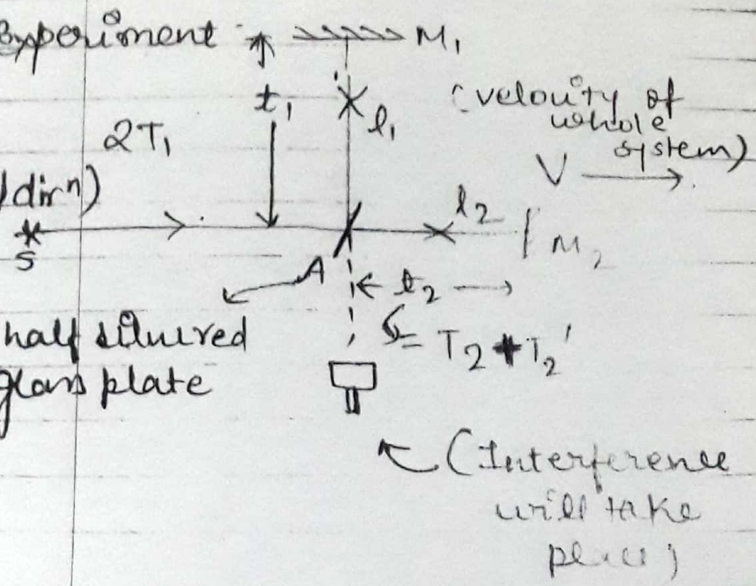
12/8/17

Special theory of Relativity

Michealson and Morley Experiment



$CT_2 = l_2 + VT_2$ (Time period during forward dir^n)

$$T_2 = \frac{l_2}{(C-V)}$$

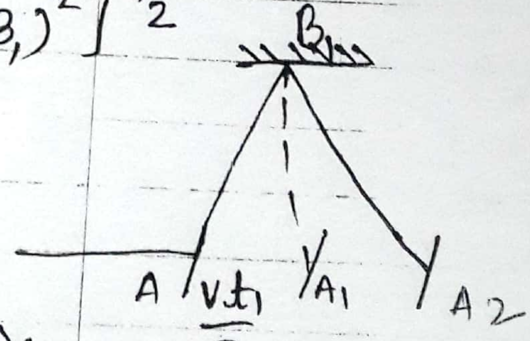
$$T_2' = \frac{l_2}{(C+V)}$$

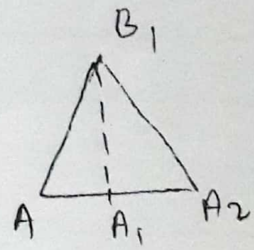
$$t_2 = T_2 + T_2' = \frac{l_2}{C-V} + \frac{l_2}{C+V}$$

$$t_2 = l_2\left(\frac{1}{C-V} + \frac{1}{C+V}\right) \Rightarrow t_2 = \frac{l_2}{c}\left(\frac{2}{1-\frac{v^2}{c^2}}\right)$$

$$AB_1 = B_1A_2 = \left[(AA_1)^2 + (AB_1)^2\right]^{1/2}$$

$$= \left(\frac{(Vt_1)^2}{2} + l_1^2\right)^{1/2}$$





$$Ct_1 = 2\left[\left(\frac{Vt_1}{2}\right)^2 + l_1^2\right]^{1/2}$$

$$\Rightarrow t_1 = \frac{2l_1}{(C^2-V^2)^{1/2}} \Rightarrow t_1 = \frac{2l_1}{C}\left[\frac{1}{\left(1-\frac{V^2}{c^2}\right)^{1/2}}\right]$$

$\Delta t = t_1 - t_2$

$$= \frac{2}{c}\left[\frac{l_1}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} - \frac{l_2}{\left(1-\frac{v^2}{c^2}\right)}\right]$$

After 90° rotation,

$$\Delta t' = \frac{2}{c}\left[\frac{l_1}{\left(1-\frac{v^2}{c^2}\right)} - \frac{l_2}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right]$$

~~Path di~~

$$\Delta t' - \Delta t = \frac{2}{c}(l_1 + l_2)\left[\frac{1}{\left(1-\frac{v^2}{c^2}\right)} - \frac{1}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right]$$

<u>Path difference</u> –

$$\Delta x = 2(l_1 + l_2)\left[\left(1 - \frac{v^2}{c^2}\right)^{-1} \left(1 - \frac{v^2}{2c^2}\right)\right]$$

$l_1 = l_2 = 10\,m$, $\lambda = 5.6 \times 10^{-7}\,m$, $\frac{v}{c} = 10^{-4}$

<u>Fringe Shift</u>,

$\Delta x = n\lambda$

$$n = \frac{2(l_1 + l_2)\left[\frac{1}{\left(1-\frac{v^2}{c^2}\right)} - \frac{1}{\left(1-\frac{v^2}{c^2}\right)^{1/2}}\right]}{\lambda}$$

$= 0.4$ fringe.

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Explanation of negative result - 1) There is no relative motion b/w the earth and ether.

2.) Length contraction

$$l = l_0 \sqrt{1 - \frac{v^2}{c^2}}$$

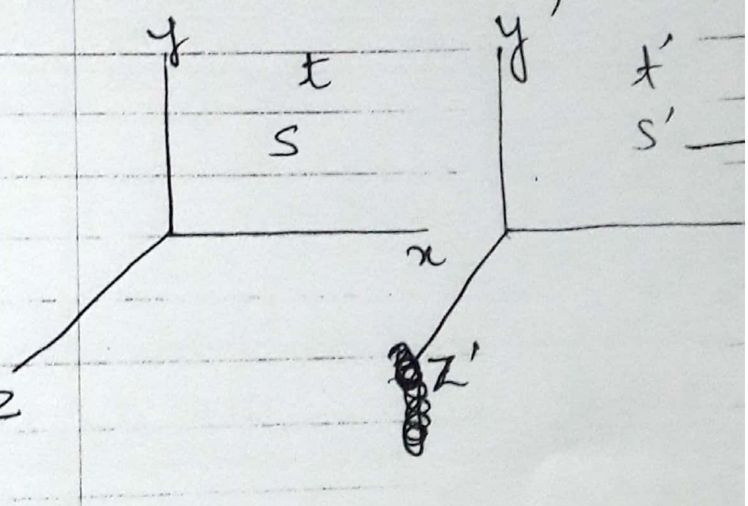
$$l_2 \longrightarrow l_2 \sqrt{1 - \frac{v^2}{c^2}}$$

<u>frame of reference</u> -

→ Inertial frame of reference - That frame where Newton's laws are valid.

~~Gatt~~

Galelian Transformation -



$x' = x - vt$

$y' = y$

$z' = z \qquad t = t'$

also,

$$v_x' = \frac{dx'}{dt} = V_x - V$$

$$V_y' = V_y$$

$$V_z' = V_z.$$

... of electricity and magnetism become ... different.

2. $c' = c - v$

Postulates of special theory of relativity-

1. The laws of the physics are the same in all inertial frames of reference.

2. The speed of light in free space has the same value in all inertial frames of reference

⇒ Lorentz transformation-

$x' = (x - vt)K$ (K is dimensionless)

$y' = y$

$z' = z$

$t' \neq t$

<u>Reason to prefer</u>-

1. Linear in x and x'
2. Simple one.
3. Easy to go back to Gallilien transformation.

$$x = K(x' + vt')$$

$$x = K[K(x - vt) + vt']$$

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$$\Rightarrow \quad x = K^2(x - vt) + Kvt'$$

$$t' = Kt + \left(\frac{1-K^2}{Kv}\right)x$$

Second postulates of special theory of relativity

says,

$$c = \frac{x}{t} \quad , \quad c = \frac{x'}{t'}$$

$$\Rightarrow \quad x = ct \quad , \quad x' = ct'.$$

$$\because \quad x' = K(x - vt)$$

$$x' = ct'$$

putting,

$$K(x - vt) = \left[Kt + \left(\frac{1-K^2}{Kv}\right)x\right]c$$

Solving for x,

$$Kx - Kvt = Kct + \frac{(1-K^2)}{Kv}cx$$

$$\left[K - \frac{(1-K^2)c}{Kv}\right]x = (Kc + Kv)t$$

$$x = \frac{(Kct + Kvt)}{K - \frac{(1-K^2)}{K}\frac{c}{v}}$$

$$x = ct\left[\frac{1 + \frac{v}{c}}{1 - \left(\frac{1}{K^2} - 1\right)\frac{c}{v}}\right]$$

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$$\Rightarrow \frac{1+\frac{v}{c}}{1-\left(\frac{1}{K^2}-1\right)\frac{c}{v}} = 1$$

$$\Rightarrow \cancel{1}+\frac{v}{c} = \cancel{1}-\left(\frac{1}{K^2}-1\right)\frac{c}{v}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{K^2}$$

$$\frac{1}{K^2} = 1-\frac{v^2}{c^2}$$

$$\boxed{K = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y' = y$$

$$z' = z.$$

$$\boxed{t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Inverse Lorentz transformation-

$$x = \frac{x' + vt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$y = y'$$

$$z = z'$$

$$t = \frac{t' + \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Space - Time-

$s^2 = x^2 + y^2 + z^2 - c^2t^2$ is invariant under Lorentz transformation.

$$x'^2 + y'^2 + z'^2 - c^2t'^2 = x^2 + y^2 + z^2 - c^2t^2$$

Length contraction:-

A rod is lying in s' frame of reference.

Proper length $(L_0) = x_2' - x_1'$

Length measured in s frame of reference

$L = x_2 - x_1$

$$x_1' = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad , \quad x_2' = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_0 = x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{L = L_0 \sqrt{1-\frac{v^2}{c^2}}}$$

$$\because \frac{v}{c} > 0$$

$$\Rightarrow \quad L < L_0$$

Time dilation -

There is a clock in s' frame of reference at position x'. An observer measures a time t_1', then another observer in s will find it t_1, similarly second measures t_2' in s' and t_2 in s.

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$x_1' = \frac{x_1 - vt}{\sqrt{1-\frac{v^2}{c^2}}} \quad , \quad x_2' = \frac{x_2 - vt}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_0 = x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{L = L_0 \sqrt{1-\frac{v^2}{c^2}}}$$

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$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t_2 = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Proper time $(t_0) = t_2' - t_1'$

$t = t_2 - t_1$

$$= \frac{t_2' + \frac{vx_1'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

$\Rightarrow \; t > t_0$

Velocity addition -

Suppose something is moving relative to both S and S' an observer in S measures its free velocity components as

$$V_x = \frac{dx}{dt}, \quad V_y = \frac{dy}{dt}, \quad V_z = \frac{dz}{dt}$$

while to an observer in S' they are

$$V_x' = \frac{dx'}{dt'}, \quad V_y' = \frac{dy'}{dt'}, \quad V_z' = \frac{dz'}{dt'}$$

By differentiating inverse lorentz transformation equation for x, y, z and 't' we get;

$$dx = \frac{dx' + v dt'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \frac{dt' + \frac{v dx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$V_x = \frac{dx}{dt} = \frac{dx' + v dt'}{dt' + \frac{v dx'}{c^2}}$$

$$V_x = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v \frac{dx'}{dt'}}{c^2}}$$

$$V_x = \frac{V_x' + v}{\left(1 + \frac{v V_x'}{c^2}\right)}$$

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$$v_y = \frac{dy}{dt} = \frac{dy' \cdot \sqrt{1-\frac{v^2}{c^2}}}{dt' + \frac{v\,dx'}{c^2}}$$

$$= \frac{\frac{dy'}{dt'}\sqrt{1-\frac{v^2}{c^2}}}{1 + \frac{v\frac{dx'}{dt'}}{c^2}}$$

$$v_y = \frac{v_y'\sqrt{1-\frac{v^2}{c^2}}}{1+\frac{v\,v_x}{c^2}}$$

If $v_x' = c$, $v_x = c$

$$v_x = \frac{v_x' + v}{1 + \frac{v\,v_x'}{c^2}}$$

$$v_x = \frac{c+v}{1+\frac{v\cancel{c}}{c^{\cancel{2}}}}$$

$$\boxed{v_x = c}$$

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Q. A spacecraft named as alpha moving at 0.9c w.r.t the Earth. A spacecraft beta is to pass 'α' at a relative speed at 0.5c in the same direction, what speed ~~need~~ must B have. w.r.t Earth?

0.90c

0.5c

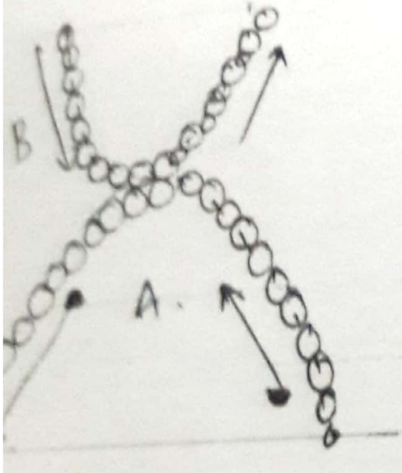
Sol $V_R = 1.4c$

wrong.

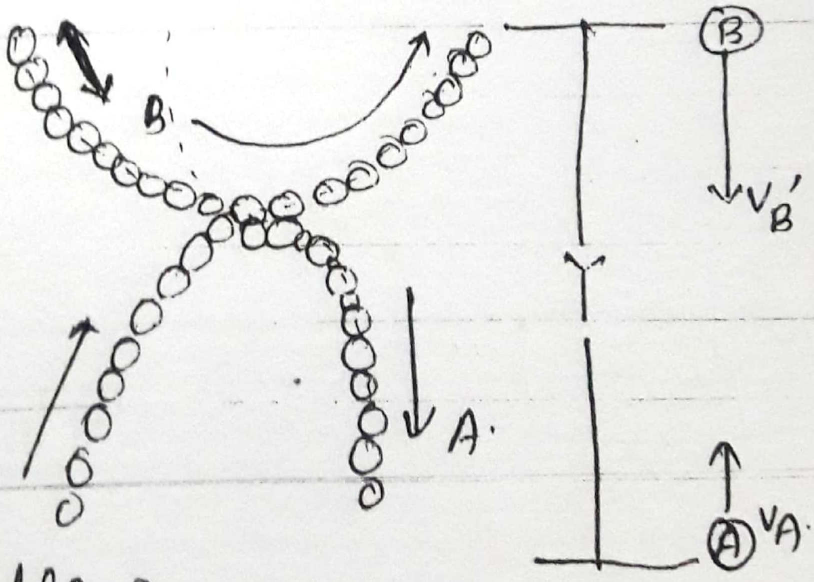
$$V_\alpha = \frac{1.40c}{1 + \frac{0.9 \times 0.5\, c^2}{c^2}} = 0.97c$$

Relativity of mass -

Rest Mass is least -



As seen from s'.



Collision as seen from 's'

A is in rest in S and B is rest in S'.
V_A is in +y direction, V_B' in -y' direction.
Everything is identical in S and S'.

$$V_A = V_B' \quad \text{—— (1)}$$

A will cover distance $\frac{Y}{2}$ before collision and $\frac{Y}{2}$ after collision. The round trip time T_0 for A as measured in S is

$$T_0 = \frac{Y}{V_A} \quad \text{—— (2)}$$

and It is is same for B in S'

$$T_0 = \frac{Y}{V_B'} \quad \text{—— (3)}$$

If linear momentum is conserved in S frame, It must be true that,

$$M_A V_A = M_B V_B \quad \text{—— (4)}$$

In S the speed $V_B = \frac{Y}{T}$ —— (5)

where, T is time required for B to make its round trip as measured in S.

In S', however, B's trip requires the time T

where $T = \frac{T_0}{\sqrt{1-\frac{v^2}{c^2}}}$ (6)

Although observers in both frames see the same event, they disagree about the length of time the particle thrown from the other frame requires to make the collision and return.

from (5)

$$V_B = \frac{Y\sqrt{1-\frac{u^2}{c^2}}}{T_0} \quad \text{—} (7)$$

from (2)

$$V_A = \frac{Y}{T_0} \quad \text{—} (8)$$

from (4)

$$M_A \times \frac{Y}{T_0} = M_B \frac{Y\sqrt{1-\frac{u^2}{c^2}}}{T_0}$$

The momentum will be conserved provided that

$$\boxed{M_A = M_B \sqrt{1-\frac{u^2}{c^2}}}$$

In S, $M_A = M_0$, $M_B = m$.

so,

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

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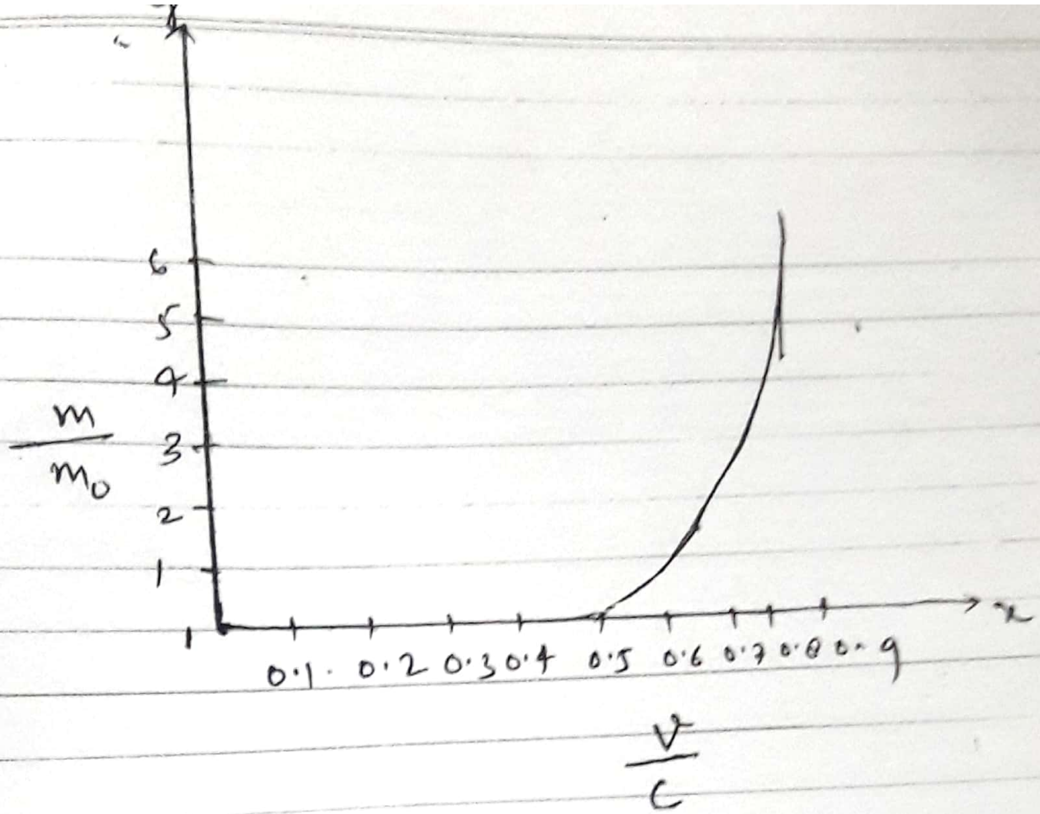

Q. find the mass of an electron whose velocity is 0.99 c? $m_0 = 9.1 \times 10^{-31}$ kg

$v = 0.99c$

$$m = \frac{9.1 \times 10^{-31}}{\sqrt{1-(.99)^2}}$$

$m = 6.45 \times 10^{-30}$ kg.

which is greater than m_0.

Relativistic momentum -

$$p = mu = \frac{m_0 v}{\sqrt{1 - \frac{u^2}{c^2}}}$$

Relativistic second law -

$$f = \frac{d(mv)}{dt} = \frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1 - \frac{u^2}{c^2}}}\right)$$

Classically

$$f = m\frac{du}{dt}.$$

Relativistically,

$$\frac{d(mu)}{dt} = m\frac{du}{dt} + v\frac{dm}{dt}.$$

<u>Mass and Energy</u> -

$$K.E = \int_0^s F \cdot ds \quad (\text{F need not be constant})$$

$$\frac{d(mu)}{dt} ds = \int_0^{mv} d(mu) \frac{ds}{dt} = \int_0^{mv} v\, d(mv)$$

$$= \int_0^{u} v \; d\left(\frac{m_0 u}{\sqrt{1 - \frac{u^2}{c^2}}}\right)$$

$$\left[\frac{m_0 u^2}{\sqrt{1-\frac{u^2}{c^2}}}\right]_0^{u} - \int_0^{u} \frac{m_0 u}{\sqrt{1-\frac{u^2}{c^2}}}\,du$$

$$1-\frac{u^2}{c^2} = t$$

$$\frac{-2u\,du}{c^2} = dt$$

$$u\,du = \frac{c^2}{-2}\,dt$$

$$\Rightarrow \left[\frac{m_0 u^2}{\sqrt{1-\frac{u^2}{c^2}}}\right]_0^{u} + \frac{m_0 c^2}{2}\int_1^{1-\frac{u^2}{c^2}} \frac{dt}{\sqrt{t}}$$

$$\Rightarrow \left[\frac{m_0 u^2}{\sqrt{1-\frac{u^2}{c^2}}}\right]_0^{u} + m_0 c^2\left(\sqrt{1-\frac{u^2}{c^2}} - 1\right)$$

$$\Rightarrow \frac{m_0 u^2}{\sqrt{1-\frac{u^2}{c^2}}} + m_0 c^2\left(\sqrt{1-\frac{u^2}{c^2}} - 1\right)$$

$$\Rightarrow \frac{m_0 u^2}{\sqrt{1-\frac{u^2}{c^2}}} + m_0 c^2\sqrt{1-\frac{u^2}{c^2}} - m_0 c^2 \Rightarrow \frac{m_0 u^2 - m_0 u^2 + m}{\sqrt{1-\frac{u^2}{c^2}}} -$$

$$\Rightarrow mc^2 - m_0 c^2$$

K.E = (Increase in mass due to relativistic motion) c^2

Also,

$$K.E = mc^2 - m_0 c^2$$

$$mc^2 = KE + m_0 c^2$$

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If we interpret ~~to~~ mc^2 as total energy the object far we see that when it is at rest and $K \cdot E = 0$, it never relays posseses the energy m_0c^2. Acc. M_0c^2 is called the rest energy of something who mass ~~ad~~ at rest is m_0

$$\boxed{E = E_0 + K \cdot E} \quad \text{where } E_0 = m_0c^2$$

~~f~~ object is moving its total energy,

$$E = mc^2 = \frac{m_0c^2}{\sqrt{1-\frac{u^2}{c^2}}}$$

<u>K·E at low speed</u> -

$$K \cdot E = mc^2 - m_0c^2 = m_0c^2\left(\frac{1}{\sqrt{1-\frac{u^2}{c^2}}} - 1\right)$$

$$= m_0c^2\left(\left(1-\frac{u^2}{c^2}\right)^{-1/2} - 1\right)$$

$$= m_0\cancel{c^2}\left(\cancel{1} + \frac{u^2}{2\cancel{c^2}} - \cancel{1}\right).$$

$$\boxed{K \cdot E = \frac{1}{2}m_0u^2}$$

...less particle -

Total Energy, $E = \frac{m_0 c^2}{\sqrt{1-\frac{u^2}{c^2}}}$

Relativistic momentum,

$$P = \frac{m_0 v}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$E^2 - p^2c^2 = m_0^2 c^4$$

$$\Rightarrow E = \sqrt{E_0^2 + p^2c^2}$$

If $m_0 = 0$, then, $\boxed{E = pc}$

$E = KE + m_0 c^2$

$m^2c^4 - m_0^2c^4$

$c^4(m^2 - m_0^2)$

$c^4\left(\frac{m_0^2}{1-\frac{v^2}{c^2}} - m_0^2\right)$

$c^4 m_0^2 \left(\cancel{1} - \cancel{1} + \frac{v^2}{c^2}\right)$

$c^2 m_0^2 v^2$

p^2c^2

...ncept of simultaneity - Two events simultaneous in one Inertial frame 'S' are not necessarily simultaneous in other frame S'

$x_1 \quad x_2 \qquad\qquad x_1' \quad x_2'$

$t_1 \quad t_2 \qquad\qquad t_1' \quad t_2'$

$$t_1 = \frac{t_1' + \frac{ux_1'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \qquad t_2 = \frac{t_2' + ux_2'}{\sqrt{1-\frac{u^2}{c^2}}}$$

$$t_2 - t_1 = \frac{(t_2' - t_1') + \frac{v}{c^2}(x_2' - x_1')}{\sqrt{1-\frac{u^2}{c^2}}} \qquad t_1 = t_2 \Rightarrow t_1' \neq t_2'$$