1 CSE512 Fall 2018 - Machine Learning - Homework 7

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Q1 Given:

$x = [1 \quad 10 \quad 20]$

$$R = \begin{pmatrix} 1 & 0 \\ 0.3 & 0.7 \\ 0 & 1 \end{pmatrix}$$

1. $$Q(\theta, \theta^{t+1}) = \sum_i \sum_c r_{ic} \log \pi_c + \sum_i \sum_c r_{ic} \log p(x_i | \theta_c)$$

2. $$\pi_c = \frac{1}{N} \sum_i r_{ic}$$

$$\pi_1 = \frac{1}{N} \sum_i r_{ic=1} = \frac{1}{3}(1+0.3+0) = \frac{1.3}{3} = 0.433$$

$$\pi_2 = \frac{1}{N} \sum_i r_{i2=c} = \frac{1}{3}(0+0.7+1) = \frac{1.7}{3} = 0.567$$

3. $$\mu_c = \frac{\sum_i r_{ic} x_i}{r_c}$$

$$\mu_1 = \frac{\sum_i r_{i1} x_i}{r_1} = \frac{(1 \times 1) + (0.3 \times 10) + (0 \times 20)}{1.3} = \frac{4}{1.3} = 3.07$$

$$\mu_2 = \frac{\sum_i r_{i2} x_i}{r_2} = \frac{(0 \times 1) + (0.7 \times 10) + (1 \times 20)}{1.7} = \frac{27}{1.7} = 15.88$$

~~E step~~:

4. $$\sigma_c^2 = \frac{\sum_i r_{ic} \times x_i^2}{r_c} - \mu_c^2$$

$$\sigma_1^2 = \frac{(1\times1) + (0.3\times100) + (0\times400)}{1.3} - 9.4249$$

$$= \frac{31}{1.3} - 9.4249 = 14.42125$$

$$\sigma_1 = \sqrt{4.42} \approx 3.79$$

$$\sigma_2^2 = \frac{(0\times1) + (0.7\times100) + (1\times400)}{1.7} - 252.1744$$

$$= \frac{70+400}{1.7} - 252.1744 = 24.296$$

$$\sigma_2 \approx 4.92911$$

E step:

$$\therefore r_{ic} = \frac{\pi_c\, p(x_i \mid \mu_c^{t+1})}{\sum_c \pi_c\, p(x_i \mid \mu_c^{t+1})} = \frac{\pi_c \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{(x_i - \mu_c^{t+1})^2}{2\sigma_c^2}\right)}{\sum_c \left(\pi_c \frac{1}{\sqrt{2\pi}\sigma_c} \exp\left(-\frac{(x_i - \mu_c^{t+1})^2}{2\sigma_c^2}\right)\right)}$$

$$\therefore r_{11} = \frac{\frac{\pi_1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x_1 - \mu_1^{t+1})^2}{2\sigma_1^2}\right)}{\frac{\pi_1}{\sqrt{2\pi}\sigma_1} \exp\left(-\frac{(x_1 - \mu_1^{t+1})^2}{2\sigma_1^2}\right) + \frac{\pi_2}{\sqrt{2\pi}\sigma_2} \exp\left(-\frac{(x_1 - \mu_2^{t+1})^2}{2\sigma_2^2}\right)}$$

$$= \frac{\frac{0.433}{(0.379\times10)} \exp\left(-\frac{(1-3.07)^2}{2\times14.42}\right)}{\frac{0.433}{(3.79)} \exp\left(-\frac{(1-3.07)^2}{2\times14.42}\right) + \frac{0.567}{4.929} \exp\left(-\frac{(1-15.88)^2}{2\times24.296}\right)}$$

$$r_{11} \approx 0.987$$

$$r_{12} = 1 - r_{11} = 1 - 0.987 = 0.013$$

$$r_{21} = \frac{\frac{\pi_1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(x_2-\mu_1^{t+1})^2}{2\sigma_1^2}\right)}{\frac{\pi_1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(x_2-\mu_1^{t+1})^2}{2\sigma_1^2}\right)+\frac{\pi_2}{\sqrt{2\pi}\sigma_2}\exp\left(-\frac{(x_2-\mu_2^{t+1})^2}{2\sigma_2^2}\right)}$$

$$= \frac{\frac{0.433}{3.79}\exp\left(-\frac{(10-3.07)^2}{2\times 14.42}\right)}{\frac{0.433}{3.79}\exp\left(-\frac{(10-3.07)^2}{2\times 14.42}\right)+\frac{(\pi_2=0.567)}{4.92}\exp\left(-\frac{(10-15.28)^2}{2\times 24.296}\right)}$$

$$\approx 0.2727$$

$$r_{22} = 1 - r_{21} = 1 - 0.2727 = 0.7273.$$

$$r_{31} = \frac{\frac{\pi_1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(x_3-\mu_1^{t+1})^2}{2\sigma_1^2}\right)}{\frac{\pi_1}{\sqrt{2\pi}\sigma_1}\exp\left(-\frac{(x_3-\mu_1^{t+1})^2}{2\sigma_1^2}\right)+\frac{\pi_2}{\sqrt{2\pi}\sigma_2}\exp\left(-\frac{(x_3-\mu_2^{t+1})^2}{2\sigma_2^2}\right)}$$

$$= \frac{\frac{0.433}{3.79}\exp\left(-\frac{(20-3.07)^2}{2\times 14.42}\right)}{\frac{0.433}{0.379}\exp\left(-\frac{(20-3.07)^2}{2\times 14.42}\right)+\frac{0.567}{4.92}\exp\left(-\frac{(20-15.88)^2}{2\times 24.296}\right)}$$

$$\approx 6.6\times 10^{-5} = (6.6\times 10^{-5})$$

$$r_{32} = 1 - r_{31} = 1 - (6.6\times 10^{-5}) = 0.999934$$

$$R = \begin{bmatrix} 0.987 & 0.013 \\ 0.2727 & 0.7273 \\ 6.6\times 10^{-5} & 0.999934 \end{bmatrix}$$