Assignment 2

Nitin Gangahar

February 2013

2.17

Rearranging coefficients we have

$$A_N = A_{N-1}(1 - \frac{4}{N} - \frac{2}{N}) + 2 \tag{1}$$

Multiplying both sides by N

$$NA_N = A_{N-1}(N-6) + 2N \tag{2}$$

The factor to divide would be $\boxed{\frac{6.5.4...1}{N.(N-1)...(N-5)}}$

Assume $N > 6$ and applying the theorem given in Chapter 2, we get

$$A_N = 2 + \sum_{j>=6}^{j<N} 2 \times \frac{j-5}{j+1} \times \frac{j-4}{j+2} \times \ldots \times \frac{N-6}{N} \tag{3}$$

This can be changed to

$$A_N = 2 + 2\sum_{j>=6}^{j<N} \frac{j! \times (n-6)!}{n! \times (j-6)!}$$

Multiplying and dividing by 6! inside the sum gives

$$A_N = 2[1 + \sum_{j>=6}^{j<N} \frac{\binom{j}{6}}{\binom{N}{6}}] \tag{4}$$

Looking up wikipedia for binomial coefficients gives us the following equation which can be used

$$\sum_{m=0}^{n} \binom{m}{k} = \binom{n+1}{k+1} \tag{5}$$

This gives

$$A_N = 2\frac{\binom{N+1}{7}}{\binom{N}{6}} \tag{6}$$

Which gives the final closed recurrence $\boxed{A_N = 2\frac{N+1}{7}}$

We can solve the initial cases $N <= 6$ manually $\square$

2.69

Solved in the forums. The solution I was getting was not specific enough.

3.20

We have the following

$$a_n = 3a_{n-1} - 3a_{n-2} + a_{n-3} n > 2 \text{and} a_0 = a_1 = 0 a_2 = 1 \tag{7}$$

First set of initial conditions and using usual method of GF gives for $N >= 2$ Second condition when used along with backward convolution can be solved and the result varies from the previous result. Hence, the initial conditions make a huge difference. $\boxed{a_n = \frac{n(3-n)}{2}}$

$\square$

3.28

By expanding with taylor series and differentiating both sides and also using the following property $\boxed{\frac{d(a^x)}{dx} = (lna)a^x}$ we get

$$\text{Required coefficient}(R) = \binom{\alpha+k-1}{k}\left(\frac{1}{\alpha} + \frac{1}{\alpha+1} + \ldots + \frac{1}{\alpha+k-1}\right)$$

and substituting $\alpha = \frac{1}{2}$

$$R = \frac{(k-\frac{1}{2})!}{k!}\left(2 + \frac{2}{3} + \ldots + \frac{2}{2k-1}\right)$$

The term $\frac{k-\frac{1}{2}!}{k!}$ can be solved to give $\frac{1}{4^n}\binom{2n}{n}$ Also the series inside can be evaluated to give the final answer for R as

$\boxed{R = \frac{1}{4^n}\binom{2n}{n}(2H_{2n} - H_n)}$