

Assignment 1

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Question 1

$$A_N = 1 + \frac{2}{N} \sum_{j=1}^N A_{j-1} \text{ for } N > 0 \quad (1)$$

Multiplying both sides by N gives

$$NA_N = N + 2 \sum_{j=1}^N A_{j-1} \text{ for } N > 0 \quad (2)$$

This holds for N-1 as

$$(N-1)A_{N-1} = (N-1) + 2 \sum_{j=1}^{N-1} A_{j-1} \text{ for } N > 1 \quad (3)$$

Subtracting the last two

$$NA_N = 1 + (N+1)A_{N-1} \quad (4)$$

Dividing both sides by $N(N+1)$ gives

$$\frac{A_N}{N+1} = \frac{1}{N(N+1)} + \frac{A_{N-1}}{N} \quad (5)$$

Iterating

$$\frac{A_N}{N+1} = \sum_{k=2}^N \frac{1}{k(k+1)} + \frac{A_1}{2} \quad (6)$$

Assuming $A_0 = 0$ as mentioned in the errata and solving the equation gives

$$\boxed{A_N = N}$$

Question 2