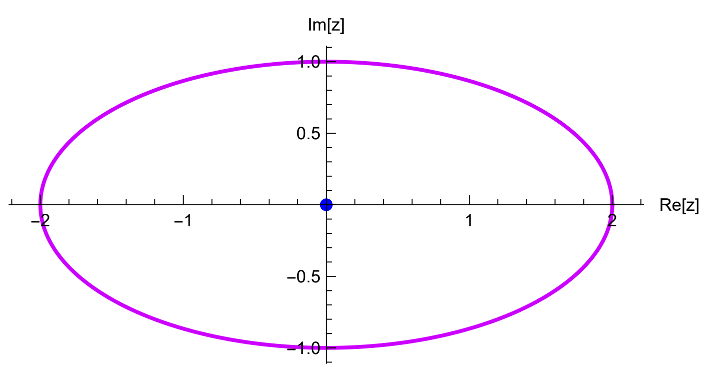
The corresponding parametric equation of the ellipse is given by
$x(t) = 2\cos(t),\ \ y(t) = \sin(t),\ \ 0 < t \leq 2\pi$.

Then, the corresponding equation of ellipse in complex domain is given by
$D = \{z(t) \mid z(t) = 2\cos(t) + i\sin(t), 0 < t < 2\pi\}$.

Graph of the given ellipse in complex plain is shown below :

In[ ]:=
```
Show[ParametricPlot[ComplexExpand[ReIm[2 * Cos[t] + i * Sin[t]]],
   {t, 0, 2 π}, AxesOrigin → {0, 0}, AxesLabel → {"Re[z]", "Im[z]"},
   PlotStyle → {Blend[{Magenta, Blue}, 0.2], Thick}],
 Graphics[{Blue, PointSize[0.02], Point[{0, 0}]}]]
```

Out[ ]=



Now we need to show the effect of rotation of this ellipse by
an angle of 30 degree .

So, $z = 2\cos t + i\sin t$ under the map f is given by

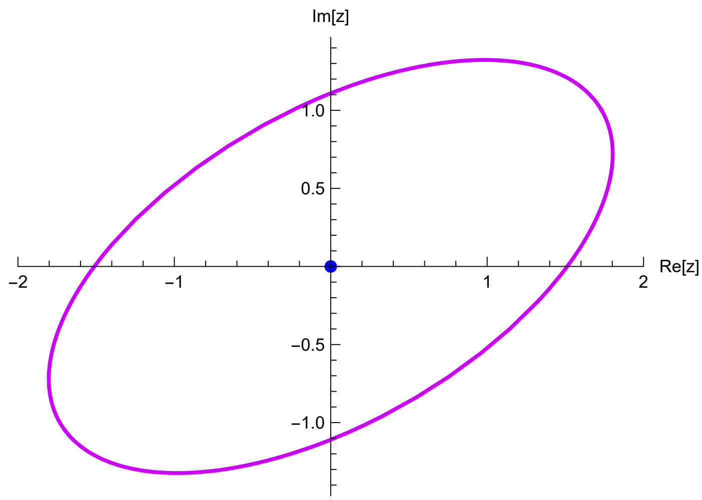
$f(z) = e^{\frac{i\pi}{6}} z$

$F(z(t)) = e^{\frac{i\pi}{6}} (2\cos t + i\sin t) = \left(\frac{\sqrt{3}}{2} + \frac{i}{2}\right)(2\cos t + i\sin t), 0 < t \leq 2\pi$.

Thus, the mapping f(z(t)) rotates the ellipse for each non zero point z through an angle of 30 ° about the origin in the counterclockwise direction . Graph of the rotated ellipse in complain plain is shown below :

In[ ]:=
```
Show[ParametricPlot[ComplexExpand[ReIm[ⅇ^(ⅈ*π/6) * (2 * Cos[t] + ⅈ * Sin[t])]],
  {t, 0, 2 π}, AxesOrigin → {0, 0}, AxesLabel → {"Re[z]", "Im[z]"},
  PlotStyle → {Blend[{Magenta, Blue}, 0.2], Thick}],
 Graphics[{Blue, PointSize[0.02], Point[{0, 0}]}]]
```

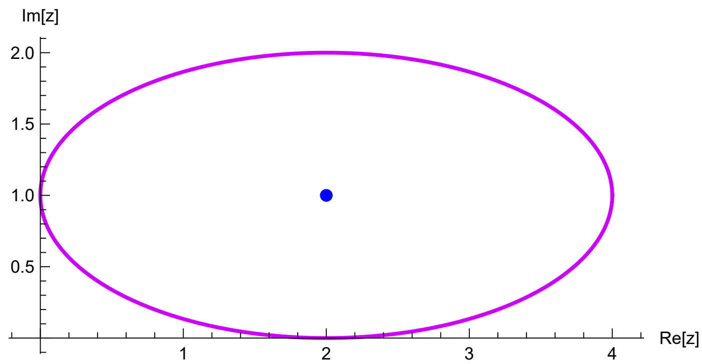
Out[ ]=



Now, we need to show the effect of shifting of the centre of
the given ellipse from $(0, 0)$ to $(2, 1)$, i.e., under the translation map
by making a parametric plot .
The corresponding mapping will be given by
$f(z) = z + 2 + i = (x + 2) + i(y + 1)$
$f(z(t)) = (2\cos t + 2) + i(\sin t + 1), 0 < t < 2\pi$.
Thus, the mapping $f(z(t))$ translates each
point z two units to the right and one unit to the upward . Graph of the translated ellipse in complex plain is shown below :

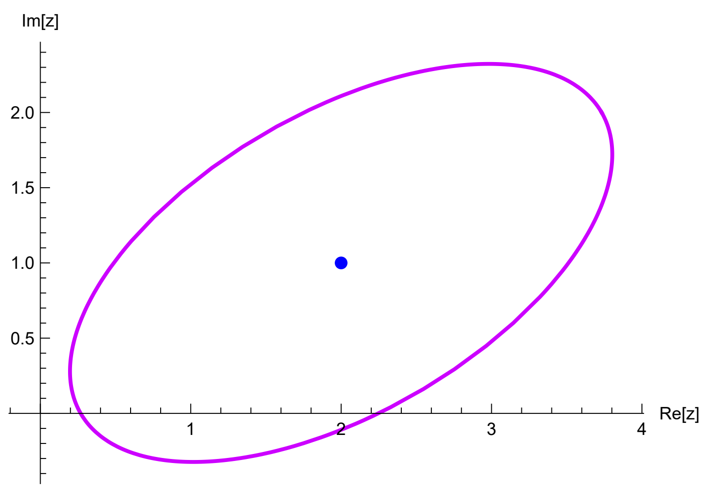
In[ ]:=
```
Show[ParametricPlot[ComplexExpand[ReIm[2 * Cos[t] + i * Sin[t] + 2 + i]],
   {t, 0, 2 π}, AxesOrigin → {0, 0}, AxesLabel → {"Re[z]", "Im[z]"},
   PlotStyle → {Blend[{Magenta, Blue}, 0.2], Thick}],
  Graphics[{Blue, PointSize[0.02], Point[{2, 1}]}]]
```

Out[ ]=



In[ ]:=
```
Show[ParametricPlot[ComplexExpand[ReIm[e^(i*π/6) * (2 * Cos[t] + i * Sin[t]) + (2 + i)]],
   {t, 0, 2 π}, AxesOrigin → {0, 0}, AxesLabel → {"Re[z]", "Im[z]"},
   PlotStyle → {Blend[{Magenta, Blue}, 0.2], Thick}],
  Graphics[{Blue, PointSize[0.02], Point[{2, 1}]}]]
```

Out[ ]=



PRACTICAL NO. - 04

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 24 Jan 2023

Show that the image of the open disk $D_1(-1-i)=\{z:|z+1+i|<1\}$ under the linear transformation $w=f(z)=(3-4i)z+6+2i$ is the open disk: $D_5(-1-3i)=\{w:|w+1-3i|<5\}$

In[ ]:=
```
ClearAll[z1, z, z2, w1];
```

In[ ]:=
```
z = x + ⅈy
```

Out[ ]=

x + ⅈy

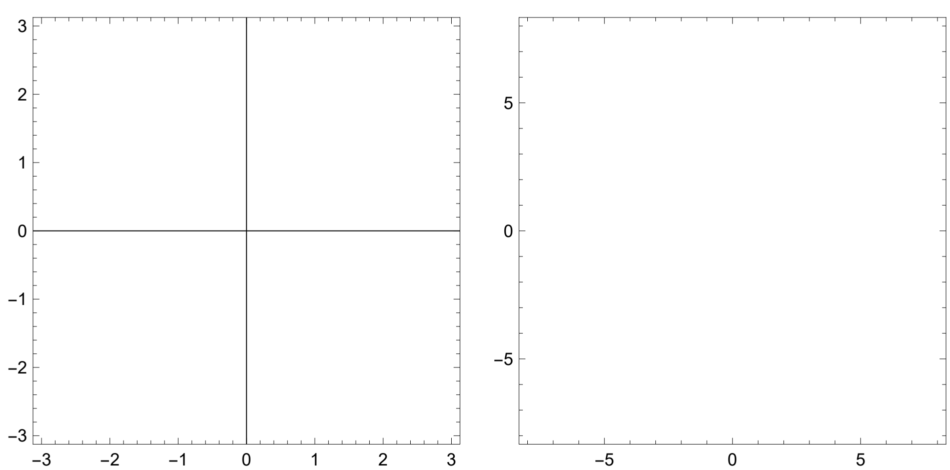
In[ ]:=
```
z2 = Solve[w1 == (3 - 4 * ⅈ) * z1 + 6 + 2 * ⅈ, z1]
```

Out[ ]=

$$\left\{\left\{z1 \to \frac{1}{25}\,(-10+3\,w1+2\,i\,(-15+2\,w1))\right\}\right\}$$

In[ ]:=
```
A1 = RegionPlot[Abs[z + 1 + ⅈ] < 1,
    {x, -3, 3}, {y, -3, 3}, BoundaryStyle → Dashed, Axes → True];
A2 = RegionPlot[Abs[(z - 6 - 2 * ⅈ) / (3 - 4 * ⅈ) + 1 + ⅈ] < 1, {x, -8, 8}, {y, -8, 8}];
GraphicsRow[{A1, A2}]
```

Out[ ]=



PRACTICAL NO. - 05

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 24 Jan 2023

Show that the image of the right half plane Re[z]=x>1 under the linear transformation w=(-1+𝕚)z-2+3𝕚 is the half plane v>u+7, where u=Re[w] etc. Plot the map.

In[ ]:=
```
ClearAll[z, z1, z2, w1, a1, a2];
```

In[ ]:=
```
z = x + i * y
```

Out[ ]=

$x + i\, y$

In[ ]:=
```
w = ComplexExpand[(-1 + i) * z + 3 * i - 2]
```

Out[ ]=

$-2 - x + i\, (3 + x - y) - y$

In[ ]:=
```
x1 = ComplexExpand[Re[w]]
```

Out[ ]=

$-2 - x - y$

In[ ]:=
```
y1 = ComplexExpand[Im[w]]
```

Out[ ]=

$3 + x - y$

In[ ]:=
```
Solve[x2 == -2 - x - y && y2 == 3 + x - y, {x, y}]
```

Out[ ]=

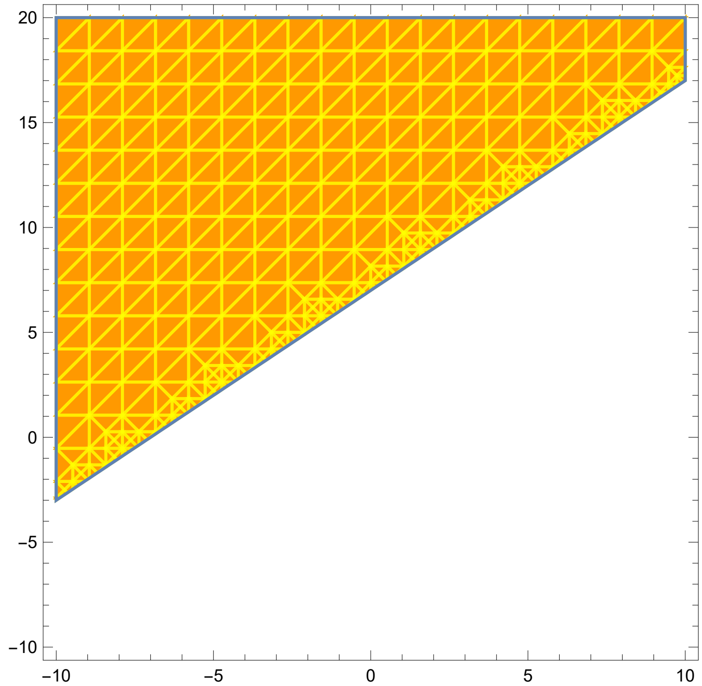
$\left\{\left\{x \to \frac{1}{2}\,(-5 - x2 + y2),\ y \to \frac{1}{2}\,(1 - x2 - y2)\right\}\right\}$

In[ ]:=
```
cont1 = RegionPlot[(-5 - x + y) / 2 > 1, {x, -10, 10}, {y, -10, 20}, PlotStyle → Red];
```

In[ ]:=
```
cont2 = RegionPlot[y > x + 7, {x, -10, 10}, {y, -10, 20}, PlotStyle → {Yellow, Opacity[0.6]}];
```

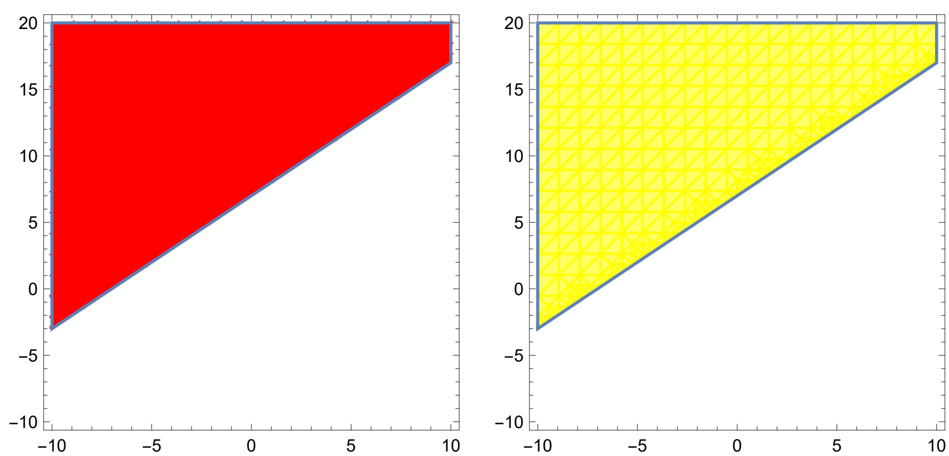
In[ ]:= `Show[cont1, cont2]`

Out[ ]=



In[ ]:= `GraphicsRow[{cont1, cont2}]`

Out[ ]=



PRACTICAL NO. - 06

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 24 Jan 2023

Show that the image of the right half plane $A = \left\{z : \mathrm{Re}[z] \geq \frac{1}{2}\right\}$ under the mapping $w = f(z) = \frac{1}{z}$ is the closed disk $\overline{D_1(1)}$ ={w:|w-1|≤1} in the w-plane

In[ ]:=
```
ClearAll[z, z1, z2, w1, a1, a2];
```

In[ ]:=
```
z = x + i * y
```

Out[ ]=

$x + i\, y$

In[ ]:=
```
w = ComplexExpand[1 / z]
```

Out[ ]=

$$\frac{x}{x^2 + y^2} - \frac{i\, y}{x^2 + y^2}$$

In[ ]:=

$$x1 = \frac{x}{x^2 + y^2};$$

$$y1 = -\frac{y}{x^2 + y^2};$$

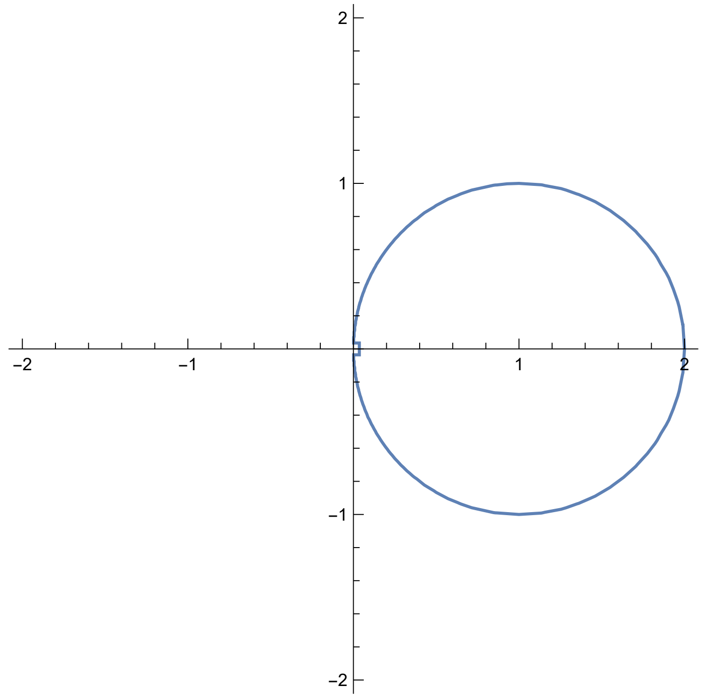
```
Solve[x2 - x1 == 0 && y2 - y1 == 0, {x, y}]
```

Out[ ]=

$$\left\{\left\{x \to \frac{x2}{x2^2 + y2^2}, y \to -\frac{y2}{x2^2 + y2^2}\right\}\right\}$$

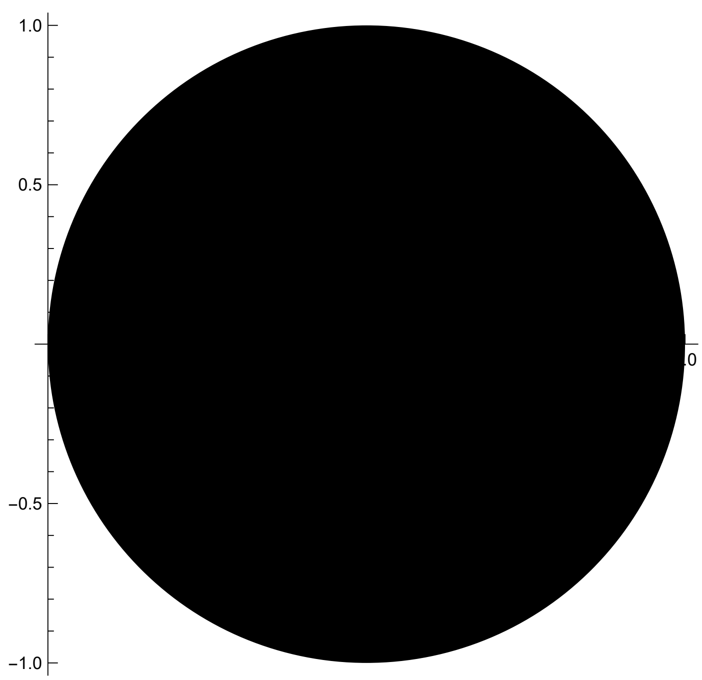
In[ ]:=
```
cont1 = ContourPlot[x1 == 1 / 2, {x, -2, 2}, {y, -2, 2}, Frame → False, Axes → True]
```

Out[ ]=



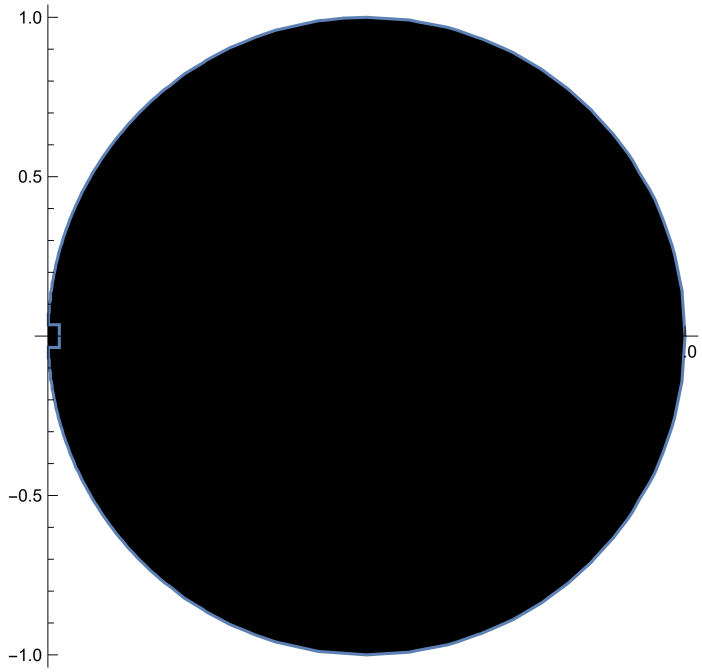
In[ ]:=
```
cont2 = Graphics[Disk[{1, 0}], Frame → False, Axes → True, PlotRange → Automatic]
```

Out[ ]=



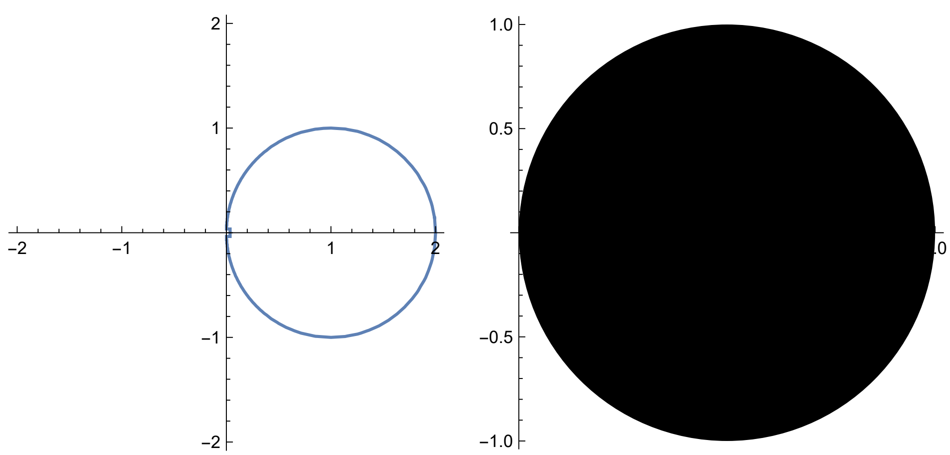
In[ ]:= Show[cont2, cont1]

Out[ ]=



In[ ]:= GraphicsRow[{cont1, cont2}]

Out[ ]=



PRACTICAL NO. - 07

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 31 Jan 2023

Make a plot of the vertical lines x=a, for $a = -1, -\frac{1}{2}, \frac{1}{2}, 1$. Find the plot of this grid under the mapping $w = f(z) = \frac{1}{z}$.

In[ ]:=
```
ClearAll;
```

In[ ]:=
```
z = x + ⅈ * y
```

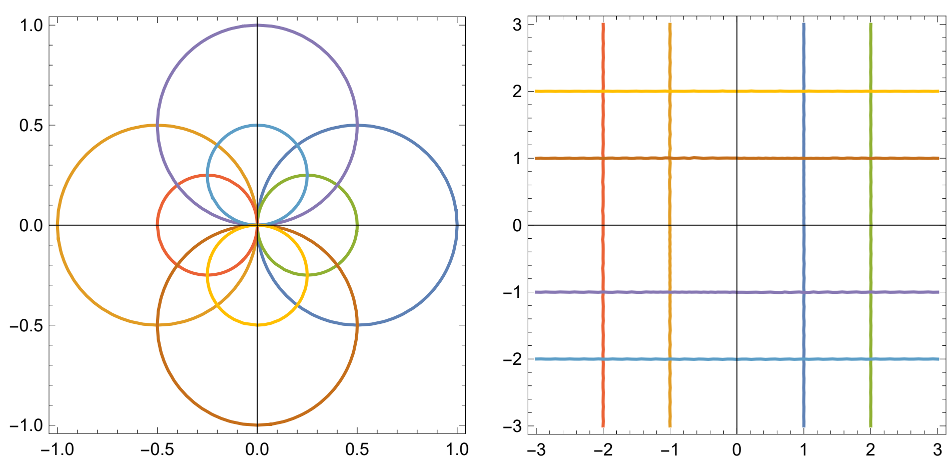
Out[ ]=
```
x + ⅈ y
```

In[ ]:=
```
a1 = ContourPlot[
   {Abs[z - 1 / 2] == 1 / 2, Abs[z + 1 / 2] == 1 / 2, Abs[z - 1 / 4] == 1 / 4, Abs[z + 1 / 4] == 1 / 4,
    Abs[z - I / 2] == 1 / 2, Abs[z + I / 2] == 1 / 2, Abs[z - I / 4] == 1 / 4, Abs[z + I / 4] == 1 / 4
   }, {x, -1, 1}, {y, -1, 1}, Axes → True];
```

In[ ]:=
```
a2 = ContourPlot[{Abs[(1 / z) - 1 / 2] == 1 / 2,
    Abs[(1 / z) + 1 / 2] == 1 / 2, Abs[(1 / z) - 1 / 4] == 1 / 4, Abs[(1 / z) + 1 / 4] == 1 / 4,
    Abs[(1 / z) - I / 2] == 1 / 2, Abs[(1 / z) + I / 2] == 1 / 2, Abs[(1 / z) - I / 4] == 1 / 4,
    Abs[(1 / z) + I / 4] == 1 / 4}, {x, -3, 3}, {y, -3, 3}, Axes → True];
```

In[ ]:=
```
GraphicsRow[{a1, a2}]
```

Out[ ]=



PRACTICAL NO. - 08

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 07 Feb 2023

POLYGON PATH

Find a parametrization of the polygonal path c = c_1+ c_2 + c_3 from -1+i to 3-i, where c_1 is the line from -1+i to -1, c_2 is the line from -1 to 1+i and c_3 is the line from 1+i to 3-i. Make a plot of this path .

Here, the parametrization of c_1 (which is a line passing from -1+i to -1) is given by

c_1(t) : z (t) = (-1+i)*(1-t) + (-1)*t,   0≤ t≤1
= -1+i+t-i*t-t
= -1+i*(1-t),   0≤t≤1

Now the parametrization of c_2 (which is a line passing from -1 to 1+i) is given by

c_2(t) : z (t) = (-1)*(1-t)+(1+i)*t,   0≤t≤1
= t-1+t+i*t
= (2*t-1)+i*t,   0≤t≤1

Now the parametrization of c3 (which is a line passing from 1+i to 3-i) is given by

c_3(t) : z (t) = (1+i)*(1-t)+(3-i)*t,   0≤t≤1
= 1-t+i-i*t+3*t-i*t
= (1+2*t)+i*(1-2*t),   0≤t≤1

The required parametrization of the polygon path c = c_1+c_2+c_3 is given by c (t) = c_1(t)+c_2(t)+c_3(t)
where,

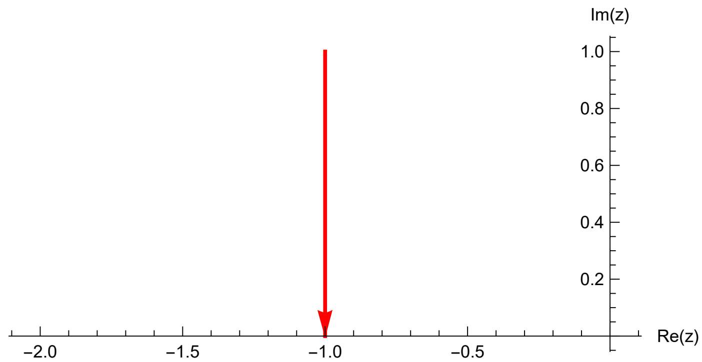
c_1(t) : z(t) = -1 +i(1-t),   0≤t≤1
c_2(t) : z(t) = (2*t-1)+i*t,   0≤t≤1
c_3(t) : z(t) = (1+2*t)+i*(1-2*t),   0≤t≤1

Thus, c (t) = c_1(t)+c_2(t)+c_3(t)
= -1+i*(1-t)+(2*t-1)+i*t+(1+2*t)+i*(1-2*t)
= (4*t-1)+i*(2-2*t),   0≤t≤1

In[ ]:=
```
c1[t_] = ComplexExpand[(-1 + I) * (1 - t) + (-1) * t]
```
Out[ ]=
```
-1 + i (1 - t)
```

In[ ]:=
```
P1 = Show[ParametricPlot[ReIm[c1[t]], {t, 0, 1}, PlotStyle → {Red, Thick},
    AxesLabel → {"Re(z)", "Im(z)"}], Graphics[{Red, Arrow[{{-1, 0.75}, {-1, 0}}]}]]
```

Out[ ]=



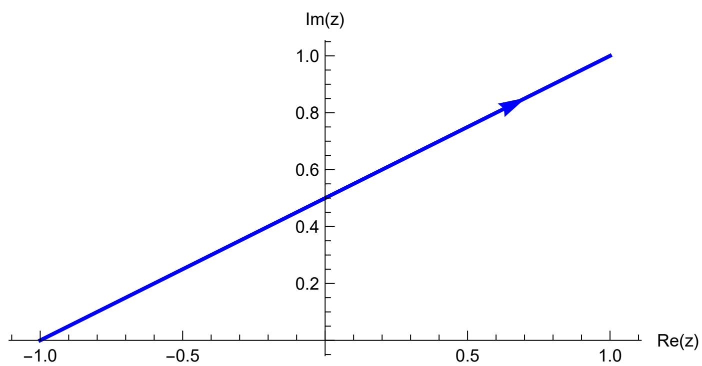
In[ ]:=
```
c2[t_] = ComplexExpand[(-1) * (1 - t) + (1 + I) * t]
```

Out[ ]=

$-1 + (2 + i)\, t$

In[ ]:=
```
P2 = Show[ParametricPlot[ReIm[c2[t]], {t, 0, 1}, PlotStyle → {Blue, Thick},
    AxesLabel → {"Re(z)", "Im(z)"}], Graphics[{Blue, Arrow[{{-1, 0}, {0.7, 0.85}}]}]]
```

Out[ ]=



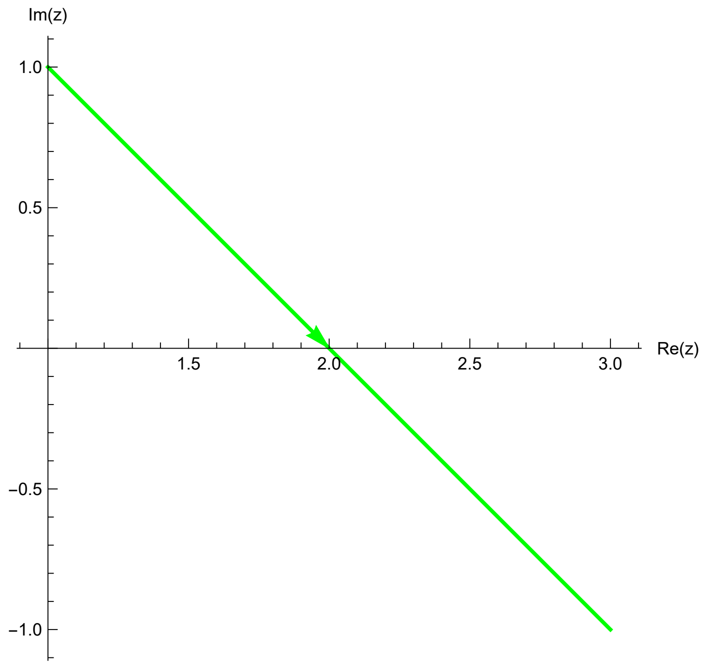
In[ ]:=
```
c3[t_] = ComplexExpand[(1 + I) * (1 - t) + (3 - I) * t]
```

Out[ ]=

$1 + i\,(1 - 2\,t) + 2\,t$

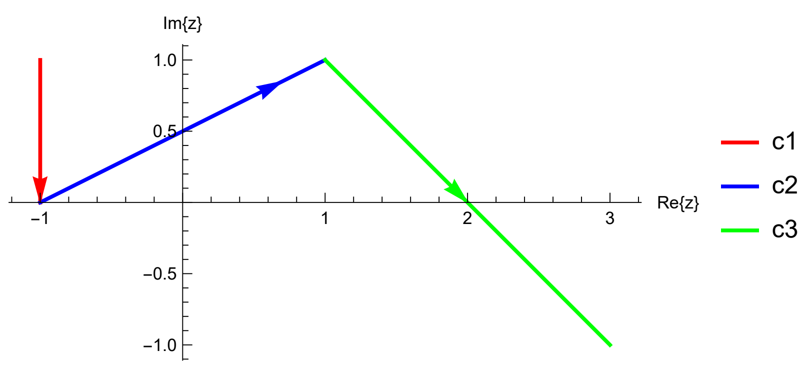
In[ ]:=
```
P3 = Show[ParametricPlot[ReIm[c3[t]], {t, 0, 1}, PlotStyle → {Green, Thick},
   AxesLabel → {"Re(z)", "Im(z)"}], Graphics[{Green, Arrow[{{1, 1}, {2, 0}}]}]]
```

Out[ ]=



In[ ]:=
```
Show[ParametricPlot[{ReIm[c1[t]], ReIm[c2[t]], ReIm[c3[t]]},
  {t, 0, 1}, PlotStyle → {{Red, Thick}, {Blue, Thick}, {Green, Thick}},
  PlotLegends → {"c1", "c2", "c3"}, AxesLabel → {"Re{z}", "Im{z}"}],
 Graphics[{Red, Arrow[{{-1, 0.75}, {-1, 0}}]}],
 Graphics[{Blue, Arrow[{{-1, 0}, {0.7, 0.85}}]}],
 Graphics[{Green, Arrow[{{1, 1}, {2, 0}}]}]]
```

Out[ ]=



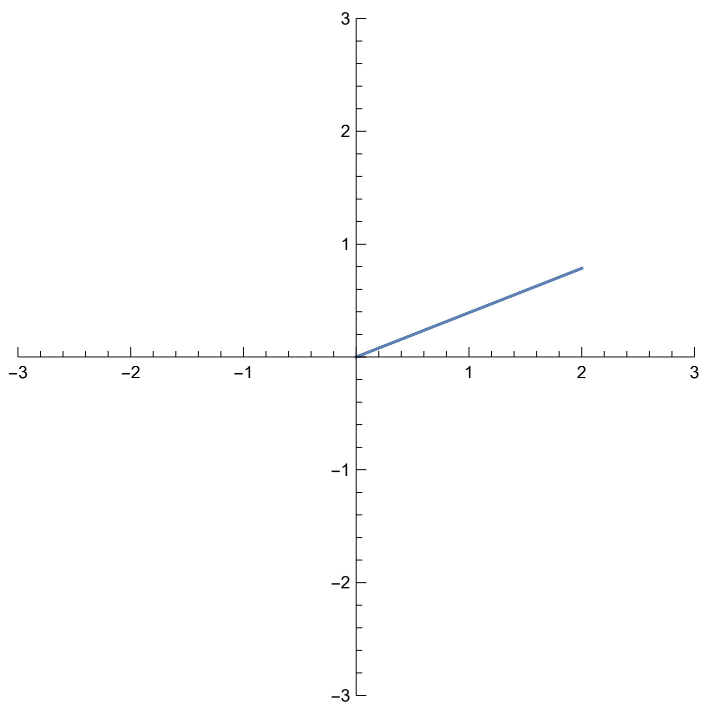
PRACTICAL NO. - 09

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 14 Feb 2023

Plot the line segment "L" joining the point A = 0 to B = $2 + \frac{\pi}{4}i$ and give an exact calculation of line integral L of e^zdz

In[ ]:=
```
L = ListLinePlot[{{0, 0}, {2, π / 4}},
  PlotRange → {{-3, 3}, {-3, 3}}, AspectRatio → Automatic]
```

Out[ ]=



In[ ]:=
```
ClearAll[z, w, t]
```

In[ ]:=
```
f[z_] := ⅇ^z
z1 = 0;
z2 = 2 + (π / 4) * ⅈ;
Integrate[f[z], {z, z1, z2}]
```

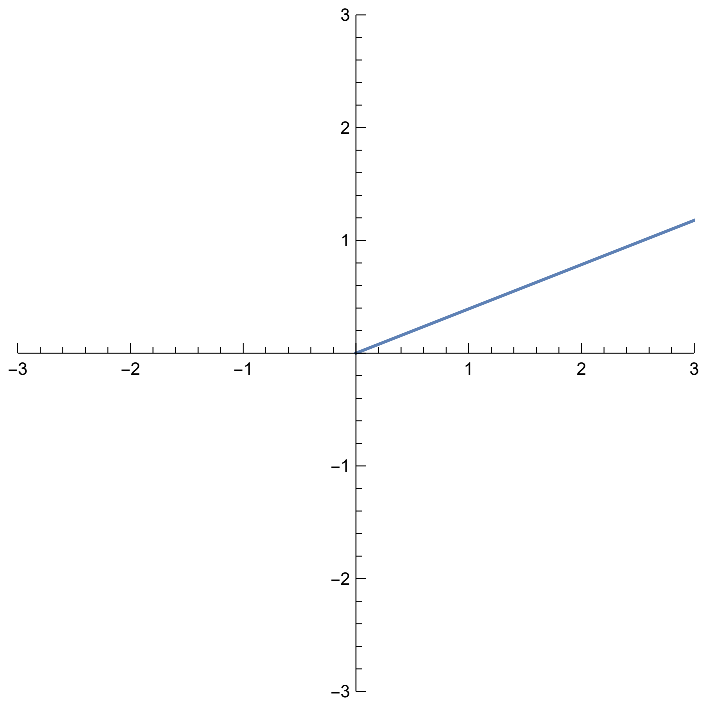
Out[ ]=

$-1 + (-1)^{1/4} e^2$

In[ ]:=
```
a = ArcTan[π / 8];
w[t_] := t * Cos[a] + ⅈ * Sin[a];
```

In[ ]:=
```
L1 = ParametricPlot[{t * Cos[a], t * Sin[a]},
  {t, 0, 4}, PlotRange → {{-3, 3}, {-3, 3}}, AspectRatio → Automatic]
```

Out[ ]=



In[ ]:=
```
Solve[u * Sin[a] == π / 4, {u}]
```

Out[ ]=

$$\left\{\left\{u \to \frac{\sqrt{64+\pi^2}}{4}\right\}\right\}$$

In[ ]:=
```
Integrate[f[w[t]] * D[w[t], t], {t, 0, Sqrt[64 + π^2]/4}]
```

Out[ ]=

$$e^{\frac{i\pi}{\sqrt{64+\pi^2}}}\left(-1+e^2\right)$$

PRACTICAL NO. - 10

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 21 Feb 2023

Plot the semicircle “C” with radius 1 centered at z=2 and evaluate the contour integral $\int_C \frac{1}{z-2}\, dz$

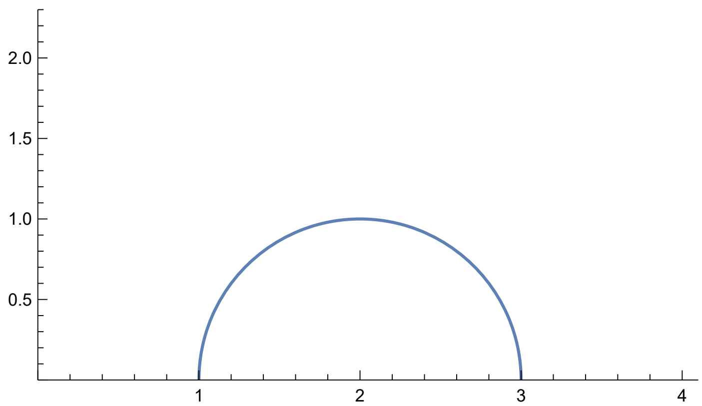
In[ ]:=
```
ClearAll[f, w, z, t]
```

In[ ]:=
```
z = x + i * y
```

Out[ ]=
```
x + i y
```

In[ ]:=
```
L = ParametricPlot[{2 + Cos[t], Sin[t]}, {t, 0, π},
  PlotRange → {{0, 4.1}, {0, 2.3}}, AspectRatio → Automatic]
```

Out[ ]=



In[ ]:=
```
f[z_] := 1 / (z - 2)
w[t_] := 2 = Cos[t] + i * (Sin[t])
```

In[ ]:=
```
Integrate[f[w[t]] * D[w[t], t], {t, 0, π}]
```

Set: Cannot assign to raw object 2.

Set: Cannot assign to raw object 2.

Out[ ]=
```
Log[3]
```

In[ ]:=

$i\pi$ if $2\,\mathrm{Re}\left[\mathrm{ArcTan}\left[i + \sqrt{1 + i^2}\right]\right] > \pi \;||\; \mathrm{Re}\left[\mathrm{ArcTan}\left[i + \sqrt{1 + i^2}\right]\right] < 0 \;||\; \mathrm{ArcTan}\left[i + \sqrt{1 + i^2}\right] \notin \mathbb{R}$

Out[ ]=

$i\pi$ if condition

PRACTICAL NO. - 11

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 28 Feb 2023

Show that $\int_{C_1} z\, dz = \int_{C_2} z\, dz = 4+2i$ where C_1 is the line segment from -1-i to 3+i and C_2 is the portion of the parabola $x = y^2$+2y joining -1-i to 3+i. Make plots of two contours C_1 and C_2 joining -1-i to 3+i.

Here, f (z) = z .

C_1 is the line segment from -1-i to 3+i . Parametrization of z over C_1 is given by
z1 (t) = (1-t)*(-1-i)+t*(3+i), 0≤t≤1
= (-1+4t)+i(-1+2t), 0≤t≤1

C_2 is the portion of the parabola $x = y^2$+2y joining -1-i to 3+i. Parametrization of z over C_2 is given by
z2 (t) = t^2+2t+i*t, -1≤t≤1.

In[ ]:=
```
f[z_] := z
```

In[ ]:=
```
z1[t_] = ComplexExpand[(1 - t) * (-1 - i) + t * (3 + i)]
```

Set: Tag Integer in 0[t_] is Protected.

Out[ ]=

$-1 + 4t + i(-1 + 2t)$

In[ ]:=
```
z2[t_] = ComplexExpand[t^2 + 2 * t + i * t]
```

Set: Tag Plus in $\left(2 + \frac{i\pi}{4}\right)$[t_] is Protected.

Out[ ]=

$(2 + i)t + t^2$

PRACTICAL NO. - 12

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 14 Mar 2023

Use ML Inequality to show that $\left|\int_C \frac{1}{z^2+1}\,dz\right| \le \frac{1}{2\sqrt{5}}$, where C is the straight line segment from 2 to 2+2i.
While Solving, represent the distance from the point z to the points i and -i, respectively, i.e., |z-i| and |z+i| on the complex plane C

M.L. Inequality
Let C denote a contour of length L and suppose f(z) is a piecewise continuous function on C.
If M is a non-negative constant such that |f(z)| ≤ M, ∀ z ∈ C.
Then, $\int_C f(z)\,dz \le$ M.L.

Here, f(z)=$\frac{1}{z^2+1}$, Parameterization of a line segment from 2 to 2+i is
z(t)=(1-t)2+t(2+i) ,   0≤t≤1
=2+it ,   0≤t≤1.
Here , the length of the v-curve C is L =$\int_0^1 |\, z'(t) \,|\, dt = \int_0^1 dt = 1$.

In[ ]:=
```
ClearAll[z, w, t]
```

In[ ]:=
```
f[z] = 1/(z^2 + 1)
```

Out[ ]=
$$\frac{1}{1+z^2}$$

In[ ]:=
```
z[t_] = ComplexExpand[(1 - t) * 2 + t * (2 + i)]
```

Out[ ]=
$2 + i\, t$

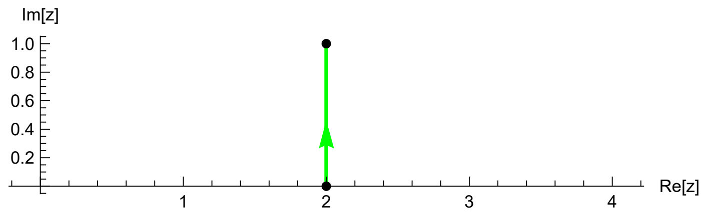
In[ ]:=
```
z'[t]
```

Out[ ]=
i

In[ ]:=
```
Abs[z'[t]]
```

Out[ ]=
1

In[ ]:=
```
Show[ParametricPlot[ReIm[z[t]], {t, 0, 1}, AxesLabel → {"Re[z]", "Im[z]"},
   PlotStyle → {Green, Thick}], Graphics[{Green, Arrow[{{2, 0.4}, {2, 0.45}}]}],
  Graphics[{Black, PointSize[0.015], Point[{2, 0}], Point[{2, 1}]}]]
```
Out[ ]=



In[ ]:=
```
L = Integrate[Abs[z'[t]], {t, 0, 1}]
```
Out[ ]=

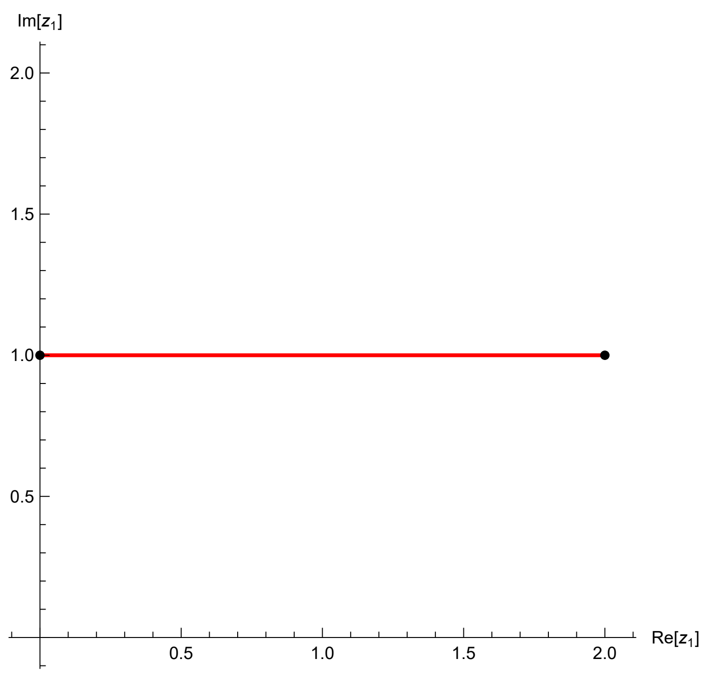
1

In[ ]:=
```
ClearAll[z, t]
```

In[ ]:=
```
z1[t_] = ComplexExpand[(1 - t) * i + t * (2 + i)]
```
Out[ ]=

$i + 2t$

In[ ]:=
```
Show[ParametricPlot[ReIm[z1[t]], {t, 0, 1},
   AxesLabel → {"Re[z1]", "Im[z1]"}, PlotStyle → {Red, Thick}],
  Graphics[{Black, PointSize[0.015], Point[{0, 1}], Point[{2, 1}]}]]
```
Out[ ]=

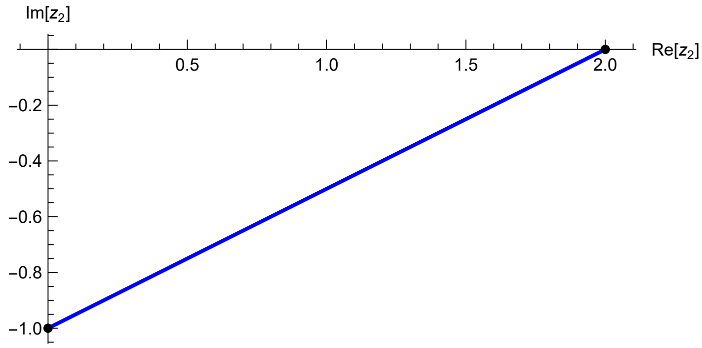


In[ ]:=
```
z2[t_] = ComplexExpand[(1 - t) * (-i) + t * 2]
```
Out[ ]=

$i(-1 + t) + 2t$

In[ ]:=
```
Show[ParametricPlot[ReIm[z2[t]], {t, 0, 1},
  AxesLabel → {"Re[z2]", "Im[z2]"}, PlotStyle → {Blue, Thick}],
 Graphics[{Black, PointSize[0.015], Point[{0, -1}], Point[{2, 0}]}]]
```

Out[ ]=



PRACTICAL NO. - 13

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 21 Mar 2023

Show that $\int_C \frac{1}{2\, z^{1/2}}\, dz$ where $z^{1/2}$ is the principal branch of the square root function and C is the line segment joining 4 to 8+6i , Also plot the path of integration.

Here, f[z] = $\frac{1}{2\, z^{1/2}}$ where $z^{1/2}$ is the principal branch of square root function.

Line segment C passing through 4 to 8 + 6i is parametrized by

$z(t) = ((1 - t)\, 4 + t\, (8 + 6i)), \qquad (0 \leq t \leq 1)$

$\quad = 4 + 4\, t + i 6t, \qquad (0 \leq t \leq 1)$

For $z(t) = re^{it}, \qquad (0 \leq t \leq 2\,\pi)$

$f(z(t)) = \frac{1}{2\left(re^{it}\right)^{1/2}} = \frac{1}{\sqrt{r}}\, e^{-it/2}, \qquad (0 \leq t \leq 2\,\pi)$

where $(0 \leq t \leq 2\,\pi)$ is the principal branch of square root function .

In[ ]:=
```
f[z_] := 1/(2 * z^(1/2))
```

In[ ]:=
```
z[t_] = ComplexExpand[(1 - t) * 4 + t * (8 + 6 * i)]
```
Out[ ]=
$4 + (4 + 6\, i)\, t$

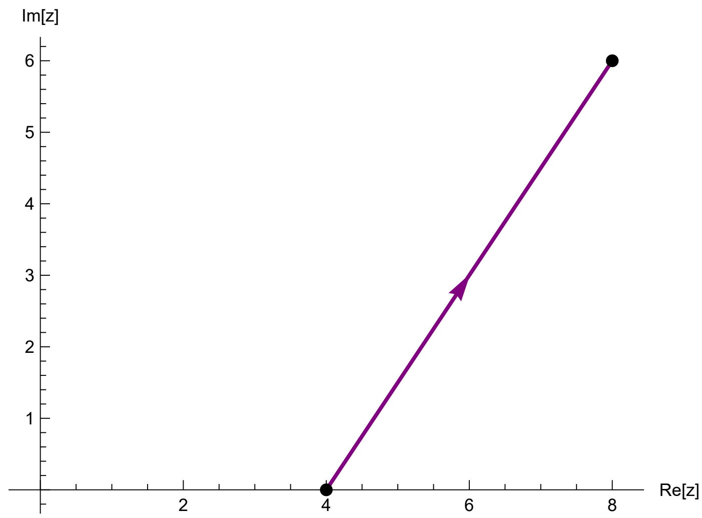
In[ ]:=
```
z'[t]
```
Out[ ]=
$4 + 6\, i$

In[ ]:=
```
f[z[t]]
```
Out[ ]=
$$\frac{1}{2\,\sqrt{4 + (4 + 6\, i)\, t}}$$

In[ ]:=
```
Integrate[f[z[t]] * z'[t], {t, 0, 1}]
```
Out[ ]=
$1 + i$

In[ ]:=
```
Show[ParametricPlot[ReIm[z[t]], {t, 0, 1},
  AxesLabel → {"Re[z]", "Im[z]"}, AxesOrigin → {0, 0}, PlotStyle → {Purple, Thick}],
 Graphics[{Purple, Arrow[{{4, 0}, {6, 3}}]}],
 Graphics[{Black, PointSize[0.020], Point[{4, 0}], Point[{8, 6}]}]]
```

Out[ ]=



PRACTICAL NO. - 14

Name - Nitin Ahlawat
Roll No. - 20HMT3224
Date - 28 Mar 2023

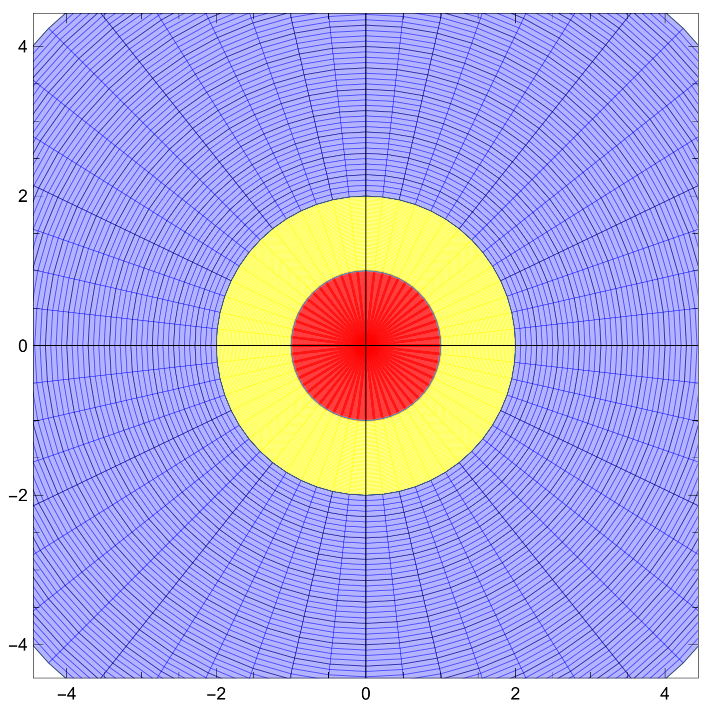
Find and plot three different Laurent Series representation for the function $f(z) = \frac{3}{2+z-z^2}$ involving powers of z.

In[ ]:=
```
f[z_] = 3/(2 + z - z^2)
```
Out[ ]=

$$\frac{3}{2+z-z^2}$$

In[ ]:=
```
z /. Solve[2 + z - z^2 == 0, z]
```
Out[ ]=
```
{-1, 2}
```

In[ ]:=
```
Show[ParametricPlot[{r * Cos[t], r * Sin[t]},
   {r, 0, 1}, {t, 0, 2 * π}, PlotStyle → Red, Mesh → None],
 ParametricPlot[{r * Cos[t], r * Sin[t]},
   {r, 1, 2}, {t, 0, 2 * π}, PlotStyle → Yellow, Mesh → None],
 ParametricPlot[{r * Cos[t], r * Sin[t]}, {r, 2, 6},
   {t, 0, 2 * π}, PlotStyle → Blue, Mesh → Full], PlotRange → 4]
```
Out[ ]=



In[ ]:=
```
Print["Taylor series expansion of f in |z|<1"]
Apart[f[z]]
```

Taylor series expansion of f in |z|<1

Out[ ]=

$$\frac{1}{1+z^2}$$

In[ ]:=
```
Print["Laurent series expansion of f in 1<|z|<2"]
Series[1 / (2 - z), {z, 0, 10} + Series[1 / (1 + z), {z, Infinity, 10}]]
```

Laurent series expansion of f in 1<|z|<2

General: $z+\frac{1}{z}-\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3-\left(\frac{1}{z}\right)^4+\left(\frac{1}{z}\right)^5-\left(\frac{1}{z}\right)^6+\left(\frac{1}{z}\right)^7-\left(\frac{1}{z}\right)^8+\frac{\ll 2\gg}{z^9}+O\left[\frac{1}{z}\right]^{11}$ is not a valid variable.

General: $z+\frac{1}{z}-\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3-\left(\frac{1}{z}\right)^4+\left(\frac{1}{z}\right)^5-\left(\frac{1}{z}\right)^6+\left(\frac{1}{z}\right)^7-\left(\frac{1}{z}\right)^8+\frac{\ll 2\gg}{z^9}+O\left[\frac{1}{z}\right]^{11}$ is not a valid variable.

Out[ ]=

$$\text{Series}\left[\frac{1}{2-z},\left\{z+\frac{1}{z}-\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3-\left(\frac{1}{z}\right)^4+\left(\frac{1}{z}\right)^5-\left(\frac{1}{z}\right)^6+\left(\frac{1}{z}\right)^7-\left(\frac{1}{z}\right)^8+\left(\frac{1}{z}\right)^9-\left(\frac{1}{z}\right)^{10}+O\left[\frac{1}{z}\right]^{11},\right.\right.$$
$$\frac{1}{z}-\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3-\left(\frac{1}{z}\right)^4+\left(\frac{1}{z}\right)^5-\left(\frac{1}{z}\right)^6+\left(\frac{1}{z}\right)^7-\left(\frac{1}{z}\right)^8+\left(\frac{1}{z}\right)^9-\left(\frac{1}{z}\right)^{10}+O\left[\frac{1}{z}\right]^{11},$$
$$\left.\left.10+\frac{1}{z}-\left(\frac{1}{z}\right)^2+\left(\frac{1}{z}\right)^3-\left(\frac{1}{z}\right)^4+\left(\frac{1}{z}\right)^5-\left(\frac{1}{z}\right)^6+\left(\frac{1}{z}\right)^7-\left(\frac{1}{z}\right)^8+\left(\frac{1}{z}\right)^9-\left(\frac{1}{z}\right)^{10}+O\left[\frac{1}{z}\right]^{11}\right\}\right]$$

In[ ]:=
```
Print["Laurent series expansion of f in |z|>2"]
```

Laurent series expansion of f in |z|>2

In[ ]:=
```
Series[f[z], {z, Infinity, 10}]
```

Out[ ]=

$$\left(\frac{1}{z}\right)^2-\left(\frac{1}{z}\right)^4+\left(\frac{1}{z}\right)^6-\left(\frac{1}{z}\right)^8+\left(\frac{1}{z}\right)^{10}+O\left[\frac{1}{z}\right]^{11}$$